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AMWRPN: Ambiguity Measure Weighted Risk Priority Number Model for Failure Mode and Effects Analysis

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ABSTRACT The relative importance of each risk factor in failure mode and effects analysis (FMEA) should be addressed properly. Intuitively, in the assessments coming from the FEMA experts, there exists a potential judgement on which risk factor has a higher weight for the FMEA item. Based on this cognition and perspective, a new ambiguity measure weighted risk priority number (AMWRPN) for FMEA is proposed. AMWRPN takes into consideration of the relative weight of different risk factors by measuring the ambiguity degree of the experts' assessments. If the assessment of an expert has a certain belief on the judgement, then the relative importance of the corresponding risk factor will be higher than the uncertain one; and vice versa. The ambiguity measure (AM) in the framework of the Dempster–Shafer evidence theory (DST) has been used to construct the exponential weight of each risk factor in AMWRPN. In comparison with the weight factor basing on fuzzy sets theory or other theories in the DST framework, the AM-based weight factor for uncertainty measure of the subjective assessment ensures the internal coordination of the proposed method. An application of the proposed method in aircraft turbine rotor blade verifies the effectiveness of the new risk priority number model.

INDEX TERMS Failure mode and effects analysis (FMEA), risk priority number (RPN), Dempster-Shafer evidence theory (DST), ambiguity measure.

I. INTRODUCTION

As a typical tool of potential risk modelling and management, failure mode and effects analysis (FMEA) is widely used in practical applications such as medical treatment [36], [62], nuclear industry [54], software engineering [39] and so on [21]. In general, five typical steps are included in FMEA process, including (1) identifying a FMEA team, (2) defining the scopes and customers of a FMEA process, (3) identifying potential failure modes and effects, (4) assessing FMEA items by ranking and (5) making recommendations on the action of each potential failure modes [25]. Among the processes of FMEA approach, the subjective assessments on the ranking of risk levels and the classical risk priority number (RPN) model are sometimes not that efficient for a variant range of practical applications [18], [19], [29].

The subjective assessments information in FMEA processes can be regarded as uncertain information in practical applications. Thus, it should be modelled reasonably by a proper method for the following procedures of uncertain information processing. In the previous researches, apart from FMEA processes, uncertain information should be well addressed in practical applications such as logistics and supply chain networks [2], [20], reliability analysis and assessment [13], [38], [65], [66], [68], pattern recognitions [32], [71], clinical diagnosis [56], control strategy designing [3], [50] and so on [15], [37], [48]. Many theories and methods have been used to deal with uncertainty in information processing, including probability theory [11], [41], information entropy [43], [52], fuzzy sets theory [1], [40], [64], Dempster-Shafer evidence theory (DST) [5], [42], [44], belief functions [30], [31], [72], random sets theory [33], [55], rough sets theory [17], [23] and so on [12], [34], [51], [67]. Among these methods, DST is widely used in uncertain information processing, especially for uncertain information fusion [9], [35], [49], [53]. However, how to measure the uncertain degree in the framework of DST while doing information fusion is an open issue. Many measures for uncertainty modelling and quantification have been proposed, including specificity measure [10], [61], the measure of aggregate uncertainty (AU) [16], the ambiguity measure (AM) [22], Deng entropy [7], the interval probability-based uncertainty measure [58], the improved belief entropy [70], and so on [8], [24], [46]. All these measures have some advantages according to previous researches [7], [16], [22], [58]. However, some properties of the aforementioned measures are still need further research. For example, the limitation of some typical uncertainty measures in DST framework is solved by [7], while the shortcoming of the measure in [7] is discussed in [70]. It should be noted that, in comparison with other uncertainty measures, AM has some advantages. AM satisfies all the five requirements for AU measures including probability consistency, set consistency, the range of AU, subadditivity and additivity. In addition, AM allows for sensitivity changes in evidence and distinguishes discord and non-specificity better than AU and some other uncertainty measures. In this paper, AM will be adopted to measure the uncertain degree of experts' subjective assessment in the DST framework.

Many researches focus on applying the methods of uncertain information processing to improve the efficiency of classical RPN models in FMEA process [4], [25], [27], [28], [57]. In [47], [63], [69], Dempster-Shafer evidence theory (DST) is modified and adopted to aggregate the subjective assessments of the FMEA items coming from multiple experts in a FMEA team. The mean value of RPN (MVRPN) is proposed as a new RPN in [63], where the Dempster's combination rule in DST is modified to fuse the belief structure of FMEA experts' uncertain assessments. In [47], the MVRPN method is improved by adding a new process of generating basis probability assignment (BPA) to improve the conflict information fusion method in [63]. The concept of generalized combination rule in [6] is adopted to model and fuse potential uncertain risk factor in a FMEA process in [69], which is accomplished by the generalized evidential risk priority number (GERPN). In [14], DST and the intuitionistic fuzzy sets theory are combined to model the uncertain information in FMEA. None of the above methods handle the weight factor among different risk factors by the uncertainty measure in the framework of DST. Thus, a new RPN model model named ambiguity measure weighted risk priority number (AMWRPN) is proposed to model the relative importance of each risk factor in the FMEA item.

In AMWRPN approach, AM is applied to measure the relative importance of each risk factor by the FMEA expert. The AM-based weight factor is an objective factor for quantification of uncertain degree coming from the expert. If an expert assesses the risk factor with a belief structure, which means uncertainty in information representation; then, the corresponding uncertain degree modelled by AM-based weight factor will have an effect on the relative weight of the corresponding risk factor. Intuitively, the higher the AM value, the bigger the corresponding uncertain degree. If the uncertain degree of a risk factor by an expert is very high, the relative weight of the corresponding risk factor should be a low value. The AMWRPN can decrease the same value of RPN in comparison with the DST-based methods in [47], [63], which is accomplished by modelling the relative importance of the risk factors. In addition, the AM-based weight factor is more reasonable than the Pignistic probability-based weight factor in [69] and the evidence distance-based weight factor of FMEA experts in [14], because the AM is especially designed for uncertainty measure in DST framework. For uncertain information modelled in DST framework, the uncertainty measure AM coming from DST framework ensures the internal coordination in comparison with those methods based on fuzzy set theory and so on [14], [25]. It should be noted that, although fuzzy set theory is popular among previous researches, it is usually adopted for uncertain assessments modelling instead of weight factor calculation of RPN in FMEA approach [25], [27], [29], because fuzzy set theory is a powerful tool in modelling uncertain information with linguistic variables. Above all, the AMWRPN is reasonable and applicable for building a more accurate method in modelling and processing the weight of risk factors based on the original subjective assessments of FMEA experts in the DST framework.

The rest of this paper is organized as follows. The preliminaries are introduced in Section II. In Section III, a new ambiguity measure weighted risk priority number approach for FMEA model, named AMWRPN, is proposed. Then, the AMWRPN-based FMEA approach is used to analyze a case study of the aircraft turbine rotor blade in Section IV. Section V is the conclusion of this paper.

II. PRELIMINARIES

A. DEMPSTER-SHAFER EVIDENCE THEORY

Some basic definitions in DST are presented as follows [5], [42].

Definition 1: Assume that $\Omega=\{\theta_1,\theta_2,\ldots,\theta_i,\ldots,\theta_N\}$ is a nonempty set with N mutually exclusive and exhaustive events, Ω is the **frame of discernment (FOD)**. The power set of Ω consists of 2^N elements denoted as follows:

$$2^\Omega=\left\{\begin{array}{l}\emptyset,\{\theta_1\},\{\theta_2\},\ldots,\{\theta_N\},\{\theta_1,\theta_2\},\\ \ldots,\{\theta_1,\theta_2,\ldots,\theta_i\},\ldots,\Omega\end{array}\right\}. \tag{1}$$

Definition 2: A **mass function** m is a mapping from the power set 2^Ω to the interval [0,1]. m satisfies:

$$m(\emptyset)=0,\ \sum_{A\in\Omega} m(A)=1. \tag{2}$$

If $m(A)>0$, then A is called a **focal element**. $m(A)$ indicates the support degree of the evidence on the proposition A.

Definition 3: The **body of evidence (BOE)**, also known as **basic probability assignment (BPA)** or **basic belief assignment (BBA)**, is defined as the focal sets and the corresponding mass functions:

$$(\Re, m) = \left\{ \langle A, m(A) \rangle : A \in 2^{\Omega}, m(A) > 0 \right\}. \tag{3}$$

where $\Re$ is a subset of the power set 2^{Ω}.

Definition 4: A BPA m can also be represented by the **belief function** Bel or the **plausibility function** Pl, defined as follows:

$$Bel(A) = \sum_{\phi \neq B \subseteq A} m(B), \quad Pl(A) = \sum_{B \cap A \neq \phi} m(B). \tag{4}$$

Definition 5: In DST, 2 independent mass functions m_1 and m_2 can be fused with **Dempster's rule of combination**:

$$m(A) = (m_1 \oplus m_2)(A) = \frac{1}{1-k} \sum_{B \cap C = A} m_1(B) m_2(C), \tag{5}$$

where k is a normalization factor defined as follows:

$$k = \sum_{B \cap C = \emptyset} m_1(B) m_2(C). \tag{6}$$

B. FAILURE MODE AND EFFECTS ANALYSIS

Failure mode and effects analysis (FMEA) is a widely used tool for potential risk analysis and risk identification in product design (DFMEA), system management (SFMEA), process management (PFMEA) and so on. One of the most important issues in applying FMEA method is determining the risk priorities of failure modes based on the risk priority number (RPN) model [25].

Definition 6: In FMEA, the **risk priority number (RPN)** is defined as follows [26], [60]:

$$RPN = O \times S \times D, \tag{7}$$

where O means the probability of the occurrence of a FMEA item, S means the severity degree if a failure happens with respect to the corresponding FMEA item, and D is the probability of a potential FMEA item being detected.

Generally, each risk factor is divided into 10 ranking levels from 1 to 10 [60]. For more information about how a FMEA item is assessed, please refer to [14], [26], [59], [60].

C. AMBIGUITY MEASURE

Ambiguity measure (AM) is proposed by Jousselme *et al.*, which satisfies some necessary requirements of uncertainty measures in DST framework including probability consistency, set consistency, the range of AU, subadditivity and additivity [16], [22].

Definition 7: AM is defined as follows [22]:

$$AM(m) = -\sum_{x \in X} BetP_m(x) \log_2 (BetP_m(x)), \tag{8}$$

where $BetP_m$ is the pignistic probability distribution of the mass function m [45], denoted as follows:

$$BetP_m(A) = \sum_{B \subseteq X} m(B) \frac{|A \cap B|}{|A|}, \tag{9}$$

where $|A|$ means the cardinality of the set A.

III. AMBIGUITY MEASURE WEIGHTED RISK PRIORITY NUMBER FOR FMEA

In order to handle the relative weight of each risk factor in FMEA model, a new RPN model is proposed based on the AM in the DST framework. Firstly, the uncertain degree of the assessments which is expressed as BPAs in the DST framework is measured by the AM. After that, the relative weight of each risk factor will be modelled as an exponential weight factor of O, S and D respectively based on the results of the uncertainty measure. Finally, the AMWRPN can be calculated based on the Definition 8.

Definition 8: Among n ($n \geq 1$) independent experts in a FMEA team, assume that each team member has an equal weight on final assessments, the **ambiguity measure weighted risk priority number (AMWRPN)** for each failure mode is defined as follows:

$$AMWRPN = \sum_{i=1}^{n} \frac{1}{n} O_i^{e^{-AM(O_i)}} \times S_i^{e^{-AM(S_i)}} \times D_i^{e^{-AM(D_i)}}, \tag{10}$$

where $AM(\cdot)$ is the ambiguity degree of an expert with respect to the corresponding risk factor. $e^{-AM(\cdot)}$ is the relative weight of each risk factor assessed by the same expert; $e^{-AM(\cdot)}$ means the uncertainty and ambiguity on the assessments of the corresponding risk factor. O_i, S_i and D_i are the aggregated assessment rating values of each risk factor O, S and D assessed by the ith expert.

According to the definition of Eq.(8), the ambiguity measure of each risk factor by the ith expert can be calculated as follows:

$$\begin{aligned} AM(O_i) &= -\sum_{O_i \in A \subseteq X} BetP_m(A) \log_2 (BetP_m(A)), \\ AM(S_i) &= -\sum_{S_i \in A \subseteq X} BetP_m(A) \log_2 (BetP_m(A)), \\ AM(D_i) &= -\sum_{D_i \in A \subseteq X} BetP_m(A) \log_2 (BetP_m(A)), \end{aligned} \tag{11}$$

where A is the proposition corresponding to the required risk factor. X is the frame of discernment of risk factors, and $X = \{O, S, D\}$. $BetP_m(A)$ is the pignistic probability distribution of a mass function $m(A)$. The aggregated assessment rating value of each risk factor O_i, S_i and D_i by the ith expert can be calculated as follows:

$$\begin{aligned} O_i &= \sum_{j=1}^{10} R_j m_j(O_i), \\ S_i &= \sum_{j=1}^{10} R_j m_j(S_i), \\ D_i &= \sum_{j=1}^{10} R_j m_j(D_i), \end{aligned} \tag{12}$$

where $j = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$, R_j is the rating value assessed by FMEA experts ($R_1 = 1, R_2 = 2, \ldots, R_{10} = 10$).

$m_j(O_i)$, $m_j(S_i)$ and $m_j(D_i)$ are the mass functions of the corresponding rating values assessed by the ith expert.

It should be noted that, in AMWRPN, the number of experts is not a matter, because the weight factor of the risk factor comes from the assessment of an expert itself. In other words, the assessment from different experts has no effect on the other FMEA item's weight factor. Of course, the current research does not consider the weights of experts. This is because the relative importance of FMEA expert has limited impact in some cases. For example, all the decision maker are top experts, in this case, the weight factor can be assigned the same value. Furthermore, we argue that the objective weights of criteria can come from the assessments because it is hidden information consisted in the subjective assessments of experts.

IV. APPLICATION AND DISCUSSION

A. APPLICATION

Five typical steps are included in a FMEA process related to the calculation of the RPNs.

- Step 1. FMEA experts give assessment on each FMEA item.
- Step 2. In DST framework, the subjective assessments coming from FMEA experts are modeled as BPAs. AM is applied to measure the uncertainty of the assessments.
- Step 3. Calculate the RPNs based on the proposed AMWRPN model.
- Step 4. Rank FMEA items based on the AMWRPN.
- Step 5. Actions on FMEA items based on the priorities of AMWRPN.

The case study in [47] is adopted to verify the effectiveness and some superiorities of the proposed method in this paper.

Step 1. In the adopted case study, the assessments of FMEA experts on each FMEA item in detail are presented in [63].

Step 2. The improved method for generating BPA of experts' assessments in [47] can express the conflicting evidence effectively, thus the constructed BPAs of evaluation information in [63] is used to verify the efficiency of AMWRPN. Take the first FMEA item (denoted as *fmea*1) as an example, of which the BPAs are shown in Table 1. With AMWRPN, for *fmea*1, the ambiguity measure of each risk factor by Expert 1 can be calculated with Eq.(11), shown as follows:

$$\begin{aligned}
AM(O_1) &= -\sum_{O_1 \in A \subseteq X} BetP_m(A)\log_2(BetP_m(A)) \\
&= -0.4\log_2 0.4 - 0.6\log_2 0.6 = 0.9710, \\
AM(S_1) &= -\sum_{S_1 \in A \subseteq X} BetP_m(A)\log_2(BetP_m(A)) \\
&= -0.1\log_2 0.1 - 0.8\log_2 0.8 - 0.1\log_2 0.1 = 0.9219, \\
AM(D_1) &= -\sum_{D_1 \in A \subseteq X} BetP_m(A)\log_2(BetP_m(A)) \\
&= -0.1\log_2 0.1 - 0.8\log_2 0.8 - 0.1\log_2 0.1 = 0.9219.
\end{aligned} \tag{13}$$

The aggregated assessment rating value of each risk factor by Expert 1 can be calculated with Eq.(12), shown as follows:

$$\begin{aligned}
O_1 &= \sum_{j=1}^{10} R_j m_j(O_1) \\
&= R_3 m_3(O_1) + R_4 m_4(O_1) \\
&= 3 \times 0.4 + 4 \times 0.6 = 3.6000, \\
S_1 &= \sum_{j=1}^{10} R_j m_j(S_1) \\
&= R_6 m_6(S_1) + R_7 m_7(S_1) + R_8 m_8(S_1) \\
&= 6 \times 0.1 + 7 \times 0.8 + 8 \times 0.1 = 7.0000, \\
D_1 &= \sum_{j=1}^{10} R_j m_j(D_1) \\
&= R_1 m_1(D_1) + R_2 m_2(D_1) + R_3 m_3(D_1) \\
&= 1 \times 0.1 + 2 \times 0.8 + 3 \times 0.1 = 2.0000.
\end{aligned} \tag{14}$$

Similarly, with Eq.(11) and Eq.(12), the ambiguity measure and aggregated assessment rating value of *fmea*1 by Expert 2 and Expert 3 can be calculated respectively, the results are presented in Table 2.

Step 3. Calculate the AMWRPN values. According to the definition of AMWRPN in Eq.(10) as well as the calculation results of Step 2, the AMWRPN of *fmea*1 can be calculated as follows:

$$\begin{aligned}
AMWRPN &= \sum_{i=1}^{3} \frac{1}{3} {O_i}^{e^{-AM(O_i)}} \times {S_i}^{e^{-AM(S_i)}} \times {D_i}^{e^{-AM(D_i)}} \\
&= \frac{1}{3}\left(3.6^{0.9710} \times 7.0^{0.9219} \times 2.0^{0.9219}\right) \\
&\quad + \frac{1}{3}\left(3.1^{0.4690} \times 7.0^{0.9219} \times 2.0^{0.9219}\right) \\
&\quad + \frac{1}{3}\left(3.2^{0.7219} \times 7.0^{0.9219} \times 2.0^{0.9219}\right) \\
&= 5.1551.
\end{aligned} \tag{15}$$

Applying the aforementioned computation process of AMWRPN method to the other 16 failure modes in [47], and the AMWRPN values of all the 17 failure modes (denoted as *fmea*1, *fmea*2, ..., *fmea*17) can be calculated, and the results are presented in Table 3. For the convenience of comparison,

TABLE 1. BPAs of experts' assessment information for *fmea*1 (adopted from [47]).

Risk Factor	Expert 1	Expert 2	Expert 3
O	$m(3)=0.4$, $m(4)=0.6$.	$m(3)=0.9$, $m(4)=0.1$.	$m(3)=0.8$, $m(4)=0.2$.
S	$m(6)=0.1$, $m(7)=0.8$, $m(8)=0.1$.	$m(6)=0.1$, $m(7)=0.8$, $m(8)=0.1$.	$m(6)=0.1$, $m(7)=0.8$, $m(8)=0.1$.
D	$m(1)=0.1$, $m(2)=0.8$, $m(3)=0.1$.	$m(1)=0.1$, $m(2)=0.8$, $m(3)=0.1$.	$m(1)=0.1$, $m(2)=0.8$, $m(3)=0.1$.

TABLE 2. AM and aggregated rating values of each expert for *fmea*1.

$fmea1$	Expert 1	Expert 2	Expert 3
$AM\,(\cdot)$	$AM\,(O_1) = 0.9710$ $AM\,(S_1) = 0.9219$ $AM\,(D_1) = 0.9219$	$AM\,(O_2) = 0.4690$ $AM\,(S_2) = 0.9219$ $AM\,(D_2) = 0.9219$	$AM\,(O_3) = 0.7219$ $AM\,(S_3) = 0.9219$ $AM\,(D_3) = 0.9219$
Rating	$O_1 = 3.6000$ $S_1 = 7.0000$ $D_1 = 2.0000$	$O_2 = 3.1000$ $S_2 = 7.0000$ $D_2 = 2.0000$	$O_3 = 3.2000$ $S_3 = 7.0000$ $D_3 = 2.0000$

TABLE 3. RPN of the compressor rotor blade with different methods.

FMEA Item	AMWRPN	MVRPN [63]	Improved MVRPN [47]	GERPN [69]
$fmea1$	5.1551	42.56	42.56	3.4910
$fmea2$	5.3174	64.00	64.05	3.9994
$fmea3$	3.8684	30.00	30.00	3.1069
$fmea4$	3.3302	18.00	17.97	2.6205
$fmea5$	1.6529	4.17	3.14	1.6095
$fmea6$	5.0964	60.00	60.00	3.9143
$fmea7$	3.3567	21.00	21.00	2.7586
$fmea8$	3.2975	15.00	15.00	2.4660
$fmea9$	8.3797	78.92	79.57	4.2881
$fmea10$	5.0964	60.00	60.00	3.9143
$fmea11$	4.7399	50.00	50.00	3.6836
$fmea12$	5.0973	50.00	50.00	3.6836
$fmea13$	4.9447	50.00	50.00	3.6836
$fmea14$	5.4187	60.00	60.04	3.9143
$fmea15$	5.9509	42.00	42.09	3.4756
$fmea16$	3.7560	23.88	23.86	2.8794
$fmea17$	5.0554	30.05	30.05	3.1089

the RPN values with other methods are also listed in Table 3. According to the calculation results presented in Table 3, only AMWRPN can distinguish all the 17 FMEA items in the sense of RPN-based priorities. The priorities based on MVRPN, the improved MVRPN and GERPN have repeated RPN values among some FMEA items. This feature indicates that AMWRPN is more accurate in modelling and processing the ambiguity information of the subjective assessments from experts.

Step 4. Rank FMEA items based on the AMWRPN. FMEA analysis with RPN should be applied to each independent component or system. According to [63], FMEA items *fmea*1, *fmea*2, *fmea*3, *fmea*4, *fmea*5, *fmea*6, *fmea*7 and *fmea*8 are for the compressor rotor blade; *fmea*9, *fmea*10, *fmea*11, *fmea*12, *fmea*13, *fmea*14, *fmea*15, *fmea*16 and *fmea*17 are for the turbo rotor blade. The priorities based on AMWRPN as well as the methods in [47], [63], [69] are presented in Fig.1. It should be noted that the methods in [47], [63], [69] has the same priorities for all the 17 FMEA items. This is because that the improved method in [47] mainly focus on expressing the conflicting BPAs more compatible for the fusion method in [63]. Both the improved MVRPN and GERPN focus on applying the experiment data and results in [63] for verifying the effectiveness of the corresponding methods. The AMWRPN steps farther by analyzing the ambiguity degree of each piece of evidence for a more accurate process of uncertain information processing. Fig.1 shows the AMWRPN-based priorities for the compressor rotor blade is *fmea*2 $\succ$ *fmea*1 $\succ$ *fmea*6 $\succ$ *fmea*3 $\succ$ *fmea*7 $\succ$ *fmea*4 $\succ$ *fmea*8 $\succ$ *fmea*5, and the AMWRPN-based priorities for the turbo rotor blade is *fmea*9 $\succ$ *fmea*15 $\succ$ *fmea*14 $\succ$ *fmea*12 $\succ$ *fmea*10 $\succ$ *fmea*17 $\succ$ *fmea*13 $\succ$ *fmea*11 $\succ$ *fmea*16, where "$\succ$" denotes a higher priority.

Step 5. Actions on FMEA items based on the priorities of AMWRPN. For practical engineering, attentions and the finite resource should be assigned to the FMEA item with a higher priority according to the AMWRPN. In this example, the FMEA item *fmea*2 for the compressor rotor blade as well as the *fmea*9 for the turbo rotor blade has the highest priority, thus these FMEA items should be well addressed preferentially; et cetera.

B. DISCUSSION

In general, the priorities based on AMWRPN is consistent with the methods in [47], [63], [69]. Among all the 8 FMEA items of the compressor rotor blade, *fmea*5 has the smallest value and *fmea*2 has the biggest value by AMWRPN, which is the same with MVRPN, GERPN and the improved MVRPN. However, there exists a little difference for *fmea*1 and *fmea*6. For *fmea*6, it is obvious that there is no uncertainty among different risk factors according to the FMEA experts' assessments in [63]. While the uncertainty of FMEA experts' assessments for *fmea*1 is captured and well addressed by the AMWRPN, which contributes to the increasing of its risk level.

The largest and smallest RPN values for the turbo rotor blade by AMWRPN are the same with those of MVRPN, GERPN and the improved MVRPN. The divergence of belief structures in *fmea*15 leads to a higher risk level according to the AMWRPN, which suggests unstable assessment may lead

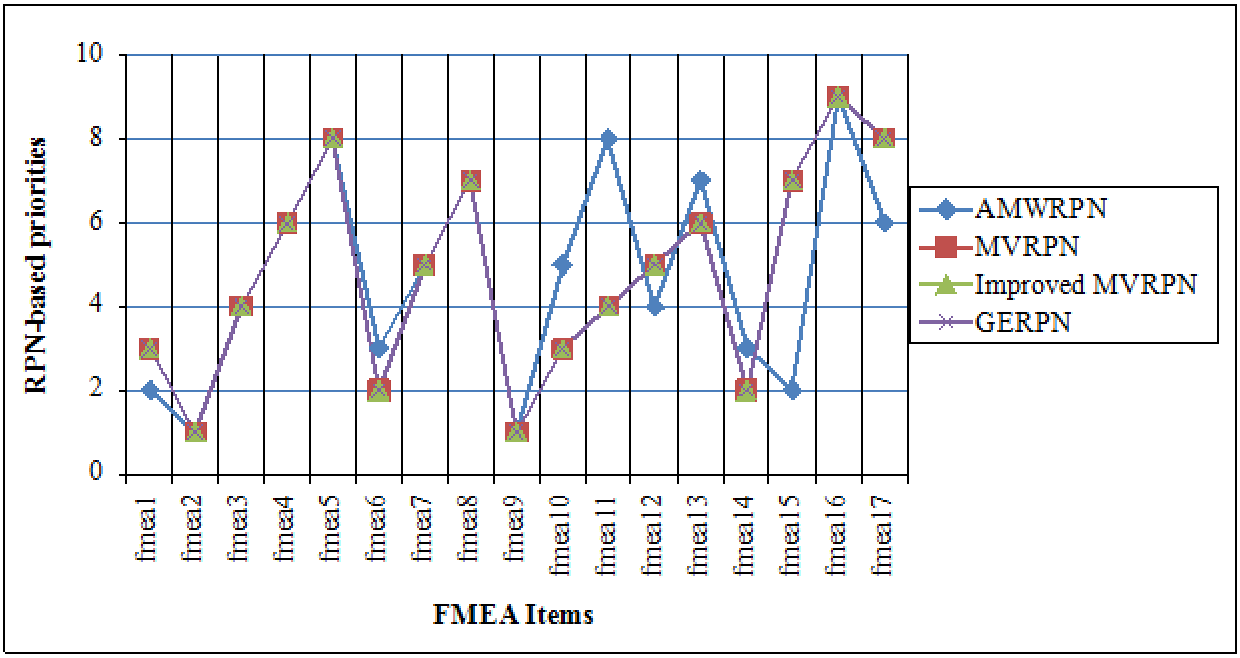

FIGURE 1. Priorities based on AMWRPN, MVRPN, Improved MVRPN and GERPN.

to a higher risk level. The differences of the rankings among different experts for *fmea*12 also leads to a higher risk level according to the proposed method. The discordance among the experts measured by the AM leads to a higher priority of *fmea*17. In comparison with the methods in [47], [63], [69], the similarity and stability of the belief structures result in a lower risk level for *fmea*10 and *fmea*11 according to the AMWRPN. This is reasonable because it means consistency among different experts. If all the experts assess a FMEA item similarly, which means the knowledge of this FMEA item among each experts is almost complete; thus, the risk level is under control. Other FMEA items' priorities change along with the aforementioned FMEA items. The feature of sensitivity of the proposed method on the almost certain subjective judgement contributes to a higher risk level for *fmea*1, *fmea*15 and *fmea*17, which should be overcome in practical applications.

According to the AMWRPN, the relative importance of each risk factor, which is the open issue for existed FMEA methods in the DST framework, e.g. in the MVRPN [63] and the improved MVRPN [47], has been handled by the weight factor of each risk factor based on the AM. The AM-based weight factor for uncertainty modelling in the DST framework ensures the internal coordination, because the AM is especially proposed for uncertainty measure in the DST [22]. For engineering applications, the practical implication of the study is almost the same with the other methods in the framework of Dempster-Shafer evidence theory. The only difference is the step of calculating the RPNs. The classical RPN will be replaced by the AMWRPN, which can be accomplished by software program easily. According to the working experience in an automotive engineering institute in China, the limitation of the AMWRPN exists not in choosing the AMWRPN model; in DST framework, the limitation of using this study in practical field exists in choosing a proper way to generate BPAs for applying the AMWRPN model. Because, in practical implications, most of the engineers have been used to classical assessments with the 10 level marks method. In the following work, how to generate BPAs with a simple way will be also taken into consideration.

V. CONCLUSIONS

A new RPN model named AMWRPN for FMEA approach is proposed in this paper. AMWRPN models the potential priority judgement of each risk factor in FMEA as the exponential weight factor of each risk factor. The uncertainty measure AM in the framework of DST is used to measure the ambiguity degree of FMEA experts' subjective assessments.

The novelty of the proposed method can be summarized as follows. On the one hand, the weight factor of different risk factors is based on the AM which is a typical uncertainty measure in the framework of DST, which ensures the internal coordination of the proposed method in comparison with those weight factors basing on fuzzy sets theory; on the other hand, compared with other DST-based RPN models, the proposed approach can be more accurate in processing the uncertain assessments of FMEA experts modelled in DST framework. A case study in aircraft turbine rotor blade verifies the efficiency of the new RPN model.

The ongoing work of this paper is applying the proposed method to many more practical engineering problems. Also, the current research does not consider the weights of experts, as well as other potential risk factors apart from the O, S and D. In some cases, these conditions are key issues which should be handled cautiously. Thus, in the following work, we will apply appropriate methods to determine the decision makers' weights, where the ordered weighted averaging (OWA) operator will be taken into consideration.

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