

Review

# A Review of Demand Models for Water Systems in Buildings including a Bayesian Approach

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Received: 24 July 2018; Accepted: 10 August 2018; Published: 13 August 2018



**Abstract:** Instantaneous flow rate estimation is essential for sizing pipes and other components of water systems in buildings. Although various demand models have been developed in line with design and technology trends, most water supply system designs are routinely and substantially over-sized to keep failure risks to a minimum. Three major types of demand models from the literature are reviewed in this paper: (1) deterministic approach; (2) probabilistic approach; and (3) demand time-series approach. As findings show some widely used model estimates are much larger than the field measurements, this paper proposes a Bayesian approach to bridge the gap between model-based and field-measured values for the probable maximum simultaneous water demand. The proposed approach is flexible to adopt estimates as its prior values from a wide range of existing water demand models for determining the Bayesian coefficients for reference models, codes, and design standards with relevant measurement data. The approach provides a useful method not only for evaluating the corresponding demand values from various design references, but also for responding to the call for sustainable building design.

**Keywords:** probable maximum simultaneous demand; water systems; deterministic models; probabilistic models; water demand time series; Bayesian estimates

## 1. Introduction

Estimation of instantaneous flow rates is essential for sizing pipes and other components in a building water system [1]. Flow rate models have been developed to determine the design flow rate (i.e., probable maximum simultaneous demands), while striking a balance between energy, costs, and health concerns. Design criteria have also been judgmentally established to ensure the immediate provision of water services at an allowable failure rate [2]. As most water supply system designs are routinely and substantially over-sized to keep failure risks to a minimum, innovative water-efficient design concepts and features have been more recently introduced to respond to the call for sustainable built environment [3,4]. Mazumdar et al. [5] suggested reducing the confidence levels in Hunter's binomial probability function [2] for better descriptions of water-saving appliances. However, suitable confidence levels for various applications have not yet been derived based on field measurement data and that increases the need for re-evaluating the theoretical basis, as well as design practices for practical pipe sizing [6]. Furthermore, measurements that rely on an opportunistic time series of flows make data validation complex and difficult [7].

Water demand modelling has come a long way since 1940 [8]. The first statistical models were introduced in the 1940s, and statistical sub-models were developed in the 1970s. Although time-series simulations have been in use since 2000, the existing approaches are not uniform and do not finally

converge on single estimates. In order to improve the accuracy of demand estimates, estimating model updates require the acquisition of long-term, high-quality measurements of actual water demand.

This paper reviews three major types of demand models for water systems in buildings and proposes a Bayesian approach to bridge the gap between model estimates and field measurements. By applying the Bayesian techniques, demand estimates can be progressively updated and continuously improved as increasingly more data becomes available. The research findings will both enrich our knowledge of water demands and advance the development of optimal water supply network designs. Sizing a piping network using demand models is also discussed with energy loss and cost implications.

## 2. Classification of Demand Models for Water Systems in Buildings—Deterministic, Probabilistic, and Simulation Approaches

Figure 1 illustrates a schematic structure based on Carson’s definition [8] for modelling design flow rate. As the basic level models (e.g., Level 1) are associated with more descriptions of the basic demand parameters, they usually offer higher-resolution flow demands and more flexible applications. The results obtained from these models can be used to develop expressions that describe models for specific engineering design applications (e.g., Levels 1 and 2). Different levels of application result in different approaches to design flow rate estimation.

Deterministic (Level 3) models estimate the design flow rates by summing the ‘flows’ or ‘units’ of all fixtures on the system and then multiply this value by a simultaneous factor ( $\leq 1$ ), or by using empirical formulas. As the allowable failure rates in these models are constant, the design flow rates are fixed values. These models are commonly used in design guides and standards for specific system designs.

Probabilistic (Level 2) models employ parametric probability distributions to estimate the likelihoods of different numbers of fixtures in simultaneous use. This approach uses the statistical nature of water appliances and flow rates and, together with an allowable failure rate, gives flow rate estimates corresponding to selected levels of adequacy or performance. However, as the time patterns in the flow rates are not primarily included in this approach, more probability parameters may be required to resolve the daily, weekly, diurnal, or seasonal patterns.

Simulation (Level 1) models attempt to model individual uses via Monte Carlo sampling from the cumulative frequency distributions of user demand parameters. These models can tackle time-dependent variables that are dependent on one another.

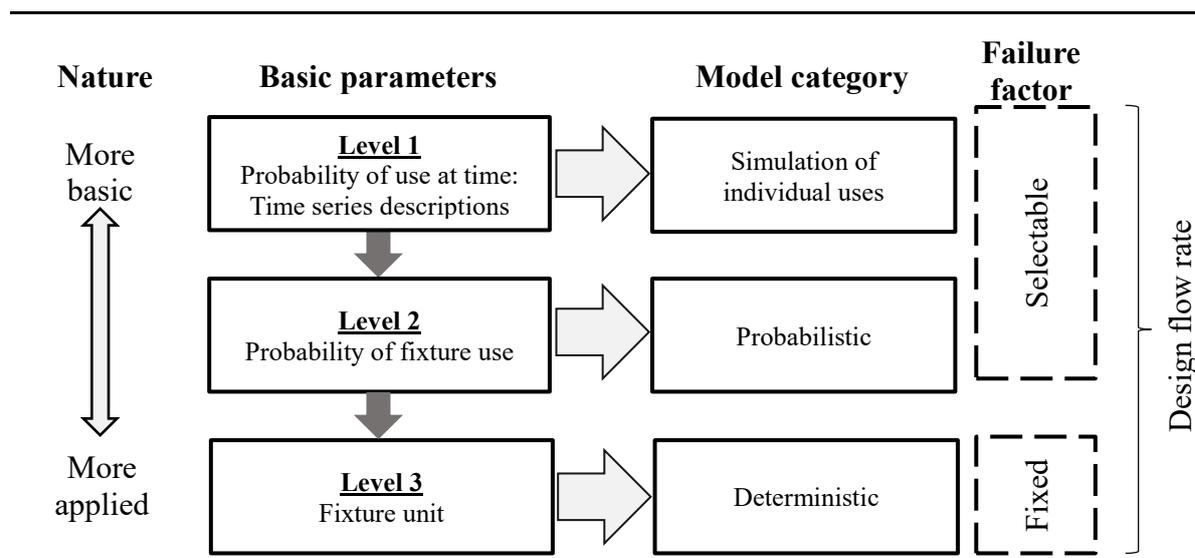


Figure 1. Demand models for water systems in buildings.

## 2.1. Deterministic Approach

Konen and Goncalves [7] reviewed a number of design flow determination standards and guidelines and concluded that deterministic equations and/or graphs developed for pipe sizing were mainly based on Rydberg's model, Hunter's model [2], or expressions (or curves) that relate the sum of unitary flow rates (or fixtures) to the design flow rate. Deterministic approaches (Figure 1) in selected codes and standards available are given below.

The analytical formula of design flow rate  $q_s$  by Rydberg [7] is the sum of three components, namely, the standard flow rate of the largest water fixture  $q_{i,\max}$ , the mean flow rate of the other fixtures, and the risk term for the random variation of the mean flow rate of the other fixtures. It can be considered the first deterministic model based on probabilistic support. A formula of design flow rate  $q_s$  used for dimensioning of supply pipes in Scandinavian countries is given by the following equation, where  $p_i$  is the probability of average water flow from each fixture  $q_{i,\mu}$ ,  $q_i$  is the nominal flow rate of fixture  $i$ , and  $k$  is the constant for a selected failure factor,

$$q_s = q_{i,\max} + p_i \left( \sum q_i - q_{i,\max} \right) + k \sqrt{q_{i,\mu} p_i \left( \sum q_i - q_{i,\max} \right)} \quad (1)$$

A number of deterministic curves in design guides are based on Hunter's probabilistic model or its modification [2,9]. Using Hunter's method, the design flow rate can be determined by Equation (2), where  $k$  ( $=1.8226$ ) is the constant at an allowable failure rate  $\lambda = 1\%$  during daily rush hours,  $M_0$  is the total number of installed fixtures, and  $p_0$  is the probability at which each fixture is operated [10].

$$q_s = q_0 \left[ M_0 p_0 + k \sqrt{2 M_0 p_0 (1 - p_0)} \right] \quad (2)$$

As fixtures of the same type are assumed in Equation (2), a fixture unit approach was adopted to approximate the design flow rate for a main pipe, which supplies appliances of different types, and to characterize the  $i$ -th appliance by the number of fixtures ( $p_i, q_i$ ),  $i = 1, 2, 3, \dots, M$ . The design flow rate, which is equivalent to the flows produced by a number of fixture units  $U_i$ , is determined by the following expressions, where  $q_f$  ( $=10 \text{ L}\cdot\text{s}^{-1}$ ) is the selected reference probable maximum demand produced by each appliance type [11,12], and  $m$  is the number of fixtures that produce the reference design flow rate,

$$q_s = q_0 \sum_i U_i p_0 + q_0 k \sqrt{2 \sum_i U_i p_0 (1 - p_0)} \quad (3)$$

$$U_i = \frac{m_0}{m_i}; \quad i = 1, 2, 3, \dots \quad (4)$$

$$q_0 \left[ m_0 p_0 + k \sqrt{2 m_0 p_0 (1 - p_0)} \right] = q_i \left[ m_i p_i + k \sqrt{2 m_i p_i (1 - p_i)} \right] = q_f \quad (5)$$

The design flow rate  $q_s$  is affected by the choice of  $q_f$ . Mui and Wong [11] estimated that the variations in  $q_s$  for a  $q_f$  range of 1 to  $250 \text{ L}\cdot\text{s}^{-1}$  could be up to 12%. According to the plumbing services design guides [13–15], the probability of use and the design flow rate of a unitary fixture  $M$  for a water supply system are 0.0282 and  $0.15 \text{ L}\cdot\text{s}^{-1}$ , respectively. Design flow rates  $q_s$  for residential fixtures can be approximated by the below expression,

$$q_s = 0.048 M^{0.72}; \quad M \geq 30 \quad (6)$$

Table 1 displays the constants for relating the sum of unitary flow rates  $\sum M_i q_i$  to the design flow rate  $q_s$  as defined in a Deutsches Institut für Normung DIN (the German Institute for Standardization) standard [7].

$$q_s = k_1 + k_2 \left( \sum M_i q_i \right)^{k_3} \quad (7)$$

In DIN-1998 W308 norm (German), there is an expression by Malan [16] for the design flow rate of a residential building, where  $U$  is fixture (loading) units of appliances,

$$q_s = 0.25 \sqrt{\sum U} \tag{8}$$

**Table 1.** Constants for relating the sum of unitary flow rates to the design flow rate at pipes (Deutsches Institut für Normung DIN standard).

Applicable Range	$k_4 \leq \sum M_i q_i \leq 20 \text{ L}\cdot\text{s}^{-1}$								$\sum M_i q_i > 20 \text{ L}\cdot\text{s}^{-1}$		
	$q_i$ for $k$ Value	$q_i < 0.5 \text{ L}\cdot\text{s}^{-1}$				$q_i \geq 0.5 \text{ L}\cdot\text{s}^{-1}$				All $q_i$	
Occupancy	$k_1$	$k_2$	$k_3$	$k_4$	$k_1$	$k_2$	$k_3$	$k_4$	$k_1$	$k_2$	$k_3$
Residences	-0.14	0.682	0.45	1.0	-0.70	1.7	0.210	1.0	-0.70	1.7	0.21
Offices	-0.14	0.682	0.45	1.0	-0.70	1.7	0.210	1.0	0.48	0.4	0.54
Hotels	-0.14	0.698	0.50	0.1	0	1.0	0.366	1.0	-1.83	1.08	0.50
Shopping centres	-0.12	0.698	0.50	0.1	0	1.0	0.366	1.0	-6.64	4.3	0.27
Hospitals	-0.12	0.698	0.52	0.1	0	1.0	0.366	1.0	1.25	0.25	0.65
Schools	-3.41	4.400	0.27	1.5	-3.41	4.4	0.270	1.5	11.5	-22.5	-0.50

Mambourg [17] presented an expression in France (Règles DTU 60.11) for design flow rate calculations,

$$q_s = \frac{0.8 \sum M_i q_i}{\sqrt{\sum M_i - 1}} \tag{9}$$

Konen and Goncalves [7] also presented some curves (adopted in Portugal) for design flow rate calculations,

$$q_s = \begin{cases} 0.5548(\sum M_i q_i)^{0.4958} & 0 < \sum M_i q_i \leq 3 \\ 0.5244(\sum M_i q_i)^{0.5462} & 3 < \sum M_i q_i \leq 44 \\ 0.2023(\sum M_i q_i)^{0.7982} & 44 < \sum M_i q_i \leq 500 \end{cases} \tag{10}$$

An expression of design flow rate as stated in Brazilian Standard NBR 5606 is given below [7],

$$q_s = 0.3 \sqrt{\sum M_i q_i} \tag{11}$$

Konen and Goncalves [7] suggested some new values for the unitary fixtures based on the probabilistic formulation developed by Hunter [2] and proposed curves for relating  $10 \leq M \leq 10,000$  water closet (WC) flush tanks and  $5 \leq M \leq 1000$  WC flush valves to the design flow rate,

$$q_s = \frac{1}{15.85} \left( 3.54 \times 10^{-10} M^3 - 1.66 \times 10^{-5} M^2 + 0.1263 M + 10.9 \right) \tag{12}$$

$$q_s = \frac{1}{15.85} \left( 8.5 \log^2 M + 11.5 \log M + 15 \right) \tag{13}$$

Murakawa [18] developed a loading unit method to estimate the design flow rate. Instead of using an overall average by fixture type, the time-dependent average of simultaneous use in the peak period is used for each fixture type. A design flow rate estimated this way is comparatively lower than that by the Hunter’s method (i.e.,  $q_{s,0}$ ) [2], and it is expressed by Equation (14), where  $k_1$  and  $k_2$  are the constants as given in Table 2.

$$q_s = k_1 q_{s,0} + k_2 q_{s,0}^2 \tag{14}$$

**Table 2.** Regression constants for approximating design flow rates.

Reference	$k_1$	$k_2$	Correlation Coefficient
[7]	0.5879	0.8683	0.9971
[18]	0.4819	0.0033	0.9976
[19]	1.0270	1.2266	0.9969
[20]	0.05	0.71	0.9989
[21]; $M_i q_i \geq 30 \text{ L}\cdot\text{s}^{-1}$	0.2283	0.5906	0.9626
[21]; $M_i q_i < 30 \text{ L}\cdot\text{s}^{-1}$	$0.11q_{\max}^2 - 0.5q_{\max} + 0.53$	$-0.36q_{\max}^2 + 1.66q_{\max} - 1.23$	0.9989

In a Japanese code established by The Society of Heating, Air-conditioning and Sanitary Engineers of Japan (SHASE) SHASE-S (or HASS) 206, a set of procedures were also introduced by Murakawa [19] to determine the design flow rate using the sum of the maximum load of an appliance in a group of different sanitary appliances and the half load of the other appliances within that group. The design flow rates determined by Konen and Goncalves [7] and Murakawa [19] were compared with those by the Hunter’s probabilistic approach  $q_{s,0}$  [2]. Using the curves presented in original papers and the constants listed in Table 2, approximations can be given by,

$$q_s = k_1 q_{s,0}^{k_2} \tag{15}$$

In Europe, a harmonized European Union (EU) Standard [21] supersedes the previous version [20]. BS6700 [20] allocates the loading units to appliances instead of mapping the total loading units for the pipe to the simultaneous demand. The design flow rate can be approximated using Equation (16) and constants from Table 2. BSEN806-3 [21] defines that one loading unit is equivalent to a draw-off flow rate of  $0.1 \text{ L}\cdot\text{s}^{-1}$  and the design flow rate can be mapped to the total loading units for the water supply pipe section. With the constants given in Table 2, the design flow rates can be approximated by the below expressions, where  $q_{\max}$  is the maximum flow rate of a single appliance unit,

$$q_s = k_1 (\sum M_i q_i)^{k_2}; M_i q_i \geq 30 \text{ L}\cdot\text{s}^{-1} \tag{16}$$

$$q_s = \exp[k_1 \sum (M_i q_i) + k_2]; M_i q_i < 30 \text{ L}\cdot\text{s}^{-1}; 0.2 \text{ L}\cdot\text{s}^{-1} \leq q_1 \leq 1.5 \text{ L}\cdot\text{s}^{-1} \tag{17}$$

Wong and Mui [22] presented a deterministic curve of design flow rate for collective residential drainage appliances. The constants, displayed in Table 3, were determined from a survey research study of 597 apartments selected among 14 high-rise residential buildings in Hong Kong. Constants from other design guides are listed in the table for comparison [13–15,23,24].

$$q_s = k_0 + k_1 (\sum M)^{k_2} \tag{18}$$

**Table 3.** Regression constants for the design flow rates in drainage stacks.

Reference	$q_s \text{ (L}\cdot\text{s}^{-1})$	$M$	$k_0$	$k_1$	$k_2$
[22]	0.23	70–157	0.9	0.0061	1
		158–6500	0	0.073	0.64
[13–15]	0.34	70–157	0.9	0.0061	1
		158–6500	0	0.073	0.64
[13–15,23]	1	All	0	0.5	0.5
[24]	0.6	100–20,000	2.3895	0.0622	0.6659

### 2.2. Probabilistic Approach

The design flow rate  $q_s$  can be determined from the probability density function of flow rates  $q$  for all connected appliances, provided that the probability does not exceed the design flow rate.  $P(q > q_s)$  is determined at an allowable maximum failure rate  $\lambda$  selected,

$$P(q > q_s) = \lambda \tag{19}$$

Hunter [2] applied binomial distribution to estimate the probability  $P(M_s < M)$  of simultaneous operation of  $M_s$  appliances out of  $M$  identical appliances, which are connected to a common supply pipe. At a constant appliance flow rate  $q$ , the design flow rate  $q_s$  is determined by the discharge probability of an appliance  $p$  as expressed below, where  $\tau_1$  is the discharge period and  $\tau_2$  is the period of time between two consecutive uses,

$$q_s = qM_s; P(M \geq M_s) = \sum_{M_s}^M C_{M_s}^M p^{M_s} (1 - p)^{M - M_s}; p = \frac{\tau_1}{\tau_2} \tag{20}$$

The time period between two consecutive uses  $\tau_2$  can be related to the user queue as expressed by the following equation [25], where  $n_a$  is the number of appliances serving a group of users at an arrival rate  $\gamma$ ,

$$\tau_2 = \frac{n_a}{\gamma} \tag{21}$$

In order to address the design flow rates for different types of appliances, Webster [26] applied a generalized binomial distribution to determine the simultaneous operation of  $M_{j,s}$  appliances out of  $M_j$  appliances in a group of identical appliances  $j$  (i.e., out of  $j$  independent groups). The probability of  $M_{j,s}$  is given by Equation (22), where  $p_j$  is the probability of an appliance  $j$  operating in the peak period.

$$P(M_{j,s}) = \prod_{j=1}^J \left[ C_{M_{j,s}}^{M_j} p_j^{M_{j,s}} (1 - p_j)^{M_j - M_{j,s}} \right]; p_j = \frac{\tau_{1,j}}{\tau_{2,j}} \tag{22}$$

The flow rates  $q$  through the common supply pipe when  $M_{j,s}$  appliances are operating simultaneously can be calculated using Equation (23), and the design flow rate  $q_s$  is determined from  $P(q > q_s)$  in Equation (19).

$$q_s = \sum_j q_j M_{j,s} \tag{23}$$

This approach is further developed for a pipe with different flow rates  $q_j$  from  $N_j$  associated probabilities  $p_j$  during the peak period [27,28], where  $p_0$  is the probability of zero flow rate for appliances  $M_{s,0}$ ,

$$P(M_{j,s}) = \frac{p_0^{M_{s,0}} p_j^{M_{s,j}}}{M_{s,0}! M_{s,j}!} \tag{24}$$

Murakawa [19] followed Hunter’s approach [2] and suggested a Poisson distribution (instead of a binomial distribution),

$$P(M \geq M_s) = \sum_{M_s}^M \frac{e^{-Mp} (Mp)^{M_s}}{M_s!} \tag{25}$$

Ilha et al. [29] developed an open model for  $\tau_1$ ,  $\tau_2$ , and  $q$  for appliances  $M$ . All of the four parameters are described by parametric distributions. The simultaneously operating appliances  $M_s$  are described by a beta-binomial distribution  $B$  as shown in Equations (26)–(30) below, where  $\tau_1$  and  $\tau_2$

can be represented by Erlang or exponential distributions; the probability  $p$  is beta distributed;  $q$  and  $q_s$  are given by gamma distributions; and  $k_1, k_2$ , and  $M_s$  are the distribution parameters,

$$P(p) = \frac{\Gamma(\tau_2 + \tau_1)}{\Gamma(\tau_1)\Gamma(\tau_2)} p^{\tau_1-1} (1 - p)^{\tau_2-1} \tag{26}$$

$$P(M_s) = C_{M_s}^M \frac{B(M_s + \tau_1, M - M_s + \tau_2)}{B(\tau_1, \tau_2)} \tag{27}$$

$$q_s = \sum_j q_j M_{j,s} \tag{28}$$

$$P(x = \tau_1, \tau_2) = \begin{cases} \frac{x^{M_s-1} e^{-x/k_2}}{k_2^{M_s} (M_s-1)!} \\ \lambda e^{-\lambda x} \end{cases} \tag{29}$$

$$P(q > q_s) = \int_{q_s}^{\infty} \frac{q^{k_1-1} e^{-q/k_2}}{k_2^{k_1} \Gamma(k_1)} dq \tag{30}$$

Alitchkov [30,31] proposed that the design flow rate can be determined by the cumulative normal distribution of simultaneous flow rates  $q_s$  at time  $t$  over a period of one year, where  $M$  is total number of fixtures and  $\sigma^2$  is the variance of flow rate which serves as a stochastic component,

$$P(q > q_s) = \frac{1}{\sigma\sqrt{2\pi}} \int_{q_s}^{\infty} \exp\left(-\frac{(q(t) - q_{s,\mu})^2}{2\sigma^2}\right) dq; q \sim q(\mu, \sigma^2) \tag{31}$$

### 2.3. Deterministic Model Simplification

Relationships of various distributions are illustrated in Figure 2. Design flow rates suggested at an acceptable failure rate  $\epsilon$  can be given by the following expression, with a constant  $k$  accounting for various distributions adopted [10,29],

$$q_s = \mu + k\sigma; q \sim q(\mu, \sigma^2) \tag{32}$$

This approach was adopted in some design guidelines as a form of deterministic equation. One possibility is to use the normal approximation for the binomial distribution to estimate peak loads directly [10]. Below is an expression by Wistort [32] for the direct estimation of the 99th percentile of the flow rates from  $j$  appliance types,

$$q_s = \mu + k\sigma; \mu = \sum_j q_j M_j p_j; \sigma = \sqrt{\sum_j q_j^2 M_j (p_j - p_j^2)} \tag{33}$$

By taking  $p_0$  ( $=0$  for  $M_p > 5$ ) as the expected number of operating fixtures in a collection of  $j$  fixture groups, it can be rewritten in a dimensionless variation [32,33],

$$q_s = \left(1 + \frac{k\sigma}{\mu}\right) \frac{\sum_j M_j p_j}{1 - p_0} \tag{34}$$

$$p_0 = \prod_j (1 - p_j)^{M_j} \tag{35}$$

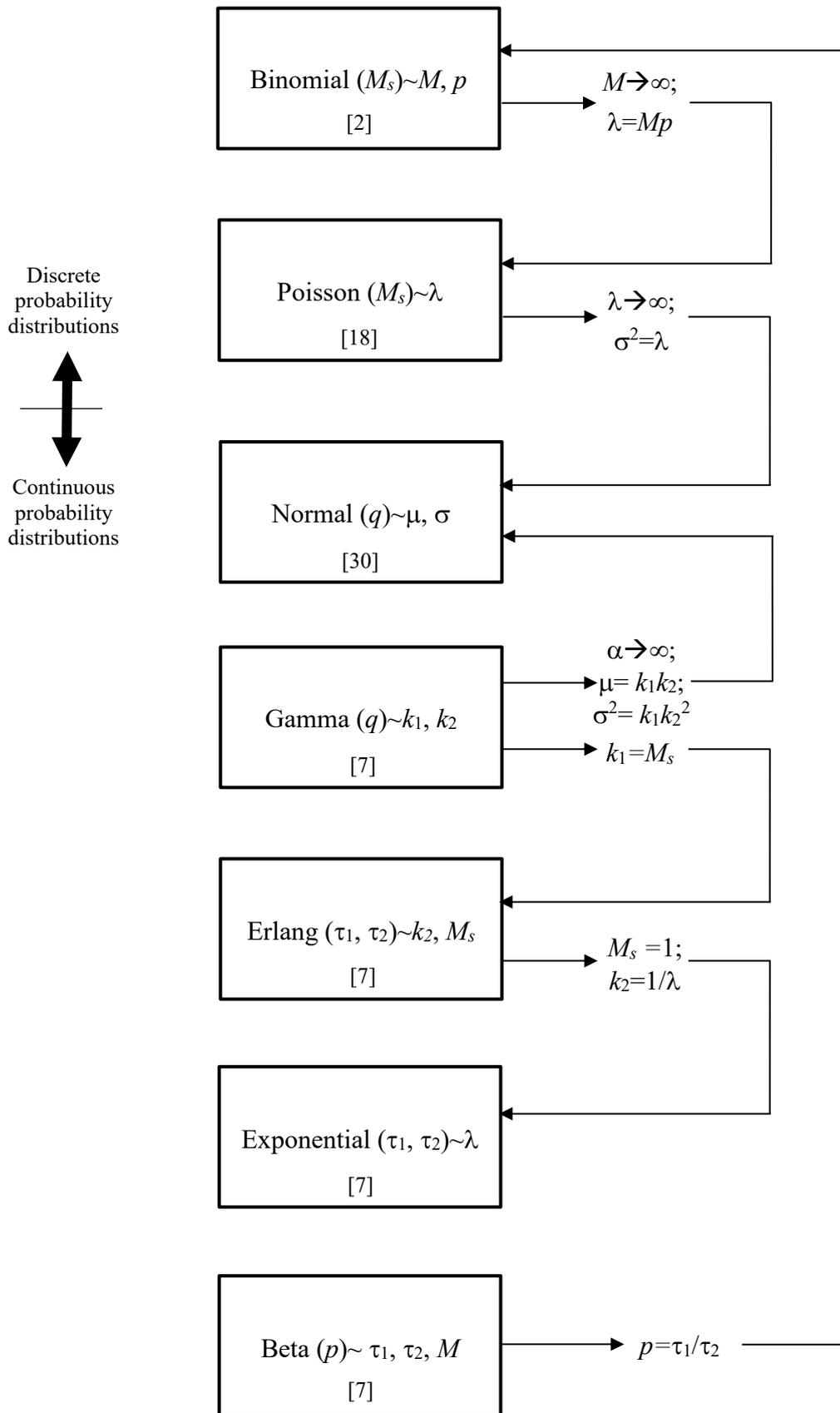
Theoretically, the design flow rate tends towards the discharge probability  $p$  when the number of appliances is increasing. Taking Hunter’s equation [2] for illustration, Equation (2) can be expressed

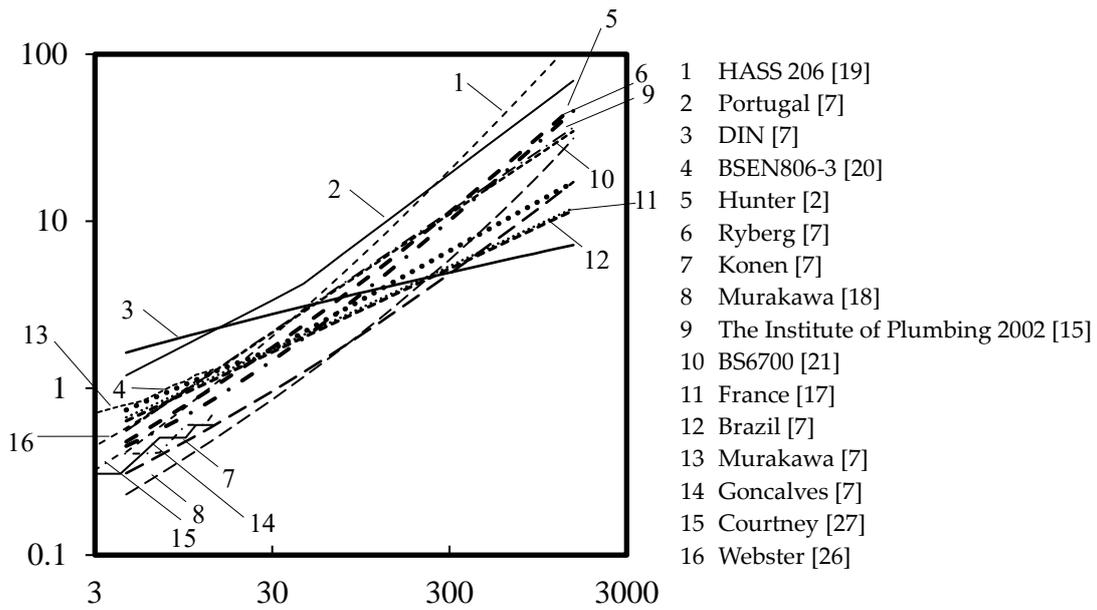
by the fractional design flow rate  $q_s^*$ , that is, the design flow rate can be divided by the maximum flow rate,

$$q_s^* = \frac{q_0 \left[ M_0 p_0 + k_\lambda \sqrt{2M_0 p_0 (1 - p_0)} \right]}{q_0 M_0} = p_0 + \frac{k}{M_0} \sqrt{2M_0 p_0 (1 - p_0)} \quad (36)$$

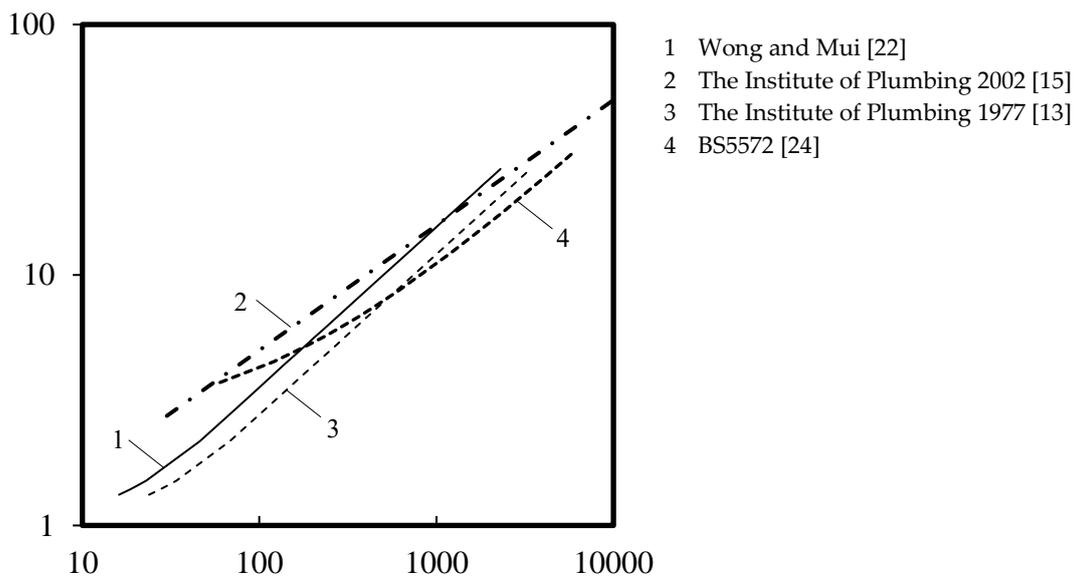
$$\lim_{M_0 \rightarrow \infty} q_s^* = \lim_{M_0 \rightarrow \infty} \left( p_0 + \frac{k_\lambda}{M_0} \sqrt{2M_0 p_0 (1 - p_0)} \right) = p_0 \quad (37)$$

The fixture flow rates determined from various deterministic and probabilistic models cannot be compared directly as different models assume different flow rates and demand probabilities for the reference fixtures installed. Figure 3 plots the design flow rates  $q_s$  as a function of the sum of fixture flow rates  $M_0 q_0$  based on various deterministic and probabilistic models for residential water supply and drainage systems. Although large variations of the predicted design flow rates (i.e., 1–10 times) can be seen in the figure, all model estimates show consistent trends. The fractional design flow rates  $q_s^*$  from various deterministic models are normally distributed ( $p > 0.05$ , w/s test). Figure 4 illustrates the average fractional design flow rate estimates against the maximum flow rates, within an estimate range of 0.005–0.36 L·s<sup>-1</sup>. The results show a decreasing trend of fractional design flow rates from 0.15 to 0.025 L·s<sup>-1</sup> against an increasing sum of the sum of fixture flow rates from 4.5 to 1500 L·s<sup>-1</sup>.



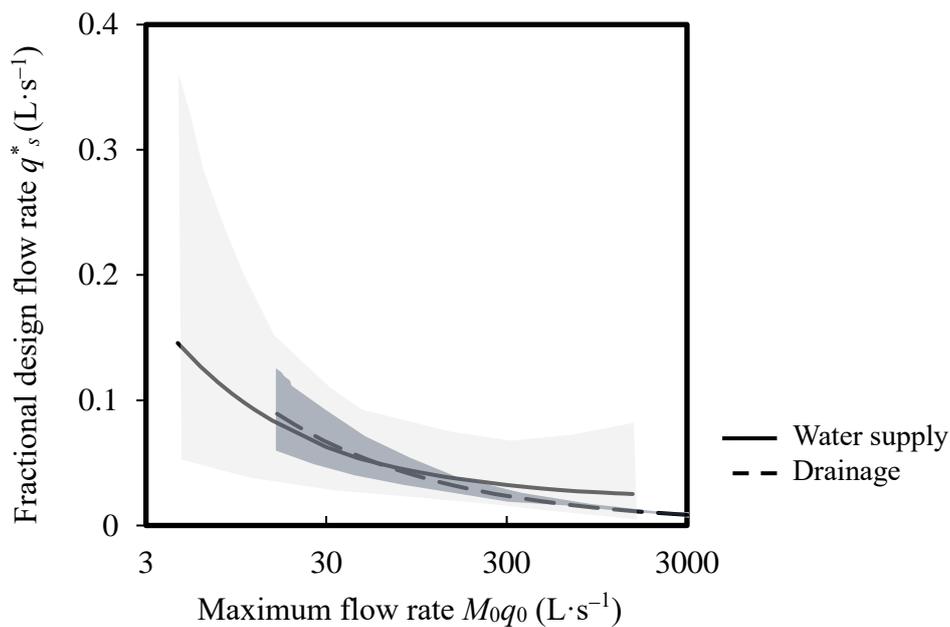


(a)



(b)

**Figure 3.** Design flow rates of residential fixture units. (a) water supply; (b) drainage.  $x$ -axis: maximum flow rate  $M_0q_0$  ( $L \cdot s^{-1}$ ).  $y$ -axis: design flow rate  $q_s$  ( $L \cdot s^{-1}$ ).



**Figure 4.** Model estimated fractional design flow rates against maximum flow rates. (Shaded area indicated one standard deviation plus and minus the average estimates).

#### 2.4. Simulation and Time Series Approach

Design flow rates can be determined from instantaneous demand time series via Monte Carlo sampling techniques [27]. In the Monte Carlo simulations, a large number of pseudo-random uniform numbers  $u_i$  are obtained from the intervals (0, 1) to map numeric values  $x_{s,i}$  to model parameters  $x_s$  as described by the following probability mass or density function,

$$x_{s,i} = \int_{-\infty}^{u_i} \tilde{x}_s dx_s \tag{38}$$

Studies show that for appliances  $i$  (with different usage patterns) installed in the same washroom, the probable maximum simultaneous water demands can be determined using the Monte Carlo sampling techniques even when the appliances are not operating simultaneously [34,35]. The design flow rate  $q_s$  for all washrooms  $j$  with demands  $q$  and an allowable failure factor  $\lambda$  (=1%) is expressed by the following equation, where  $q_i$  is the water demand in rush hour  $\tau$ ,  $\tau_0$  is the time period without demand, and  $\tau_1$  is the time period with demand,

$$P(q > q_s) = \lambda; q = \sum_{ij} q_{ij}(t); t \in \tau = \tau_0 + \sum_i \tau_{1,i} \tag{39}$$

With known demand probability and demand flow rate in each hour, daily demand time series can also be made up using Monte Carlo simulations [36]. Mui and Wong [37] proposed a time series model constructed this way to determine the occurrence and duration of drainage demands from random and intermittent appliance discharges.

Simulation procedures for the demand time series can be programmed for the ease of use [38]. Once the time series are obtained, descriptive statistic quantities of instantaneous demands (e.g., maximum and average values with various failure factors) in any integrating periods can be computed. A number of research works have been done for this purpose. Rathnayaka et al. [39] reviewed some tools for generating end-use data for residential water systems. Duncan and Mitchell [40] developed a model that simulates household water demands for a range of end uses and aggregates multi-year demand sequences

generated at 1-min time steps to a time series. Thyer et al. [41] presented a probabilistic behavioural model to simulate household water use at 1-min time steps. SIMDEUM, developed by Blokker et al. [42,43], is a water demand end-use model that combines various water use behavioural patterns with the knowledge of appliance types to predict water use on a micro scale (at 1-s time steps). In order to optimize the inflow rate of a tank water supply system, Wong et al. [44] integrated a demand time series.

Figure 5 shows the core calculation procedures for constructing the time series of simultaneous demands. Sub-models of key parameters (e.g., number of appliance demands within a time period, flow rate, demand duration, etc.) can be included either through various approximations of parametric distribution functions, surveyed frequency distributions, or further physical relationships. The queuing models by Goncalves and Alves da Graca [25] and Mui and Wong [45] for sanitary appliances in congested use and a fuzzy algorithm by Oliverira et al. [46] for demand start time and duration calculations are a few good examples.

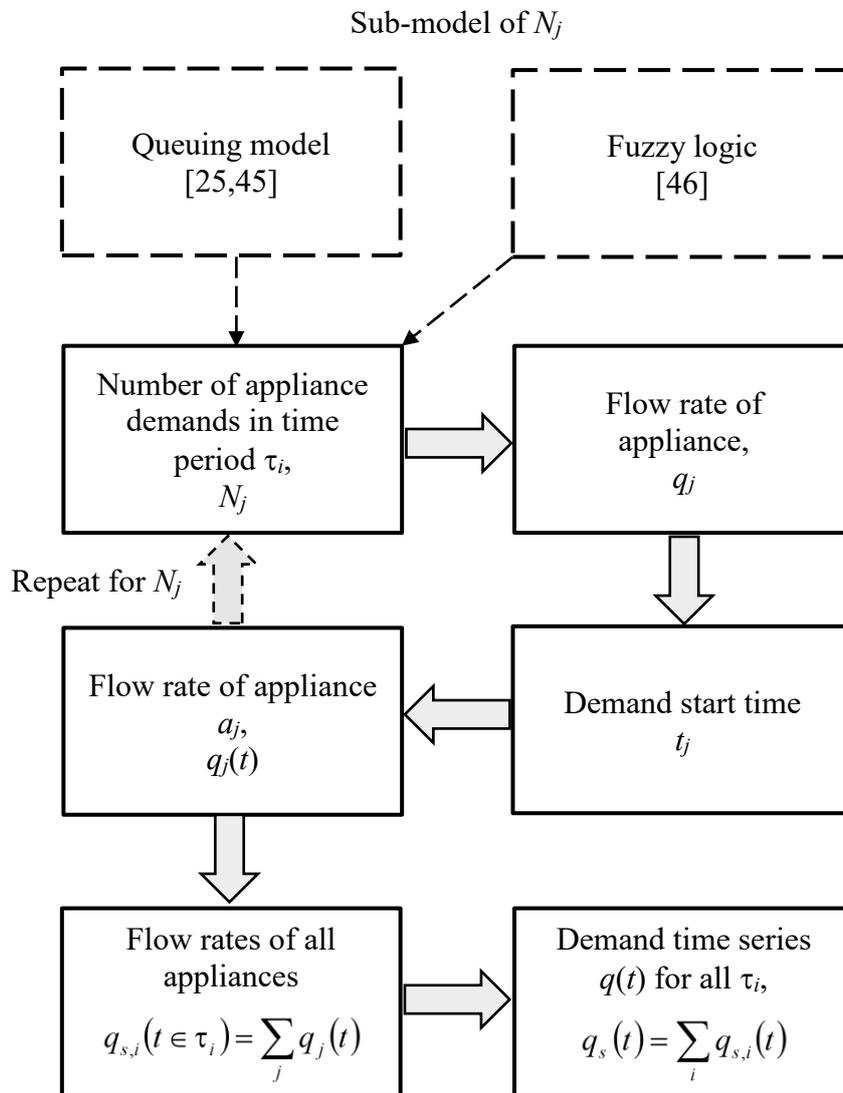


Figure 5. Simulations for time series of simultaneous demands.

For simulations of the simultaneous demands, a time-series is subdivided into a number of time partitions  $\tau_i$ ,  $i = 1, 2, 3, \dots$ , with a number of demands  $N$  of appliances  $j = 1, 2, 3, \dots$ ; and each demand has a time variant demand  $q_j(t)$  for each operation and a uniformly distributed demand start

time  $t_j$  in  $\tau_i$  [36]. Using Monte Carlo sampling for the fractional demand start time  $u$  from a uniform distribution function, the demand start time in the time partition is given by,

$$t_j = u\tau_i \quad (40)$$

The flow rates from all appliances in the time partition and for all time partitions are given by,

$$q_s(t) = \sum_i q_{s,i}(t) \quad (41)$$

$$q_{s,i}(t) = \sum_j q_j(t) \quad (42)$$

Asano et al. [47] studied time series of instantaneous demands in an office building and compared the maximum flow rate predictions made by a linear multivariate equation with those made by a neural network. Although the neural network approach could give a better prediction, no physical explanation was given in that study. In latter studies by Murakawa et al. [48,49], time series of instantaneous maximum flow rates modelled by Monte Carlo simulation techniques were used to estimate the design flow rates for office buildings and restaurants.

### 3. Bayesian Approach

Most water supply system designs are routinely and substantially over-sized as the prospect of system failure is commercially and professionally unimaginable [4,50]. It was opportunistic to determine the probable maximum simultaneous demands in measurements. Usage patterns of water appliances associated with occupancy, replacement of newer appliances, and lifestyle changes in installations added uncertainty to data quality of the maximum demands in long-term measurement. Indeed, there were insufficient long-term measurements available in open literature to establish promising design flow rates for all water installations in buildings. Regarding the most appropriate choice of design flow rate for sustainable development in buildings, there is no conclusive evidence that favors either model or measurement outcome.

#### 3.1. Measurement Data

Vrana et al. [51] studied the peak flow rates measured in 12 ( $n = 12$ ; number of residents = 12–168) water supply systems for residential buildings in the Czech Republic and compared them with the design flow rates given in four design guidelines, namely, CSN75-5455 (Czech), EN806-3 (British), W3 (Swiss), and DIN1988-300 (German). Reportedly, the measured rates were fractions of the design flow rates: 0.173–0.483 for CSN, 0.2–0.568 for EN, 0.262–0.684 for W3, and 0.256–0.692 for DIN. The fraction values appeared to be normally distributed ( $p \geq 0.1$ , Shapiro-Wilk test); except for CSN ( $p = 0.04$ ,  $t$ -test for correlation), where no significant correlation between predicted and measured values was found ( $p > 0.05$ ,  $t$ -test for correlation).

Pieterse-Quirijns et al. [52] and Blokker et al. [53] investigated the flow rates measured on a per second basis for the hot and cold water supply systems in two offices (255–2000 employees), two business hotels (80–192 rooms), and two nursing homes (124–260 beds). The measurement periods ranged from 28 to 47 days, and the measurement results showed that the peak flow rates were only fractions (0.417–0.755) of the design flow rates given in existing guidelines.

In a research project by Malan [16], daily peak domestic water demands were gauged in a 134-unit apartment building for 12 consecutive calendar days. The measured peak demand was  $8.77 \text{ L}\cdot\text{s}^{-1}$  and that was equal to 46.6% of the estimated value given in the German design guide W308. That project included 166 WC cistern inlet valves, 536 taps (15-mm), and 268 taps (20-mm).

Murakawa et al. [48] reported that, from a continuous measurement period of 14 months, the instantaneous maximum flow rate of a water supply system, which served 21 restaurants

(with a total of 1932 seats), was  $8.8 \text{ L}\cdot\text{s}^{-1}$ . The design flow rate of the system was  $10.4 \text{ L}\cdot\text{s}^{-1}$  [13–15,54]. According to the studies by Takata et al. [55] and Murakawa et al. [48], the instantaneous maximum flow rate recorded for the water supply system in a 14-story office building (total floor area =  $18,256 \text{ m}^2$ ) was  $3.2 \text{ L}\cdot\text{s}^{-1}$ , while the system design flow rate was  $11.8 \text{ L}\cdot\text{s}^{-1}$  [13–15].

In yet another study, Murakawa et al. [56] investigated the water flow rates for 16 residential buildings (95–910 flats). The maximum simultaneous flow rates, recorded from measurements made at 1.2-min intervals for one year, were presented as a function of loading units. A total of 29 measurement data sets ( $n = 29$ ) were given as a fraction ( $\alpha = 0.276\text{--}0.522$ ) of a design flow rate range of 2.9 to  $65 \text{ L}\cdot\text{s}^{-1}$  [18,56]. The fraction values were assumed to be normally distributed ( $p \geq 0.1$ , Shapiro–Wilk test), and there was no significant correlation between the fractions  $\alpha$  and the loading units ( $p = 0.75$ ,  $t$ -test for correlation).

Recently, a Bayesian approach has been proposed to bridge the gap between model estimates and field measurements for the probable maximum simultaneous water demand [57]. Bayes’ theorem, which relates the conditional and marginal probabilities of stochastic events A and B (where B has a non-vanishing probability), asserts that the probability of an event A given by event B depends not only on the relation between events A and B, but also on the marginal probability of occurrence of each event. This theory can be applied to a sample size not large enough for decision-making purposes, yet relevant enough for statistical analysis. Its general formulation and various applications are available in the literature [54,58]. Studies applied Bayesian analysis to improve understanding of the downtime characteristics of water installations [59]. Factor weights contributed to water pipe conditions were evaluated with Bayesian inference [60]. A Bayesian network was used as an aid to integrated water resource planning, accounting various considerations of environmental, economic, social, and political impacts, as well as inputs from stakeholders in decision making process [61]. In this section, the proposed approach predicts the probable maximum simultaneous demand for the total fixtures installed using the readily available model predictions (event A) and the measurements from a compatible installation (event B).

Given a measured (maximum) value  $q_m^* \sim N(\mu, \sigma^2)$ , the posterior estimate of a fractional design flow rate  $q_{s,1}^* \sim N(\mu_1, \sigma_1^2)$  is expressed by the following Bayesian rules [62], where  $q_{s,0}^* \sim N(\mu_0, \sigma_0^2)$  is the prior estimate of the fractional design flow rate;  $p$  is the probability;  $\mu$  and  $\sigma^2$  are the mean and variance of a normal distribution function, respectively;  $\mu$  and  $\mu_0$  are the best estimates of the fractional measured value and design value  $q_m^*$  and  $q_{s,0}^*$ , respectively,

$$p(q_{s,1}^* | q_m^*) = p(q_{s,0}^*) p(q_m^* | q_{s,0}^*) \tag{43}$$

$$\sigma_1^2 = (\sigma_0^{-2} + \sigma^{-2})^{-1}; \mu_1 = \mu_0 \sigma_0^{-2} / (\sigma_0^{-2} + \sigma^{-2}) + \mu \sigma^{-2} / (\sigma_0^{-2} + \sigma^{-2}) \tag{44}$$

In these rules, the weightings are proportional to their respective variances, and the posterior mean is a weighted average of the prior mean and the measured value given. This posterior mean can be characterized by the ratio of standard deviations and expressed as a parameter  $\beta$ .

$$\beta^2 = \sigma^2 / \sigma_0^2 \tag{45}$$

Given a measurement maximum flow rate  $\mu$  is significantly different from a prior belief of the maximum (design) flow rate  $\mu_0$  that  $|\mu_0 - \mu| > \varepsilon$ , where  $\varepsilon$  is a cut-off value of the acceptable error.

Suppose repeatedly measurements given the same value  $\mu$  that an acceptable choice of design flow rate  $q_s^* \sim \mu_n$ . Denote  $X = \sigma_0^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \beta^2 / (1 + \beta^2)$  and  $Y = \mu \sigma^{-2} / (\sigma_0^{-2} + \sigma^{-2}) = \mu / (1 + \beta^2)$ , posterior estimates  $\mu_1, \mu_2, \dots, \mu_n$  are given below,

$$\mu_1 = \mu_0 X + Y, \mu_2 = \mu_0 X^2 + XY + Y, \dots, \mu_n = \mu_0 X^n + Y (X^{n-1} + X^{n-2} + \dots + X + 1) \tag{46}$$

$$\mu_n = \mu_0 X^n + Y (1 - X^n) / (1 - X) \tag{47}$$

It is noted for Equation (47)  $\mu_n \rightarrow \mu$  when  $n \rightarrow \infty$ . Taking  $n$  is a finite number of the repeated observations such that the  $n$ -th estimate shows no significant difference from measured acceptance, that is,  $|\mu_n - \mu| \leq \varepsilon$ , and  $\beta^2$  can be determined below,

$$\mu_0 X^n + Y(1 - X^n)/(1 - X) = \mu + \varepsilon \tag{48}$$

$$\mu_0 \left( \frac{\beta^2}{1 + \beta^2} \right)^n + \left( \frac{\mu}{1 + \beta^2} \right) \left( \frac{1 - \left( \frac{\beta^2}{1 + \beta^2} \right)^n}{1 - \left( \frac{\beta^2}{1 + \beta^2} \right)} \right) = \mu + \varepsilon \tag{49}$$

$$\beta^2 = c_r^{1/n} / (1 - c_r^{1/n}); c_r = \varepsilon (\mu_0 - \mu)^{-1} \tag{50}$$

The constant  $c_r$  is the ratio of acceptable error to the difference between the prior value  $\mu_0$  and the measured value  $\mu$ . Therefore, the weighting parameter  $\beta$  can be expressed by the target sample size, the acceptable error of the estimate, the measured value, and the prior estimate.

For the simplicity of application of the Bayesian calculated results with the prior estimated values, a ratio of measured value to the prior estimate is defined below,

$$\alpha = q_m^* / q_{s,0}^* \tag{51}$$

The target sample size  $n_\infty$ , for repeated measures of the maximum flow rate, is a finite value that is deemed sufficient to adopt the measured maximum flow rate for design calculations with acceptable error in the final estimate  $\varepsilon_\infty$ .

$$\varepsilon_\infty = |q_{s,\infty}^* / q_{m,\infty}^* - 1| \tag{52}$$

Figure 6 illustrates the examples of  $\beta$  (ranged between 0.5 and 4) for a sample size  $n$  with errors  $\varepsilon$  and  $\alpha$  (ranged between 0.4 and 0.7). With the predetermined values for  $\beta \sim \beta(n_\infty, \varepsilon_\infty, \alpha)$ , the posterior estimate of the fractional design flow rate for measured  $q_m^*$  and a sample size  $n$  is given by,

$$q_{s,i}^* = (q_{s,i-1}^* + q_m^* \beta^{-2})(1 + \beta^{-2})^{-1}; i = 1, 2, 3, \dots n \tag{53}$$

### 3.2. Bayesian Coefficient

The posterior design flow rate estimates for a sample size  $q_{s,n}$  can be determined by multiplying the corresponding prior design flow rate estimates  $q_{s,0}$  and a Bayesian coefficient  $\alpha_n$ ,

$$q_{s,n} = \alpha_n q_{s,0} \tag{54}$$

Figure 7 exhibits the Bayesian design flow rates estimated for some residential buildings with target sample sizes  $n_\infty = 50$  and 200,  $\alpha = 0.5215$ ,  $n = 29$ , and  $\varepsilon_\infty = 0.05$ . Estimates made by Murakawa [16] are shown for comparison. For the cases with a sample size close to the target sample size, the posterior estimates are very close to the ones made by Murakawa and approaching the measured maximum loading units. For a larger target sample size, the posterior estimates are closer to the prior values than the measured ones. Table 4 summarizes the coefficients  $\alpha_n$  for the office and restaurant cases. Correction to the estimated design flow rate is unnecessary for any case in which the difference between prior and measured values is insignificant, the coefficient  $\alpha_n$  is close to unity or the sample size is small (e.g., the restaurant case).

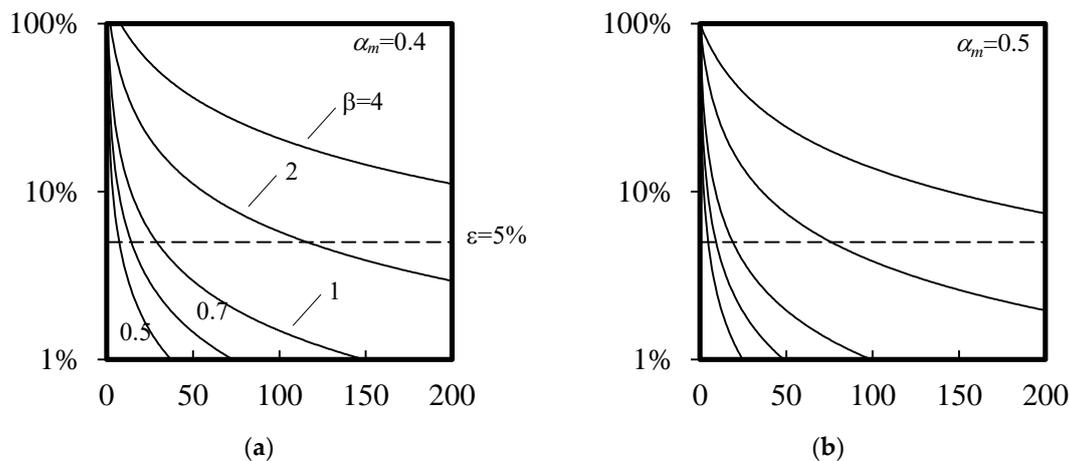
Figure 8 plots the measured maximum flow rates against the estimates made by SIMDEUM [36]. As can be seen, good predictions were made by SIMDEUM with only a few underpredictions. Figure 8 also graphs the Bayesian estimates for  $n_\infty = 3$  and 20, where the Bayesian correction factors applied. When  $n_\infty = 3$ , the Bayesian estimates are very close to the predictions made by SIMDEUM; when  $n_\infty = 20$ , a target sample size of 10% can be achieved that the corrected Bayesian estimates are closer to the original design estimates.

**Table 4.** Bayesian coefficients of design flow rates  $\alpha_m$ .

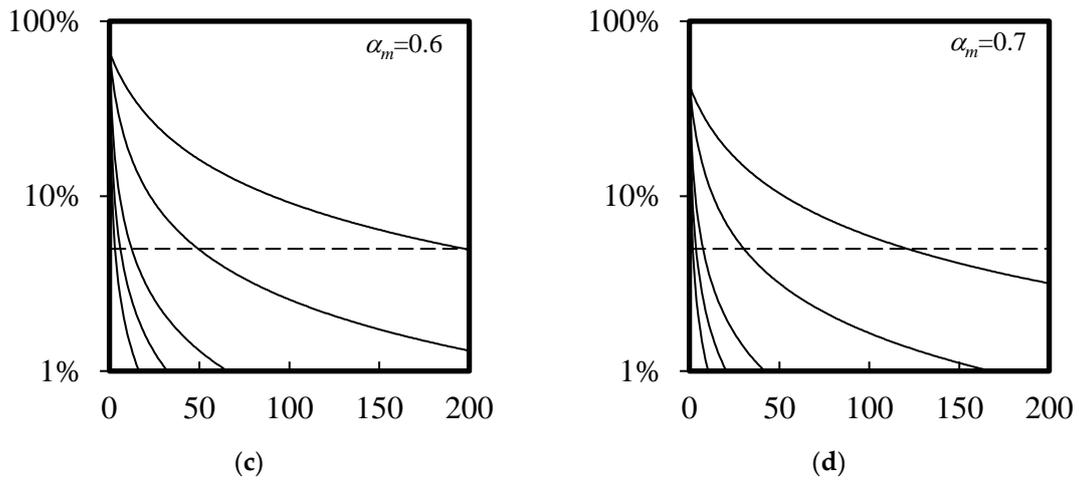
Building Type and Location	Sample Size $n$	Prior Estimated Design Flow Rate $q_{s,0}$ ( $L \cdot s^{-1}$ )	Measured (Maximum) Fraction $\alpha_m$	$\alpha_m$ (Reference Design Guide)		
				$n_\infty = 50$	$n_\infty = 100$	$n_\infty = 200$
<i>Czech Republic</i>				<i>CSN75-5455 (Czech)</i>		
Residential	11	1.09–3.79	0.483	0.571	0.633	0.715
<i>British</i>				<i>EN806-3 (British)</i>		
Residential	11	0.95–3.80	0.568	0.666	0.728	0.801
<i>Swiss</i>				<i>W3 (Swiss)</i>		
Residential	11	0.80–2.34	0.684	0.796	0.843	0.895
<i>German</i>				<i>DIN1988-300 (German)</i>		
Residential	11	0.88–2.24	0.692	0.799	0.850	0.901
<i>Netherlands</i>				<i>Dutch guidelines</i>		
Office	2	1.1–4.0	0.579–0.755	0.956	0.976	0.994
Hotel (cold)	2	1.5–1.8	0.437–0.567	0.844	0.904	0.946
Hotel (hot)	2	0.71–1.17	0.416–0.441	0.725	0.817	0.891
Nursing home	2	1.5–3.2	0.385–0.571	0.800	0.874	0.927
<i>South Africa</i>				<i>W308 (German)</i>		
Residential	1	18.8	0.466	0.837	0.904	0.947
<i>Japan</i>				<i>Loading unit (Japanese)</i>		
Residential	29	2.9–65	0.522	0.565	0.602	0.695
Office	1	11.8	0.271	0.627	0.750	0.847
Restaurant	1	10.4	0.846	0.992	0.996	0.998

Figure 9 shows the measured maximum flow rates against the estimates made by the design guidelines CSN75-5455, EN806-3, W3, and DIN1988-300. It also graphs the Bayesian estimates for  $n_\infty = 13$  (i.e., a sample size very close to the target sample size), 50, and 200. The figure demonstrates that Bayesian coefficients can significantly improve prediction quality (Figure 9b).

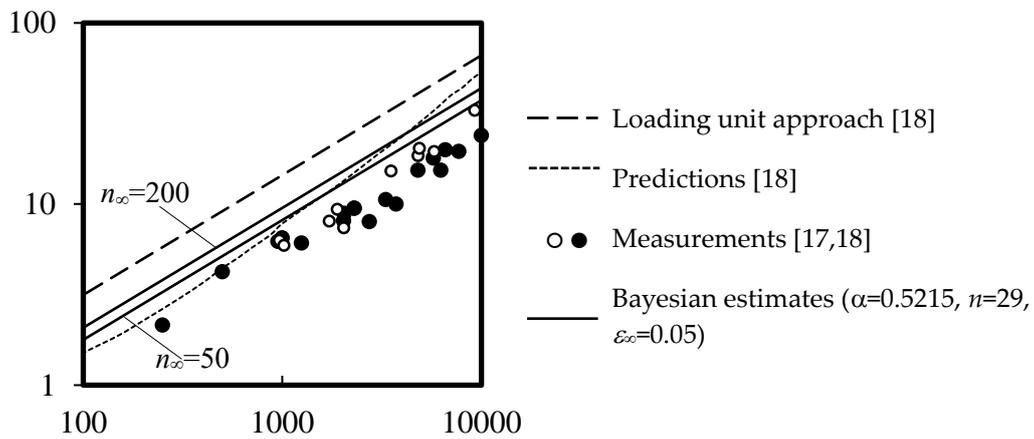
The Bayesian coefficients of design flow rates for various buildings and design guidelines are summarized in Table 4. As the proposed Bayesian approach, without any conceptual assumptions of failure factors as required in many probabilistic models (e.g., Hunter’s model), gives results comparable to those obtained through design guidelines as well as small-scale measurements, existing systems can be retained with only minor modifications to the updated measurement results.



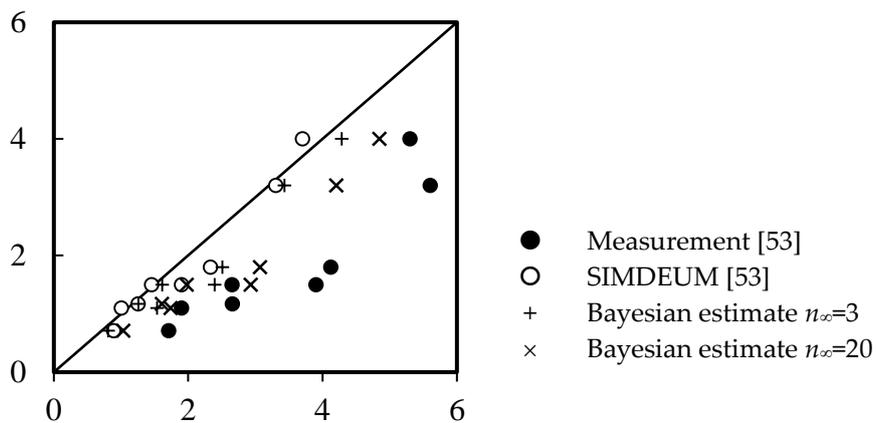
**Figure 6.** Cont.



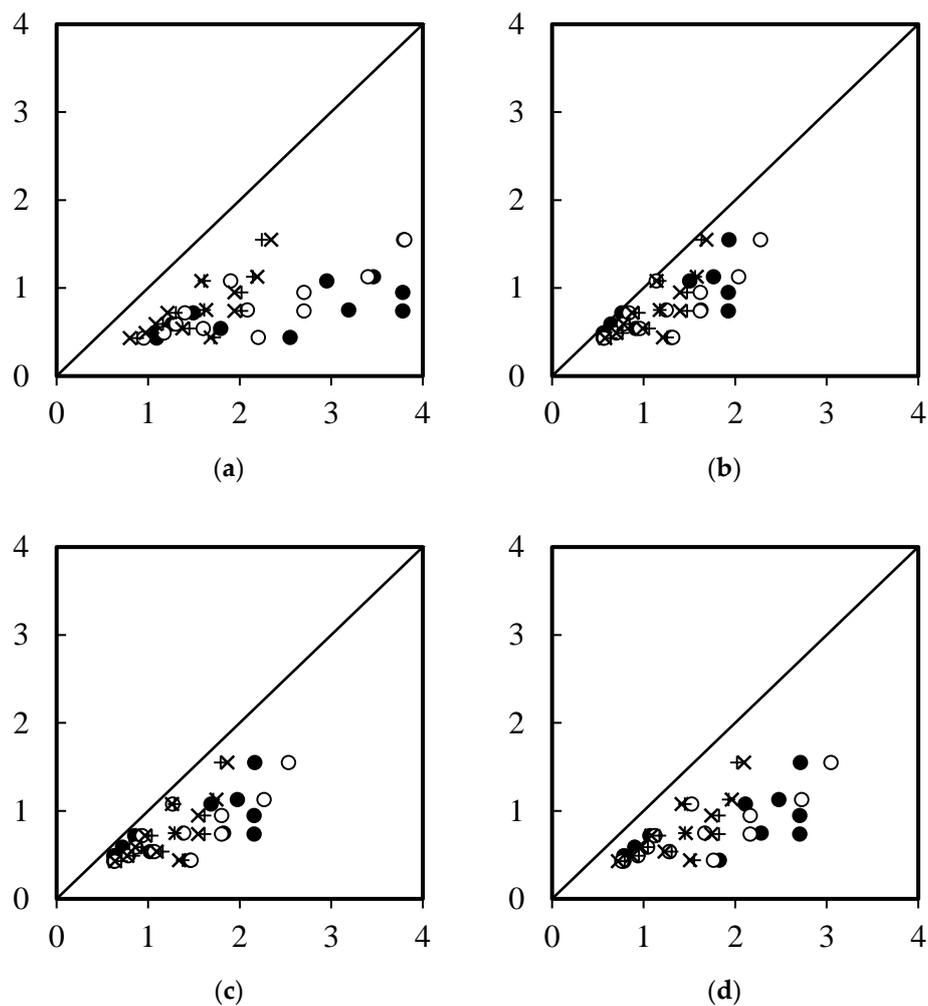
**Figure 6.** Posterior error estimates. (a)  $\alpha_m = 0.4$ ; (b)  $\alpha_m = 0.5$ ; (c)  $\alpha_m = 0.6$ ; and (d)  $\alpha_m = 0.7$ . *x*-axis: sample size *n*. *y*-axis: error  $\epsilon$  (%).



**Figure 7.** Design flow rates for some residential buildings in Japan. *x*-axis: loading unit *U*. *y*-axis: design and measurement flow rate  $q_s$  ( $L \cdot s^{-1}$ ).



**Figure 8.** Design flow rates for offices, business hotels, and nursing homes. *x*-axis: design flow rate  $q_s$  ( $L \cdot s^{-1}$ ). *y*-axis: measured maximum flow rate  $q_m$  ( $L \cdot s^{-1}$ ).



**Figure 9.** Design flow rates for Czech residential buildings. (a) Original estimates; (b) Bayesian estimates for  $n_{\infty} = 13$ ; (c) Bayesian estimates for  $n_{\infty} = 50$ ; (d) Bayesian estimates for  $n_{\infty} = 200$ .  $x$ -axis: design flow rate  $q_s$  ( $L \cdot s^{-1}$ ).  $y$ -axis: measured maximum flow rate  $q_m$  ( $L \cdot s^{-1}$ ). *Design references:* ●: CSN75 (Czech); ○: EN806 (British); +: W3 (Swiss); ×: DIN1988 (German).

#### 4. Demand Sizing, Energy Loss Minimization, and Cost Implications

A pipe size that is based on demand estimates has cost implications. Construction, maintenance, and remedy (repairing) costs of water supply systems for some high-rise buildings in Hong Kong are correlated with the ratio of pipe surface area to pipe length [63]. The costs  $\eta$  for pipe construction, maintenance, and remedy are given by the following expression, where  $D$  is the average pipe diameter, with constants  $k_0$  and  $k_1$  as exhibited in Table 5,

$$\eta \sim k_0 D^{2k_1} \quad (55)$$

For a high-rise tank water system that supplies water to 600 residential WC cisterns as described by Wong et al. [44], downsizing the supply pipe from a diameter of 67 mm to 54 mm can reduce the pipe construction, maintenance, and remedy costs by 32%, 79%, and 94%, respectively.

**Table 5.** Constants  $k_0$  and  $k_1$  for water systems in buildings.

System	Cost	Commercial Buildings		Residential Buildings	
		$k_0$	$k_1$	$k_0$	$k_1$
Water supply	Construction	1	0.33	1	0.91
	Maintenance	1	0.38	1	3.62
	Remedy	1	1.29	1	6.50
Drainage	Construction	0.67	0.18	0.62	1.69
	Maintenance	1	5.83	1	1.74
	Remedy	1	1.29	1	6.50

As energy is consumed to compensate the pressure loss due to pipe friction, pipe sizes should be chosen to minimize energy losses in a piping network. According to an energy loss optimization method by Mui et al. [64], for the probabilistic demands (from  $p = 0$  to 1) at the branch pipes (of identical radius) in a basic T-shaped piping network of constant volume, the optimal radius ratio of the centre pipe is in the range from  $2^{1/7}$  to  $2^{3/7}$ . This method can be extended to a tree-shaped piping network where a centre pipe is used to feed a number of paired branch pipes (i.e., a number of T-shaped piping networks as arranged in Figure 10), and the optimal radius ratio  $\phi_p$  is given by [65],

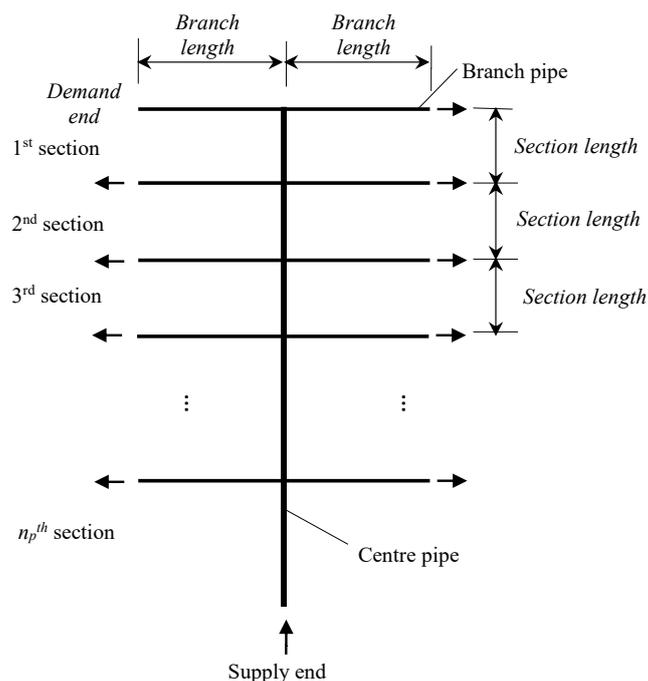
$$\phi_p = \begin{cases} (1 + n_p)^{1/7} & p < 1 \\ \left(2 \sum_{i=1}^{n_p} (2i)^2\right)^{1/7} & p = 1 \end{cases} \quad (56)$$

To demonstrate the implications of the volume constraint, an example using eight pairs of WC cisterns was presented by the authors of [65]. Two cases, case 1 with medium demands and case 2 with high demands, were illustrated. The demand flow rate was  $0.1 \text{ L}\cdot\text{s}^{-1}$  with a branch pipe diameter of 16 mm (determined by the Hunter's method [2]) and a flow velocity of  $1 \text{ m}\cdot\text{s}^{-1}$ . While the pipe radius ratios were  $16 \text{ mm}/8 \text{ mm} = 2$  and  $20 \text{ mm}/8 \text{ mm} = 2.5$ , the demand probabilities were 0.1 and 0.2 (corresponding to public water closets in offices and shopping malls [13–15]) for cases 1 and 2, respectively. Suppose choices of pipe radius were available, the existing designs would consume 12% (case 1) or 43% (case 2) more pipe friction energy than the optimal cases. In other words, with appropriate demand-controlled pump operations, this pipe sizing approach can offer pumping energy savings potentials for the two cases. For the optimal pipe radius ratios of 1.70 and 1.86 for demand probabilities 0.1 and 0.2, with the constraint of an equal pipeline volume, the corresponding centre/branch pipe radii are 15.5 mm/9.1 mm and 19 mm/10.2 mm, respectively.

In commercial building settings, the cost implications for sizing pipes with minimum energy loss were estimated according to the lengths of centre and branch pipes in a ratio of 1:1 using Equation (55). The results, shown in Table 6, indicate that the proposed sizing method will reduce energy losses, but with additional costs generated by larger pipe sizes in compared with those sizes given in some practices [13–15]).

**Table 6.** Implications of sizing a tree-shaped piping network (with branch demand flow rates of  $0.1 \text{ L}\cdot\text{s}^{-1}$ ) with energy loss minimization [61,62].

Case	Demand Probability	Average Pipe Diameter	Energy Loss	Costs		
				Construction	Maintenance	Remedy
(1)	0.1	+2.5%	−11%	+0.8%	+0.9%	+3.0%
(2)	0.2	+4.3%	−30%	+1.4%	+1.6%	+5.6%



**Figure 10.** A tree-shaped piping network (A  $n_p$ -section T-shaped piping network).

## 5. Discussion

Overestimation of the probable maximum water demands being made simultaneously at the different water outlets in a network leads to inefficient use of water and energy resources. Without accurate demand estimates, it is impossible to optimize water systems in the quest for a sustainable built environment.

In the above sections, by reviewing all major demand models and corresponding datasets with the additional dimension of predictive Bayesian input information, a better understanding of the relationships between the various types of demand and other attributes influencing the behaviour of water systems in buildings is gained. This outcome can enhance future statistical model development and measurement data analysis. The proposed Bayesian approach could result an immediate improvement in demand estimation and recommendations for existing design practice. As existing model estimates and latest demand values actually observed can be synthesized to provide up to date best available information, when implementing the proposed Bayesian model updating approach, very little modification to existing design standards/guidelines/approaches is required. Examples showing how a new dataset of water demands derived from some unit applications could be made, the new dataset that conforms better with the needs for updating and developing statistical models. Development of new technical scheme that is capable of synthesizing non-uniform demand estimates made by various methods, with new data available, are thus recommended. The scheme will determine and provide adjustments to existing practice in relation to accurate urban water demand forecasting.

## 6. Conclusions

Most water supply system designs are routinely and substantially over-sized to keep errors to a minimum. This paper reviewed three major types of demand models for sizing pipes and other components in a building water supply system. In order to bridge the gap between model estimates and field measurements for the probable maximum simultaneous water demand, a Bayesian approach was proposed. The proposed approach provides a useful method not only for evaluating the corresponding demand values from various design references, but also for reducing energy losses in pipes though with a bigger price tag.

**Author Contributions:** L.-T.W. and K.-W.M. contributed equally to the study, developed the Bayesian model and wrote the manuscript.

**Funding:** The work described in this paper was partially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region (HKSAR), China (PolyU 5272/13E), and three other grants from The Hong Kong Polytechnic University (GYBA6, GYM64, GYBFN).

**Conflicts of Interest:** The authors declare no conflicts of interest.

## Nomenclature

$B()$	Beta-binomial distribution
$C$	Binomial coefficient
$D$	Pipe diameter
$i, j$	$i, j = 1, 2, 3, \dots$ as defined
$k$	Constant
$M$	Number of fixtures
$m, m_i$	Number of fixtures, number of fixtures of fixture $i$ (given a reference flow rate $q_f$ )
$N$	Number of demands
$N()$	Normal distribution
$n$	Sample size
$n_a$	Number of appliances serving a group of users
$n_p$	Number of T-networks in a tree-shaped network
$P()$	Probability
$p, p_i, p_j$	Probability, probability of operating fixture $i$ , probability of operating fixture group $j$
$Q$	Flow rate
$q, q_i$	Flow rate, flow rate of fixture $i$
$q_s, q_m$	Design flow rate, measured maximum flow rate
$q_f$	Probable maximum demand
$t$	Time
$U, U_i$	Fixture (loading) unit, fixture unit of fixture $i$
$u$	Uniform random number between 0 and 1
$x$	Dummy variable as defined
$X, Y$	Dummy variable as defined
<i>Greek</i>	
$\Gamma()$	Gamma function
$\alpha$	Bayesian constant (fractional flow rate of design value)
$\beta$	Ratio of measured to predicted standard deviations
$\varepsilon$	Error
$\lambda$	Failure rate
$\phi$	Optimal pipe diameter ratio
$\gamma$	Arrival rate
$\eta$	Cost
$\mu$	Mean
$\sigma$	Variance
$\tau$	Time period
$\tau_0, \tau_1, \tau_2$	Time period of no demand, of a demand, between two consecutive demands

*Subscripts*

0, 1, 2, ...	Of conditions 0, 1, 2, ... as defined
<i>f</i>	Of reference
<i>i, j</i>	Of <i>i, j</i> as defined
<i>m</i>	Of measured value
max	Of maximum
<i>p</i>	Of probability
<i>s</i>	Of design condition
∞	Of target value

*Superscripts*

*	Fractional value
~	Probability density function

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