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Evolution and Competition, a Game Theoretical Analysis of Heterogeneous Wireless Networks in Unlicensed Spectrum

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ABSTRACT Wireless community networks (WCNs) are emerging as an alternative to provide wireless offloading before the deployment of 5G network. However, it is not clear about its development prospects, e.g., how to understand the WCN technology affects the adoption of users, and the competition in a communication market coexisting different wireless technologies, i.e., LTE in unlicensed spectrum. To this end, we envision three distinct development stages of WCN: evolution, regulation, and competition. Specially, we first study the evolution of self-organizing WCN in which the WCN services expands as WCN participants increases. We propose a dynamic model and show the existence of unique equilibrium point at which the fraction of subscribers does not change. Next, we turn to investigate the impact of regulation on a commercial WCN by introducing an operator. The users in commercial WCN can be divided into two parts: insiders and outsiders, according to whether contributing connectivity into the community. We discuss the economic interactions with regard to insiders, outsiders, and the operator, and derive the equilibrium adoption of various users and the optimal price of the WCN operator. We then investigate a competitive duopoly market that coexists a WCN provider (WCNP) and an LTE-U provider (LP). We model the economic relationships among the users, WCNP and LP as a tripartite game, in which the two providers choose their market shares independently. We derive a sufficient condition that guarantees the existence of equilibrium in the market competition game.

INDEX TERMS Wireless community networks, game theory, evolution, unlicensed spectrum.

I. INTRODUCTION

Thanks to the rapid development of wireless technologies, users now could enjoy the ubiquitous connectivity freely, i.e., no limit at home but connectivity everywhere. On the other hand, the increased mobile devices also lead to an exponential growth data traffic that is expected to continue increasing by 1000 times in the upcoming five years [1]. Such rapid growth of traffic demands are straining the limited cellular network capacity. According to traditional capacity expansion methods, network technology upgrades may be prohibitively expensive for additional spectrum acquisition, and will be soon outpaced by the increasing traffic demands.

WiFi technology is a natural alternative to meet the continuously increasing data traffic. There are several reasons favoring WiFi offloading, e.g., WiFi utilizes unlicensed spectrum without conflicting with cellular networks. The latest WiFi technology is continuously converging with the specifications of 5G wireless networks that can provide a very high data rate. Due to these advantages, several WiFi modes emerged as a helpful supplement for cellular networks, e.g., WiFi Community Networks(WCN). WCN allows users to share their wireless access by contributing their wireless router, in exchange for either compensation or reciprocation. Roaming users without contributing their own wireless access can also easily obtain connectivity by paying small fee. Consequently, the coverage of WCN will grow organically as more users participate in the network. WCN opens up the possibility of converting the organical WiFi allies to commercial connectivity service.

Recently, some WCN services are currently operated by competitive vendors so as to provide a commercial data service. FON [1] has built a WiFi sharing community by selling their customized infrastructure, which authenticates the accessing users and connects to a FON server through the Internet. In China, WCN service has became a business opportunity in recent years, there emerged many WCN operators, e.g., treebear [2], nextwifi [3]. According to iResearch 2016 Report [4], the commercial Wi-Fi market in China will amount to CNY3.26 billion at a growth rate of 77.8% by 2018. It is believed that WiFi will be an important component before the deployment of 5G technology.

Despite these advances, other issues occur before large scale deployment of commercial WCN [1]. For instance, it is not clear that commercial WCN with operator always performs better than the organical mode. On the other hand, another important aspect of the 5G vision, namely LTE in unlicensed spectrum (LTE-U), has recently emerged as a significant competitor of WiFi. Most operators believe LTE-U will seamlessly enhance the capacity of cellular networks, as well as energy efficiency [9]. LTE-U operators would like to hold obscure attitudes toward WCN services, since WCN could be an opponent of existing LTE-U technology. One issue that must be considered by the operators of WCN and LTE-U is how to coexist in unlicensed spectrum. Specially, if commercial WCN services can indeed attract more potential users to participate in the community, and can serve as an opponent of existing centralized networks operating in unlicensed spectrum, how the price war will evolve in the case of multi-operators scenario, where a WCN Provider(WCNP) and a LTE-U Provider(LP) coexist in a duopoly market and fight for users by choosing the appropriate access price. What's the impact of competition between the operators it bring on users? These issues are highly relevant, but so far lack of literatures verify them in theory. Recent studies [5]–[7] have proposed game theory to study the problem of LTE-WiFi coexistence. Most of these solutions argued the feasibility of using public WiFi for offloading cellular traffic. Few of literatures on the other hand provide a comprehensive solution for the evolution of WCN services, as well as the pricing war between LTE-U and WiFi.

The evolution of WCN exhibits different features with the involvement of various parties, which can be depicted by several stages. However, it is not clear about its development prospects, e.g., how to understand the WCN technology affects the adoption of users, and the competition in a communication market coexisting different wireless technologies, i.e., LTE in unlicensed spectrum (LTE-U). In this paper, we envision three serial and distinct stages of the WCN epoches: evolution, regulation and competition. Specially, we first analyze the evolution stage of WCN where users of the WCN participate in a self-organizing mode. In this mode, users share their connectivity and enjoy the same benefits in return when themselves roaming. By developing a discretized model, we first show that for any coverage function, there exists a unique equilibrium of the user subscription where the fraction of users keeps stable. For comparison, we next investigate the impact of regulation on a commercial WCN by introducing an operator. The users in commercial WCN can be divided into two parts: insiders and outsiders, which can be differentiated by whether contributing connectivity to WCN. We discuss the economic interactions with regard to the three parties, i.e., insiders, outsiders and the operator. We derive the equilibrium adoption of users and the optimal price of the operator. We show that the adoption level of insiders and outsiders will both increase by introducing an operator. Nevertheless, outsiders may access internet from commercial WCN for free due to the strong network externality. Then, we investigate a duopoly market which includes a WCNP and LTE-U, i.e., LP. Specifically, we model the economic interactions among users, WCNP and LP as a three-stage Stackelberg game. we derive the optimal price pair of the two providers and the corresponding equilibrium market share in face of the optimal price pair. We show that the price of LTE-U services will decrease due to the competition between the two operators. We also verify that the competitive environment does not bring any loss in terms of social welfare. We believe that studying the evolution of WCN we introduce here is a primal need, because it is a very important aspect of nowadays wireless network. The proposed results can be applied to develop qualitative pricing guidelines for successful WCN service deployments.

II. RELATED WORK

As an alternative to cellular networks, wireless community networks have emerged as a low-cost approach to provide high-speed wireless data service in unlicensed spectrum. A neighborhood self-organizing WCN was considered in [8] and [9]. In [8], this type of network built by people according to Microsoft system. That is, in addition to providing an Internet accessing, users can form wireless community networks with their own wireless access points exploiting the large amount of unutilized connectivity, which could further facilitate people's access to the Internet. Reference [9] provides a simpler solution where users could share their bandwidth by installing client software on their computer. These applications are useful but have their limitations, since users must join the community by sharing their own access points first. Moreover, such applications brought a lot of potential safety problems [10].

Recently, some wireless community networks are currently operated by some competitive vendors so as to provide a commercial data service. FON [1] has built a WiFi sharing community by selling their customized infrastructure, which authenticates the accessing users and connects to a FON server through the Internet. However, the FON solution requires users to upgrade to a new WiFi router provided by FON and this could be an obstacle for further growth. Some research institutions have also shown their interests in this subject, for example, a world wide WiFi community network funded by Google and Skype [11]. It is not clear whether wireless community networks can indeed emerge as a viable alternative to cellular wireless network providers [12].

LTE-U technology is proposed to meet continuously increasing data traffic requirements of emerging wireless services in recent years. LTE-U enables LTE base stations (BSs) to exploit the readily available unlicensed spectrum, and could take full advantage of LTE, e.g., backhaul management, resource allocation and interference management. However, little literature has been raised in term of the co-existence of LTE-U and Wi-Fi networks in unlicensed spectrum. In [13], a MAC mechanisms is proposed to investigate coexistence of LTE-U and Wi-Fi. [14] presented a physical layer framework to study the interations of LTE-U and Wi-Fi networks. It shows that the proposed power control mechanism can improve the performance of both types of networks. These works focused more on the resource allocation problems at WiFi and LTE-U networks. To capture the dynamic interations between WiFi and LTE-U networks, new approaches should be developed and used to address the new challenges in these networks.

Our work applies game theory to study the issues in WCN. Game theory has been quite recently introduced in telecommunication networks. For instance, Jin *et al.* [15] studied technology adoption and competition between incumbent and emerging network technologies. Reference [16] investigated market dynamics emerging when next-generation networks and conventional networks coexist, by applying a market model that consists of content providers, Internet Services Providers(ISP) and users. He and Walrand [17] proposed a pricing algorithm in a differentiated services environment based on the cost of providing different levels of services. These models characterizing various technologies was oversimplified to capture their influences on the users. A model closely related to ours is in [18], the pricing competition between WiMAX and WiFi community is investigated, but the externality is QoS instead of coverage. In [19], the economic interactions between users, ISPs and community providers are investigated. The authors proposed a model of the global wireless community concept as a Stackelberg game of two levels and constructed the respective payoff functions of each player. Others works related to our work is [20], [21] in which the authors examined the evolution of network sizes in wireless social community networks. A key assumption, based on which equilibrium was derived, is that a WCN provides a higher QoS to each user as the number of subscribers increases. These works mostly focus on the competitions between the providers, regardless of the decisions of users, which is also a dominant factor determining the equilibrium of market.

III. SELF-ORGANIZING WCN

Consider a WCN in a self-organizing mode, the participants share their connectivity and in turn they can enjoy the same benefits when themselves roaming. In this respect, the roaming coverage of WCN services can be expanded with the increasing of participants number. Assume the potential users that have possibility to participate in WCN are normalized. For ease of exposition, we use $x \in (0, 1)$ to denote the adoption level, i.e., the fraction of users that have joined in WCN.

The users are rational, they make participating decision by their expected utility from participating WCN. The utility of participants consists of two parts, one is home utility gained from surfing Internet at home, the other is roaming utility derived from roaming outside through WCN. Considering the characteristics of netizen, we assume the home utility as a stage function related to the users' available bandwidth. Note that we assume home utility is a constant, namely the basic home utility denoted by λ_0, if the available bandwidth is larger than a bandwidth threshold. Otherwise, the home utility decreases with the decreasing of available bandwidth. Let W_0 denote the standard bandwidth offered by ISP, and $V(x)$ denote the volume of roaming traffic incurred by the WCN affecting home users. $V(x)$ is a negative roaming traffic that increases with the adoption level x. $W = W_0 - V(x)$ is the available bandwidth. Thus, the home utility function can be represented as follow

$$\lambda(x) = \begin{cases} \lambda_0, & W \geq \bar{W} \\ \dfrac{W \cdot \lambda_0}{\bar{W}}, & W < \bar{W} \end{cases} \tag{1}$$

where $\bar{W}$ is the bandwidth threshold.

On the other hand, roaming utility depends on the coverage of WCN, which is closely related to the adoption level. We use $g(x)$ to represent the services coverage, which grows with the level of adoption x. Users are heterogeneous. Each user is characterized by a non-negative real value $\alpha \in [0, 1]$, which represents the possibility the user roams outside. A high α corresponds to a user that frequently roams and a user with a high value of α is able to derive a relatively large utility from a same level of service's coverage. In contrast, it derives less utility from accessing internet at home. Hence α also can be interpreted as its valuation on the WCN service's coverage. Specifically, when a user participate in WCN, its utility is given by

$$u(\alpha) = (1 - \alpha)\lambda(x) + \alpha \cdot g(x) \tag{2}$$

where $\alpha \cdot g(x)$ represents the utility they derive from getting connectivity through home base while roaming, which accounts for the effect of heterogeneity in the roaming characteristics of users, i.e., low α or sedentary users derive comparatively little benefits from being able to connect through other users' home base.

The users are rational, they make participating decision by evaluating the utility deriving from participating in the WCN. As the adoption level changes, individual user's utility varies, which affects its own adoption decision. User would like to quit the WCN if its utility is less than that at home, which can be mathematically represented as

$$U(\alpha) \geq (1 - \alpha)\lambda_0 \tag{3}$$

For analytical tractability, we assume $g(x)$ is an increasing and continuously differentiable function. In addition, the users' valuation of services'coverage follows a probability distribution whose probability density function (PDF) $f(\alpha)$ is strictly positive and continuous on $[0, \beta]$. The cumulative density function (CDF) is given by $F(\alpha) = \int_0^{\alpha_m} f(\alpha)d\alpha$, where α_m is the marginal user whose utility equals to zero.

This assumption can be considered as an expression of user diversity in terms of the valuation of service's coverage. When the users' valuation of coverage is sufficiently diverse, its distribution can be described by a continuous positive PDF on a certain interval as in [22]. Note that the lower bound on the interval is set as zero to simplify the analysis, and this will be the case when there is enough diversity in the users' valuation of coverage so that there are nonsubscribers for any positive price. Note that according to the assumption, the adoption level of users can be represented as $x = 1 - F(\alpha_m)$. In next subsection, we study the dynamics of user subscription in self-organizing mode.

A. EVOLUTION IN SELF-ORGANIZING WCN

In the organizing WCN, users make adoption decision by taking consideration of the services' coverage and the extra traffic coming from the other users. Note that the service's coverage varies with the changing of fraction of participants, which eventually leads to the changing of users' utility. To describe the dynamics of user decision, we construct a discrete-time model where the time periods are divided into several periods indexed $t = 1, 2, \ldots n$. We assume that the adoption decisions at period $t + 1$ are driven by the expectation of user's utility which are a function of the period t. Specifically, at each period $t + 1$, each user holds a belief on its expected utility, that is, users consider the utility gained from the previous period t as its expected utility of current period. Then, users makes adoption decisions independently based on these history information, e.g., the adoption level at period t x_t and the services coverage $g(x_t)$.

Therefore, the user with valuation α will prefer to participant in the WCN at period $t + 1$ if and only if its expected utility derived from WCN is no less than that at home, i.e.

$$u_{t+1}(\alpha) = (1 - \alpha)\lambda(x) + \alpha \cdot g(x_t) \geq (1 - \alpha)\lambda_0 \tag{4}$$

From Eq. 4, we can get the user with marginal valuation that is indifferent with the WCN's service at period $t + 1$ by setting $u_{t+1}(\alpha) = 0$

$$\alpha^m_{t+1} = \frac{\lambda_0 - \lambda(x_t)}{\lambda_0 - \lambda(x_t) + g(x)} \tag{5}$$

where $\lambda(x_t)$ is the home utility indicated in Eq. 1.

Notice that the user with marginal valuation α^m_{t+1} is a function of x_t, in other words, the adoption level of system at period $t + 1$ is associated with that at period t. This implies that the fraction of users of WCN evolves following a sequence x_t. To characterize this dynamic state of system, i.e., specifying the adoption level at period $t + 1$, we use the following equation to capture the evolution of user's adoption level.

$$x_{t+1} = H(x_t) = 1 - F(\alpha_m) \tag{6}$$

Given Eq. 4, we can verify the definition of equilibrium point as follows: x^*_t is an equilibrium point of the adoption level in WCN if it satisfies $x^*_{t+1} = H(x^*_t) = x^*_t$. It implies that once an equilibrium point is reached, the fraction of users remains the same and stabilizes in the long run. The following Proposition establishes the existence and uniqueness of an equilibrium point.

Proposition 1: Starting from an initial state of zero adoption, a non-zero adoption level of users is possible.

Proof: To investigate the existence of equilibrium point, we first define

$$K(x_t) = H(x_t) - x_t \tag{7}$$

Note that x^*_t is an equilibrium point if and only if it is a root of Eq. 7. Hence, it suffices to show that $K(x_t)$ has a unique root on its domain.

Note that $F(.)$ is differentiable on $x_t \in (0, 1)$ with $F'(.) = f(.)$. By applying the first order optimality condition to $K(x_t)$ with respect to x_t, we have

$$K'(x_t) = H(x_t) - 1 \tag{8}$$

Since $\lambda(x_t)$ is a stage function, we can consider two cases.

Case 1 ($W > \bar{W}$ and $\lambda(x_t) = \lambda_0$) Eq. 8 can be further reduced to

$$K'(x_t) = \frac{\lambda_0(-g'(x_t))}{(\lambda_0 - g(x))^2} - 1 \tag{9}$$

Notice that $g'(x_t) < 0$, we can verify that $K'(x_t) < 0$ which implies that $K'(x_t)$ is strictly decreasing.

Case 2 ($W < \bar{W}$), $\lambda(x_t)$ is a decreasing function of x_t. $\lambda(x_t) > 0$ and $\lambda'(x_t) < g'(x_t)$, hence $K'(x_t) < 0$. Thus, $K'(x_t)$ is strictly decreasing.

Therefore, $K(x_t)$ is strictly decreasing in $(0, 1)$. Next, we note that $K(0) = 1 - F(\frac{\lambda_0}{\lambda_0}) > 0$ and $K(1) = -F(\frac{\lambda(1)}{\lambda(1) - g(1)}) < 0$.

Since $K(x_t)$ is continuous on $x_t \in (0, 1)$, we can obtain a unique root of $K(x_t)$ on $x_t \in (0, 1)$ by applying the intermediate value theorem. □

Proposition 1 verifies that an unique equilibrium point can be obtained from any start adoption point. However, it does not provide an explicit expression. Generally, the users with high value of α, i.e., frequently roaming users, would be stay at WCN unless the adoption is large enough. In order to provide a closed-form expression of the equilibrium point, we give below a simple realization of coverage function $g(x)$ with the above outcome. The coverage function $g(x)$ is linearly-increasing $g(x) = kx - b$, for all $x \in (0, 1)$ and $b > 0$.

For ease of expression, we further assume that the user with high α will bring a comparatively large volume of traffic. Hence, the traffic varies with the per-unit fraction of user Δx

is $\Delta V = \alpha \cdot \Delta x$. The total amount of roaming traffic generated by the roaming users can be represented as $V = \int_{\alpha_m}^{1} \alpha dx$. It should be noted that only the user with valuation that is larger than α_m will stay in the system.

More specially, recall that the adoption level of users is $x = 1 - F(\alpha_m)$, if we consider $F(x)$ is a uniform distribution in $x_t \in (0, 1)$, we get $x = 1 - \alpha_m$. Thus, $V = \int_{\alpha_m}^{1} \alpha dx$ can be further rewritten as $V(x_t) = \int_{1-x_t}^{1} \alpha d\alpha$.

We can estimate the volume of traffic in the WCN as

$$V(x_t) = -\frac{x_t^2}{4} + x_t + \frac{1}{4} \tag{10}$$

With a linearly-increasing coverage function and the volume of traffic indicated in Eq. 10, we can obtain a simple closed-form expression of the equilibrium point. Specifically, with $g(x) = kx - b$ and $V = -\frac{x_t^2}{4} + x_t + \frac{1}{4}$, for $x_t \in (0, 1)$, by setting $H(x_t^*) = x_t^*$, the equilibrium point of the adoption level of users that are dynamics in WCN can be expressed as follows:

$$x_t^* = \frac{(\sqrt{(k^2\bar{W}^2 - 4\bar{W}\lambda_0^2 - 4k\bar{W}\lambda_0 + 4W_0\lambda_0^2)} + k\bar{W})}{2\lambda_0} \tag{11}$$

So far, our equilibrium analysis verifies the existence of a stable point of the adoption level in the WCN. We can also employ geometry based argument for a more explicit explanation. As illustrated in Fig 1, there are serveal intersections that are possible in the entire adoption range, but only a stable equilibrium can be reached independent of initial point. In this case, the adoption level of WCN proceeds monotonically towards the equilibrium, either increasing or decreasing depending on the value of the initial adoption level.

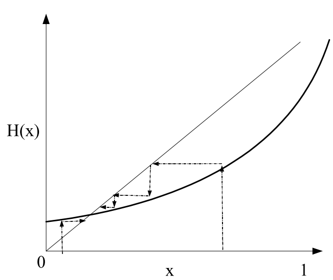

FIGURE 1. Equilibrium illustration.

IV. REGULATION IN WCN

We proceed to investigate the impact of regulation on evolution of WCN with an operator. The WCN operator gathers the connectivity of insiders and provides paid wireless access to the normal users without wireless router to expand the influence of WCN. We assume the WCN operator provides a cheaper option comparing with the other access technologies, e.g., LTE-U. Users prefer to connect WCN if available.

The users in WCN with operator can be divided into two categories, insiders and outsiders. Insiders refer to the participants that possess access equipment at home. They are the contributors of WCN, in return, they will get compensation from operator to subsidize their dissipative bandwidth costs. The outsiders are the normal users without AP router. They join WCN by taking account of the utility deriving from the roaming experience. We assume the outsiders that have possibility to subscribe to the WCN services are also normalized. Let $y_t \in (0, 1)$ denote the adoption level of outsiders at period t. Therefore, the payoff function of insiders can be rewritten as

$$u(\alpha) = (1 - \alpha)\lambda'(x_t) + \alpha \cdot g(x_t) + m \cdot V_2(y_t) \tag{12}$$

where

$$\lambda'(x) = \begin{cases} \lambda_0, & W \geq \bar{W} \\ \dfrac{W_0 - V_1(x_t) - V_2(y_t)}{\bar{W}}\lambda_0, & W < \bar{W} \end{cases} \tag{13}$$

Note that $V_1(x_t)$ and $V_2(y_t)$ are roaming traffic caused by the insiders and outsiders at period t, respectively. $m \cdot V_2(y_t)$ represents the compensation that insider could derive from the operator.

Meanwhile each outsider make subscription decision according to the accessing price and its valuation on the services coverage. Thus, the payoff function of outsiders can be represented as

$$u_o = \theta \cdot g(x_t) - p \tag{14}$$

where $\theta \in [0, 1]$ is the coverage preference, which represents the roaming characteristics of outsiders, p is the subscription fee.

In this WCN with operator, given access price p, both insiders and outsiders independently make adoption decision according to the previous system state. These interactions may be extraordinarily complex as the adoption level of users and services' coverage change. Next, we investigate the equilibrium point in this WCN with operator.

A. EQUILIBRIUM IN WCN WITH OPERATOR

Building upon the presented payoff functions, we concentrate on investigating the equilibrium point of the WCN with operator. In this mode, the operator has the power to determine the internet access price of outsiders. On the other hand, the adoption level of insiders determines the coverage of WCN services. Given coverage and access price, the outsiders would subscribe to the WCN operator at period t only if their utility is greater than zero. We first seek the equilibrium adoption level of outsiders. The following lemma clarify the adoption level of outsiders

Lemma 1: For a given access price p and adoption level of insiders, the fraction of outsiders which are willness to subscribe the WCN services is

$$y_t = 1 - F(\frac{p}{g(x_t)}) \tag{15}$$

where x_t is the adoption level of insiders at period t.

Proof: We can get the insider with marginal valuation that is indifferent with the WCN services by setting $u(\alpha) = 0$

$$\alpha_m = \frac{\lambda'(x_t) + m \cdot V_2(y_t)}{D - g(x_t)} \tag{16}$$

The adoption level of insiders at period $t+1$ can be represented as

$$x_{t+1} = H(x_t) = 1 - F(\frac{\lambda'(x_t) + m \cdot V_2(y_t)}{D - g(x_t)}) \tag{17}$$

Notice that the adoption level of outsider is $y_{x_t} = 1 - F(\frac{p}{g(x_t)})$, by substituting $y(x_t)$, we can convert Eq. 17 to a function of x_t. □

Similarly, if applying linearly-degrading coverage functions into Eq. 15, we can obtain a closed-form expression, i.e., the equilibrium point of insiders' adoption level dynamics in WCN with operator can be expressed as a function of p as follows:

$$x_t^* = \frac{\sqrt{mp+k^2m^2+4k^2mp-2km^2p+\bar{W}\cdot k+m^2p^2-4\cdot W_0\cdot\lambda_0^2}}{2(\lambda_0-km)} \tag{18}$$

Comparing the equilibrium adoption level of x^* with that in the self-organizing WCN indicated in Eq. 11, we can verify that the adoption level of insiders increases as operator is involved, which implies that the existence of operator is benefit to enhance the market sharing of WCN.

B. REVENUE MAXIMIZATION WITH OPERATOR

We next investigate the optimal price of operator. WCN operator receives revenue from the outsiders who pay for the services. At the same time, operator has to undertake amount of cost for providing such services. Generally, the cost of the operator is mainly composed of two parts, one is the operating cost that incurred by supporting another request of insiders. The other is the cost that the payment for subsidizing the insiders' dissipative bandwidth costs. For simplicity, we use c_1 and c_2 to denote the marginal operating cost and marginal compensation, respectively. In addition, WCN operator could obtain some external profits from each subscription of outsiders due to networks' externality effects, e.g., portal advertisements. It is reasonable to assume the external profits is related to the involved users. We use b to denote the marginal profit. The operator's revenue function R_r then can be represented as:

$$R_r = p \cdot y + b \cdot y - c_1 \cdot y - c_2 \cdot y \tag{19}$$

where y is the fraction of insiders and outsiders, respectively. $p \cdot y$ corresponds to the profits collected from outsiders.

Notice that the equilibrium adoption of insiders and outsiders depicted in Eq. 18 is a function of price p. We can interpret the equilibrium adoption of outsiders as a demand function, which represents the changes of outsiders' adoption with price p. For ease of presentation, we use $y = D_p(p)$ to denote the outsiders' demand function with respect to price p. In addition, we also introduce the concept of price elasticity, which describes the degree that the fraction of y varies with the price p. The outsiders' price elasticity are denoted as $E_p = \frac{D'_p(p)}{y/p}$, where $D'_p(p)$ is the first order derivative of the demand function $D_p(p)$ with respect to p.

Given the demand function of outsiders, the problem for operator is how to carefully set the price so as to maximize its revenue. Since the operator's revenue function is depicted in Eq. 19, the optimal price provided by operator that achieves its maximal revenue should satisfy

$$p^* \in \arg\max_p\{(p * y - c_1 \cdot y - c_2 \cdot y - b(y))\} \tag{20}$$

By solving the above problem, we can obtain the optimal price of operator. The detailed analysis is depicted in Lemma 2.

Lemma 2: Suppose the network externality function is linear function, then optimal variables solving the revenue maximization of problem in Eq. 20 satisfies $p^* = \frac{c_1+c_2-b}{1+\frac{1}{E_p}}$.

Proof: Note that the adoption level of outsiders y is a function of p. Considering the constraint $y = d_p(p)$, we apply the first-order condition with respect to p in Eq.(19), then we have

$$\frac{\partial R_r}{\partial p} = y + p \cdot d'_p(p) - c_1 \cdot d'_p(p) - -c_2 \cdot d'_p(p) - b \cdot d'_p(p) = 0 \tag{21}$$

Recall that the demand elasticity is $E_p = \frac{d'(p)}{y/p}$, Eq. (21) can be further simplified and we obtain $p^* = \frac{c_1+c_2-b}{1+\frac{1}{E_p}}$. □

Due to the existence of the external effective coefficient b, we can verify that the optimal price is possibly very low. When the network externality is large, the optimal price may even equal to zero by taking into account the fact that the marginal cost c_1 and c_2 are always very low. In other words, The outsiders may enjoy the WCN's service free. The extra traffic cost is able to compensate by the operator from the network externality.

V. COMPETITION IN THE DUOPOLY MARKET

When a WCN operator has developed to a certain phase, e.g., possessing plenty of user resources, it can be considered as an independent network service provider. On the other hand, it also has to face price war from competitors, e.g., LTE-U provider, since users can find a better combination of coverage and price among the services providers in the market. In this section, we will study a duopoly market with two competing networks services providers, which corresponds to WCN Provider (WCNP) and LTE-U Provider (LP).

Formally, consider a network service area coexists a WCNP and a LP. LP typically uses a licensed spectrum to provide the ubiquitous wireless access. We use g_L to denote the coverage of LP. Generally, we have $g_L = 1$ which indicates it is full coverage, and q is the corresponding subscription fee.

Hence the payoff function of the outsider if it subscribes the service of LP can be represented as:

$$u_o = \theta \cdot g_L - q \tag{22}$$

LP collects profits from the subscribers who are wiliness to pay for the LTE-U service. We use z_t to denote the market share of LP at period t. Meanwhile LP has to undertake a certain cost which is denoted as c_3. The term c_3 is used to deploy and maintain base stations, to acquire the spectrum licenses, etc. Thus, the payoff function of LP is:

$$R_{LP} = p \cdot z_t - c_3 \cdot z_t - b \cdot z_t \tag{23}$$

where b is network externality.

With the two providers operating in the duoplay market, each user has three possible choices at each time instant: subscribe to LP, subscribe to WCNP, and subscribe to neither. Users subscribe to a wireless network operator based on the provided coverage and the subscription fee, as well as their valuation on the services' coverage. A fraction of users with small valuation will refrain from subscribing to any operator, because they are not satisfied with the available coverage or the subscribing price. We follow the dynamic model in which the users make subscription decisions to maximize their expected profits at discrete time period t. In addition, we assume the switch cost involved in subscription decisions between two providersis zero comparing with the subscription fee [13].

In this duoplay market, LP provides full coverage service with price q while WCNP offers service with coverage $g(x_t)$ with price p. Users dynamically make subscription decisions based on their expected utility in two services. At the equilibrium state of system, the market shares of the two providers are uniquely determined when the fraction of users subscribing to each provider no longer changes. Thus, the equilibrium point of the user subscription in the duopoly market satisfies

$$H_1(x_t^*, y_t^*) = x_t^*, H_2(x_t^*, y_t^*) = y_t^* \tag{24}$$

We can characterize the existence and uniqueness of an equilibrium point and provide equations characterizing it in the following Proposition.

Proposition 2: For any non-negative price pair $\{p, q\}$, there exists a unique equilibrium point y_t^*, z_t^* of the user subscription dynamics in the duopoly market. Moreover, y_t^*, z_t^* satisfies: if $\frac{p}{g(1)} < \frac{q}{g_L}$

$$\begin{aligned} y_t^* &= F(\frac{q-p}{g_L - g(x_t)}) - F(\frac{p}{g(x_t)}), \\ z_t^* &= 1 - F(\frac{q-p}{g_L - g(x_t)}) \end{aligned} \tag{25}$$

else

$$y_t^* = 0, z_t^* = 1 - F(\frac{q}{g_L}). \tag{26}$$

Proof: A user v subscribes to the LTE-U service if its payoff with LP is positive and strictly greater than its payoff with WCN. Mathematically, if $\frac{p}{g(1)} < \frac{q}{g_L}$, we can express this inequality which depicts the set of user that will prefer subscribing to the LP as follows:

$$\theta_v \cdot g_L - q \geq \theta \cdot g_L - q\theta_v \cdot g_L - q \geq 0 \tag{27}$$

Given the price pair (p, q), the market share of LP can be obtained

$$z_t^* = 1 - F(\frac{q-p}{g_L - g(x_t)}) \tag{28}$$

Likewise, we can also get the set of user that will subscribe to the WCN as follows:

$$\theta_v \cdot g_L - q \geq \theta \cdot g_L - q\theta_v \cdot g_L - q \geq 0 \tag{29}$$

Hence, it can be shown that the equilibrium market share of WCNP is

$$y_t^* = F(\frac{q-p}{g_L - g(x_t)}) - F(\frac{p}{g(x_t)}) \tag{30}$$

On the other hand, if $\frac{p}{g(1)} > \frac{q}{g_L}$, we can verify that $F(\frac{q-p}{g_L - g(x_t)}) < F(\frac{p}{g(x_t)})$ which indicates that no users subscribe to the WCN services. The corresponding market share of the two providers is

$$z_t^* = 1 - F(\frac{q}{g_L}), \quad y_t^* = 0 \tag{31}$$

□

Proposition 2 shows that the equilibrium point depends on the relative values of access price and service coverage. Specifically, if $\frac{p}{g(1)} > \frac{q}{g_L}$, i.e., the price per coverage of WCN service is always larger than or equal to that of LTE-U services, then no users subscribe to WCN service at the equilibrium state. On the other hand, if WCNP offers a smaller price per coverage than LP does, i.e.,$\frac{p}{g(1)} > \frac{q}{g_L}$, then both providers are able to get a positive fraction of subscribers. It indicates that WCNP should set its service price carefully by taking consideration its limited service coverage and the price of its opponent. Note that the price of WCN service should be always lower than that of LP, since its service coverage is less than that of LTE-U service, otherwise, WCNP may loss most of market share.

A. REVENUE MAXIMIZATION IN DUOPOLY MARKET

We now study the revenue maximization in the duopoly market by formulating the dynamic interactions involved in duopoly market as a tripartite game. In this game, LP and WCNP have the power to determine their access price pair $\{p, q\}$. We consider LP as the leader of the duopoly market, where WCNP is the follower, which is consistent with the fact existing in current market. In what follows, the users determine independently which kind of services they subscribe, in response to the price pair $\{p, q\}$ given by the two providers. It should be noted that we assume the price of WCN service is less than that of LP, where both provider can attract a positive market share. Combining the interactions among the three parties, we can model the tripartite game as a dynamic three-stage Stackelberg game. In this game, WCNP, LP and users, seek to maximize their own revenues by determining the price pair $\{p, q\}$. We can derive the equilibrium strategy profile for the three-stage Stactkelberg game by applying the concept of backward induction, which works as follows:

Firstly, in the third stage, for a given price pair $\{p, q\}$ of the duopoly market, the users take the price pair $\{p, q\}$ given by LP and WCNP as input, and decide the corresponding adoption of two providers. Then back to the second stage, WCNP, acting as a leader of the users, is aware of its market share. WCNP can choose its optimal price p^* by expecting the best response of users. The game then rolls back to the first stage, LP seeks to maximize its profit by adjusting the price q of LTE-U services. Note that the market share of two providers are functions of the price pair $\{p, q\}$. The market share of providers can also be considered as the corresponding demand of users in face of the prices pair $\{p, q\}$. For ease of presentation, we use $y_t = D_p(p, q)$ and $z_t = D_q(p, q)$ to denote the services demand function of WCNP and LP, respectively. For later use, we introduce the concept of price elasticity, which describes the degree that the users' bandwidth consumption x varies with the prices. The users' price elasticity with respect to the price p are denoted as $E_p = \frac{D_p'(p)}{x/p}$, where $D_p'(p)$ is the first order derivative of the demand function $D_p(p)$ with respect to p. We can obtain the users' price elasticity E_q with respect to the price q likewise.

Finally, the equilibrium strategy profile $\{p^*, q^*, y^*, z^*\}$ of the dynamic three-stage game can be derived. The detailed analysis is listed in Proposition 3.

Proposition 3: There exists a subgame perfect equilibrium for the tripartite game in the duopoly market and the equilibrium strategy profile $\{p^*, q^*\}$ *satisfies:*

$$p^* = \frac{c_1 + c_2 - b}{1 + \frac{1}{E_p}}, \quad q^* = \frac{\beta\delta + (1-\beta)\gamma}{1 - a},$$
$$x^* = (\frac{\sigma}{p^*})^{\frac{1}{a}}. \tag{32}$$

Proof: First, for the given price pair $\{p, q\}$ of two providers, the market share of WCNP is $y_t = D_p(p) = F(\frac{q-p}{g_L - g(x_t)}) - F(\frac{p}{g(x_t)})$ As the revenue function is indicated in Eq. 19, by applying the first order optimality condition with respect to p, we can obtain the optimal price p^* in face of the price q given by LP

$$p^* = \frac{c_1 + c_2 - b}{1 + \frac{1}{E_p}} \tag{33}$$

Notice that the optimal price p^* is function of q, by expecting the WCNP's best pricing strategy $p = f(q)$, we can get the market share of LP in term of price q by substituting $p = f(q)$ into the revenue function of LP. By applying the first order optimality condition with respect to q, we can obtain the optimal price q^* given by LP

$$q^* = \frac{c_3 - b}{1 + \frac{1}{E_q}} \tag{34}$$

Therefore, the corresponding market share of WCNP and LP are $z_t = 1 - \frac{q^* - p^*}{g_l - g(x_t)}$ and $g_t = 1 - \frac{q^* - p^*}{g_l - g(x_t)} - \frac{p^*}{g(x_t)}$, respectively. □

VI. EVALUATION

In this section, we conduct several numerical evaluations to illustrate the insight obtained from previous theoretical analysis. In particular, we concentrate on analyzing the changing of participants' revenue.

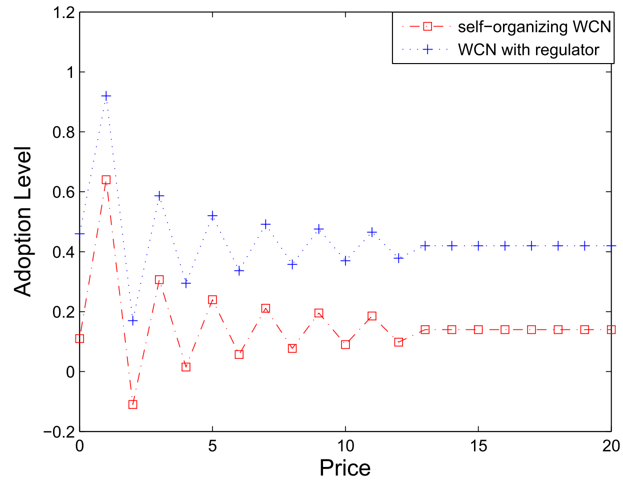

FIGURE 2. Dynamic adoption in WCN.

We first investigate the dynamic adoption level of users in WCN. Fig. 2 shows two different evolution of adoption level in self-organizing mode and regulating mode. We can observe that the equilibriums fraction of users in operator mode is equal to 0.42, which is larger than that in self-organizing mode. We can identify that the existence of operator in WCN is benefit to attract more users to subscribe the WCN service, which is consistent with our previous analysis.

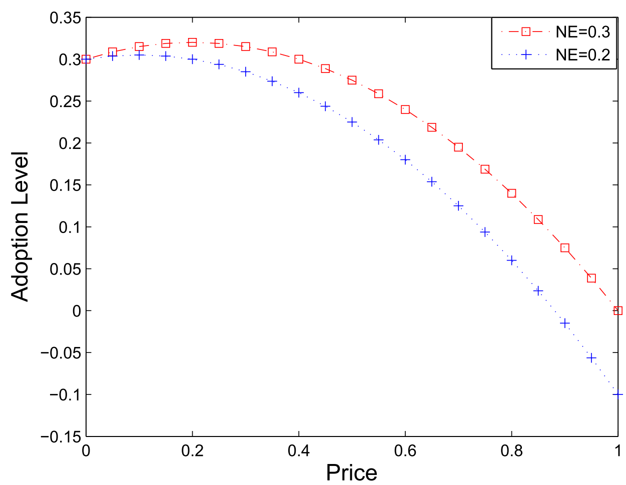

FIGURE 3. The impact of service price *p* on WCN operator's revenue.

We proceed to study the impact of service price on the WCN operator's revenue in various network externity. From Fig. 3, the regulator's revenue first increases and then decreases. There exists a maximum revenue when $p = 0.23$. We can notice that the larger network externality is, the lower service price will be. This result implies that WCN operator has an incentive to decrease its services price to attract more users to engage in the community, as the benefit from the network externality could cover its incurred cost.

We next study the convergence of adoption level in the duopoly market. We let the initial adoption level of LP and WCNP start from a nonzero value. As shown in Fig. 4, we observe that the adoption level of WCNP decreases due to the competition between the WCNP and LP. The reason behind this phenomenon is that users with high valuations of coverage tend to choose LP, which provides a better coverage services.

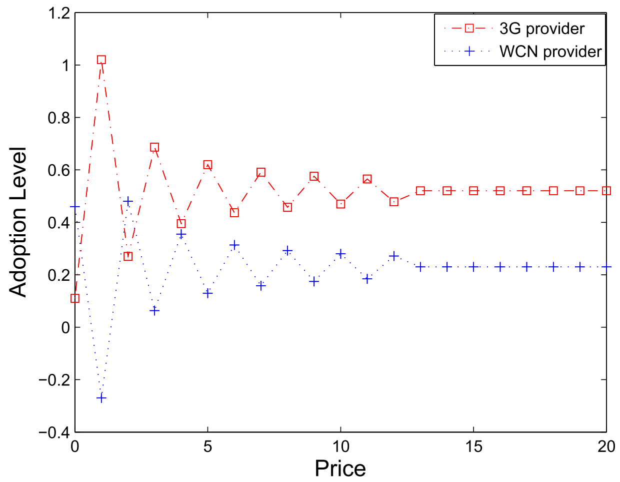

FIGURE 4. The adoption level in the duopoly market.

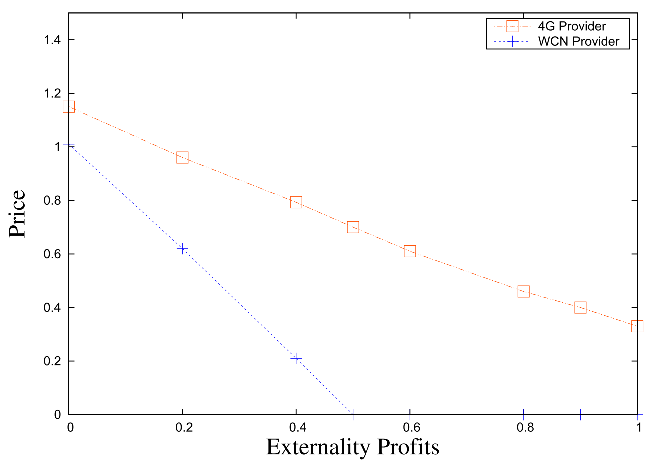

FIGURE 5. The effect of the externality effects on prices.

As indicated in Sec V, the networks externality effects b is the marginal profit, which is associated with users adoption. A large value of b implies that operators are able to get more externality profits per-traffic from data monetization, i.e., ads profits. Here we demonstrate how the network externality factor contribute to the price of operators. As can be observed from Fig. 5, the price of operators is gradually decreased along the increasing value of b. In particular, the price of WCN operator could be zero as b increases. The intuition behind this phenomenon is that the increasing of b is able to cover the cost of operators, which leads to a declining price. This could be easily understood why some internet companies sell some of internet services cheaply or even free, e.g., google and youtube services, since they could get enough return from externality effects in fashion of ads profits per-traffic.

VII. CONCLUSION AND FUTURE WORK

In this paper, we investigate three distinct stages of development of the WCN: evolution, regulation and competition. We first develop a discretized model for the self-organizing WCN and show the existence of unique equilibrium point at which the fraction of subscribers does not change. Next, we discuss the economic interactions with regard to the involved parties and derive the equilibrium adoption of users in WCN with operator. We also study the optimal price of the regulator that maximizes the operator's revenue. Finally, we analyze a duopoly market existing LP who provides its subscribers with a ubiquitous service coverage. We model the interactions among the users, WCNP and LP as a tripartite game and derive a sufficient condition that guarantees the existence of at least one pure Nash equilibrium in the market competition game.

This study is intended to be a first step towards the understanding of economic interactions in user-provided Wi-Fi application and ISP-supported LTE-U networking frameworks. An interesting future work worthy of attention is to refine the revenue flow of participants, e.g., introducing more sophisticated user categories. Moreover, the internal competition among multiple regulators can be also taken into consideration.

REFERENCES

[1] FON. (2016). *FON WiFi Sharing Community*. [Online]. Available: https://fon.com/

[2] Treebear. (Oct. 2016). *Treebear WiFi Sharing Community*. [Online]. Available: http://www.treebear.cn

[3] NextWiFi. (Nov. 2016). *NextWiFi Sharing Community*. [Online]. Available: http://www.nextwifi.com

[4] IResearch. (Apr. 2016). *Iresearch WiFi Sharing Center*. [Online]. Available: www.iresearch.com.cn

[5] R. Mahindra, H. Viswanathan, K. Sundaresan, M. Y. Arslan, and S. Rangarajan, "A practical traffic management system for integrated LTE-WiFi networks," in *Proc. 20th Annu. Int. Conf. Mobile Comput. Netw.*, 2014, pp. 189–200.

[6] A. M. Voicu, L. Simić, and M. Petrova, "Coexistence of pico- and femto-cellular LTE-unlicensed with legacy indoor Wi-Fi deployments," in *Proc. IEEE Int. Conf. Commun. Workshop (ICCW)*, Jun. 2015, pp. 2294–2300.

[7] C. Hasan, M. K. Marina, and U. Challita, "On LTE-WiFi coexistence and inter-operator spectrum sharing in unlicensed bands: Altruism, cooperation and fairness," in *Proc. 17th ACM Int. Symp. Mobile Ad Hoc Netw. Comput.*, 2016, pp. 111–120.

[8] M. Research. (2008). *Self-Organizing Neighborhood Wireless Mesh Networks*. [Online]. Available: http://research.microsoft.com/mesh/

[9] (Oct. 2014). *Whisher WiFi Sharing Community*. [Online]. Available: http://www.whisher.com/

[10] X. Ai, V. Srinivasan, and C.-K. Tham, "Wi-Sh: A simple, robust credit based Wi-Fi community network," in *Proc. IEEE INFOCOM*, Apr. 2009, pp. 1638–1646.

[11] F.ISP. (Jan. 2017). *Community of Free Subscribers*. [Online]. Available: http://www.free.fr
[12] S. Frattasi, H. Fathi, A. Gimmler, F. H. P. Fitzek, and R. Prasad, "Designing socially robust 4G wireless services," *IEEE Technol. Soc. Mag.*, vol. 25, no. 2, pp. 51–64, Jul. 2006.
[13] F. M. Abinader *et al.*, "Enabling the coexistence of LTE and Wi-Fi in unlicensed bands," *IEEE Commun. Mag.*, vol. 52, no. 11, pp. 54–61, Nov. 2014.
[14] A. M. Voicu, L. Simić, and M. Petrova, "Coexistence of pico- and femto-cellular LTE-unlicensed with legacy indoor Wi-Fi deployments," in *Proc. IEEE ICCW*, Jun. 2015, pp. 2294–2300.
[15] Y. Jin, S. Sen, R. Guérin, K. Hosanagar, and Z.-L. Zhang, "Dynamics of competition between incumbent and emerging network technologies," in *Proc. NetEcon*, 2008, pp. 49–54.
[16] J. Musacchio and D. Kim, "Network platform competition in a two-sided market: Implications to the net neutrality issue," in *Proc. TPRC Conf. Commun., Inf., Internet Policy*, 2009, pp. 1–17.
[17] L. He and J. Walrand, "Pricing and revenue sharing strategies for Internet service providers," in *Proc. IEEE INFOCOM*, Mar. 2005, pp. 205–216.
[18] A. Zemlianov and G. de Veciana, "Cooperation and decision-making in a wireless multi-provider setting," in *Proc. INFOCOM*, Mar. 2005, pp. 386–397.
[19] M. H. Manshaei, P. Marbach, and J.-P. Hubaux, "Evolution and market share of wireless community networks," in *Proc. Int. Conf. Game Theory Netw. (GameNets)*, May 2009, pp. 508–514.
[20] M. H. Manshaei, J. Freudiger, M. Felegyhazi, P. Marbach, and J.-P. Hubaux, "On wireless social community networks," in *Proc. IEEE INFOCOM*, Apr. 2008, pp. 1552–1560.
[21] G. Biczók, L. Toka, A. Vidacs, and T. A. Trinh, "On incentives in global wireless communities," in *Proc. 1st ACM Workshop User-Provided Netw., Challenges Opportunities*, 2009, pp. 1–6.
[22] C.-K. Chau, Q. Wang, and D.-M. Chiu, "On the viability of Paris metro pricing for communication and service networks," in *Proc. IEEE INFOCOM*, Mar. 2010, pp. 1–9.

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