The Impact of Consumer Fairness Seeking on Distribution Channel Selection: Direct Selling vs. Agent Selling

Zelong Yi

Department of Transportation Economics and Logistics Management, College of Economics, Shenzhen University, Shenzhen, Guangdong, China, yizl@szu.edu.cn

Yulan Wang

Faculty of Business, Hong Kong Polytechnic University, Hong Kong, yulan.wang@polyu.edu.hk

Yun Liu

Shanghai Lixin University of Accounting and Finance, Shanghai, China, dayaguai@gmail.com

Ying-Ju Chen

Department of ISOM, School of Business and Management & Department of IELM, School of Engineering, Hong Kong University of Science and Technology, imchen@ust.hk

Consumers seek for not only base functionalities of products they buy but also fairness in transactions. In this work, we investigate how such fairness-seeking behavior affects a manufacturer's distribution channel structure selection. Specifically, the manufacturer can sell the product directly to consumers (named *direct selling*) or via a middleman retailer (named *agent selling*). The manufacturer then decides which distribution channel to adopt with an aim to maximize his profit. Under a newsvendor framework, the distribution channel structure endogenizes the procurement cost and thus impacts consumers' fairness perception and willingness to pay. Interestingly, we show that it may be in the manufacturer's best interest to downward decentralize his distribution channel by adopting agent selling when consumers are extremely fairness-minded. However, when the consumer's fairness concern is weak, direct selling is preferred by the manufacturer. We further show that the above results qualitatively hold when we take into consideration the downstream competition, the dominance of the manufacturer in retail pricing and the heterogeneity of the consumers in their fairness seeking.

Keywords: Fairness Seeking; Behavioral Modelling; Operations-Marketing Interface; Distribution Channel

1. Introduction

Nowadays, due to more and more news reports and media coverage on inequality in transactions, consumers are increasingly concerned about fairness. When they purchase a product, consumers would intentionally compare their payoffs with the sellers' profits and might be reluctant to buy when the business transaction is deemed to be inequitable. Consumers are sensitive to fairness partially because unfairness widely exists in practice – "*Some people are convinced, for example, that pharmaceutical companies make obscenely high profits on patent-protected drugs, that gasoline prices are exorbitant and determined more by industry collusion than market forces, that restaurants gouge patrons by selling wine at exorbitant prices.*"[1] As stated by Bolton et al. (2003), "*... there is a general perception that prices are unfair, and that companies – not just retailers, but firms in general – make a lot of profit.*" When fairness is taken into account, it could be the case that consumers would rather sacrifice their own monetary payoffs and sometimes even give up the transaction or boycott the brand to punish the greedy seller (Kahneman et al. 1986). Naturally, for consumers with fairness concerns, their willingness to pay and corresponding utility gained from the product get lowered if any inequality is perceived in the transaction (Guo 2015; Guo and Jiang 2016).[2]

The existing literature has shown that consumers' concern about fairness can play a critical role in both consumers' and firms' strategies (Bolton and Ockenfels 2000; Cui et al. 2007; Chen and Cui 2013; Wu and Niederhoff 2014). Along this research line, the focal research point is how to set a "fair" price. However, the concern of price fairness may have a greater impact on the firm operations, not only on the firm profitability which is constrained by the fear of perceived exploitation, but also on the firm strategy such as the distribution channel selection. Specifically, consider the following two distribution channels: one, *direct selling* where the manufacturer sells his products directly to consumers; and two, *agent selling* where the manufacturer downward decentralizes his distribution channel and sells via a middleman retailer. Both distribution structures are commonly observed in the business practice. For example, Apple, Zara and Eureka Forbes sell their products directly by themselves while Land's End and Levi Strauss normally intend to invite intermediary agents to distribute their products in end markets.

[1] See "Pricing and Fairness: Do Your Customers Assume You Are Gouging Them?" posted at `http://www.inc.com/articles/2002/09/24612.html` for more details.

[2] Fairness seeking is also related to a growing body of literature studying the impacts of consumer social preferences on the firm strategy and profitability (Becker 1991; Charness and Rabin 2002; Lim and Ho 2007; Wu et al. 2008; Amaldoss and Jain 2008, 2010; Lim 2010; Tereyagoglu and Veeraraghavan 2012).

In our paper, we intend to address the following research questions: Will a manufacturer prefer agent selling over direct selling when facing the fairness-seeking consumers? And under what conditions? To shed light on this question, we propose a modified newsvendor model which could be adopted to characterize either direct selling or agent selling. The seller (that is, the manufacturer under direct selling or the retailer under agent selling) decides on both the product' stocking quantity and retail price. The retail price is then publicly announced and observable to the consumers. However, the stocking quantity and the wholesale price (the procurement cost borne by the retailer under agent selling) are unobservable to consumers. Moreover, those endogenized firms' decisions hinge upon the structure of distribution channel. Nevertheless, consumers can rationally anticipate the firms' optimal decisions. Consequently, the fairness-seeking consumers' valuation towards the product is also endogenized and distribution-channel dependent.

To address the aforementioned research questions, the rational expectation hypothesis (Muth 1961; Stokey 1981; Su 2007; Su and Zhang 2008; Lai et al. 2010; Tereyagoglu and Veeraraghavan 2012) is adopted, where economic outcomes are self-fulfilled and consistent with people's expectation. Under our specific setting, given the expectation of the stocking quantity and the wholesale price, fairness-seeking consumers make their buy-or-not-buy decisions. And given the expectation of consumers' reservation buying price, the seller makes decisions on pricing and stocking. Furthermore, every player's expectation is consistent with actual outcomes. We then derive the rational expectations equilibrium under both direct selling and agent selling. The equilibrium results suggest that when consumers become more inequality averse, the manufacturer shall be hurt in either case, since the selling price should be lowered so that the transaction can be perceived fairer.

We further find that when consumers are fairness concerned, either distribution channel structure can be preferred by the manufacturer, a result that reconfirms the coexistence of both structures in practice. The manufacturer's preference over the distribution channel structure critically depends on the degree of consumers' fairness seeking. When consumers are not so fairness-concerned, direct selling is preferred by the manufacturer as it can eliminate the double marginalization effect. As we may notice, many luxury brands such as Chanel, Celine, Hermes, and Dior adopt direct selling and sell their products exclusively in their own stores. Consumers who are keen on these brands usually show great brand loyalty and care little about fairness. They just go directly to their exclusive stores and purchase. The fact that these brands adopt the direct-selling manner is consistent with our modeling results. However, when consumers are extremely fairness-concerned, the manufacturer prefers agent selling over direct selling. This is because under agent selling, the augmented wholesale price as a result of double marginalization induces consumers to perceive a fairer trade when buying from the retailer. The effect of increased fairness perception can dominate the effect of lowered ordering quantity. Consequently, consumers are more willing to buy (even at a higher retail price) and the manufacturer is better off. For example, in the hotel industry, especially in the budget hotel segment, consumers easily switch to other hotels of the same type if they perceive a hotel's price to be unfair. As a result, even though hotels are capable of selling rooms via their own channels, the whole hotel industry relies heavily on efficient and convenient online travel agents or third-party websites to sell rooms.

When told that adding an intermediary retailer to the distribution channel can improve a manufacturer's profit, most people would argue that the intermediary retailer must possess some specific skills that the manufacturer lacks (e.g., knowledge about local cultures, accessibility to consumers, lower delivery costs, special retail locations, etc.), or the market competition is highly intensive (McGuire and Staelin 1983; Liu and Tyagi 2011), or consumers are strategic and forward-looking (Su and Zhang 2008). However, here we show that even when the aforementioned factors/reasons do not exist, under certain conditions, a manufacturer can still benefit from including an intermediary agent (retailer) in his distribution channel. This is due to the endogenous effect of distribution channel structures on the rational expectation of fairness-seeking consumers about firms' profits as well as their willingness to pay.

To check the robustness of our results, we extend the baseline model in the following aspects. First, we consider the situation in which the consumers care about fairness between the manufacturer and themselves. We find that in such a case agent selling cannot outperform direct selling in terms of the manufacturer's profitability. However, the distribution channel as a whole can be benefited by agent selling. Second, we examine the impacts of downstream competition by incorporating multiple retailers into our model. We show that the number of retailers does not affect the manufacturer's profitability. Third, in the baseline model, whoever sells the products determines the retail price, i.e., the manufacturer under direct selling and the retailer under agent selling. However, in reality it could be true that the manufacturer is the dominant player in the industrial chain and decides the retail price even under agent selling (e.g., the manufacturer-suggested retail price). We show that all the baseline results remain intact under in a case. Fourth, in the real life, it is possible that some consumers are not concerned with fairness while the others are. We show that if the majority of consumers have no fairness concerns, direct selling is always preferred by the manufacturer. Only when the proportion of fairness-seeking consumers is higher than a threshold can agent selling be preferable over direct selling. Fifth, our main results are derived under a uniform demand distribution. To check the result robustness regarding the demand distribution, we also conduct numerical experiments under other distributions such as the normal and Poisson distributions. We find that the main results still hold.

The rest of this paper is organized as follows. Section 2 reviews the most related studies. We present the model formulation and assumptions in Section 3. In Section 4, we derive the rational expectations equilibria for the firms as well as fairness-seeking consumers under both direct selling and agent selling, and analyze the manufacturer's optimal channel structure selection. In Section 5, we discuss the issues such as that consumers care about the manufacturer's profitability, the existence of competition among multiple downstream retailers, the manufacturer endowed with the retail-pricing power, the mixture of consumers with and without fairness concerns, and general demand distribution. Concluding remarks are provided in Section 6. All the proofs are relegated to the online appendix.

2. Literature Review

This paper belongs to the growing body of literature that studies the impacts of fairness concerns on firms' performance and strategic decisions. One strand of literature studies peer-induced fairness, where agents' perception of fairness is closely related to the peers' payoffs (Campbell 1999; Kukar-Kinney et al. 2007; Ho and Su 2009; Chen and Cui 2013). For example, through experimental studies, Haws and Bearden (2006) identify the potential negative effects associated with price differentiation under dynamic pricing practices when consumers value price fairness.

Our paper is also closely related to another strand of literature that studies distributional fairness, where one party' perception of fairness depends on the payoffs of their business partner. Scholars in sociology, psychology and marketing have found that this type of fairness can play an important role in firms' transactions and decisions (Kumar et al. 1995; Cui et al. 2007; Wu and Niederhoff 2014). Fehr and Schmidt (1999) build an inequity-aversion model in which the players' utilities are negatively affected by the inequitable monetary payoffs. Cui et al. (2007) study the effect of fairness consideration in a conventional dyadic channel. They show that channel coordination can be achieved via a simple wholesale price contract when members are fairness-minded. Wu and Niederhoff (2014) consider a channel under a newsvendor setting where both the supplier and the retailer have fairness concerns. They show that suppliers may upward or downward adjust the wholesale price relative to the profit-maximizing one, and the supplier's fairness preference has a stronger effect on the channel's overall performance than that of the retailer. Guo (2015) studies the optimal selling strategies of a monopolistic seller by assuming a fraction of buyers to be fairness-seeking and initially uninformed about the seller's variable cost. He shows that the seller's *ex ante* profit may unexpectedly increase when more buyers are fairness concerned. Assuming consumers are fairness concerned but the firm cost information is unobservable, Guo and Jiang (2016) investigate a firm's optimal quality and pricing decisions. They show that the optimal product quality may be non-monotone in the degree of consumer inequity aversion. Moreover, they find that consumer fairness-seeking may hurt consumers but benefit an efficient firm.

Our work is also closely related to the strand of literature that investigate the channel structure: centralized distribution channel versus decentralized distribution channel (Jeuland and Shugan 1983; Moorthy 1987; Coughlan and Wernerfelt 1989; Iyer 1998; Cachon 2003; Zhao et al. 2009). The pioneer work of McGuire and Staelin (1983) identifies conditions under which competing manufacturers can be better off with distribution channel decentralization. They show that for less substitutable products, manufacturers should sell them directly; while for more competitive products, manufacturers should adopt a decentralized distribution channel. Moorthy (1988) extends the conditions for distribution channel extension to the case of multiple products with strategic interactions. By building a two-period model, Desai et al. (2004) investigate when channel decentralization can be beneficial for durable-goods manufacturers. Su and Zhang (2008) find that when consumers are strategic, a decentralized channel with a simple wholesale price contract may outperform a centralized one. From perspectives of downstream firms, Liu and Tyagi (2011) show that upward distribution channel extension can add values to competing firms when they are able to endogenize their product positioning.

In this paper, we intend to investigate the interaction between the consumer fairness concern and the distribution channel design. We supplement the aforementioned studies by showing that consumer fairness-seeking behavior can significantly impact a firm's distribution channel design, and a decentralized channel can outperform a centralized one in the presence of strong fairness-minded consumers.

3. Model Setup

Consider a monopolistic manufacturer (he) sells his products to consumers with fairness concerns. A transaction occurs when the consumer decides to pay the price to buy the product. Each consumer has a one-unit demand with a monetary value $v > 0$. The manufacturer can either sell products directly by himself, named *direct selling* or downward extend his distribution channel by relying on an intermediary retailer (her) to distribute and sell his products, named *agent selling*. If the manufacturer adopts direct selling, he needs to determine both the retail price p_d and the production quantity Q_d before the demand realization. However, if the manufacturer adopts agent selling, the channel now consists of a manufacturer and a retailer. The manufacturer needs to decide the wholesale price ω while the downstream retailer needs to decide how much to order from the manufacturer – the stocking quantity Q_a – and the retail price p_a before the demand realization.

We assume that the manufacturer incurs a constant marginal cost of production, c per unit. If the realized demand exceeds the stocking quantity, the sale is foregone. But if the stocking quantity exceeds demand, any unsold inventory can be salvaged at price s. Without loss of generality, $0 < s < c < v$ is assumed to ensure that overstocking is costly to the seller (i.e., $s < c$) and the materialization of transactions (i.e., $c < v$). Both the manufacturer and the retailer are risk-neutral and self-interested profit-maximizers. We deliberately assume away the retailer's advantages in selling experience, know-how in the local market, logistics cost, etc.

As the business contract between the manufacturer and the retailer is private and confidential, the wholesale price, *endogenously* decided by the manufacturer considering other parties' responses, is not directly observable to the fairness-minded consumers. However, consumers hold full information about the product cost c and salvage price s, which are *exogenously* provided by outside market environment and cannot be directly affected by the firms.[3] They also well know the distribution channel structure including who (the manufacturer or the retailer) makes the pricing and ordering decisions. In such a way, consumers' willingness to pay is endogenized and directly influenced by the distribution channel.

The fairness-minded consumers care about not only their monetary payoff but also about the equity of the transaction. Specifically, a consumer's monetary payoff from the transaction is $v - p_d$ under direct selling and $v - p_a$ under agent selling. In addition, the fairness-minded consumers also compare the product' selling price against its underlying costs to evaluate the transaction fairness so as to decide whether to buy the product or not. Then, the fairness-minded consumers would compare their own consumer surplus against the seller's profit margin,[4] and they are willing to buy the products if they are more "fairly" priced (not necessarily higher monetary payoffs). That is, the

[3] This kind of information can be easily accessed via some professional websites for many physical goods. Some information is even common knowledge to both the firms and the consumers. For example, Alibaba, the Chinese e-commerce company, provides both B2B (business-to-business) and B2C (business-to-consumer) sales services, and its official website covers cost information on varied categories of materials and products. Take the following link as an illustration: `http://www.alibaba.com/Consumer-Electronics_p44?nc=y`. It provides cost information of consumer electronics. For public listed firms, this kind of information may also be disclosed in their annual reports, from which some expert consumers can get knowledge about product cost.

[4] Taking the pharmaceutical industry as an example, it is quite often for big manufacturers to sell off manufacturing rights to smaller manufacturers, who may then raise the drug prices. Consumers who seek fairness in transactions oftentimes blame the intermediary small manufacturers for the high prices of drugs. Our model is consistent with such practical observations. We would like to thank the senior editor for providing us this example.

consumer's net purchase utility takes into account both the monetary payoff from the transaction and the fairness concern. And the purchase utility of the fairness-seeking consumer is as follows:

$$\textit{Direct Selling:} \quad U = \underbrace{v - p_d}_{\text{monetary payoff}} - \underbrace{\lambda \cdot \max\{(p_d - c) - (v - p_d), 0\}}_{\text{unfairness perception induced by disadvantageous inequality}}; \tag{1}$$

$$\textit{Agent Selling:} \quad U = \underbrace{v - p_a}_{\text{monetary payoff}} - \underbrace{\lambda \cdot \max\{(p_a - \varepsilon_\omega) - (v - p_a), 0\}}_{\text{unfairness perception induced by disadvantageous inequality}}, \tag{2}$$

where ε_ω is the consumers' rational expectation over the wholesale price, the "amount of unfairness" is measured by the difference between the seller's profit margin and the consumer's surplus, and the parameter $\lambda > 0$ represents the degree of consumer disadvantageous inequality aversion (which reflects consumers' sensitivity to the transaction unfairness).[5] A larger λ implies a stronger fairness concern. And the consumer's utility is negatively affected by the perception of unfairness. Such construction of the utility function can be also found in Fehr and Schmidt (1999) and Guo (2015).

The potential market size is random. Let the random market size be $D_\varepsilon = A + D$, where $A > 0$ is a constant term and D is a random term. That is, under both distribution channels (direct selling and agent selling), the market has at least A consumers. These A consumers can be regarded as loyal ones to the brand and they enter the market for sure. Such market size information is well known to all the parties (the manufacturer, the retailer and the consumers). In the basic model, D is assumed to follow the uniform distribution between 0 and $\overline{D} > 0$, i.e., $D \sim U(0, \overline{D})$. Consequently, the probability density function $f(D) = 1/\overline{D}$ and the cumulative distribution function $F(D) = D/\overline{D}$, where $0 \le D \le \overline{D}$. In the extension, we also consider other distribution functions such as the normal and Poisson distributions to check the result robustness.

4. Results and Analysis

The main purpose of this paper is to characterize the manufacturer's optimal choice over distribution channel selection when consumers are sensitive to fairness. In what follows, we first analyze the direct-selling case where the manufacturer sells by himself, and subsequently we study the counterpart case where agent selling is adopted. Based on these analyses, a constructive comparison between direct selling and agent selling is delivered. By doing so, we identify specifics with which one distribution channel is preferred over the other by the manufacturer.

[5] These two utility functions can be modified by multiplying by the inventory availability probability. However, such modification does not change the main results because it does not impact consumers' willingness to pay, and the firms' wholesale price, retail price and ordering quantity decisions.

4.1. Direct Selling

In the direct-selling case, the manufacturer directly sells his products to the consumers. He decides both the selling price and the production quantity before the demand is realized. Consumers decide whether or not to buy the products.

Based on the chronology above, we first describe the consumers' decision problem. Given the retail price p_d, the consumers form their reservation price r, which is unobservable to the manufacturer, and decide on whether to buy the product or not. Specifically, for each consumer, a transaction occurs if and only if the consumer can enjoy a non-negative utility, that is,

$$U = v - p_d - \lambda \cdot \max\{(p_d - c) - (v - p_d), 0\} \geq 0.$$

Specifically, at the reservation price r, $U(r) = v - r - \lambda \cdot \max\{(r - c) - (v - r), 0\} = 0$ holds. Now suppose that the reservation price r leads to a higher monetary payoff to the consumers than to the manufacturer, i.e., $r - c < v - r$. Then, we have $U = v - r = 0$ and $r = v$, which however contradicts the assumption $r - c < v - r$ (because $c > v$ is counterfactual). Hence, there must be

$$r - c \geq v - r.$$

As $U = v - r - \lambda \cdot ((r - c) - (v - r)) = 0$, the reservation price can be derived as

$$r = \frac{(1+\lambda)v + \lambda c}{1 + 2\lambda} = xv + yc, \text{ where } x \equiv \frac{1+\lambda}{1+2\lambda}, y \equiv \frac{\lambda}{1+2\lambda}.$$

Furthermore, taking the limitation over the fair-seeking parameter λ, we have $\lim_{\lambda \to 0} r = v$ and $\lim_{\lambda \to \infty} r = (v + c)/2$. The former describes a situation where consumers have no fairness concern at all and are willing to accept a ceiling price at the product's monetary value; the latter considers another extreme where consumers are extremely concerned about the fairness of the transaction and consequently, the reservation price is such that the monetary payoffs are split equally between the manufacturer and the consumer (i.e., $r - c = v - r = (v + c)/2$).[6]

Next, we consider the manufacturer's pricing and stocking quantity decision. Specifically, if the selling price is higher than the reservation price, the demand will be zero. Thus, the optimal selling

[6] Note that, a consumer would feel guilty if he or she earns more surplus than the manufacturer does in one transaction. In this situation, the utility function becomes $U = v - p_d - \lambda \cdot \max\{(p_d - c) - (v - p_d), 0\} - \lambda' \cdot \max\{(v - p_d) - (p_d - c), 0\}$, where λ' measures the consumers' extent of being guilty (Fehr and Schmidt 1999; Guo 2015). It is straightforward to see that, in equilibrium, the manufacturer would never charge a price making consumers guilty (i.e., $p_d - c \geq v - p_d$ or $p_d \geq (v + c)/2$ must be satisfied). If that happened (i.e., $p_d - c < v - p_d$ or $p_d < (v + c)/2$), the consumers experience a negative utility due to feelings of the guilt and the utility function becomes $U = v - p_d - \lambda' \cdot ((v - p_d) - (p_d - c))$. Then, the manufacturer could simply charge a price that is a bit higher to benefit both parties – increasing his marginal profit and decreasing consumers' perception of the guilt. In such a way, the utility function will eventually reduce to the current form in our model: $U = v - p_d - \lambda \cdot \max\{(p_d - c) - (v - p_d), 0\}$.

price is to charge a price equal to the consumers' reservation price r. Consequently, all A loyal consumers choose to buy the products. Thus, the manufacturer shall stock at least A units of products. The manufacturer then decides his total production quantity which can be written as $Q_d = A + Q$ to maximize his profit as follows:

$$\Pi_d(p_d, Q) = A(p_d - c) + \mathsf{E}[p_d \min(D, Q) + s(Q - D)^+ - cQ],$$

where $\min(D, Q) = D - (D - Q)^+$. The following proposition summarizes the manufacturer's optimal decisions.

Proposition 1. *Under direct selling, the manufacturer's optimal stocking quantity, selling price and corresponding expected profit are, respectively,*

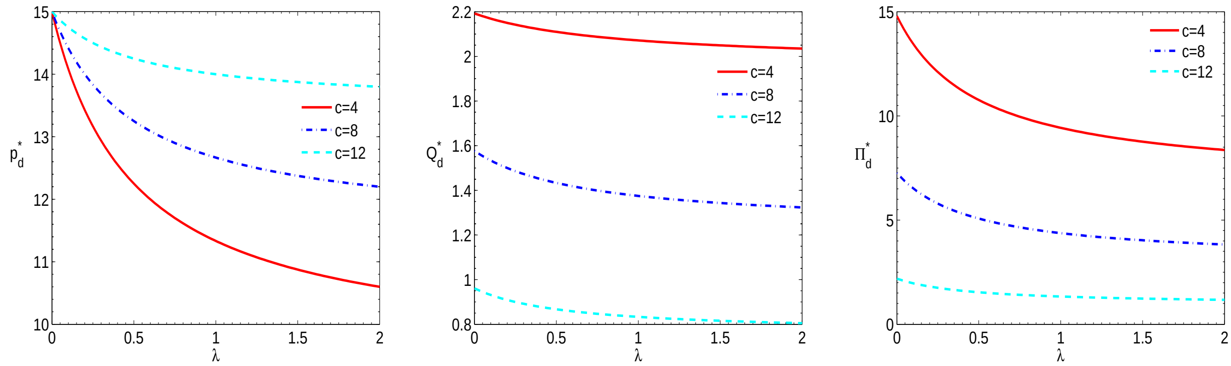
$$p_d^* = xv + yc, \; Q_d^* = A + Q^* = A + \frac{x(v-c)}{xv + yc - s}\overline{D},$$
$$\Pi_d^* = Ax(v-c) + \frac{x^2(v-c)^2}{2(xv + yc - s)}\overline{D}.$$

In the following lemma, we present how the marginal cost affects the manufacturer's profitability and his optimal decisions on price and stocking quantity.

Lemma 1. *Under direct selling, the optimal selling price p_d^* increases with the marginal cost c while the manufacturer's profit margin $p_d^* - c$ optimal stocking quantity Q_d^* and expected profit Π_d^* decrease with c.*

As stated in Lemma 1, when consumers care about fairness in a transaction, the selling price increases with the marginal cost. This stands in strict contrast to the classical setup wherein the selling price is solely driven by the consumers' valuation. In this way, we show that consumer fairness concern creates a salient linkage between the production side (cost) and the marketing side (selling price). This is because an increasing marginal cost would squeeze out the manufacturer's profit margin and makes the consumer feel fairer which induces them to willingly pay more. The reservation price is thus lifted up. As to the production (or stocking) quantity, on the one hand, a higher selling price (due to a higher cost) incentivizes the manufacturer to produce more; however, on the other hand, a higher cost also leads to a bigger loss for the unsold units. The tradeoff of these two driving forces leads to that in equilibrium, both the stocking level and the manufacturer's expected profit decrease with the marginal cost c. Figure 1 illustrates the sensitivity analysis over the marginal cost under direct selling.

Next, we derive the impact of the market random demand and consumer fairness concern on the manufacturer's profitability and his optimal decisions.



(a) (b) (c)

Figure 1 **The optimal retail price p_d^*, stocking quantity Q_d^* and manufacturer's expected profit Π_d^* under direct selling:** $s = 2$, $v = 15$, $A = 0.5$ **and** $\overline{D} = 2.0$.

Lemma 2. *Under direct selling,*

(i.) the optimal selling price p_d^* does not depend on $\overline{D}$; however, the manufacturer's optimal stocking quantity Q_d^* and expected profit Π_d^* increase with $\overline{D}$.

(ii.) the manufacturer's optimal selling price p_d^*, stocking quantity Q_d^* and expected profit Π_d^* all decrease with consumer fairness-concern parameter λ.

As shown in Proposition 1, the optimal selling price p_d^* depends only on the product's marginal production cost and monetary value as well as the consumer's fairness concern parameter. A larger $\overline{D}$ implies that the random demand is stochastically larger, and thus the manufacturer shall produce and stock more, which leads to a larger expected profit. A larger λ implies that consumers are more inequality averse and thus more sensitive to the inequality in the transaction. As a result, the manufacturer is less able to charge a higher price, which discourages him to produce as many as before. With decreased pricing power and production incentives when consumers become more fairness-seeking, the manufacturer's profit is undoubtedly negatively affected; see Figure 1 for an illustration.

4.2. Agent Selling

In the agent-selling case, the manufacturer outsources the product distribution to an intermediary retailer who then sells the products to consumers. The manufacturer acts as the Stackelberg leader deciding the wholesale price ω. The retailer (instead of the manufacturer) then decides how many units to order from the upstream manufacturer as well as the retail price p_a. The consumers form their reservation price r and make their buying decision.

We first analyze the consumers' buying decision. The consumers cannot directly observe the wholesale price ω since it is private information between the retailer and the manufacturer. However, they can make the inference about the wholesale price denoted by ε_ω. Specifically, ε_ω can be inferred from hands-on information including the retail price, the demand distribution, the market parameters, etc. Then, similar to the direct-selling case, the consumer's utility function can be written as

$$U = v - p_a - \lambda \cdot \max\{(p_a - \varepsilon_\omega) - (v - p_a), 0\},$$

where at the reservation price r, $U(r) = v - r - \lambda \cdot \max\{(r - \varepsilon_\omega) - (v - r), 0\} = 0$ holds. Suppose that the consumers receive a higher monetary payoff than the retailer at the reservation price r, i.e., $r - \varepsilon_\omega < v - r$. Then, we have $U = v - r = 0$ and $r = v$, which contradicts the assumption $r - \varepsilon_\omega < v - r$. Hence, it must be true that $r - \varepsilon_\omega \geq v - r$. Solving the consumer's utility function

$$U = v - r - \lambda \cdot ((r - \varepsilon_\omega) - (v - r)) = 0$$

yields the reservation price

$$r = \frac{(1+\lambda)v + \lambda\varepsilon_\omega}{1+2\lambda} = xv + y\varepsilon_\omega.$$

Given ω, the retailer's problem is equivalent to that of the manufacturer under direct selling except that the marginal cost c is now replaced by the wholesale price ω. Similar to the direct-selling case, the retailer forms a belief ε_r over the fairness-minded consumers' reservation price r and decides the retail price p_a subject to $p_a \leq \varepsilon_r$. For a profit-maximizing retailer, she would always charge the highest retail price that she believes the consumers can accept, i.e., $p_a = \varepsilon_r$. Note that in a rational expectations equilibrium, $\varepsilon_r = r$ and $\varepsilon_\omega = \omega$ hold. Hence, in equilibrium, $p_a = r = xv + y\omega$. That is, the optimal selling price shall equal r, consumers' reservation price, and all A loyal consumers choose to buy the products. Then, the retailer decides her optimal stocking quantity which can be written as $Q_a = A + Q$ to maximize her profit as follows:

$$\Pi_R(p_a, Q; \omega) = A(p_a - \omega) + \mathsf{E}[p_a \min(D, Q) + s(Q - D)^+ - \omega Q].$$

Likewise, the manufacturer forms a belief over the retailer's ordering quantity denoted by ε_Q. With fully anticipating the retailer's response in a rational expectations equilibrium (i.e. $\varepsilon_Q = Q(\omega)$), the manufacturer then determines the wholesale price ω to maximize his profit

$$\Pi_M(\omega) = (\omega - c)(A + \varepsilon_Q).$$

In a rational expectations equilibrium, all players would foresee others' optimal decisions precisely (Muth 1961; Stokey 1981; Su 2007; Su and Zhang 2008; Lai et al. 2010; Tereyagoglu and

Veeraraghavan 2012). Below we shall first define our rational expectations equilibrium under agent selling.

DEFINITION 1. Under agent selling, a rational expectations equilibrium $(\omega, p_a, Q, r, \varepsilon_r, \varepsilon_\omega, \varepsilon_Q)$ should satisfy the following conditions: (i) $p_a = \varepsilon_r$; (ii) $Q = arg\ \max_q \Pi_R(p_a, q; \omega)$; (iii) $\varepsilon_Q = Q$; (iv) $\omega = arg\ \max_\omega (\omega - c)(A + \varepsilon_Q)$; (v) $r = xv + y\varepsilon_\omega$; (vi) $\varepsilon_r = r$; and (vii) $\varepsilon_\omega = \omega$.
We next derive the optimal decisions of the manufacturer and the retailer, whose results are summarized in the following proposition.

PROPOSITION 2. *Under agent selling, the rational expectations equilibrium exists. In equilibrium, the optimal wholesale price charged by the manufacturer is*

$$\omega^* = \begin{cases} \frac{1}{y}\left[\sqrt{\frac{x(v-s)(xv+yc-s)\overline{D}}{x\overline{D}-yA}} - (xv - s)\right], & if\ \overline{D} > \frac{(v-s)A}{(v-c)x}; \\ v, & otherwise. \end{cases}$$

The manufacturer's expected profit is

$$\Pi_M^* = \begin{cases} \frac{1}{y^2}\left(\sqrt{(xv+yc-s)(x\overline{D}-yA)} - \sqrt{x(v-s)\overline{D}}\right)^2, & if\ \overline{D} > \frac{(v-s)A}{(v-c)x}; \\ A(v-c), & otherwise. \end{cases}$$

The retailer sets the retail price $p_a^* = xv + y\omega^*$ and orders

$$Q_a^* = A + Q^* = A + \frac{x(v-\omega^*)}{xv + y\omega^* - s}\overline{D}.$$

Next, we derive the impact of the marginal production cost on the optimal decisions and the manufacturer's profitability, which is illustrated in Figure 2.

LEMMA 3. *Under agent selling, the wholesale price ω^* and the retail price p_a^* increase with the marginal cost c while the profit margin of the retailer $(p_a^* - \omega^*)$, the optimal stocking quantity Q_a^* and the manufacturer' profit Π_M^* decrease with c.*

The result of Lemma 3 is similar to that of Lemma 1 under direct selling. The high production cost pushes up both the wholesale price and retail price. However, a higher production cost reduces the retailer's profit margin so that the transaction becomes more fairly balanced in favor of fair-seeking consumers. In addition, the profit margin reduction due to the production cost increase also dampens the retailer's ordering incentives, which consequently hurts the manufacturer.

We further study how the market factors such as random demand and consumer fairness concern affect the performance of the manufacturer and the retailer.

LEMMA 4. *Under agent selling,*

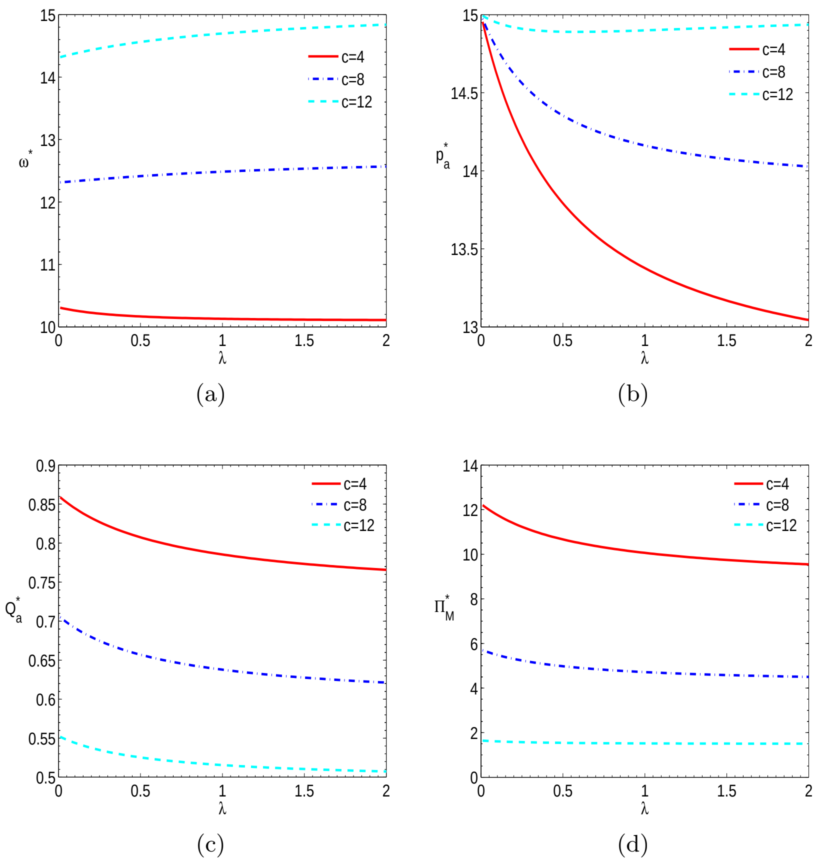


Figure 2 **The optimal wholesale price ω^*, retail price p_a^*, stocking quantity Q_a^* and the expected profit of the manufacturer Π_M^* under agent selling:** $s = 2$, $v = 15$, $A = 0.5$ **and** $\overline{D} = 4.0$.

(i.) the optimal wholesale price ω^* and retail price p_a^* decrease with $\overline{D}$ while the profit margin of the retailer ($p_a^* - \omega^*$), the optimal stocking quantity Q_a^* and the manufacturer' expected profit Π_M^* increase with $\overline{D}$;

(ii.) the manufacturer' profit Π_M^* decreases with λ.

When the upper bound of random demand increases (i.e., $\overline{D}$ increases), on the one hand, the market potential becomes larger but on the other hand, the market demand becomes more uncertain as its standard deviation also increases. Thus, the retailer lowers her retail price to mitigate the demand risk while the manufacturer reduces his wholesale price to compensate the retailer's overstocking risk. As a result of equilibrium pricing, the profit margin of the retailer actually increases as the demand becomes larger and more volatile. Correspondingly, the retailer increases her ordering quantity, which benefits the manufacturer. Again, Lemma 4 implies that a higher degree of consumer fairness concern (i.e., a largerλ) hurts the manufacturer. Our numerical examples (see Figure 2) shows that when consumers become more inequality averse, it dampens the retailer's

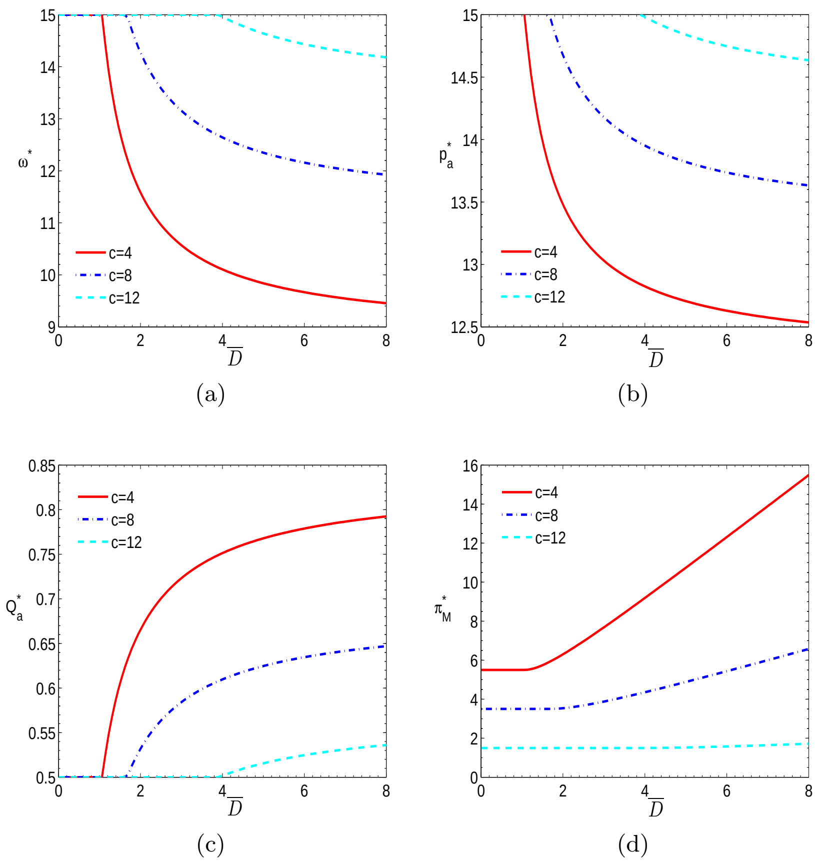


Figure 3 **The optimal wholesale price ω^*, retail price p_a^*, stocking quantity Q_a^* and the expected profit of the manufacturer Π_M^* under agent selling: $s=2$, $v=15$, $\lambda=4$ and $A=0.5$.**

ordering incentives and the retailer tends to lower her retail price so that the transaction can be perceived fairer by the consumer. Furthermore, as λ increases, the wholesale price may not necessarily decrease as it is an endogenous decision of the profit-maximizing manufacturer. The manufacturer may have the incentive to increase his wholesale price so as to reduce the retailer's profit margin such that the transaction can be perceived fairer by the fairness-seeking consumers. Consequently, an increase in λ may exert two opposite effects on the pricing decisions of the retailer and the manufacturer and thus, the equilibrium pricing does not have monotonicity; see Figure 2 for the illustration.

4.3. Comparative Statics

In this section, we compare the performance of two distribution channels analyzed in §4.1 and §4.2. First, we have the following comparison result regarding the stocking quantities in two distribution channels.

LEMMA 5. *The stocking quantity under agent selling is lower than that under direct selling, i.e.,* $Q_a^* < Q_d^*$.

Naturally, due to the double marginalization effect, the stocking quantity under agent selling is smaller than that in its direct-selling counterpart. This directly affects the consumer surplus in terms of product availability: the inventory availability under agent selling is lower than that under direct selling. Hence, when one agent is added into a distribution channel, consumers will be hurt due to a lowered chance to obtain the products.

Then, we compare the distribution channel profits under agent selling and direct selling. By noting that the retail price under agent selling can be higher than that under direct selling, we immediately obtain that the distribution channel as a whole can also be benefited by adding one more agent to the channel. We summarize this insight in the following corollary:

COROLLARY 1. *The total profit of the distribution channel under agent selling can be higher than that under direct selling.*

Given that $\omega > c$, for a fixed retail price, fairness-concerned consumers feel less disutility in a decentralized distribution channel than that in a centralized distribution channel, which further leads to a higher purchase utility. Consumers then form a higher reservation price and thus, the seller can charge a higher retail price in equilibrium. Since the demand is the same (i.e., D_ε) regardless of the distribution channel structure, the decentralized system with a higher retail price can naturally be more profitable.

Next, we identify the conditions under which the manufacturer shall prefer direct selling or agent selling when consumers exhibit fairness-seeking preference. To facilitate our analysis, define

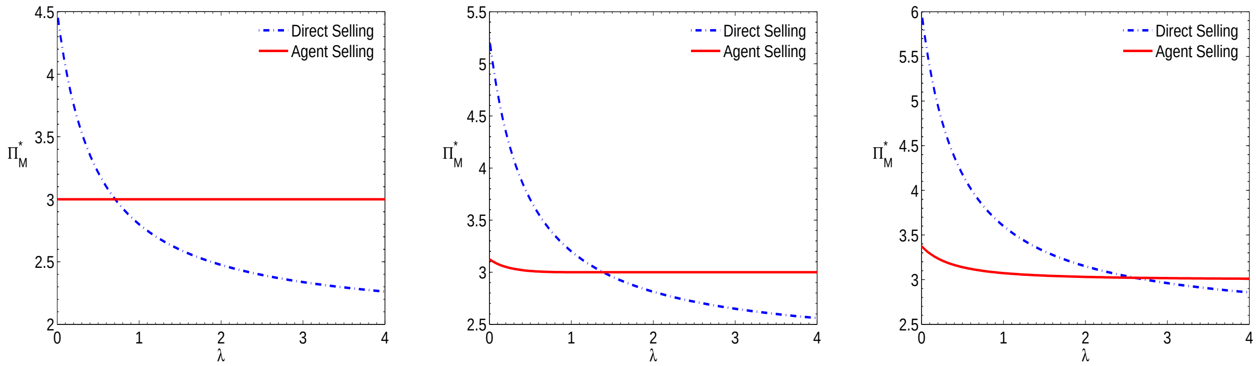
$$\widehat{\overline{D}} = \frac{2(c+v-2s)(8(c+v-2s)\sqrt{(v-c)(v-s)} - (v-c)(3c+v-4s))A}{(9c+7v-16s)(v-c)^2}.$$

PROPOSITION 3. *When the consumers are fairness concerned, depending on the magnitude of the upper bond of the market demand, we have the following result:*

(i.) When $0 < \overline{D} < \widehat{\overline{D}}$, *there exists a unique threshold* $\widehat{\lambda}$ *such that agent selling is preferred by the manufacturer over direct selling if and only if* $\lambda > \widehat{\lambda}$.

(ii.) When $\overline{D} \geq \widehat{\overline{D}}$, *direct selling always outperforms agent selling and makes the manufacturer better off.*

(iii.) When $A \to 0$, $\widehat{\overline{D}} \to 0$, *i.e., when the number of loyal consumers approaches zero, direct selling is always preferred by the manufacturer.*



(a) $\overline{D} = 1.0$ (b) $\overline{D} = 1.5$ (c) $\overline{D} = 2.0$

Figure 4 Direct selling versus agent selling regarding the manufacturer's profit: $s = 2$, $c = 8$, $v = 14$ **and** $A = 0.5$.

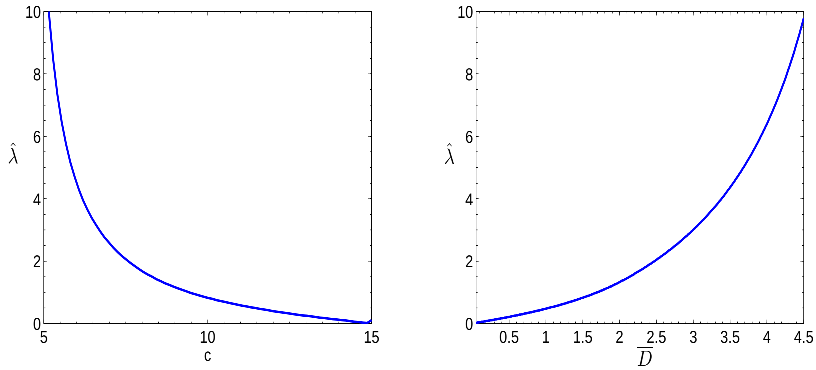
Proposition 3 shows that consumers' desire for fairness in a transaction may deviate a manufacturer's distribution strategy from direct selling to agent selling, a result consistent with the observed coexistence of both distribution channel structures in the business practice while in contrast to the result of the traditional literature without consideration of consumer behavior (e.g., benchmark models in McGuire and Staelin (1983), Su and Zhang (2008) and Liu and Tyagi (2011)). According to the traditional literature (McGuire and Staelin 1983; Su and Zhang 2008; Liu and Tyagi 2011), agent selling leads to double marginalization effect that potentially reduces the ordering quantity and hurts the manufacturer. However, when consumer fairness concern is taken into consideration, it is the double marginalization effect that gives rise to a higher level of fairness perception because of the increased product procurement cost. When λ, consumers' fairness concern, is sufficiently large (i.e., consumers are sufficiently sensitive to inequality in the transaction), the effect of increased fairness perception can dominate the impact of less ordering quantity and thus benefits the manufacturer. That is, the endogenized wholesale price can act as a strategic weapon to impact consumers' perception about the transaction's fairness and make them willing to pay more, which in turn, may improve the manufacturer's profitability. Consequently, the profit-maximizing manufacturer may downward decentralize his distribution channel and adopt agent selling. Interactions between consumers' fairness concerns and double marginalization effect eventually lead to our key result. As one example, in the entertainment industry, concert promoters usually adopt the agent-selling strategy and sell concert tickets through scalpers who function as high-price retailers. It is frequently reported that consumers blame the scalpers (but not the concert promoters) for the high ticket prices. In this context, consumers may perceive the pricing of these scalpers (instead of the concert promoters) to be unfair. Our conclusion is consistent with this practical observation.

Proposition 3 shows that demand uncertainty (which is characterized by $\overline{D}$) plays a key role in the manufacturer's distribution channel selection. Undoubtedly, without demand uncertainty, both the manufacturer and the retailer know exactly how many people would like to buy and hence the stocking decision perfectly matches the demand. As a result, the Stackelberg manufacturer is able to extract the same surplus under both distribution channels by charging a retail price $p_d^* = v$ under direct selling and a wholesale price $\omega^* = v$ under agent selling. Only when there exists demand uncertainty and consumers are fairness-minded can agent selling make the manufacturer better off. Proposition 3 shows that under demand uncertainty, when consumers are very fairness concerned, then agent selling makes the manufacturer better off; however, when such fairness concern is mild, then direct selling is preferred by the manufacturer. See Figure 4 for the illustration. Proposition 3 also implies that when the market demand is too volatile ($\overline{D} \geq \widehat{\overline{D}}$), a centralized distribution channel (that is, direct selling) shall be preferred over a decentralized distribution channel (that is, agent selling) by the manufacturer. As stated in Lemma 4, when $\overline{D}$ increases, although both the wholesale price and the retail price are lower, the profit margin of the retailer actually is larger, leading fairness-minded consumers less likely to buy the product. Consequently, the potential advantage of agent selling fades away and direct selling is thus adopted by the manufacturer.

Next, we numerically examine how the threshold of the fairness concern parameter $\widehat{\lambda}$ (at which the manufacturer is indifferent between direct selling and agent selling) changes with c, the manufacturer's production cost and $\overline{D}$, the upper bound of the random demand. As shown in Figure 5a, an increase in the production cost lowers the fairness concern threshold $\widehat{\lambda}$ and makes the agent selling more likely to be adopted by the manufacturer. Recall that under agent selling, an increase in the marginal production cost would lead to a higher wholesale and this effect becomes stronger when the marginal production cost is larger. This makes agent selling more likely to be preferred by the manufacturer even when the consumer fairness concern is not so strong (i.e., a smaller λ). Figure 5b shows that when the upper bound of random demand $\overline{D}$ increases, the fairness concern threshold $\widehat{\lambda}$ becomes larger and it is less likely for the manufacturer to sell through an agent. Recall that under agent selling, an increase in $\overline{D}$ would lead to a lower wholesale price for the manufacturer but a higher profit margin for the retailer. Thus, to induce the manufacturer adopt agent selling requires that consumers are extremely fairness concerned, that is, a larger λ.

5. Extensions

Below we extend our basic model to the following five cases. One, consumers care about fairness between the manufacturer and themselves. Two, multiple retailers participate in product distri-



(a) (b)

Figure 5 **(a) The relationship between $\widehat{\lambda}$, the threshold of the fairness concern parameter and c, the marginal production cost: $s = 2$, $v = 15$, $A = 0.5$ and $\overline{D} = 1.5$. (b) The relationship between $\widehat{\lambda}$, the threshold of the fairness concern parameter and $\overline{D}$, the upper bound of the random demand: $s = 2$, $c = 10$, $v = 15$ and $A = 0.5$.**

bution. Three, the manufacturer decides both the wholesale price and the retail price. Four, the market is composed of two types of consumers with and without fairness concern. Five, we consider other more general demand distributions such as the normal and Poisson distributions to check whether the results under the uniform distribution is robust.

5.1. Fairness Concern about the Manufacturer's Profit under Agent Selling

In the decentralized case of the baseline model, we have assumed that consumers care about fairness only between the retailer and themselves. It is also possible that what concerns consumers is the fairness between the manufacturer and themselves. In this subsection, we consider such possibility and study the manufacturer's distribution channel structure selection when consumers behave so. Again, under agent selling, the manufacturer is the Stackelberg leader and sets the wholesale price ω. The retailer (instead of the manufacturer) posts the retail price p_a and determines how many units to order from the upstream manufacturer, i.e., Q_a. The consumers form their reservation price r and make their buying decision.

For the consumers, they face the similar utility function as that in the baseline model except that they are now concerned about the difference between the manufacturer's profit margin and their own surplus. Again, the consumers cannot directly observe the wholesale price ω because it is a price contract between the retailer and the manufacturer but they can make inferences about the wholesale price denoted by ε_ω. The consumer's utility function can then be written as

$$U = v - p_a - \lambda \cdot \max\{(\varepsilon_\omega - c) - (v - p_a), 0\} \geq 0,$$

where at the reservation price r, $U(r) = v - r - \lambda \cdot \max\{(\varepsilon_\omega - c) - (v - r), 0\} = 0$ holds. Suppose that the consumers receive a higher monetary payoff than the manufacturer at the reservation price r, i.e., $\varepsilon_\omega - c < v - r$. Then, we have $U = v - r = 0$ and $r = v$, which contradicts the assumption $\varepsilon_\omega - c < v - r$. Hence, it must be true that $\varepsilon_\omega - c \geq v - r$. Solving $U = v - \varepsilon_\omega - \lambda \cdot ((\varepsilon_\omega - c) - (v - r)) = 0$ yields the reservation price

$$r = \frac{(1+\lambda)v - \lambda(\varepsilon_\omega - c)}{1+\lambda} = v - \frac{y}{x}(\varepsilon_\omega - c).$$

Given ω, the retailer's problem is equivalent to that under the baseline agent-selling model. Similarly, the retailer forms a belief ε_r over the fairness-minded consumers' reservation price r and decides the retail price p_a subject to $p_a \leq \varepsilon_r$. For a profit-maximizing retailer, she would always charge the highest retail price that she believes the consumers can accept, i.e., $p_a = \varepsilon_r$. Note that in a rational expectations equilibrium, $\varepsilon_r = r$ and $\varepsilon_\omega = \omega$ hold. Hence, in equilibrium, $p_a = r = v - y/x(\omega - c)$. That is, the optimal selling price shall equal r and all A loyal consumers choose to buy the products. Then, the retailer decides her optimal stocking quantity which can be written as $Q_a = A + Q$ to maximize her profit as follows:

$$\Pi_R(p_a, Q; \omega) = A(p_a - \omega) + \mathsf{E}[p_a \min(D, Q) + s(Q - D)^+ - \omega Q].$$

Likewise, the manufacturer holds a belief over the retailer's ordering quantity denoted by ε_Q. Again, fully anticipating the retailer's response in a rational expectations equilibrium (i.e. $\varepsilon_Q = Q(\omega)$), the manufacturer maximizes his profit by deciding the wholesale price ω,

$$\Pi_M(\omega) = (\omega - c)(A + \varepsilon_Q).$$

Below, we define our rational expectations equilibrium under this setting.

Definition 2. When consumers care about the fairness between the manufacturer and themselves, a rational expectations equilibrium $(\omega, p_a, Q, r, \varepsilon_r, \varepsilon_\omega, \varepsilon_Q)$ should satisfy the following conditions: (i) $p_a = \varepsilon_r$; (ii) $Q = arg\ \max_q \Pi_R(p_a, q; \omega)$; (iii) $\varepsilon_Q = Q$; (iv) $\omega = arg\ \max_\omega (\omega - c)(A + \varepsilon_Q)$; (v) $r = v - \frac{y}{x}(\varepsilon_\omega - c)$; (vi) $\varepsilon_r = r$; and (vii) $\varepsilon_\omega = \omega$.

Then, we can obtain the following results.

Proposition 4. *Under agent selling, when the consumers care about fairness between the manufacturer and themselves, the rational expectations equilibrium exists. In equilibrium, the optimal wholesale price charged by the manufacturer is*

$$\omega^* = \begin{cases} \frac{x}{y}\left[(v - s) - \sqrt{\frac{(v-s)(x(v-s)+y(c-s))\overline{D}}{Ay+\overline{D}}}\right] + c, & if\ \overline{D} > \frac{(xv+yc-s)A}{v-c}; \\ xv + yc, & otherwise. \end{cases}$$

The manufacturer's expected profit is

$$\Pi_M = \begin{cases} \frac{x}{y^2}\left(\sqrt{(xv+yc-s)\overline{D}} - \sqrt{(v-s)(Ay+\overline{D})}\right)^2, & if\ \overline{D} > \frac{(xv+yc-s)A}{(v-c)}; \\ Ax(v-c), & otherwise. \end{cases}$$

The retailer sets the retail price $p_a^* = v + y/x(\omega^* - c)$ *and orders*

$$Q_a^* = A + \frac{x(v-\omega^*) - y(\omega^* - c)}{x(v-s) - y(\omega^* - c)}\overline{D}.$$

Next, we compare the performance of the two distribution channels in terms of the manufacturer's profitability.

Lemma 6. *When the consumers show the fairness concern between the manufacturer and themselves, the manufacturer cannot benefit from selling through an intermediary retailer.*

Different from that in the baseline agent-selling model, the manufacturer now cannot be better off via agent selling when the consumers show the fairness concern between the manufacturer and themselves. In the baseline model, consumers concern about the retailer's payoff under agent selling. In order to alleviate consumers' hesitation to buy induced by their fairness-seeking behavior, the manufacturer adjusts the wholesale price to *indirectly* affect the retailer's pricing and the consumers' willingness to pay. However, in the current model, consumers concern about the manufacturer's payoff. Thus, the manufacturer needs to reduce his wholesale price to *directly* influence the consumers' buying decision. Under such circumstances, compared with direct selling, agent selling will makes the manufacturer worse off due to double marginalization effect.

The next lemma shows that although the manufacturer cannot benefit from the downward distribution channel extension, the distribution channel as a whole can be better off.

Lemma 7. *When the consumers care about fairness between the manufacturer and themselves, the total profit of the distribution channel under agent selling can be higher than that under direct selling.*

Lemma 7 reveals the possibility of the performance improvement for the whole distribution channel under agent selling when consumers seek for fairness between the manufacturer and themselves. Such fairness-seeking behavior, as expected, lowers consumers' overall valuation and their willingness to buy the product. It forces the manufacturer to lower his wholesale price to mitigate this effect. Due to such price cutting, the wholesale price can be much lower than the manufacturer's direct-selling price such that the consumers could feel fairer under agent selling than that under direct selling. This leads to that consumers are willing to pay more and the retailer tends to order more. Thus, the whole distribution channel could be more profitable.[7]

Next, we further extend our model by considering the situation where consumers are fairness concerned about the profits of both the retailer and the manufacturer. For example, oil companies usually adopt the agent selling strategy and sell their products such as gasoline and low sulphur diesel to drivers via service stations and third parties who have their own service spots. Meanwhile, it is noted that their consumers are very concerned about fairness. According to a survey conducted by Consumer Reports National Research Center (aftermarketNews, June 27, 2008), quite a large portion of respondents blamed the oil companies for the high gas prices. In this context, consumers may perceive the pricing actions of these companies (all those in the distribution channel) to be unfair. That is, they may impose weights on the transaction fairness with respect to both the retailer (say, β) and the manufacturer (say, $1-\beta$). Thus, for each consumer, a transaction occurs if and only if the consumer's utility

$$U = v - p_a - \lambda \cdot \{\beta \max\{(p_a - \varepsilon_\omega) - (v - p_a), 0\} + (1-\beta)\max\{(\varepsilon_\omega - c) - (v - p_a), 0\}\} \geq 0.$$

Note that, when $\beta = 1$, it degenerates to that in the baseline model and when $\beta = 0$, it degenerates to that in the model examined above. Then, we can derive the following result.

LEMMA 8. *When consumers exhibit β portion of fairness concern about the retailer's profit and $1-\beta$ portion of fairness concern about the manufacturer's profit, the manufacturer can be better off by selling through an intermediary retailer if β is sufficiently large.*

By Lemma 8, the manufacturer may prefer agent selling over direct selling when consumers impose mixed concerns about the profits of the manufacturer and the retailer. Such preference critically hinges on the weights allocated to the concerns over the payoffs of the manufacturer and the retailer. Agent selling is preferable when consumers show great concerns about the retailer's payoff. If so, the manufacturer just needs to *indirectly* affect the retailer's decision by slightly adjusting the wholesale price. This finding deepens our understanding about the impact of the consumers' fairness-seeking behavior on the manufacturer's distribution channel selection.

[7] It is worth noting that under agent selling, consumers may also concern about the fairness between the payoff of the whole distribution channel and that of their own. In such a case, consumers hold the utility function $U = v - p_a - \lambda \cdot \max\{(p_a - c) - (v - p_a), 0\}$, which takes exactly the same form as that under direct selling. Thus, the optimal retail price and ordering quantity as well as the total channel profit remain the same as those under the direct-selling case. As such, the manufacturer's profit (as part of the total channel profit) must be lower than that under direct selling.

The empirical evidence of a large β can be captured in the entertainment industry. In this industry, concert promoters usually sell tickets through scalpers (agent selling) who sometimes charge higher-than-regular prices if the concert is popular. Here, consumers feel unfair (a large β) and blame the scalpers instead of the performers for making exorbitant profits. In contrast, a small β is usually observed in the cosmetics industry. Cosmetics companies such as Estee Lauder, Dior and Lancome often sell their products directly on their official websites or in their branded stores. Although consumers (especially, husbands) may think the prices of the cosmetic products unreasonably high compared to their costs, rarely consumers blame these companies (a small β). These evidences are consistent with our model's prediction that agent (direct, respectively) selling is preferable if β is sufficiently large (relatively small, respectively).

5.2. When There Exists Downstream Competition

In the baseline agent-selling model, we assume that the channel consists of one manufacturer and one retailer. In practice, it is possible that the manufacturer sells products through multiple intermediary retailers. In this part, we examine this scenario by considering n symmetric retailers competing in price. The manufacturer first designs a menu of wholesale prices $\{\omega_i\}$ for each retailer i, and then each retailer i makes her ordering decision $\{Q_{ai}\}$ and posts her retail price $\{p_{ai}\}$, $i = 1, 2, ..., n$. A consumer is sensitive to whether the transaction between the retailer and herself is considered (un)fair.

Note that the manufacturer may be prohibited from charging different wholesale prices to different retailers. For example, price discrimination by manufacturers is governed by the Robinson-Patman Act in the United States of America and Article 102 of the Treaty on the Functioning of the European Union in the Europe (Baldwin 1987; Herweg and Muller 2014). We shall consider the situation in which the manufacturer is forced to charge all the retailers a uniform wholesale price, i.e., $\omega_i = \omega_j$ for any $i \neq j$. We then relax this constraint and consider another situation in which the manufacturer may charge different retailers different wholesale prices. The following proposition holds in both situations and shows the robustness of our central result: the manufacturer can benefit from the multi-agent participation in distributing his products:

Proposition 5. *The manufacturer's profit under agent selling is the same regardless of the number of retailers and he can still be benefited from selling through multiple retailers.*

By Proposition 5, the manufacturer cannot further increase his profitability by introducing multiple intermediary retailers into his distribution channel. One may conjecture that the downstream competition reduces the retail price, which in turn reduces the wholesale price and thus increases the ordering quantity. This may improve the manufacturer's profitability. However, here the symmetric retailers equally share the random demand, and they also equally bear the inventory risk. Thus, the manufacturer does not need to adjust his wholesale price. As a result, the retail price and the total ordering quantity remain the same as those in the baseline model. Taking into consideration the extra expenses incurred by involving more retailers, we would suggest the manufacturer to sell only through one intermediary retailer.

We then further consider another situation where the manufacturer can sell his products both directly and through the intermediary agents. Such multichannel encroachment is widely observed in practice. Under such a setting, n symmetric retailers compete against the manufacturer in the end market. The manufacturer first decides whether to encroach the downstream market by selling directly. If the manufacturer decides "not to encroach", the setting reduces to the model discussed above. If the manufacturer decides "to encroach", he announces his selling pricing, p_d, and makes the inventory decision, Q_d. He then designs a menu of wholesale prices $\{\omega_i\}$ for each retailer i, and then each retailer i makes her ordering decision $\{Q_{ai}\}$ and posts her retail price $\{p_{ai}\}$, $i = 1, 2, ..., n$. The consumers, who are sensitive to the (un)fairness of the transaction between the seller from whom they buy and themselves, make their corresponding purchase decisions.

The following proposition summarizes the manufacturer's preference over multichannel encroachment, direct selling and agent selling.

PROPOSITION 6. *Regarding the manufacturer's profitability, multichannel encroachment is always dominated by either direct selling or agent selling. When* $0 < \overline{D} < \widehat{\overline{D}}$*, there exists a unique threshold* $\widehat{\lambda}$ *such that agent selling is preferred by the manufacturer over direct selling if and only if* $\lambda > \widehat{\lambda}$*. Otherwise, the manufacturer prefers direct selling.*

Proposition 6 shows that when consumers are fairness-seeking, multichannel distribution is never preferred by the manufacturer. This broadens our understanding of the manufacturer's distribution channel selection when consumers are fairness concerned.

5.3. When Manufacturer Fully Dominates

In the base agent-selling model, we assume that the wholesale price and the retail price are determined by the manufacturer and the retailer, respectively. While this assumption is empirically reasonable, it is worth noting that in some situations the upstream manufacturer is dominant and also holds the right to decide the retail price (such as the manufacturer-suggested retail price). In this section, we consider such a scenario where the manufacturer is endowed with both pricing power and re-examine the manufacturer's distribution channel selection.

Under agent selling, the consumer's buy-or-not-buy decision remains the same as that described in §4.2. Similarly, the manufacturer (instead of the retailer) forms a belief ε_r over r, the reservation price of the fairness-minded consumers and sets $p_a = \varepsilon_r$. Since $\varepsilon_r = r$ holds in a rational expectations equilibrium, all A loyal consumers buy the product.

As to the retailer, given the wholesale price ω and the retail price p_a, she determines how many units to order from the upstream manufacturer which can be written as $Q_a = A + Q$ to maximize her profit

$$\Pi_R(Q;\omega,p_a) = A(p_a - \omega) + \mathsf{E}[p_a \min(D,Q) + s(Q-D)^+ - \omega Q].$$

Likewise, the manufacturer holds a belief over the retailer's ordering quantity denoted by ε_Q. With fully anticipating the retailer's response in a rational expectations equilibrium (i.e., $\varepsilon_Q = Q(\omega, p_a)$), the manufacturer decides the wholesale price ω to maximize his profit

$$\Pi_M(\omega) = (\omega - c)(A + \varepsilon_Q).$$

Below we present the definition of a rational expectations equilibrium in this game.

Definition 3. A rational expectations equilibrium $(\omega, p_a, Q, r, \varepsilon_r, \varepsilon_\omega, \varepsilon_Q)$ in this game should satisfy the following conditions: (i) $p_a = \varepsilon_r$; (ii) $Q = arg\ \max_q \Pi_R(q;\omega,p_a)$; (iii) $\varepsilon_Q = Q$; (iv) $\omega = arg\ \max_\omega (\omega - c)(A + \varepsilon_Q)$; (v) $r = xv + y\varepsilon_\omega$; (vi) $\varepsilon_r = r$; and (vii) $\varepsilon_\omega = \omega$.

From the above definition we can immediately obtain $p_a^* = xv + y\omega^*$. That is, the retail pricing decision of the manufacturer remains exactly the same as that of the retailer as stated in §4.2, because both of them can rationally and correctly anticipate the consumers' belief and behavior. Thus, no matter who decides the retail price, the optimal decisions (quantity and wholesale price) are the same as those in §4.2. Hence, all the aforementioned results hold here.

Corollary 2. *When both the wholesale price and the retail price are set by the manufacturer under agent selling, the equilibrium outcomes are the same as those stated in Proposition 2. Consequently, the manufacturer' preference over the two distribution channels remain the same as that stated in Proposition 3.*

5.4. Consumer Heterogeneity

In the baseline model, we assume that all the consumers are fairness-minded; however in reality, maybe only a proportion of them seek for fairness in a transaction while the others do not. Here, we take into account such heterogenous consumer behavior and examine how this heterogeneity affects the manufacturer's optimal choice over the distribution channel selection. Specifically, we assume that in the market, α portion of consumers are fairness-seeking when making their purchase decision, whose utility function remains the same as those stated in (1) and (2). The remaining $1-\alpha$ portion of consumers are indifferent to fairness, whose utility function can be written as

$$U = v - p_d \text{ (direct selling); } U = v - p_a \text{ (agent selling).}$$

Obviously, a fairness-minded consumer expects a lower retail price than a consumer without fairness concern. The seller, that is, the manufacturer (under direct selling) or the retailer (under agent selling), knows the consumers' composition. With such information, the seller could either charge a high selling price so as to target only the consumers without fairness concern or charge a low price to cover both types of consumers. Let

$$\hat{\alpha}_d \equiv 1 - \frac{x\,(v-s)\left(2\,(cy+vx-s)\,A + (v-c)\,x\overline{D}\right)}{(cy-s+vx)\left(2\,(v-s)\,A + (v-c)\,\overline{D}\right)},$$

the following proposition summarizes the optimal decisions of the manufacturer under direct selling.

Proposition 7. *When α portion of consumers are fairness-seeking, under direct selling,*

(i.) when $\alpha > \hat{\alpha}_d$, the manufacturer enjoys a higher profit by adopting a low-pricing strategy. The optimal selling price, production quantity and corresponding profit of the manufacturer are the same as those stated in Proposition 1.

(ii.) when $\alpha < \hat{\alpha}_d$, the manufacturer enjoys a higher profit by adopting a high-pricing strategy. The optimal selling price, production quantity and corresponding profit of the manufacturer are, respectively,

$$p_d^* = v;\; Q_d^* = (1-\alpha)A + \frac{(1-\alpha)(v-c)}{v-s}\overline{D};$$
$$\Pi_d^* = (1-\alpha)(v-c)A + \frac{(1-\alpha)(v-c)^2}{2(v-s)}\overline{D}.$$

Next, let

$$\hat{\alpha}_a = \begin{cases} 1 - \frac{4(v-s)\overline{D}}{y^2}\frac{\left(\sqrt{(v-s)x\overline{D}}-\sqrt{(xv+yc-s)(x\overline{D}-yA)}\right)^2}{\left((v-s)A+(v-c)\overline{D}\right)^2}, & if\ \overline{D} > \frac{(v-s)A}{(v-c)x}; \\ 1 - \frac{4(v-c)(v-s)\overline{D}A}{\left((v-s)A+(v-c)\overline{D}\right)^2}, & if\ \frac{(v-s)A}{v-c} < \overline{D} \le \frac{(v-s)A}{(v-c)x}; \\ 0, & otherwise. \end{cases}$$

We list the optimal decisions of the manufacturer and the retailer under agent selling as follows:

Proposition 8. *When α portion of consumers are fairness-seeking, under agent selling,*

(i.) when $\alpha > \hat{\alpha}_a$, the manufacturer enjoys a higher profit under a low-pricing strategy. The channel optimal decisions and the manufacturer's profit are the same as those stated in Proposition 2.

(ii.) when $\alpha < \hat{\alpha}_a$, the manufacturer enjoys a higher profit under a high-pricing strategy. The optimal wholesale price and corresponding profit of the manufacturer are, respectively,

$$\omega^* = \begin{cases} \frac{(v-s)A+(v-c)\overline{D}}{2\overline{D}} + c, & if\ \overline{D} > \frac{(v-s)A}{v-c}; \\ v, & otherwise. \end{cases}$$
$$\Pi_M^* = \begin{cases} \frac{(1-\alpha)\left((v-s)A+(v-c)\overline{D}\right)^2}{4(v-s)\overline{D}}, & if\ \overline{D} > \frac{(v-s)A}{v-c}; \\ (v-c)(1-\alpha)A, & otherwise. \end{cases}$$

The retailer sets the retail price $p_a^* = v$ and orders

$$Q_a^* = (1-\alpha)A + \frac{v-\omega^*}{v-s}(1-\alpha)\overline{D}.$$

Propositions 7 and 8 imply that when α, the proportion of fairness-minded consumers is sufficiently large (i.e., $\alpha > \max\{\hat{\alpha}_d, \hat{\alpha}_a\}$), the low-pricing strategy is always preferred by the manufacturer and the performances of both distribution channels remain the same as those in the baseline model when all consumers are concerned about fairness. In such a scenario, the fairness-minded consumers occupy a large market share; failing to take into consideration the consumer's fairness concern will hurt the manufacturer's profitability. However, when α is sufficiently small (i.e., $\alpha < \min\{\hat{\alpha}_d, \hat{\alpha}_a\}$), a high-pricing strategy benefits the manufacturer, under which only the consumers without fairness concern are served. The optimal distribution channel choice goes back to that under the traditional literature, that is, direct selling (centralized distribution) outperforms agent selling (decentralized distribution). The following lemma summarizes the above discussion.

Lemma 9. *When α portion of consumers are fairness-seeking,*

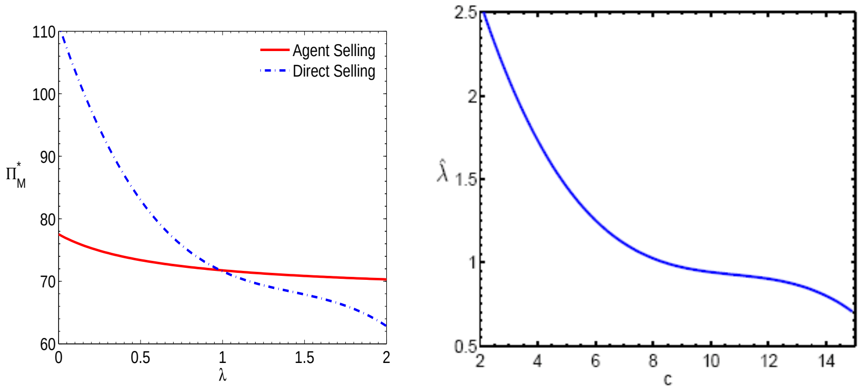
(i.) if $\alpha > \max\{\hat{\alpha}_d, \hat{\alpha}_a\}$, there exists a unique threshold $\widehat{\lambda}$ such that agent selling generates a higher profit for the manufacturer than direct selling if and only if the fairness concern parameter $\lambda > \widehat{\lambda}$ and $0 < \overline{D} < \widehat{\widehat{D}}$.

(ii.) if $\alpha < \min\{\hat{\alpha}_d, \hat{\alpha}_a\}$, compared to agent selling, direct selling is always preferred by the manufacturer.

5.5. General Demand Distribution

In the main context, for simplicity and tractability, we have essentially assumed that the random term in the demand function follows a uniform distribution between 0 and $\overline{D}$, i.e., $D \sim U(0, \overline{D})$. In this section, we relax this constraint and examine the situations where the random demand follows either a normal or Poisson distribution. We then numerically examine whether the main insights continue to hold under these two general distributions.

Through our numerical studies, we find that under both normal and Poisson distributions, the stocking quantity under agent selling is significantly lower than that under direct selling, a result



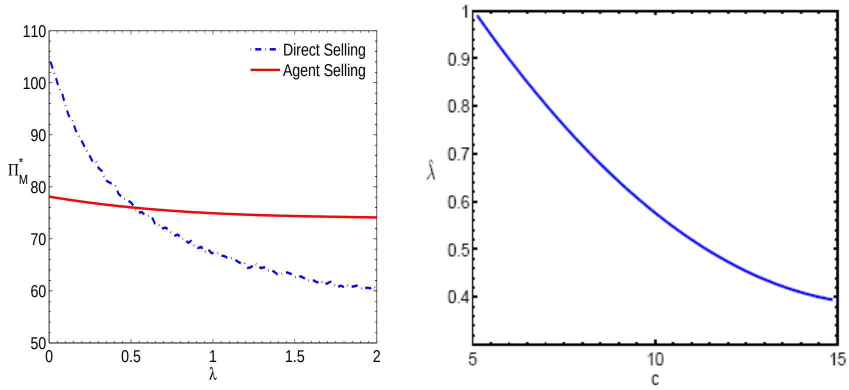
(a) (b)

Figure 6 **(a) Direct selling versus agent selling regarding the manufacturer's profit:** $c = 8$; **(b) The relationship between** $\widehat{\lambda}$**, the threshold of the fairness concern parameter and** c**. In this example,** $s = 2$, $v = 15$**, and the demand distribution is** $N(\mu = 20, \sigma^2 = 25)$**.**

consistent with that in Lemma 5. This is due to the double marginalization effect. Moreover, as the consumer's fairness concern becomes stronger (i.e., a larger λ), the manufacturer's preference over the distribution channel structure switches from direct selling to agent selling, see Figures 6a (under a normally-distributed demand) and 7a (under a Poisson distributed demand) for the illustration. In addition, the threshold of the fairness concern parameter $\widehat{\lambda}$ at which the manufacturer is indifferent between direct selling and agent selling decreases with the manufacturer's production cost c; see the illustration of Figures 6b (under a normally-distributed demand) and 7b (under a Poisson distributed demand). This result is also consistent with that under the uniform distribution. Our numerical studies show that the main results derived under a uniform distribution are robust and they continue to hold under both the normal and Poisson distributions. All the analytical and numerical analysis show that the consumer's fairness perception has a strong impact on the manufacturer's distribution channel structure selection. Ignoring such consumer behavior will lead to the suboptimal performance of the manufacturer.

6. Conclusion

In this study, we consider a setting where consumers' purchase decisions are affected by both monetary payoffs and their fairness concerns. We then unravel how such fairness-seeking behavior interacts with the firms' pricing and stocking decisions under two distribution channel structures, direct selling and agent selling. By comparing the manufacturer's profits under these two structures, we intend to answer which channel structure is preferred by the manufacturer under what conditions when facing the fairness-seeking consumers. Specifically, building upon a modified newsvendor



(a) (b)

Figure 7 **(a) Direct selling versus agent selling regarding the manufacturer's profit:** $c = 10$**; (b) The relationship between** $\widehat{\lambda}$**, the threshold of the fairness concern parameter and** c**. In this example,** $s = 5$**,** $v = 15$**, and the demand distribution is** $Poisson(\Lambda = 25)$**.**

model, we derive the rational expectations equilibria among the channel members under both distribution channel structures.

Interestingly, we show that when consumers have strong fairness concern in a transaction, the manufacturer may prefer agent selling and downward outsourcing his distribution to an intermediary retailer . Most existing voices in favor of agent selling are mainly due to the manufacturer's lack of skills or capabilities to sell in a local market. However, our study presents another angle: consumers' fairness-seeking behavior may lead to the preference of agent selling over direct selling. This is because agent selling inflates the seller's procurement cost of the product and mitigates consumers' fairness concerns. From this aspect, we reveal an uncovered kernel favoring agent selling from the aspect of consumer behavior. Our findings highlight the importance of incorporating the consumer's fairness concern into the distribution channel design. Robustness of main results are well confirmed under different extensions.

Our work is extended to five aspects and we find that first, when consumers show the fairness concern over the manufacturer's profit, the manufacturer cannot benefit from agent selling but the whole distribution channel can. Second, when the consumers are fairness-seeking, the number of retailers in the decentralized distribution channel has no impact on the manufacturer's profitability. Third, in the decentralized distribution channel, whether the retail price is determined by the retailer or the manufacturer has no impact on the channel performance. Fourth, our main results still hold as long as the fraction of fairness-minded consumers is large enough. And last, the manufacturer's preference over the decentralized distribution channel remains intact under more general demand distributions.

In this work, as in most existing behavior literature, we assume a symmetric information setting between the distribution channel parties and the consumers. One potential direction for future research is to consider a situation of asymmetric information. For example, the seller holds private information about, for instance, the marginal cost of the product. As a result, whether to downward outsource the distribution has a signalling effect – at least partially conveying product cost information to consumers. From this aspect, we may explore how such asymmetric information influences the consumers' fairness perception, the distribution channel parties' pricing and stocking decisions, the distribution channel design and parties' profitability.

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Online Appendix

The Impact of Consumer Fairness Seeking on Distribution Channel Selection: Direct Selling vs. Agent Selling

Proof of Proposition 1:

Given s, c and v, the highest price that the manufacturer is able to charge satisfies the following indifference equation:

$$U = v - p_d^* - \lambda \cdot \max\{(p_d^* - c) - (v - p_d^*), 0\} = 0,$$

which immediately implies

$$p_d^* = \frac{(1+\lambda)v + \lambda c}{1+2\lambda} = xv + yc,$$

where $x \equiv (1+\lambda)/(1+2\lambda)$ and $y \equiv \lambda/(1+2\lambda)$.

Given p_d^*, the manufacturer faces a newsvendor problem and decides the stocking quantity to maximize his profit, i.e.,

$$\begin{aligned}
\Pi_d(Q) =& A(p_d^* - c) + \mathsf{E}[p_d^* \min(D, Q) + s(Q-D)^+ - cQ] \\
=& A(p_d^* - c) + p_d^* \mathsf{E}[D] - p_d^* \mathsf{E}[(D-Q)^+] + s\mathsf{E}[(Q-D)^+] - cQ \\
=& A(p_d^* - c) + p_d^* \mathsf{E}[D] - p_d^* \int_Q^{\overline{D}} (D-Q) f(D) dD + s \int_0^Q (Q-D) f(D) dD - cQ.
\end{aligned}$$

It can be easily shown that $\Pi_d(Q)$ is concave and the first-order condition is

$$\frac{\partial \Pi_d(Q)}{\partial Q} = p_d^*(1 - F(Q)) + sF(Q) - c = 0.$$

Hence, the optimal newsvendor ordering quantity shall satisfy

$$F(Q^*) = \frac{p_d^* - c}{p_d^* - s} = \frac{(1+\lambda)(v-c)}{(1+\lambda)v + \lambda c - (1+2\lambda)s}.$$

Thus, under direct selling,

$$Q^* = \frac{p_d^* - c}{p_d^* - s}\overline{D} = \frac{(1+\lambda)(v-c)}{(1+\lambda)v + \lambda c - (1+2\lambda)s}\overline{D} = \frac{x(v-c)}{xv + yc - s}\overline{D} \text{ and } Q_d^* = A + Q^*.$$

For the manufacturer's profit, we can show that

$$\begin{aligned}
\Pi_d^* =& A(p_d^* - c) + \mathsf{E}[p_d^* \min(D, Q^*) + s(Q^* - D)^+ - cQ^*] \\
=& A(p_d^* - c) + (p_d^* - s) \int_0^{Q^*} D f(D) dD + p_d^* Q^* [1 - F(Q^*)] + sQ^* F(Q^*) - cQ^* \\
=& A(p_d^* - c) + (p_d^* - s) \int_0^{Q^*} D f(D) dD + (p_d^* - c) Q^* - (p_d^* - s) Q^* \frac{p_d^* - c}{p_d^* - s} \\
=& A(p_d^* - c) + (p_d^* - s) \frac{(Q^*)^2}{2\overline{D}} \\
=& A \frac{(1+\lambda)(v-c)}{1+2\lambda} + \frac{1}{2} \frac{(1+\lambda)^2 (v-c)^2}{1+2\lambda} \frac{\overline{D}}{(1+\lambda)v + \lambda c - (1+2\lambda)s} \\
=& Ax(v-c) + \frac{1}{2} x(v-c)^2 \frac{x}{xv + yc - s} \overline{D}.
\end{aligned}$$

Proof of Lemma 1:

By Proposition 1, we have

$$\begin{aligned}
\frac{dp_d^*}{dc} &= \frac{d}{dc}\left(\frac{(1+\lambda)v+\lambda c}{1+2\lambda}\right) = \frac{\lambda}{1+2\lambda} > 0;\\
\frac{d(p_d^*-c)}{dc} &= -\frac{1+\lambda}{1+2\lambda} < 0;\\
\frac{dQ_d^*}{dc} &= \frac{d}{dc}\left(A + \frac{(1+\lambda)(v-c)}{(1+\lambda)v+\lambda c-(1+2\lambda)s}\overline{D}\right) = (s-v)\,\frac{2\lambda^2+3\lambda+1}{\left(v-s+(c+v-2s)\,\lambda\right)^2}\overline{D} < 0;\\
\frac{d\Pi_d^*}{dc} &= \frac{d}{dc}\left(Ax(v-c)+\frac{1}{2}x(v-c)^2\frac{x}{xv+yc-s}\overline{D}\right) = \frac{1}{2}x^2\,(c-v)\,\frac{2\,(cy+vx-s)+(v-c)\,y}{(cy+vx-s)^2}\overline{D} - Ax < 0.
\end{aligned}$$

Proof of Lemma 2:

By Proposition 1, we can easily have $dp_d^*/d\overline{D} = 0$ since p_d^* is independent of $\overline{D}$. In addition, we can show

$$\begin{aligned}
\frac{dQ_d^*}{d\overline{D}} &= \frac{d}{d\overline{D}}\left(A + \frac{(1+\lambda)(v-c)}{(1+\lambda)v+\lambda c-(1+2\lambda)s}\overline{D}\right) = \frac{(1+\lambda)(v-c)}{(1+\lambda)(v-s)+\lambda(c-s)} > 0;\\
\frac{d\Pi_d^*}{d\overline{D}} &= \frac{x^2(v-c)^2}{2\,(xv+yc-s)} > 0;\\
\frac{dp_d^*}{d\lambda} &= \frac{d}{d\lambda}\left(\frac{(1+\lambda)v+\lambda c}{1+2\lambda}\right) = \frac{c-v}{(1+2\lambda)^2} < 0;\\
\frac{dQ_d^*}{d\lambda} &= \frac{d}{d\lambda}\left(A + \frac{(1+\lambda)(v-c)}{(1+\lambda)v+\lambda c-(1+2\lambda)s}\overline{D}\right) = \frac{(c-v)\,(c-s)}{\left((1+\lambda)v+\lambda c-(1+2\lambda)s\right)^2}\overline{D} < 0;\\
\frac{d\Pi_d^*}{d\lambda} &= \frac{d}{d\lambda}\left(A\frac{(1+\lambda)(v-c)}{1+2\lambda} + \frac{1}{2}\frac{(1+\lambda)^2(v-c)^2}{1+2\lambda}\frac{\overline{D}}{(1+\lambda)v+\lambda c-(1+2\lambda)s}\right)\\
&= -\frac{v-c}{(2\lambda+1)^2}A - \frac{(\lambda+1)\,(v-c)^2\left((3\lambda+1)\,c+(1+\lambda)\,v-(4\lambda+2)\,s\right)}{2\,(2\lambda+1)^2\left((1+\lambda)v+\lambda c-(1+2\lambda)s\right)^2}\overline{D}\\
&< 0.
\end{aligned}$$

Proof of Proposition 2:

We first prove the existence of the rational expectations equilibrium under this setting. To do so, we first show that the conditions given in Definition 1 are on a rational expectations equilibrium path. In a rational expectations equilibrium, the retailer will foresee the consumer's optimal choice and form a correct belief over the consumer's reservation price, i.e., $\varepsilon_r = r = xv + y\varepsilon_\omega$. For a profit-maximizing retailer, she will charge the highest price the consumer can accept, that is, $p_a = \varepsilon_r = r$. Although the retailer does not observe ε_ω, she can bet on $\varepsilon_\omega = \omega$ as long as she believes the consumer has the rational expectation. After observing the retail price $p_a = xv + y\omega$, the rational consumer will, in turn, update his or her information set and derive the wholesale price at $\varepsilon_\omega = \omega$ even if he or she got the wrong inference in the first place. This process verifies the retailer's belief that the consumer is rational and able to infer the "correct" wholesale price. So the conditions $p_a = \varepsilon_r$, $\varepsilon_r = r$, $r = xv + y\varepsilon_\omega$ and $\varepsilon_\omega = \omega$ are on the rational expectations equilibrium path. This process also confirms that ε_ω is a function of p_a, i.e., $\varepsilon_\omega(p_a)$.

Since the consumer will update his or her information set after observing the retail price, a natural question then follows: can the retailer seize the chance to manipulate the consumer's belief so as to make a higher profit? In other words, will the retailer be motivated to deviate from the equilibrium path? The answer is no. Let us assume for a given wholesale price ω, the retailer deviates the retail price to $p_a^{new} = xv + y(\omega + \epsilon)$, where $\epsilon > 0$. In this case, the consumer will be misled and take $\varepsilon_\omega^{new} = \omega + \epsilon$ as the real wholesale price. However, the manufacturer will see through the retailer's trick by her optimal decision on the ordering quantity Q, which maximizes the following profit function,

$$\Pi_R(p_a^{new}, Q; \omega) = A(p_a^{new} - \omega) + E[p_a^{new} \min(D, Q) + s(Q - D)^+ - \omega Q].$$

Then, the optimal ordering quantity

$$Q^{updated}(\omega) = \frac{p_a^{new} - \omega}{p_a^{new} - s}\overline{D} = \frac{x(v-\omega) + y\epsilon}{xv + y\omega - s + y\epsilon}.$$

This shows that if the retailer manipulates a higher retail price, she will order a higher stocking quantity for the purpose of profit maximization. Then, the manufacturer will respond to this updated order and adjust the optimal wholesale price. Intuitively, a higher stocking quantity at the original wholesale price can be transformed into a shift-out of the demand curve, which will push up the wholesale price in equilibrium. Mathematically, we can derive the updated optimal wholesale price $\omega^{*updated}$ by maximizing the manufacturer's profit,

$$\Pi_M(\omega) = (\omega - c)(A + Q^{updated}(\omega)) = (\omega - c)\left[A + \frac{x(v-\omega) + y\epsilon}{y\omega + xv - s + y\epsilon}\overline{D}\right].$$

Expecting a higher updated wholesale price, the retailer will adjust her stocking quantity accordingly if she still sticks to p_a^{new}. The manufacturer will then respond by changing the wholesale price. This process repeats until it reaches the rational expectations equilibrium that satisfies $\omega^{new} = \omega + \epsilon$ (from the beginning we have known that $p_a^{new} = xv + y(\omega + \epsilon)$, $\omega^{new} = \omega + \epsilon$ and $\varepsilon_\omega^{new} = \omega + \epsilon$ are on the rational expectations equilibrium path). That means the manufacturer drains off the retailer's additional profit and the retailer will not be better off by deviating from the equilibrium path. So, the existence of the equilibrium is guaranteed.

Note that, for agent selling, the retailer's problem is equivalent to the manufacturer's problem under direct selling, except that the marginal cost is replaced by the wholesale price ω. Then, by Proposition 1 and Definition 1, in the rational expectations equilibrium, the retailer's optimal decisions must be as follows:

$$\begin{aligned}
p_a^*(\omega) =& \frac{(1+\lambda)v + \lambda\omega}{1+2\lambda};\\
Q^*(\omega) =& \frac{p_a^* - \omega}{p_a^* - s}\overline{D} = \frac{(1+\lambda)(v-\omega)}{(1+\lambda)v + \lambda\omega - (1+2\lambda)s}\overline{D};\\
Q_a^*(\omega) =& A + Q^* = A + \frac{(1+\lambda)(v-\omega)}{(1+\lambda)v + \lambda\omega - (1+2\lambda)s}\overline{D} = A + \frac{x(v-\omega)}{xv + y\omega - s}\overline{D}.
\end{aligned}$$

Consequently, the manufacturer decides the wholesale price ω to maximize his profit as follows:

$$\begin{aligned}\Pi_M(\omega) =& (\omega-c)(A+Q^*(\omega))\\ =&(\omega-c)\left[A+\frac{(1+\lambda)(v-\omega)}{(1+\lambda)v+\lambda\omega-(1+2\lambda)s}\overline{D}\right]\\ =&(\omega-c)\left[A+\frac{x(v-\omega)}{y\omega+xv-s}\overline{D}\right]\\ =&-\left[\frac{1}{y^2}(x\overline{D}-yA)(y\omega+xv-s)+\frac{x}{y^2}((xv-s)+yv)\overline{D}\frac{xv+yc-s}{y\omega+xv-s}\right]\\ &+\frac{1}{y^2}(x\overline{D}-yA)(xv+yc-s)+\frac{x}{y^2}((xv-s)+yv)\overline{D}.\end{aligned}$$

It can be easily shown that $\Pi_M(\omega)$ is concave in ω, and the optimal wholesale price is

$$\omega^*=\begin{cases}\frac{1}{y}\left(\sqrt{\frac{(v-s)(xv+yc-s)x\overline{D}}{x\overline{D}-yA}}-(xv-s)\right), & if\ \overline{D}>\frac{(v-s)A}{(v-c)x};\\ v, & otherwise.\end{cases}$$

Correspondingly, we have

$$\Pi_M^*=\begin{cases}\frac{1}{y^2}\left(\sqrt{(xv+yc-s)(x\overline{D}-yA)}-\sqrt{(v-s)x\overline{D}}\right)^2, & if\ \overline{D}>\frac{(v-s)A}{(v-c)x};\\ A(v-c), & otherwise.\end{cases}$$

Proof of Lemma 3:

By Proposition 2, we have

$$\omega^*=\begin{cases}\frac{1}{y}\left(\sqrt{\frac{(v-s)(xv+yc-s)x\overline{D}}{x\overline{D}-yA}}-(xv-s)\right), & if\ \overline{D}>\frac{(v-s)A}{(v-c)x};\\ v, & otherwise.\end{cases}$$

which implies

$$\frac{d\omega^*}{dc}=\begin{cases}\frac{1}{2}\sqrt{\frac{(v-s)x\overline{D}}{(xv+yc-s)\left(x\overline{D}-yA\right)}}, & if\ \overline{D}>\frac{(v-s)A}{(v-c)x};\\ 0, & otherwise,\end{cases}\geq 0.$$

Then, we have

$$\begin{aligned}\frac{dp_a^*}{dc}&=\frac{d}{dc}\left(\frac{(1+\lambda)v+\lambda\omega^*}{1+2\lambda}\right)=\frac{\lambda}{1+2\lambda}\frac{d\omega^*}{dc}\geq 0;\\ \frac{d(p_a^*-\omega^*)}{dc}&=-\frac{1+\lambda}{1+2\lambda}\frac{d\omega^*}{dc}\leq 0.\end{aligned}$$

In addition,

$$\frac{dQ_a^*}{dc}=\frac{d}{dc}\left(\frac{(1+\lambda)(v-\omega^*)}{(1+\lambda)v+\lambda\omega^*-(1+2\lambda)s}\overline{D}+A\right)=-\frac{(1+\lambda)(1+2\lambda)(v-s)\overline{D}}{((1+\lambda)v+\lambda\omega^*-(1+2\lambda)s)^2}\frac{d\omega^*}{dc}\leq 0.$$

By noting that

$$\Pi_M^*=(\omega^*-c)\left(A+\frac{(1+\lambda)(v-\omega^*)}{(1+\lambda)v+\lambda\omega^*-(1+2\lambda)s}\overline{D}\right),$$

from the Envelop Theorem, we can immediately have

$$\frac{d\Pi_M^*}{dc} = -A - \frac{(1+\lambda)(v-\omega^*)}{(1+\lambda)v+\lambda\omega^*-(1+2\lambda)s}\overline{D} \leq 0.$$

Proof of Lemma 4:

By Proposition 2, we have

$$\omega^* = \begin{cases} \frac{1}{y}\left(\sqrt{\frac{(v-s)(xv+yc-s)x\overline{D}}{x\overline{D}-yA}} - (xv-s)\right), \ if \ \overline{D} > \frac{(v-s)A}{(v-c)x}; \\ v, \qquad\qquad\qquad\qquad\qquad\qquad otherwise. \end{cases}$$

which implies

$$\frac{d\omega^*}{d\overline{D}} = \begin{cases} -\frac{\sqrt{x(v-s)(cy-s+vx)}}{2\left(x\overline{D}-Ay\right)\sqrt{\left(x\overline{D}-Ay\right)\overline{D}}}A, \ if \ \overline{D} > \frac{(v-s)A}{(v-c)x}; \\ 0, \qquad\qquad\qquad\qquad\qquad otherwise, \end{cases} \leq 0.$$

We can show that

$$\begin{aligned} \frac{dp_a^*}{d\overline{D}} &= \frac{d}{d\overline{D}}\frac{(1+\lambda)v+\lambda\omega^*}{1+2\lambda} = \frac{\lambda}{1+2\lambda}\frac{d\omega^*}{d\overline{D}} \leq 0; \\ \frac{d(p_a^*-\omega^*)}{d\overline{D}} &= -\frac{1+\lambda}{1+2\lambda}\frac{d\omega^*}{d\overline{D}} \geq 0; \\ \frac{dQ_a^*}{d\overline{D}} &= \frac{d}{d\overline{D}}\left(\frac{(1+\lambda)(v-\omega^*)}{(1+\lambda)v+\lambda\omega^*-(1+2\lambda)s}\overline{D} + A\right) \\ &= -\frac{(1+\lambda)(1+2\lambda)(v-s)\overline{D}}{((1+\lambda)v+\lambda\omega^*-(1+2\lambda)s)^2}\frac{d\omega^*}{d\overline{D}} + \frac{(1+\lambda)(v-\omega^*)}{(1+\lambda)v+\lambda\omega^*-(1+2\lambda)s} \\ &\geq 0. \end{aligned}$$

By noting that

$$\Pi_M^* = A(\omega^*-c) + \frac{(1+\lambda)(v-\omega^*)(\omega^*-c)}{(1+\lambda)v+\lambda\omega^*-(1+2\lambda)s}\overline{D},$$

from Envelop Theorem, we can immediately have

$$\frac{d\Pi_M^*}{d\overline{D}} = \frac{(1+\lambda)(v-\omega^*)(\omega^*-c)}{(1+\lambda)v+\lambda\omega^*-(1+2\lambda)s} \geq 0; \ \frac{d\Pi_M^*}{d\lambda} = -\frac{(\omega^*-s)(v-\omega^*)(\omega^*-c)}{((1+\lambda)v+\lambda\omega^*-(1+2\lambda)s)^2}\overline{D} \leq 0.$$

Proof of Lemma 5:

Recall that $Q_d^* = A + x(v-c)\overline{D}/(xv+yc-s)$ and $Q_a^* = A + x(v-\omega^*)\overline{D}/(xv+y\omega^*-s)$. By noting that $c < \omega^*$, we have $Q_a^* < Q_d^*$.

Proof of Proposition 3:

The manufacturer needs to compare Π_M^* and Π_d^* to decide on whether he should adopt agent selling or direct selling.

By continuity of $\Pi_M^*(\lambda)$ and $\Pi_d^*(\lambda)$, to show our results, we first need to show that $\lim_{\lambda\to 0}\Pi_M^*(\lambda) < \lim_{\lambda\to 0}\Pi_d^*(\lambda)$ and $\lim_{\lambda\to\infty}\Pi_M^*(\lambda) > \lim_{\lambda\to\infty}\Pi_d^*(\lambda)$. Note that $\lim_{\lambda\to 0} x = 1$ and $\lim_{\lambda\to 0} y = 0$. Thus, we have

$$\lim_{\lambda\to 0}\Pi_d^*(\lambda) = \lim_{\lambda\to 0}\left[Ax(v-c) + \frac{1}{2}(v-c)^2\frac{x^2}{xv+yc-s}\overline{D}\right] = A(v-c) + \frac{(v-c)^2}{2(v-s)}\overline{D}.$$

When $\lambda \to 0$, we can show that

$$\begin{aligned}
&\lim_{\lambda\to 0}\frac{1}{y}\left(\sqrt{\frac{(v-s)(xv+yc-s)x\overline{D}}{x\overline{D}-yA}}-(xv-s)\right)\\
&=\lim_{\lambda\to 0}\frac{(2\lambda+1)}{\lambda}\left(\sqrt{\frac{(v-s)((1+\lambda)\,v+\lambda c-(2\lambda+1)\,s)\,(1+\lambda)\,\overline{D}}{\big((1+\lambda)\,\overline{D}-\lambda A\big)\,(2\lambda+1)}}-\frac{(1+\lambda)\,v}{1+2\lambda}+s\right)\\
&=\lim_{\lambda\to 0}\frac{d}{d\lambda}\left((2\lambda+1)\left(\sqrt{\frac{(v-s)((1+\lambda)\,v+\lambda c-(2\lambda+1)\,s)\,(1+\lambda)\,\overline{D}}{\big((1+\lambda)\,\overline{D}-\lambda A\big)\,(2\lambda+1)}}-\frac{(1+\lambda)\,v}{1+2\lambda}+s\right)\right)\\
&=\frac{v+c}{2}+\frac{A(v-s)}{2\overline{D}}.
\end{aligned}$$

Thus, under agent selling,

$$\lim_{\lambda\to 0}\omega^*=\begin{cases}\frac{v+c}{2}+\frac{A(v-s)}{2\overline{D}}, & if\ \overline{D}>A\frac{v-s}{v-c};\\ v, & otherwise.\end{cases}$$

Furthermore, we have

$$\lim_{\lambda\to 0}p_a^*=v.$$

Correspondingly, we can derive that

$$\lim_{\lambda\to 0}\Pi_M^*(\lambda)=\begin{cases}\frac{\left((v-s)A+(v-c)\overline{D}\right)^2}{4\overline{D}(v-s)}, & if\ \overline{D}>\frac{v-s}{v-c}A;\\ A(v-c), & otherwise.\end{cases}$$

Thus, we have

$$\lim_{\lambda\to 0}\Pi_M^*(\lambda)-\lim_{\lambda\to 0}\Pi_d^*(\lambda)=\begin{cases}-\frac{(v-c)A}{2}-\frac{(v-c)^2\overline{D}}{4(v-s)}+\frac{A^2(v-s)}{4\overline{D}}\equiv K_1, & if\ \overline{D}>\frac{v-s}{v-c}A;\\ -\frac{(v-c)^2}{2(v-s)}\overline{D}, & otherwise.\end{cases}$$

Note that

$$\frac{d}{d\overline{D}}K_1=\frac{d}{d\overline{D}}\left(-\frac{(v-c)A}{2}-\frac{(v-c)^2\overline{D}}{4\,(v-s)}+\frac{A^2(v-s)}{4\overline{D}}\right)=\frac{A^2\,(s-v)^2+\overline{D}^2\,(c-v)^2}{4\overline{D}^2\,(s-v)}<0$$

and when $\overline{D}=\frac{v-s}{v-c}A$, $K_1=\frac{A(c-v)}{2}<0$. Thus, $K_1<0$ for $\overline{D}>\frac{v-s}{v-c}A$.

Therefore, $\lim_{\lambda\to 0}\Pi_M^*(\lambda)-\lim_{\lambda\to 0}\Pi_d^*(\lambda)<0$. Hence, we prove that $\lim_{\lambda\to 0}\Pi_M^*(\lambda)<\lim_{\lambda\to 0}\Pi_d^*(\lambda)$.

We further notice that $\lim_{\lambda\to\infty}x=1/2$ and $\lim_{\lambda\to\infty}y=1/2$. Thus, we have

$$\lim_{\lambda\to\infty}\Pi_d^*(\lambda)=\lim_{\lambda\to\infty}\left[Ax(v-c)+\frac{1}{2}x(v-c)^2\frac{x}{xv+yc-s}\overline{D}\right]=\frac{1}{2}A(v-c)+\frac{(v-c)^2}{4(v+c-2s)}\overline{D}.$$

Note that

$$\begin{aligned}
\lim_{\lambda\to\infty}\Pi_M^*(\lambda)&=\lim_{\lambda\to\infty}\left[(\omega^*-c)\left(A+\frac{x(v-\omega^*)}{xv+y\omega^*-s}\overline{D}\right)\right]\\
&=\lim_{\lambda\to\infty}\left[(\omega^*-c)\left(A+\frac{v-\omega^*}{\omega^*+v-2s}\overline{D}\right)\right]\\
&=\lim_{\lambda\to\infty}\left[-\left((\overline{D}-A)(\omega^*+v-2s)+2\overline{D}(v-s)\frac{v+c-2s}{\omega^*+v-2s}\right)+(\overline{D}-A)(v+c-2s)+2\overline{D}(v-s)\right].
\end{aligned}$$

Again, it can be easily shown that $\lim_{\lambda\to\infty}\Pi_M^*(\lambda)$ is concave. By $\lim_{\lambda\to\infty}\omega^* \le \lim_{\lambda\to\infty} p_a^* = \lim_{\lambda\to\infty}(v+\omega^*)/2$, we have $\lim_{\lambda\to\infty}\omega^* \le v$. Therefore, there must exist an optimal wholesale price as follows:

$$\lim_{\lambda\to\infty}\omega^* = \begin{cases} \sqrt{\frac{2\overline{D}(v-s)(v+c-2s)}{\overline{D}-A}} - (v-2s), & if\ \overline{D} > 2A\frac{v-s}{v-c}; \\ v, & otherwise. \end{cases}$$

And the corresponding profit is

$$\lim_{\lambda\to\infty}\Pi_M^*(\lambda) = \begin{cases} \left(\sqrt{(\overline{D}-A)(v+c-2s)} - \sqrt{2\overline{D}(v-s)}\right)^2, & if\ \overline{D} > 2A\frac{v-s}{v-c}; \\ A(v-c), & otherwise. \end{cases}$$

Then, we have

$$\lim_{\lambda\to\infty}\Pi_M^*(\lambda) - \lim_{\lambda\to\infty}\Pi_d^*(\lambda) = \begin{cases} -\frac{(v-c)A}{2} - \frac{(v-c)^2}{4(v+c-2s)}\overline{D} + (\sqrt{(\overline{D}-A)(v+c-2s)} - \sqrt{2\overline{D}(v-s)})^2, & if\ \overline{D} > 2A\frac{v-s}{v-c}; \\ \frac{(v-c)A}{2} - \frac{(v-c)^2}{4(v+c-2s)}\overline{D}, & otherwise. \end{cases}$$

When $\overline{D} \le 2A(v-s)/(v-c)$, we have

$$\begin{aligned} \lim_{\lambda\to\infty}\Pi_M^*(\lambda) - \lim_{\lambda\to\infty}\Pi_d^*(\lambda) &= \frac{(v-c)A}{2} - \frac{(v-c)^2}{4(v+c-2s)}\overline{D} \\ &\ge \frac{1}{2}A(v-c) - \frac{(v-c)^2}{4(v+c-2s)}\cdot 2A\frac{v-s}{v-c} \\ &= \frac{1}{2}A(v-c) - \frac{1}{2}A\frac{(v-c)(v-s)}{v+c-2s} \\ &= \frac{(v-c)(c-s)}{2(v+c-2s)}A \\ &> 0. \end{aligned}$$

When $\overline{D} > 2A(v-s)/(v-c)$, we have

$$\begin{aligned} \lim_{\lambda\to\infty}\Pi_M^*(\lambda) - \lim_{\lambda\to\infty}\Pi_d^*(\lambda) &= -\frac{(v-c)\,A}{2} - \frac{(v-c)^2}{4(v+c-2s)}\overline{D} + (\sqrt{(\overline{D}-A)(v+c-2s)} - \sqrt{2\overline{D}(v-s)})^2 \\ &= -\frac{A(v-c)}{2} - \frac{(v-c)^2\overline{D}}{4\,(v+c-2s)} + (\overline{D}-A)(v+c-2s) + 2\overline{D}(v-s) \\ &\quad -2\sqrt{2\overline{D}\,(v-s)\,(\overline{D}-A)(v+c-2s)} \\ &= \frac{(3c^2-24cs+18cv+32s^2-40sv+11v^2)\,\overline{D} - 2A\,(c-2s+v)\,(c-4s+3v)}{4\,(c-2s+v)} \\ &\quad -2\sqrt{2\overline{D}\,(v-s)\,(\overline{D}-A)(v+c-2s)}. \end{aligned}$$

We can show that

$$\frac{d}{d\overline{D}}\left(\lim_{\lambda\to\infty}\Pi_M^*(\lambda) - \lim_{\lambda\to\infty}\Pi_d^*(\lambda)\right) = \frac{(3c^2-24cs+18cv+32s^2-40sv+11v^2)}{4\,(c-2s+v)} - \sqrt{\frac{2(v-s)\,(c-2s+v)}{\overline{D}\,(\overline{D}-A)}}\,(A-2\overline{D})$$

$$\frac{d^2}{d\overline{D}^2}\left(\lim_{\lambda\to\infty}\Pi_M^*(\lambda) - \lim_{\lambda\to\infty}\Pi_d^*(\lambda)\right) = -\frac{\sqrt{2D\,(v-s)\,(D-A)\,(c-2s+v)}}{2D^2\,(A-D)^2}A^2 < 0.$$

Thus, $\lim_{\lambda\to\infty}\Pi_M^*(\lambda)-\lim_{\lambda\to\infty}\Pi_d^*(\lambda)$ is concave in $\overline{D}$.

Furthermore, it can be shown that

$$\left[\lim_{\lambda\to\infty}\Pi_M^*(\lambda)-\lim_{\lambda\to\infty}\Pi_d^*(\lambda)\right]\Big|_{\overline{D}=\frac{2A(v-s)}{(v-c)}}=\frac{1}{2}A\,(v-c)\,\frac{c-s}{c-2s+v}>0.$$

Therefore, there exists a unique threshold $\widehat{\overline{D}}$ such that $\lim_{\lambda\to\infty}\Pi_M^*(\lambda)-\lim_{\lambda\to\infty}\Pi_d^*(\lambda)>0$ if and only if $2A(v-s)/(v-c)<\overline{D}<\widehat{\overline{D}}$. We then derive $\widehat{\overline{D}}$ by letting $\lim_{\lambda\to\infty}\Pi_M^*(\lambda)-\lim_{\lambda\to\infty}\Pi_d^*(\lambda)=0$, that is,

$$\begin{aligned}K_2 &\equiv \frac{(3c^2-24cs+18cv+32s^2-40sv+11v^2)\,\overline{D}-2A\,(c-2s+v)\,(c-4s+3v)}{4\,(c-2s+v)}\\ &=2\sqrt{2\overline{D}\,(v-s)\,(\overline{D}-A)(v+c-2s)}.\end{aligned}$$

Note that

$$\begin{aligned}K_2 &> \frac{(-12cs+10cv-20sv+c^2+16s^2+5v^2)\,\overline{D}}{4\,(c-2s+v)}\\ &> \frac{(-12c^2+10cv-20cv+c^2+16c^2+5v^2)\,\overline{D}}{4\,(c-2s+v)}\\ &= \frac{5\,(c-v)^2\,\overline{D}}{4\,(c-2s+v)}\\ &> 0,\end{aligned}$$

as

$$\frac{d}{ds}\left(-12cs+10cv-20sv+c^2+16s^2+5v^2\right)=32s-12c-20v<0.$$

Solving $K_2=2\sqrt{2\overline{D}\,(v-s)\,(\overline{D}-A)(v+c-2s)}$, we can derive the threshold point

$$\widehat{\overline{D}}=2A\,(c-2s+v)\,\frac{-\,(v-c)\,(3c-4s+v)+8\,(c-2s+v)\,\sqrt{(v-c)\,(v-s)}}{(v-c)^2\,(9c+7v-16s)}.$$

Hence, $\lim_{\lambda\to\infty}\Pi_M^*(\lambda)>\lim_{\lambda\to\infty}\Pi^*(\lambda)$ if and only if $\overline{D}<\widehat{\overline{D}}$. Moreover, when $A\to 0$, $\widehat{\overline{D}}\to 0$.

Also note that both $\Pi_M^*(\lambda)$ and $\Pi_d^*(\lambda)$ are continuous functions of λ. Therefore, when $\overline{D}<\widehat{\overline{D}}$, by Intermediate Value Theorem, there exists at least one $\widehat{\lambda}$ such that agent selling generates a higher profit than direct selling if $\lambda>\widehat{\lambda}$.

Next, we prove that the threshold $\widehat{\lambda}$ is unique. For direct selling, we have

$$\begin{aligned}\frac{\partial\Pi_d^*}{\partial\lambda} &= \frac{d}{d\lambda}\left(\frac{(1+\lambda)\,(v-c)A}{1+2\lambda}+\frac{(\lambda+1)^2\,(c-v)^2\,\overline{D}}{2\,(2\lambda+1)\,(v-s+c\lambda-2s\lambda+v\lambda)}\right)\\ &= -\frac{(v-c)A}{(1+2\lambda)^2}-\frac{1}{2}\,\frac{(c-v)^2\,(\lambda+1)\,((3\lambda+1)\,c+(\lambda+1)\,v-(2+4\lambda)\,s)\,\overline{D}}{(2\lambda+1)^2\,((1+\lambda)\,v+c\lambda-(1+2\lambda)\,s)^2}\\ &= -\frac{(v-c)A}{(1+2\lambda)^2}-\frac{1}{2}(v-c)^2\overline{D}\frac{2x(xv+yc-s)+x^2(c-v)}{(xv+yc-s)^2(1+2\lambda)^2}\\ &< 0.\end{aligned}$$

For agent selling, from Lemma 4, we have derived that

$$\frac{\partial \Pi_M^*}{\partial \lambda} = \frac{-(v-\omega^*)(\omega^*-s)(\omega^*-c)}{(xv+y\omega^*-s)^2(1+2\lambda)^2}\overline{D} \le 0.$$

Note that, at the threshold $\lambda = \widehat{\lambda}$, we shall have $\Pi_d^*(\widehat{\lambda}) = \Pi_M^*(\widehat{\lambda})$, that is,

$$\left[Ax(v-c) + \frac{1}{2}(v-c)^2\overline{D}\frac{x^2}{xv+yc-s}\right]\Big|_{\lambda=\widehat{\lambda}} = \left[A(\omega^*-c) + \frac{x(v-\omega^*)(\omega^*-c)}{xv+yw^*-s}\overline{D}\right]\Big|_{\lambda=\widehat{\lambda}}.$$

Then, we have

$$\begin{aligned}
\frac{\partial \Pi_d^*}{\partial \lambda}\Big|_{\lambda=\widehat{\lambda}} =& \left[-\frac{A(v-c)}{(1+2\lambda)^2} - \frac{1}{2}(v-c)^2\overline{D}\frac{2x(xv+yc-s)+x^2(c-v)}{(xv+yc-s)^2(1+2\lambda)^2}\right]\Big|_{\lambda=\widehat{\lambda}} \\
=& -\frac{1}{(1+2\lambda)^2}\left[A(v-c) + \frac{1}{2}(v-c)^2\overline{D}\frac{x}{(xv+yc-s)} + \frac{1}{2}(v-c)^2\overline{D}x\frac{xv+yc-s+(c-v)x}{(xv+yc-s)^2}\right]\Big|_{\lambda=\widehat{\lambda}} \\
=& -\frac{1}{(1+2\lambda)^2}\left[\frac{A(\omega^*-c)}{x} + \frac{(v-\omega^*)(\omega^*-c)}{xv+y\omega^*-s}\overline{D} + \frac{(c-s)(v-c)^2x}{2(xv+yc-s)^2}\overline{D}\right]\Big|_{\lambda=\widehat{\lambda}}.
\end{aligned}$$

As a result, by noting that $0 < s < c \le \omega^* \le v$, we can show that

$$\begin{aligned}
\left[\frac{\partial \Pi_d^*}{\partial \lambda} - \frac{\partial \Pi_M^*}{\partial \lambda}\right]\Big|_{\lambda=\widehat{\lambda}} =& \frac{1}{(1+2\lambda)^2}\left[-\frac{A(\omega^*-c)}{x} - \frac{1}{2}(v-c)^2\overline{D}x\frac{c-s}{(xv+yc-s)^2} - \frac{x(v-\omega^*)^2(\omega^*-c)}{(xv+y\omega^*-s)^2}\overline{D}\right]\Big|_{\lambda=\widehat{\lambda}} \\
&< 0.
\end{aligned}$$

Hence, the uniqueness of $\widehat{\lambda}$ is confirmed.

Proof of Proposition 4:

The proof of the existence of this rational expectations equilibrium follows the same logic as that of the proof of Proposition 2. Thus, we omit the details for expositional brevity.

As noted, given the wholesale price ω, we have

$$p_a^*(\omega) = \frac{(1+\lambda)v - \lambda(\omega-c)}{1+\lambda} = v - \frac{y}{x}(\omega-c).$$

The retailer faces the same problem as that in the baseline model, which immediately implies that

$$Q^*(\omega) = \frac{p_a^*-\omega}{p_a^*-s}\overline{D} = \frac{x(v-\omega)-y(\omega-c)}{x(v-s)-y(\omega-c)}\overline{D} \text{ and } Q_a^*(\omega) = A + Q^* = A + \frac{x(v-\omega)-y(\omega-c)}{x(v-s)-y(\omega-c)}\overline{D}.$$

As to the manufacturer, we have

$$\begin{aligned}
\Pi_M(\omega) =& (\omega-c)\left(A + \frac{x(v-\omega)-y(\omega-c)}{x(v-s)-y(\omega-c)}\overline{D}\right) \\
=& \frac{1}{y}\left(y(\omega-c) - x(v-s) + x(v-s)\right)\left(A + \frac{\overline{D}}{y} + \frac{x(v-c)-\frac{x}{y}(v-s)}{x(v-s)-y(\omega-c)}\overline{D}\right) \\
=& \frac{1}{y}\left(y(\omega-c) - x(v-s) + x(v-s)\right)\left(A + \frac{\overline{D}}{y} - \frac{x}{y}\frac{x(v-s)+y(c-s)}{x(v-s)-y(\omega-c)}\overline{D}\right) \\
=& -\left(\frac{Ay+\overline{D}}{y^2}(x(v-s)-y(\omega-c)) + \frac{x^2(v-s)\overline{D}}{y^2}\frac{x(v-s)+y(c-s)}{x(v-s)-y(\omega-c)}\right) + \frac{x\overline{D}}{y^2}(x(v-s)+y(c-s)) \\
&+ \frac{x}{y^2}(v-s)(Ay+\overline{D}).
\end{aligned}$$

It can be easily shown that $\Pi_M(\omega)$ is concave in ω, and the optimal wholesale price

$$\omega^* = \begin{cases} \frac{x}{y}\left((v-s)-\sqrt{\frac{(v-s)(x(v-s)+y(c-s))\overline{D}}{Ay+\overline{D}}}\right)+c, \; if\ \overline{D} > \frac{(xv+yc-s)A}{v-c}; \\ xv+yc, \qquad otherwise. \end{cases}$$

Correspondingly, we have

$$\Pi_M^* = \begin{cases} \frac{1}{y^2}\left(\sqrt{x\overline{D}(x(v-s)+y(c-s))}-\sqrt{x(v-s)(Ay+\overline{D})}\right)^2, \; if\ \overline{D} > \frac{(xv+yc-s)A}{v-c}; \\ Ax(v-c), \qquad otherwise. \end{cases}$$

and

$$\Pi_R^* = \begin{cases} \frac{A}{y}\left(\sqrt{\frac{(v-s)(xv+yc-s)\overline{D}}{Ay+\overline{D}}}-(xv+yc-s)\right)+\frac{1}{2y^2}\frac{\left(\sqrt{\frac{(v-s)(xv+yc-s)\overline{D}}{Ay+\overline{D}}}-(xv+yc-s)\right)^2}{\sqrt{\frac{(v-s)(xv+yc-s)\overline{D}}{Ay+\overline{D}}}}\overline{D}, \; if\ \overline{D} > \frac{(xv+yc-s)A}{v-c}; \\ 0, \qquad otherwise. \end{cases}$$

Proof of Lemma 6:

Recall that

$$\Pi_d^* = Ax(v-c)+\frac{1}{2}\frac{x^2(v-c)^2}{xv+yc-s}\overline{D}.$$

Then, by Proposition 4, we can easily get that when $\overline{D} \le (xv+yc-s)A/(v-c)$, $\Pi_M^* < \Pi_d^*$.

When $\overline{D} > (xv+yc-s)A/(v-c)$, we first show that Π_M^* and Π_d^* monotonically increase with $\overline{D}$. Note that

$$\begin{aligned} \frac{\partial \Pi_d^*}{\partial \overline{D}} =& \frac{1}{2}\frac{x^2(v-c)^2}{xv+yc-s} > 0; \\ \frac{\partial \Pi_M^*}{\partial \overline{D}} =& \frac{\partial}{\partial \overline{D}}\left[\frac{1}{y^2}\left(x\overline{D}(xv+yc-s)+x(v-s)(Ay+\overline{D})-2\sqrt{x^2(xv+yc-s)(v-s)\overline{D}(Ay+\overline{D})}\right)\right] \\ =& \frac{1}{y^2}\left[x(xv+yc-s+v-s)-2x\sqrt{(xv+yc-s)(v-s)}\frac{1}{2}\frac{Ay+2\overline{D}}{\sqrt{Ay\overline{D}+\overline{D}^2}}\right] \\ =& \frac{1}{y^2}\left[x(xv+yc-s+v-s)-x\sqrt{(xv+yc-s)(v-s)}\sqrt{\frac{A^2y^2}{Ay\overline{D}+\overline{D}^2}+4}\right], \end{aligned}$$

which indicates that $\partial \Pi_M^*/\partial \overline{D}$ increases with $\overline{D}$. Besides, when $\overline{D} = (xv+yc-s)A/(v-c)$,

$$\begin{aligned} \frac{\partial \Pi_M^*}{\partial \overline{D}} =& \frac{1}{y^2}\left[x(xv+yc-s+v-s)-x\sqrt{(xv+yc-s)(v-s)}\frac{v-s+xv+yc-s}{v-c}\frac{A}{\frac{A}{v-c}\sqrt{(xv+yc-s)(v-s)}}\right] \\ =& 0. \end{aligned}$$

Thus, when $\overline{D} > (xv+yc-s)A/(v-c)$, we have (i) $\partial \Pi_M^*/\partial \overline{D} > 0$, (ii) $\partial^2 \Pi_M^*/\partial \overline{D}^2 > 0$, (iii) $\partial \Pi_d^*/\partial \overline{D} > 0$, and (iv) $\partial^2 \Pi_d^*/\partial \overline{D}^2 = 0$. Moreover, when $\overline{D} = (xv+yc-s)A/(v-c)$, it can be shown that $\Pi_d^* > \Pi_M^*$.

As such, to show $\Pi_d^* > \Pi_M^*$ for any $\overline{D} > (xv+yc-s)A/(v-c)$, it suffices to show $\Pi_d^* > \Pi_M^*$ when $\overline{D} \to \infty$. Specifically,

$$\begin{aligned}
\lim_{\overline{D}\to\infty} \frac{\Pi_M^*}{\overline{D}} &= \lim_{\overline{D}\to\infty} \frac{1}{y^2}\left(\sqrt{x(xv+yc-s)} - \sqrt{x(v-s)(\frac{Ay}{\overline{D}}+1)}\right)^2 \\
&= \frac{x}{y^2}\left(\sqrt{(xv+yc-s)} - \sqrt{(v-s)}\right)^2; \\
\lim_{\overline{D}\to\infty} \frac{\Pi_d^*}{\overline{D}} &= \lim_{\overline{D}\to\infty} \frac{Ax(v-c)}{\overline{D}} + \frac{1}{2}\frac{x^2(v-c)^2}{xv+yc-s} = \frac{1}{2}\frac{x^2(v-c)^2}{xv+yc-s}.
\end{aligned}$$

By $\left(\sqrt{v-s} - \sqrt{xv+yc-s}\right)^2 \geq 0$, we have $y(v-c) + 2(xv+yc-s) \geq 2\sqrt{v-s}\sqrt{xv+yc-s}$. By $x > 1/2$, we have

$$\begin{aligned}
y\sqrt{2x}(v-c) + 2(xv+yc-s) &> 2\sqrt{v-s}\sqrt{xv+yc-s}, \\
y\sqrt{2x}(v-c) &> 2\sqrt{v-s}\sqrt{xv+yc-s} - 2(xv+yc-s), \\
\frac{1}{2}\frac{xy^2(v-c)^2}{xv+yc-s} &> \left(\sqrt{xv+yc-s} - \sqrt{v-s}\right)^2, \\
\frac{1}{2}\frac{x^2(v-c)^2}{xv+yc-s} &> \frac{x}{y^2}\left(\sqrt{xv+yc-s} - \sqrt{v-s}\right)^2, \\
\lim_{\overline{D}\to\infty} \frac{\Pi_d^*}{\overline{D}} &> \lim_{\overline{D}\to\infty} \frac{\Pi_M^*}{\overline{D}}.
\end{aligned}$$

Therefore, when $\overline{D} > (xv+yc-s)A/(v-c)$, we have $\lim_{\overline{D}\to\infty} \Pi_d^* > \lim_{\overline{D}\to\infty} \Pi_M^*$. Thus, $\Pi_M^* < \Pi_d^*$ holds for any $\overline{D}$.

Proof of Lemma 7:

When $\overline{D} \leq A(xv+yc-s)/(v-c)$, it is quite straightforward to show that $\Pi_M^* + \Pi_R^* < \Pi_d^*$ by direct comparison. Then, we need to check whether $\Pi_M^* + \Pi_R^* < \Pi_d^*$ holds when $\overline{D} > A(xv+yc-s)/(v-c)$. To do so, we just need to consider the case when $\overline{D} \to \infty$ and $\lambda \to \infty$. Specifically, by Proposition 4, we have

$$\lim_{\overline{D}\to\infty} \frac{\Pi_R^*}{\overline{D}} = \lim_{\overline{D}\to\infty} \frac{1}{2y^2} \frac{\left(\sqrt{\frac{(v-s)(xv+yc-s)\overline{D}}{Ay+\overline{D}}} - (xv+yc-s)\right)^2}{\sqrt{\frac{(v-s)(xv+yc-s)\overline{D}}{Ay+\overline{D}}}} = \frac{1}{2y^2}\sqrt{\frac{xv+yc-s}{v-s}}\left(\sqrt{v-s} - \sqrt{xv+yc-s}\right)^2,$$

which combined with the results obtained in Proposition 4 leads to

$$\lim_{\overline{D}\to\infty} \frac{\Pi_R^* + \Pi_M^*}{\overline{D}} = \frac{1}{y^2}\left(x + \frac{1}{2}\sqrt{\frac{xv+yc-s}{v-s}}\right)\left(\sqrt{v-s} - \sqrt{xv+yc-s}\right)^2.$$

As $\lim_{\lambda\to\infty} x = 1/2$ and $\lim_{\lambda\to\infty} y = 1/2$, we have

$$\begin{aligned}
\lim_{\lambda\to\infty}\lim_{\overline{D}\to\infty} \frac{\Pi_R^* + \Pi_M^*}{\overline{D}} &= 4\left(\frac{1}{2} + \frac{1}{2}\sqrt{\frac{\frac{1}{2}v + \frac{1}{2}c - s}{v-s}}\right)\left(\sqrt{v-s} - \sqrt{\frac{1}{2}v + \frac{1}{2}c - s}\right)^2 \\
&= (v-c)\left(1 - \sqrt{\frac{\frac{1}{2}v + \frac{1}{2}c - s}{v-s}}\right)
\end{aligned}$$

and

$$\lim_{\lambda\to\infty}\lim_{\overline{D}\to\infty}\frac{\Pi_d^*}{\overline{D}}=\lim_{\lambda\to\infty}\frac{1}{2}\frac{x^2(v-c)^2}{xv+yc-s}=\frac{\frac{1}{8}(v-c)^2}{\frac{1}{2}v+\frac{1}{2}c-s}.$$

Then,

$$\begin{aligned}\lim_{\lambda\to\infty}\lim_{\overline{D}\to\infty}\frac{\Pi_R^*+\Pi_M^*}{\overline{D}}-\lim_{\lambda\to\infty}\lim_{\overline{D}\to\infty}\frac{\Pi_d^*}{\overline{D}}=&(v-c)\left(1-\sqrt{\frac{\frac{1}{2}v+\frac{1}{2}c-s}{v-s}}-\frac{\frac{1}{8}(v-c)}{\frac{1}{2}v+\frac{1}{2}c-s}\right)\\=&(v-c)\left(\frac{\frac{3}{8}v+\frac{5}{8}c-s}{\frac{1}{2}v+\frac{1}{2}c-s}-\sqrt{\frac{\frac{1}{2}v+\frac{1}{2}c-s}{v-s}}\right).\end{aligned}$$

Let

$$y(v)=\left(\frac{3}{8}v+\frac{5}{8}c-s\right)^2(v-s)-\left(\frac{1}{2}v+\frac{1}{2}c-s\right)^3.$$

Then, we have

$$\begin{aligned}\frac{\partial y(v)}{\partial v}&=\left(\frac{3}{8}v+\frac{5}{8}c-s\right)\left(\frac{9}{8}v+\frac{5}{8}c-\frac{7}{4}s\right)-\frac{3}{2}\left(\frac{1}{2}v+\frac{1}{2}c-s\right)^2;\\\frac{\partial^2 y(v)}{\partial v^2}&=\frac{3}{32}v+\frac{3}{16}c-\frac{9}{32}s>\frac{9}{32}(c-s)>0.\end{aligned}$$

That implies that the first-order derivative $\partial y(v)/\partial v$ increases with v. Since

$$\frac{\partial y(v)}{\partial v}>(c-s)\left(\frac{7}{4}c-\frac{7}{4}s\right)-\frac{3}{2}(c-s)^2>0,$$

we have $y(v)$ increases with v and

$$y(v)>(c-s)^3-(c-s)^3>0.$$

This immediately leads to

$$(v-c)\left(\frac{\frac{3}{8}v+\frac{5}{8}c-s}{\frac{1}{2}v+\frac{1}{2}c-s}-\sqrt{\frac{\frac{1}{2}v+\frac{1}{2}c-s}{v-s}}\right)>0.$$

Thus, we have

$$\lim_{\lambda\to\infty}\lim_{\overline{D}\to\infty}\frac{\Pi_R^*+\Pi_M^*}{\overline{D}}>\lim_{\lambda\to\infty}\lim_{\overline{D}\to\infty}\frac{\Pi_d^*}{\overline{D}}.$$

By Intermediate Value Theorem, when λ and $\overline{D}$ are sufficiently large, the distribution channel as a whole can be benefited if the manufacturer sells through an intermediary retailer.

Proof of Lemma 8:

When $\beta\to 1$, the current model reduces to the baseline model. By Proposition 3, the manufacturer can be better off by selling through an intermediary retailer if λ is sufficiently large. On the other hand, when $\beta\to 0$, the current model reduces to the one where consumers only care about the manufacturer's profit. By Lemma 6, the manufacturer is worse off by the downward distribution channel extension. Note that the manufacturer's profit hinges upon the consumers' utility function and must be a continuous function of β. Thus, by Intermediate Value Theorem, when the value of β is large enough, the current case approaches the baseline model and the manufacturer's profit under agent selling can be higher than that under direct selling.

Proof of Proposition 5:

Given the wholesale prices $\{\omega_i\}$ and retail prices $\{p_{ai}\}$, $i = 1, 2...n$, the fairness-minded consumers decide whether to buy the product and from which retailer to buy. The consumers cannot directly observe the wholesale price ω_i because it is a private contract between retailer i and the manufacturer. Nevertheless, they can make the inference about this wholesale price which is denoted by ε_{ω_i} and make the corresponding purchase decisions. A transaction with retailer i occurs if and only if the fairness-sensitive consumers can enjoy a non-negative utility, i.e.,

$$U_i = v - p_{ai} - \lambda \cdot \max\{(p_{ai} - \varepsilon_{\omega_i}) - (v - p_{ai}), 0\} \geq 0,$$

and this utility is higher than those obtained from other retailers, i.e.,

$$U_i \geq U_j, \ where\ j \neq i, i, j = 1, 2...n.$$

We note that manufacturer may be prohibited from charging different wholesale prices to different retailers. Hence, we first consider the case in which the manufacturer is forced to charge the retailers a uniform wholesale price, i.e., $\omega_i = \omega_j$ for any $i \neq j$, and then relax this constraint by considering the retailers may face different wholesale prices.

Case 1: The retailers face a uniform wholesale price

We first show that given a uniform wholesale price, consumers will be indifferent to which retailer to buy from. This can be proved by contradiction. Suppose that the consumers buy from only a set of retailers, ϕ, who provide them the highest level of utility. Then, the retailers in the complementary set $\bar{\phi}$ do not have any sales. Consequently, these retailers in $\bar{\phi}$ will strategically adjust their retail prices to provide at least the same utility level as that provided by other retailers in ϕ so as to attract some consumers. If the retailers in the complementary set $\bar{\phi}$ provide an even higher utility level to consumers, the retailers in ϕ will also strategically match this consumer utility level. Such actions repeat themselves until the equilibrium is obtained. In equilibrium, all retailers charge the same retail price, share the sales equally and make the same level of profit.

Next, we examine the stability of the equilibrium. Suppose that some retailers may further decrease their prices by *one cent* to provide a higher utility level and capture the entire market demand. However, the other retailers can also respond by cutting the retail prices to match the utility level provided by them. That is, the retailers are always able to match the utility level provided by the others. From this aspect, the retailers will always share the market equally and the equilibrium is stable: the retailers will set profit-maximizing retail prices and will not unilaterally deviate.

As a result, $(A+D)/n$ consumers choose to buy the products from retailer i, $i=1,2,..n$. Moreover, the consumer utility will be definitely extracted by the profit-maximizing retailers to a level as low as zero. Specifically, $U_i(r_i)=v-r_i-\lambda\cdot\max\{(r_i-\varepsilon_{\omega_i})-(v-r_i),0\}=0$ must hold at the reservation price r_i which leads to the reservation price

$$r_i=\frac{(1+\lambda)v+\lambda\varepsilon_{\omega_i}}{1+2\lambda}=xv+y\varepsilon_{\omega_i}.$$

Similar to that in the baseline model, retailer i forms a belief ε_{r_i} over the fairness-minded consumers' reservation price r_i and sets the retail price satisfying $p_{ai}\le\varepsilon_{r_i}$. Thus, recalling the rational expectations equilibrium concept, the optimal selling price becomes

$$p^*_{ai}=r_i=\frac{(1+\lambda)v+\lambda\omega_i}{1+2\lambda}=xv+y\omega_i.$$

Then, the profit function of retailer i becomes

$$\Pi_{Ri}(Q_i;\omega_i)=\frac{A}{n}(p^*_{ai}-\omega_i)+\mathsf{E}\left[p^*_{ai}\min\left(\frac{D}{n},Q_i\right)+s\left(Q_i-\frac{D}{n}\right)^+-\omega_iQ_i\right].$$

Hence, the optimal ordering quantity from retailer i is

$$Q^*_{ai}=\frac{A}{n}+Q^*_i=\frac{A}{n}+\frac{x(v-\omega_i)}{xv+y\omega_i-s}\frac{\overline{D}}{n}.$$

The manufacturer holds a belief over the retailers' ordering quantities denoted by ε_{Q_i}. With fully anticipating the retailers' responses in a rational expectations equilibrium (i.e., $\varepsilon_{Q_i}=Q_i(\omega_i)$, $Q_{ai}(\omega_i)=A/n+Q_i(\omega_i)$), the manufacturer decides on the wholesale price ω_i by maximizing his profit function as follows:

$$\Pi_M(\omega_i)=\sum_{i=1}^{n}(\omega_i-c)\left(\frac{A}{n}+\varepsilon_{Q_i}\right).$$

Note that in this case, the equilibrium must be symmetric: both the wholesale price and the retail price are not varied across the retailers, which are denoted as ω and p_a, respectively. Then, the demand will be equally divided among the retailers, which equals $D_\varepsilon=(A+D)/n$, and the ordering quantity for each retailer will be the same, denoted by Q_a. Besides, the retailer's problem under agent selling is equivalent to the manufacturer's problem under direct selling except that the marginal cost is replaced by the wholesale price ω. Then, similar to the baseline model, in the rational expectations equilibrium, each retailer's optimal decisions must be as follows:

$$\begin{aligned}
p^*_a(\omega)=&\frac{(1+\lambda)v+\lambda\omega}{1+2\lambda};\\
Q^*(\omega)=&\frac{p^*_a-\omega}{p^*_a-s}\frac{\overline{D}}{n}=\frac{(1+\lambda)(v-\omega)}{(1+\lambda)v+\lambda\omega-(1+2\lambda)s}\frac{\overline{D}}{n};\\
Q^*_a(\omega)=&\frac{A}{n}+Q^*=\frac{A}{n}+\frac{(1+\lambda)(v-\omega)}{(1+\lambda)v+\lambda\omega-(1+2\lambda)s}\frac{\overline{D}}{n}=\frac{A}{n}+\frac{x(v-\omega)}{xv+y\omega-s}\frac{\overline{D}}{n}.
\end{aligned}$$

Consequently, the manufacturer decides the wholesale price ω to maximize his profit as follows:

$$\begin{aligned}\Pi_M(\omega) =& n(\omega-c)(\frac{A}{n}+Q^*(\omega))\\ =&(\omega-c)\left[A+\frac{(1+\lambda)(v-\omega)}{(1+\lambda)v+\lambda\omega-(1+2\lambda)s}\overline{D}\right]\\ =&(\omega-c)\left[A+\frac{x(v-\omega)}{y\omega+xv-s}\overline{D}\right],\end{aligned}$$

which is independent of n and the same as that in the baseline model. Thus, the optimal wholesale price and the corresponding manufacturer's profit are the same as those stated in Proposition 2. Then, according to Proposition 3, compared with direct selling, the manufacturer under agent selling can still be better off by relying on multiple retailers to distribute his products.

Case 2: The manufacturer may set different wholesale prices toward different retailers

We first show that if the manufacturer charges different retailers different wholesale prices, the following result is still valid: consumers will be indifferent to which retailer to buy from. This can be proved by contradiction. Suppose that the consumers buy from only a set of retailers, ϕ, who provide them the highest level of utility. Then, the retailers in the complementary set $\bar{\phi}$ do not have any sales. Consequently, these retailers in $\bar{\phi}$ will strategically adjust their retail prices to provide at least the same utility level as that provided by other retailers in ϕ so as to attract some consumers. If the retailers in the complementary set $\bar{\phi}$ provide an even higher utility level to consumers, the retailers in ϕ will also strategically match this consumer utility level. Such actions repeat themselves until the equilibrium is obtained.

We note that the retail prices across consumers may be different. Retailers facing higher wholesale prices will set higher retail prices, which however can lead to the same level of consumer utility. The reason is as follows. The retailers in the set with the lowest wholesale price (say ϕ') may further decrease their prices by *one cent* to provide a higher utility level and capture the entire market demand. However, the retailers in the complementary set, $\bar{\phi}'$, can also respond by cutting the retail prices to match the utility level provided by the retailers in ϕ'. It is true that the retailers in $\bar{\phi}'$ need to pay higher wholesale prices and may not have enough space to cut retail prices compared with the retailers in ϕ'. However, consumers are fairness-concerned and they are willing to pay higher retail prices to buy from the retailers in $\bar{\phi}'$. Thus, the retailers in $\bar{\phi}'$ are able to match the utility level provided by the retailers in ϕ'. From this aspect, the retailers will always share the market equally and the equilibrium is stable: the retailers will set profit-maximizing retail prices and will not unilaterally deviate.

As a result, $(A+D)/n$ consumers choose to buy the products from retailer i, $i=1,2,..n$. Moreover, the consumer utility will be definitely extracted by the profit-maximizing retailers to a level as low as zero. We now show that the equilibrium must be symmetric: the manufacturer charges the same wholesale price to all the retailers even though he is not restricted to do so, and the retailers set the same retail price and ordering quantity. This can also be proved by contradiction. Suppose that the manufacturer charges different wholesale prices across the retailers and thus earns different levels of profits from each of them. Let a set of retailers, ϕ'', provide the manufacturer the highest level of profit. Then, the manufacturer makes less profit from other retailers that are in the complementary set, $\bar{\phi}''$. For a profit-maximizing manufacturer, he can easily adjust the wholesale prices for those retailers in $\bar{\phi}''$ so that he earns the same and highest profit from them as well. Hence, in equilibrium, the manufacturer will charge a unified wholesale price for all the retailers even though he is not restricted to do so. Then, all the retailers make the same decisions about the retail price and the ordering quantity, which gives the manufacturer the same level of profit. Consequently, the equilibrium outcome under this case shall be the same as that in Case 1. That is, the optimal wholesale price and the manufacturer's profit are the same as those stated in Proposition 2, and compared with direct selling, the manufacturer under agent selling can still be better off by relying on multiple retailers to distribute his products.

Proof of Proposition 6:

Under the multichannel encroachment, given the wholesale prices $\{\omega_i\}$, the retail prices $\{p_{ai}\}$ and p_d, $i=1,2...n$, the fairness-minded consumers decide whether to buy the product and from whom (the retailer or the manufacturer) to buy. The consumers cannot directly observe the wholesale price ω_i for retailer i because it is a private contract between retailer i and the manufacturer. Nevertheless, they can make inferences about the wholesale price which is denoted by ε_{ω_i} and make the corresponding purchase decisions. Specifically, the consumer utility from a transaction with the manufacturer is

$$U_m = v - p_d - \lambda \cdot \max\{(p_d - c) - (v - p_d), 0\},$$

and that with retailer i is

$$U_i = v - p_{ai} - \lambda \cdot \max\{(p_{ai} - \varepsilon_{\omega_i}) - (v - p_{ai}), 0\}.$$

The consumer buys from the manufacturer if and only if his or her utility is non-negative and higher than that obtained from the retailers, i.e.,

$$U_m \geq 0 \; and \; U_m \geq U_i, \; i = 1, 2, ..., n.$$

Similarly, the consumer buys from retailer i if and only if his or her utility is non-negative and higher than that obtained from the manufacturer and other retailers, i.e.,

$$U_i \geq 0, \;\; U_i \geq U_m \; and \; U_i \geq U_j, \; where \; j \neq i, \; i, j = 1, 2, ..., n.$$

We note that, given the wholesale prices $\{\omega_i\}$, the sellers' problem (the manufacturer and the retailers) is the same as that in Proposition 5 except that the manufacturer's cost is exactly the marginal production cost c. We also note that regulations and laws may prohibit manufacturers from charging differentiated wholesale prices to different retailers. When a unified wholesale price is set across all the retailers, it becomes a special case of our analysis.

Specifically, we will first show that consumers are indifferent to which firm– the manufacturer or any of the retailers– to buy from no matter whether the wholesale prices are uniform or varied across retailers. This can be proved by contradiction. Suppose that the consumers buy from only a set of sellers (the manufacturer or the retailers), ϕ, who provide them the highest level of utility. Then, the sellers in the complementary set $\bar{\phi}$ do not have any sales. Consequently, these sellers in $\bar{\phi}$ will strategically adjust their selling prices to provide at least the same utility level as that provided by other sellers in ϕ so as to attract some consumers. If the sellers in the complementary set $\bar{\phi}$ provide an even higher utility level to consumers, the sellers in ϕ will also strategically match this consumer utility level. Such actions repeat themselves until the equilibrium is obtained. In equilibrium, all sellers share the sales equally.

We also note that the selling prices across consumers may be different. Retailers facing higher wholesale prices will set higher retail prices, which however lead to the same level of consumer utility. The reason is as follows. The sellers in the set with the lowest wholesale price (say ϕ') may further decrease their prices by *one cent* to provide a higher utility level and capture the entire market demand. However, the sellers in the complementary set, $\bar{\phi}'$, can also respond by cutting the retail prices to match the utility level provided by the sellers in ϕ'. It is true that the sellers in $\bar{\phi}'$ need to pay higher wholesale prices and may not have enough space to cut selling prices compared with the sellers in ϕ'. However, consumers are fairness-concerned and they are willing to pay higher retail prices to buy from the sellers in $\bar{\phi}'$. Thus, the sellers in $\bar{\phi}'$ are able to match the utility level provided by the sellers in ϕ'. From this aspect, the sellers will always share the market equally and the equilibrium is stable: the sellers will set profit-maximizing retail prices and will not unilaterally deviate.

As a result, $(A+D)/(n+1)$ consumers choose to buy the products from the manufacturer and retailer i, $i=1,2,..n$, repsectively. Moreover, the consumer utility will be definitely extracted by the profit-maximizing sellers to a level as low as zero. Specifically, the consumer utility with regard to retailer i, $U_i(r_i)=v-r_i-\lambda\cdot\max\{(r_i-\varepsilon_{\omega_i})-(v-r_i),0\}=0$ must hold at the reservation price r_i, which leads to

$$r_i=\frac{(1+\lambda)v+\lambda\varepsilon_{\omega_i}}{1+2\lambda}=xv+y\varepsilon_{\omega_i}.$$

Similar to that in the baseline model, retailer i forms a belief ε_{r_i} over the fairness-minded consumers' reservation price r_i, and sets the retail price satisfying $p_{ai}\leq\varepsilon_{r_i}$. Thus, recalling the rational expectations equilibrium concept, the optimal selling price becomes

$$p^*_{ai}=r_i=\frac{(1+\lambda)v+\lambda\omega_i}{1+2\lambda}=xv+y\omega_i.$$

Similarly, the consumer utility with regard to the manufacturer, $U_m = v - p_d - \lambda \cdot \max\{(p_d - c) - (v - p_d), 0\} = 0$ must hold at the reservation price r_m which leads to

$$r_m = \frac{(1+\lambda)v + \lambda c}{1+2\lambda} = xv + yc.$$

Thus, the corresponding optimal selling price is

$$p_m^* = r_m = \frac{(1+\lambda)v + \lambda c}{1+2\lambda} = xv + yc.$$

Recall that $(A+D)/(n+1)$ consumers choose to buy the products from each seller (both the manufacturer and n retailers). To examine the manufacturer's total profit, we first consider his payoff by selling directly to $(A+D)/(n+1)$ consumers. By Proposition 1, we have

$$\Pi_M^{d*} = \frac{1}{n+1}\left[x(v-c)A + \frac{x^2(v-c)^2}{2(xv+yc-s)}\overline{D}\right],$$

which equals $1/(n+1)$ portion of the manufacturer's profit under direct selling.

Next, we consider the manufacturer's payoff by selling through n retailers. Each retailer has $(A+D)/(n+1)$ consumers to buy from her. Then, similar to the proof of Proposition 5, we can show that in equilibrium, the manufacturer charges the same wholesale price across all the retailers and the retailers charge the same retail price. And in the rational expectations equilibrium, each retailer's optimal decisions is as follows:

$$\begin{aligned}
p_a^*(\omega) =& \frac{(1+\lambda)v + \lambda\omega}{1+2\lambda};\\
Q^*(\omega) =& \frac{p_a^* - \omega}{p_a^* - s}\frac{\overline{D}}{n+1} = \frac{(1+\lambda)(v-\omega)}{(1+\lambda)v + \lambda\omega - (1+2\lambda)s}\frac{\overline{D}}{n+1};\\
Q_a^*(\omega) =& \frac{A}{n+1} + Q^* = \frac{A}{n+1} + \frac{(1+\lambda)(v-\omega)}{(1+\lambda)v + \lambda\omega - (1+2\lambda)s}\frac{\overline{D}}{n+1} = \frac{A}{n+1} + \frac{x(v-\omega)}{xv + y\omega - s}\frac{\overline{D}}{n+1}.
\end{aligned}$$

Consequently, the manufacturer decides the wholesale price ω to maximize

$$\begin{aligned}
\Pi_M^a(\omega) =& n(\omega - c)\left(\frac{A}{n+1} + Q^*(\omega)\right)\\
=& \frac{n}{n+1}(\omega - c)\left[A + \frac{(1+\lambda)(v-\omega)}{(1+\lambda)v + \lambda\omega - (1+2\lambda)s}\overline{D}\right]\\
=& \frac{n}{n+1}(\omega - c)\left[A + \frac{x(v-\omega)}{y\omega + xv - s}\overline{D}\right],
\end{aligned}$$

which is similar to that under the baseline agent-selling setting. Then, similar to that stated in Proposition 2, it can be easily shown that the manufacturer's expected profit from selling through n retailers is

$$\Pi_M^{a*} = \begin{cases} \frac{n}{n+1}\frac{1}{y^2}\left(\sqrt{(xv+yc-s)(x\overline{D} - yA)} - \sqrt{(v-s)x\overline{D}}\right)^2, & if\ \overline{D} > \frac{(v-s)A}{(v-c)x};\\ \frac{n}{n+1}A(v-c), & otherwise, \end{cases}$$

which equals $n/(n+1)$ portion of the manufacturer's profit under agent selling. Thus, under multichannel encroachment, the manufacturer's total profit becomes

$$\Pi_M^* = \Pi_M^{d*} + \Pi_M^{a*},$$

which is a weighted average of the manufacturer's profits under direct selling and agent selling with the corresponding weights $1/(n+1)$ and $n/(n+1)$, respectively. Hence, the manufacturer's profit under multichannel encroachment cannot be higher than that under either direct selling or agent selling. Specifically, according to Proposition 3, when $0 < \overline{D} < \widehat{\overline{D}}$, if $\lambda > \widehat{\lambda}$, agent selling outperforms direct selling and hence the multichannel encroachment is dominated by agent selling. By contrast, if $\lambda \leq \widehat{\lambda}$, direct selling outperforms agent selling and then the multichannel encroachment is dominated by direct selling.

Proof of Proposition 7:

Bearing in mind that the acceptable price is lower for fairness-sensitive consumers, the manufacturer will set a low price if he targets all consumers but a high price if he targets only consumers without fairness concern. We examine the manufacturer's optimal pricing decisions by comparing the following two cases.

Case 1 (charging a low price and selling to all): Given the price p_{dL}, the consumers decides whether to buy the product or not. A transaction occurs if and only if the fairness-sensitive consumers can enjoy a non-negative utility, i.e.,

$$U = v - p_{dL} - \lambda \cdot \max\{(p_{dL} - c) - (v - p_{dL}), 0\} \geq 0.$$

Specifically, $U(r) = v - r - \lambda \cdot \max\{(r - c) - (v - r), 0\} = 0$ holds at the reservation price r. For the manufacturer, the price p_{dL} is subject to $p_{dL} \leq \varepsilon_r$, where ε_r is the manufacturer's belief on the fairness-minded consumers' reservation price r. So the optimal selling price should be

$$p_{dL}^* = r = \frac{(1+\lambda)v + \lambda c}{1 + 2\lambda} = xv + yc.$$

Then, all A loyal consumers will buy the products. The manufacturer optimally plans for the production quantity $Q_{dL} = A + Q_L$ to maximize his profit:

$$\Pi_{dL}(Q_L) = A(p_{dL}^* - c) + \mathsf{E}[p_{dL}^* \min(D, Q_L) + s(Q_L - D)^+ - cQ_L].$$

Sequentially, by backward induction, the optimal stocking quantity and corresponding profit of the manufacturer can be derived as

$$Q_L^* = \frac{x(v-c)\overline{D}}{xv + yc - s}; \ \Pi_{dL}^* = Ax(v-c) + \frac{\overline{D}}{2} \frac{x^2(v-c)^2}{xv + yc - s}.$$

Case 2 (charging a high price and selling to consumers without fairness concern only): Given the price p_{dH}, the consumers without fairness concern will trade with the manufacturer if and only if their utility is non-negative, i.e.,

$$U = v - p_{dH} \geq 0.$$

Specifically, at the reservation price r, $U(r) = v - r = 0$, i.e., $r = v$. The manufacturer only cares about the consumers without fairness concern so that he will set a high price p_{dH}, subject to $p_{dH} \leq \varepsilon_r$, where ε_r is the manufacturer's belief on the reservation price of the consumers without fairness concern. So the optimal selling price is set at

$$p_{dH}^* = r = v.$$

Then, all $(1-\alpha)A$ loyal and "no-fairness-concern" consumers will buy the product. The manufacturer then decides the production quantity $Q_{dH} = (1-\alpha)A + Q_H$ to maximize his profit:

$$\Pi_{dH}(Q_H) = (1-\alpha)A(p_{dH}^* - c) + \mathsf{E}[p_{dH}^* \min((1-\alpha)D, Q_H) + s(Q_H - (1-\alpha)D)^+ - cQ_H].$$

Sequentially, the optimal stocking quantity and the corresponding profit of the manufacturer can be derived as

$$Q_H^* = \frac{v-c}{v-s}(1-\alpha)\overline{D};\ \Pi_{dH}^* = (1-\alpha)A(v-c) + \frac{\overline{D}}{2}\frac{(1-\alpha)(v-c)^2}{v-s}.$$

Then, $\Pi_{dL}^* > \Pi_{dH}^*$ requires

$$Ax(v-c) + \frac{\overline{D}}{2}\frac{x^2(v-c)^2}{xv+yc-s} > (1-\alpha)A(v-c) + \frac{\overline{D}}{2}\frac{(1-\alpha)(v-c)^2}{v-s}.$$

That is,

$$\alpha > 1 - \frac{x\,(v-s)\left(2\,(cy+vx-s)\,A + (v-c)\,x\overline{D}\right)}{(cy-s+vx)\left(2\,(v-s)\,A + (v-c)\,\overline{D}\right)} \equiv \hat{\alpha}_d.$$

Thus, $\Pi_{dL}^* > \Pi_{dH}^*$ when $\alpha > \hat{\alpha}_d$ and $\Pi_{dL}^* \leq \Pi_{dH}^*$ otherwise.

Proof of Proposition 8:

The proof of the existence of the rational expectations equilibrium shares the same logic and intuition as that of the proof of Proposition 2. Thus, we omit the details for expositional brevity. Similar to that under direct selling, the retailer will set a low price if she targets all consumers but a high price if she targets only consumers without fairness concern. We then examine the following two cases.

Case 1 (charging a low price and selling to all): Given p_{aL}, the consumers decide to buy or not. The consumers cannot directly observe the wholesale price ω_L because it is a private contract between the retailer and the manufacturer. Nevertheless, they can make inferences about the wholesale price which is denoted by ε_{ω_L}. A transaction occurs if and only if the fairness-minded consumers can enjoy a non-negative utility, i.e.,

$$U = v - p_{aL} - \lambda \cdot \max\{(p_{aL} - \varepsilon_{\omega_L}) - (v - p_{aL}), 0\} \geq 0.$$

Specifically, $U(r) = v - r - \lambda \cdot \max\{(r - \varepsilon_{\omega_L}) - (v - r), 0\} = 0$ at the reservation price r. Then, the reservation price becomes

$$r = xv + y\varepsilon_\omega.$$

Similar to the direct-selling case, the retailer forms a belief ε_r over the fairness-minded consumers' reservation price r, and sets the retail price satisfying $p_{aL} \leq \varepsilon_r$. Thus, recalling the rational expectations equilibrium concept, the optimal selling price becomes

$$p^*_{aL} = r = xv + y\omega_L.$$

As a result, all A loyal consumers choose to buy the products. Then, the retailer's profit function becomes

$$\Pi_{RL}(Q_L; \omega_L) = A(p^*_{aL} - \omega_L) + \mathsf{E}[p^*_{aL} \min(D, Q_L) + s(Q_L - D)^+ - \omega_L Q_L].$$

Hence, the optimal ordering quantity from the retailer is

$$Q^*_{aL} = A + \frac{x(v - \omega_L)}{xv + y\omega_L - s}\overline{D}.$$

The manufacturer holds a belief over the retailer's ordering quantity denoted by ε_Q. With fully anticipating the retailer's response in a rational expectations equilibrium (i.e., $\varepsilon_Q = Q_L(\omega_L)$, $Q_{aL}(\omega_L) = A + Q_L(\omega_L)$), the manufacturer decides on the wholesale price ω_L by maximizing his profit

$$\Pi_{ML}(\omega_L) = (\omega_L - c)(A + \varepsilon_Q).$$

By taking the rational expectations equilibrium, the optimal wholesale price and the corresponding profit of the manufacturer can be derived as

$$\omega^*_L = \begin{cases} \frac{1}{y}\left(\sqrt{\frac{(v-s)(xv+yc-s)x\overline{D}}{x\overline{D}-yA}} - (xv - s)\right), & if\ \overline{D} > \frac{(v-s)A}{(v-c)x}; \\ v, & otherwise. \end{cases}$$

$$\Pi^*_{ML} = \begin{cases} \frac{1}{y^2}\left(\sqrt{(xv + yc - s)(x\overline{D} - yA)} - \sqrt{(v - s)x\overline{D}}\right)^2, & if\ \overline{D} > \frac{(v-s)A}{(v-c)x}; \\ A(v - c), & otherwise. \end{cases}$$

Case 2 (charging a high price and selling to consumers without fairness concern only): Now only the $1 - \alpha$ proportion of consumers that have no fairness concern are involved in transactions. A transaction occurs if and only if the consumers without fairness concern can enjoy a non-negative utility, i.e.,

$$U = v - p_{aH} \geq 0.$$

Specifically, $U(r) = v - r = 0$ at the reservation price r. Then, the reservation price becomes $r = v$.

Similar to the direct-selling case, the retailer forms a belief ε_r over the "no-fairness-concern" consumers' reservation price r, and sets the retail price satisfying $p_{aH} \leq \varepsilon_r$. Thus, recalling the rational expectations equilibrium concept, the optimal selling price becomes

$$p^*_{aH} = r = v.$$

As a result, all $(1-\alpha)A$ loyal consumers choose to buy the products. Then, the retailer's profit function becomes

$$\Pi_{RH}(Q_H; \omega_H) = (1-\alpha)A(p^*_{aH} - \omega_H) + \mathsf{E}[p^*_{aH} \min((1-\alpha)D, Q_H) + s(Q_H - (1-\alpha)D)^+ - \omega_H Q_H].$$

Hence, the retailer's optimal ordering quantity is

$$Q^*_{aH} = (1-\alpha)A + \frac{v - \omega_H}{v - s}(1-\alpha)\overline{D}.$$

The manufacturer holds a belief over the retailer's ordering quantity denoted by ε_Q. With fully anticipating the retailer's response in a rational expectations equilibrium (i.e., $\varepsilon_Q = Q_H(\omega_H)$, $Q_{aH}(\omega_H) = A + Q_H(\omega_H)$), the manufacturer decides on the wholesale price ω_H by maximizing his profit

$$\Pi_{MH}(\omega_H) = (\omega_H - c)((1-\alpha)A + \varepsilon_Q).$$

By taking the rational expectations equilibrium, the optimal wholesale price and the corresponding profit of the manufacturer can be derived as

$$\begin{aligned}
\omega^*_H &= \begin{cases} \frac{A(v-s)+(v-c)\overline{D}}{2\overline{D}} + c, \; if \;\overline{D} > \frac{(v-s)A}{v-c}; \\ v, \qquad\qquad\qquad\quad otherwise. \end{cases} \\
\Pi^*_{MH} &= \begin{cases} \frac{(1-\alpha)\left(A(v-s)+(v-c)\overline{D}\right)^2}{4\overline{D}(v-s)}, \; if \;\overline{D} > \frac{(v-s)A}{v-c}; \\ A(v-c)(1-\alpha), \qquad\quad otherwise. \end{cases}
\end{aligned}$$

For conditions leading to $\Pi^*_{ML} > \Pi^*_{MH}$, since $(v-s)A/((v-c)x) > (v-s)A/(v-c)$, we have the following three cases:

(i.) When $\overline{D} > (v-s)A/((v-c)x)$, $\Pi^*_{ML} > \Pi^*_{MH}$ if and only if

$$\frac{1}{y^2}\left(\sqrt{(v-s)x\overline{D}} - \sqrt{(xv+yc-s)(x\overline{D}-yA)}\right)^2 > \frac{(1-\alpha)\left(A(v-s)+(v-c)\overline{D}\right)^2}{4\overline{D}(v-s)},$$

which requires

$$\alpha > 1 - \frac{4\overline{D}(v-s)}{y^2}\frac{\left(\sqrt{(v-s)x\overline{D}} - \sqrt{(xv+yc-s)(x\overline{D}-yA)}\right)^2}{\left(A(v-s)+(v-c)\overline{D}\right)^2}.$$

(ii.) When $(v-s)A/(v-c) < \overline{D} \leq (v-s)A/((v-c)x)$, $\Pi^*_{ML} > \Pi^*_{MH}$ if and only if

$$A(v-c) > \frac{(1-\alpha)\left(A(v-s)+(v-c)\overline{D}\right)^2}{4\overline{D}(v-s)},$$

which requires

$$\alpha > 1 - \frac{4\overline{D}A(v-c)(v-s)}{\left(A(v-s)+(v-c)\overline{D}\right)^2}.$$

(iii.) When $\overline{D} \leq (v-s)A/(v-c)$, $\Pi^*_{ML} > \Pi^*_{MH}$ for any $\alpha > 0$ as $A(v-c)(1-\alpha) < A(v-c)$.

To summarize, we have $\Pi^*_{ML} > \Pi^*_{MH}$ if and only if $\alpha > \hat{\alpha}_a$, where

$$\hat{\alpha}_a = \begin{cases} 1 - \frac{4\overline{D}(v-s)}{y^2}\frac{\left(\sqrt{(v-s)x\overline{D}}-\sqrt{(xv+yc-s)(x\overline{D}-yA)}\right)^2}{\left(A(v-s)+(v-c)\overline{D}\right)^2}, & if\ \overline{D} > \frac{(v-s)A}{(v-c)x}; \\ 1 - \frac{4\overline{D}A(v-c)(v-s)}{\left(A(v-s)+(v-c)\overline{D}\right)^2}, & if\ \frac{(v-s)A}{v-c} < \overline{D} \leq \frac{(v-s)A}{(v-c)x}; \\ 0, & otherwise. \end{cases}$$

Proof of Lemma 9:

The proof of (i) goes back to the proof of Proposition 3: there exists a unique $\widehat{\lambda}$ such that agent selling generates a higher profit than direct selling if and only if $\lambda > \widehat{\lambda}$ when $0 < \overline{D} < \widehat{\overline{D}}$.

Now we show the proof of (ii). In this case, we can show that

$$\begin{aligned} \Pi^*_{dH} - \Pi^*_{MH} &= (1-\alpha)A(v-c) + \frac{\overline{D}}{2}\frac{(1-\alpha)(v-c)^2}{v-s} - \frac{(1-\alpha)\left((v-s)A+(v-c)\overline{D}\right)^2}{4\overline{D}(v-s)} \\ &= \frac{(1-\alpha)}{4\overline{D}(v-s)}\left((c-v)^2\overline{D}^2 + 2A(s-v)(c-v)\overline{D} - A^2(s-v)^2\right) \\ &> \frac{(1-\alpha)}{4\overline{D}(v-s)}\left((c-v)^2\overline{D}^2 + 2(v-s)^2A^2 - A^2(s-v)^2\right) \\ &> 0. \end{aligned}$$