Intermittent Chaos in Switching Power Supplies Due to Unintended Coupling of Spurious Signals

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Abstract — This paper studies the "intermittent" chaos and subharmonics observed in switching dc/dc converters using a simple circuit model that describes possible unintended coupling of some weak spurious signal to the converter. The study shows that the signal strength and frequency of the intruding signal are vital parameters that affect the type of intermittent behavior and the period of intermittency. Simulations and experimental results are presented.

Index Terms — Switching converters, spurious signal coupling, "intermittent" chaos.

I. INTRODUCTION

It is not uncommon in the design of switching power supplies that the rather puzzling irregular (chaotic) behavior is observed intermittently between long periods of regular behavior. Noise or bad construction is the usual blame. In this paper we propose a circuit model to explain the "intermittent" chaos observed in switching dc/dc converters. The model essentially incorporates a weak periodic signal which is coupled to the converter via unintended paths (e.g., conducted or radiated EMI). We show that when the coupling is weak and the frequency of the coupled signal is different from the switching frequency, "intermittent" subharmonics occurs. As the signal strength of the coupled signal increases, "intermittent" chaos occurs. The study is supported by simulations and experiments.

II. SIMPLIFIED MODEL OF SPURIOUS SIGNAL INTRUSION

We begin with the crude but practically valid assumption that the power supply under study is not perfectly protected from intrusion of signals which are generated outside the power supply [1, 2]. The intrusion can take the form of coupling via conducted or radiated paths. Sometimes, the intruders can live on the same circuit board or be present at a very close proximity. Figure 1 (a) shows a crude model which illustrates the situation.

In order to study the effects of coupling of spurious (intruding) signals, we need a circuit model that describes the way in which the power supply is connected to the spurious signal source. One simple approach is to model the coupling
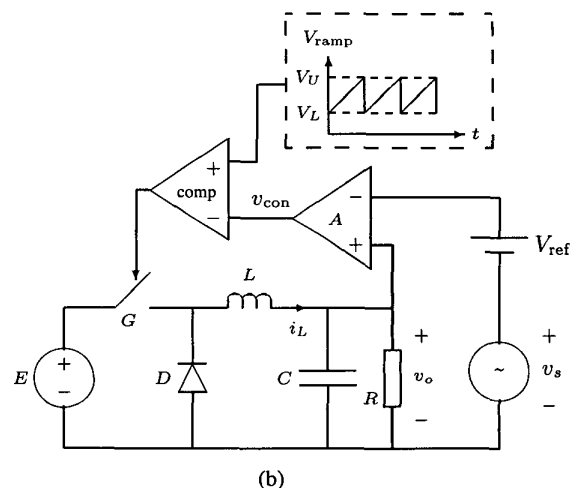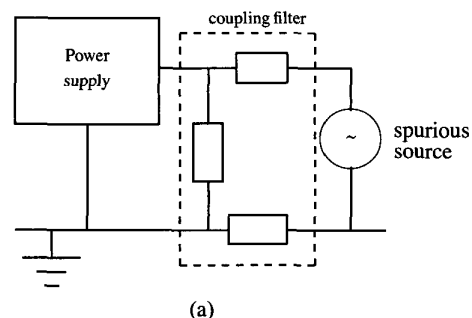

Fig. 1: A power supply coupled with spurious source. (a) Crude model; (b) circuit model showing a buck converter with an additive intruding signal v_s acted upon the reference voltage.

as an additive process which injects the spurious signal directly to the reference voltage of the power supply, as shown in Fig. 1 (b).

Suppose the effective additive source is v_s, which may be due to some oscillators or PWM generators present in the proximity. For convenience we denote the strength of the spurious source by α which is the ratio of the amplitude of v_s to V_{ref}, i.e.,

$$\alpha = \hat{v}_s / V_{ref} \tag{1}$$

where $\hat{v}_s$ is amplitude of the effective intruding source appeared behind the reference voltage of the power supply. We
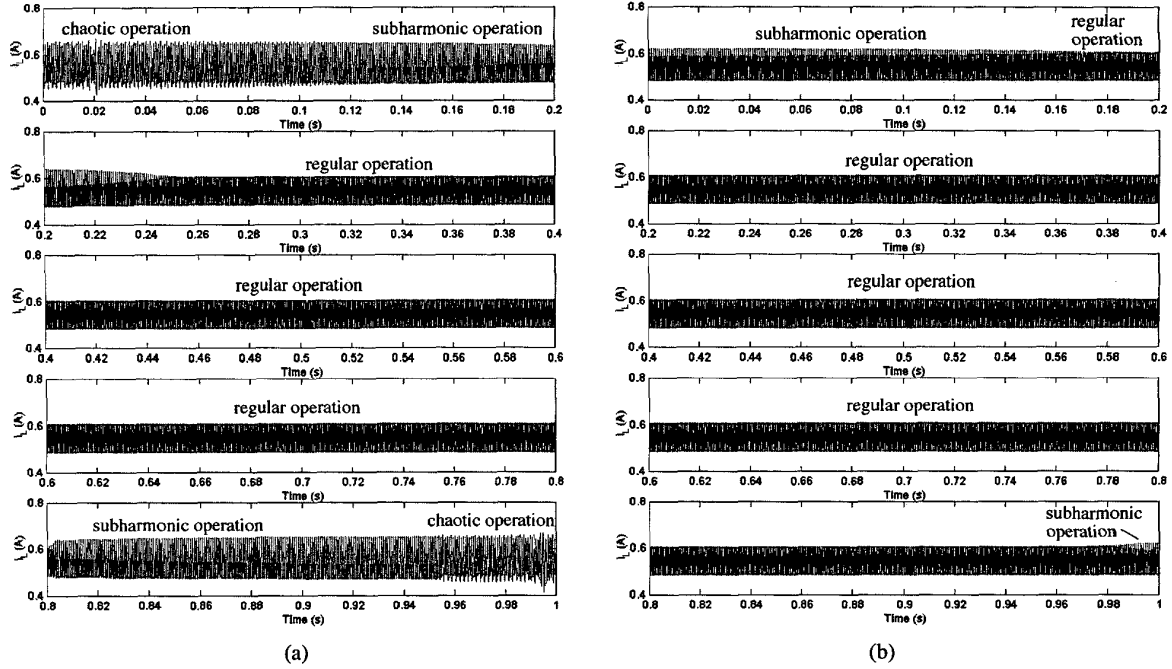
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Fig. 2: Inductor current waveform for buck converter with unintended coupling of sinusoidal intruding source for (a) $\alpha = 0.0034$ showing "intermittent" chaos, and (b) $\alpha = 0.0003$ showing "intermittent" subharmonics.

will investigate the effect of coupling the intruding source to the power supply for various levels of strengths of the intruding source as well as for different types of intruding sources (e.g., sinusoids and square pulses).

III. QUICK GLIMPSE OF "INTERMITTENT" CHAOS

When v_s is a sinusoidal signal of frequency f_s, the reference voltage is being modulated as

$$V_{\text{ref}} \mapsto V_{\text{ref}}(1 + \alpha \sin 2\pi f_s t) \qquad (2)$$

Such spurious coupling can occur in power supplies that operate in an RF environment. Suppose the switching frequency is f_o, and $f_o \neq f_s$. Obviously, we would expect the effect of the intruding source on the power supply to be periodic in $1/|f_o - f_s|$.

To get a quick glimpse of the phenomenon, let us consider the buck converter shown in Fig. 1 (b) for two particular effective signal strengths. The circuit parameters are as follows: $L = 20\,\text{mH}, C = 47\,\mu\text{F}, R = 22\,\Omega, f_o = 2500\,\text{Hz}, E = 24\,\text{V}$ and $V_{\text{ref}} = 11\,\text{V}$. The spurious signal frequency f_s is 2501 Hz. Time-domain cycle-by-cycle simulations are performed and the inductor current waveform is shown in Fig. 2. Here, we observe "intermittent" chaos and "intermittent" subharmonics for relatively strong and weak coupling, respectively.[1]

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[1] The term *intermittent chaos* has been used in the mathematics literature to describe a few special types of chaotic behavior [3]. Here, we use the term to describe a particular operation where chaos shows up periodically between regular operations. Such intermittent chaotic operation is sometimes called *breathing* in the physics literature [4].

These kinds of phenomena are commonly observed by practicing power supply engineers in their design workbenches. Here, we show that the phenomena can be explained in terms of coupling of a spurious signal through unintended paths to the power supply.

The intermittent period is related to the difference between the intruding signal frequency and the power supply's switching frequency. In this case, the intermittent period is 1 s.

IV. TIME-BIFURCATION DIAGRAMS – A CLOSER LOOK

For a clear exposition of the phenomenon, it is customary in the systems theory literature to examine the "sampled waveforms", which effectively ignore the detailed switching ripples but focus on the movement of the waveforms at periodic switching instants. Specifically we sample the waveforms at $t = nT$, where T is the switching period, i.e., $T = 1/f_o$, and the resulting plots are called *time-bifurcation diagrams* since they reflect the change of qualitative behavior as time goes.

A. Sinusoidal Intruding Source

We begin with the case of sinusoidal intruding signals as described in the previous section, i.e.,

$$v_s = \alpha V_{\text{ref}} \sin 2\pi f_s t. \qquad (3)$$

Some typical time-bifurcation diagrams are shown in Fig. 3, from which we observe the following.

• When the intruding signal strength is very weak, the power supply can still maintain its regular operation,
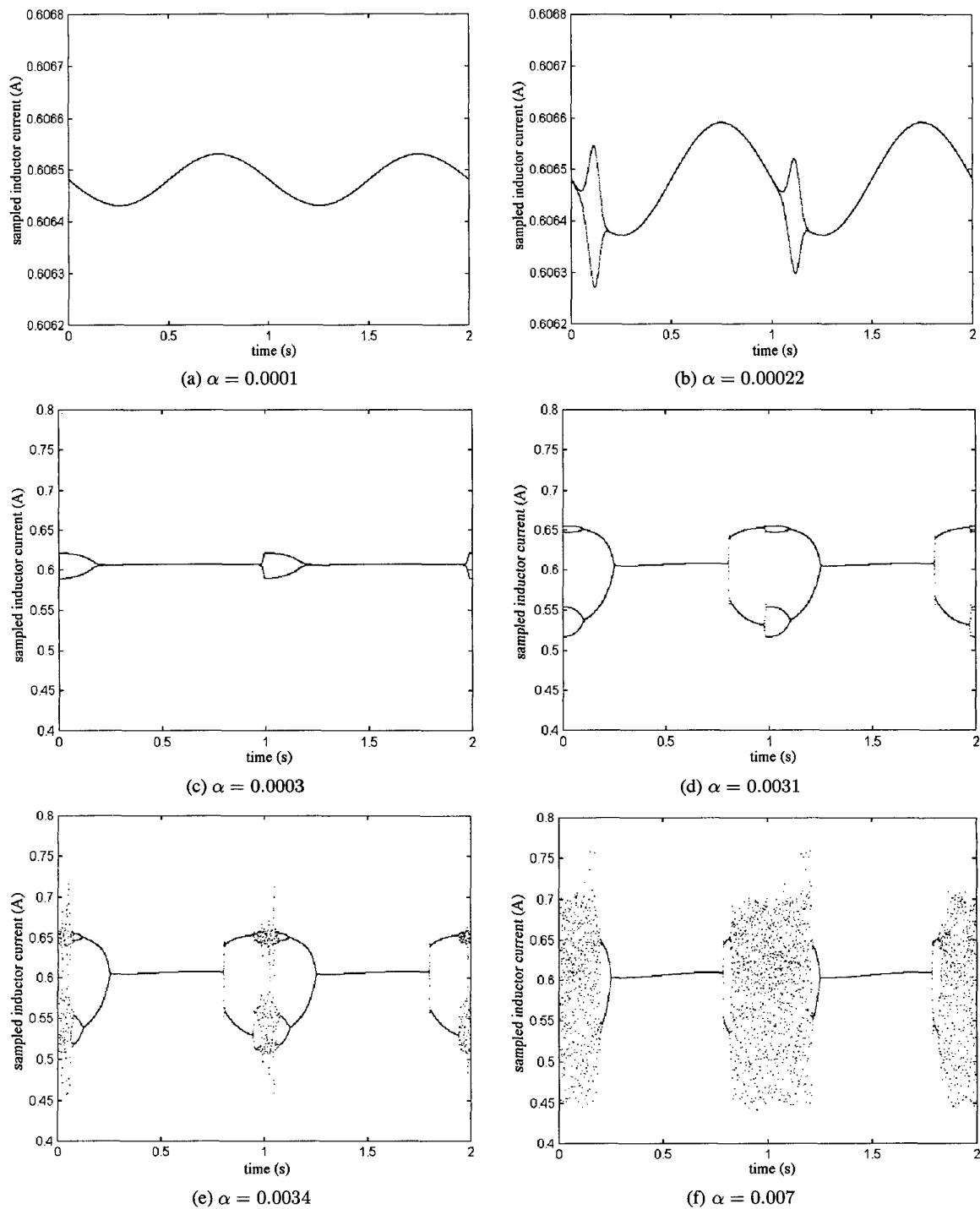
Fig. 3: Sampled inductor current waveforms (time-bifurcation diagrams) for buck converter with unintended coupling of sinusoidal intruding source for different coupling strengths. (a) Regular operation with fluctuation of average value; (b)–(d) "intermittent" subharmonics; (e)–(f) "intermittent" chaos.

though the average operating point fluctuates. The effect is not significant.

• As the intruding signal strength increases, the power supply experiences subharmonic operation intermittently with regular operation. For a relatively low intruding signal strength, period-2 subharmonics are observed intermittently with regular operation. Further increase in intruding signal strength causes period-4 subharmonics to occur intermittently with period-2 subharmonics and reg-
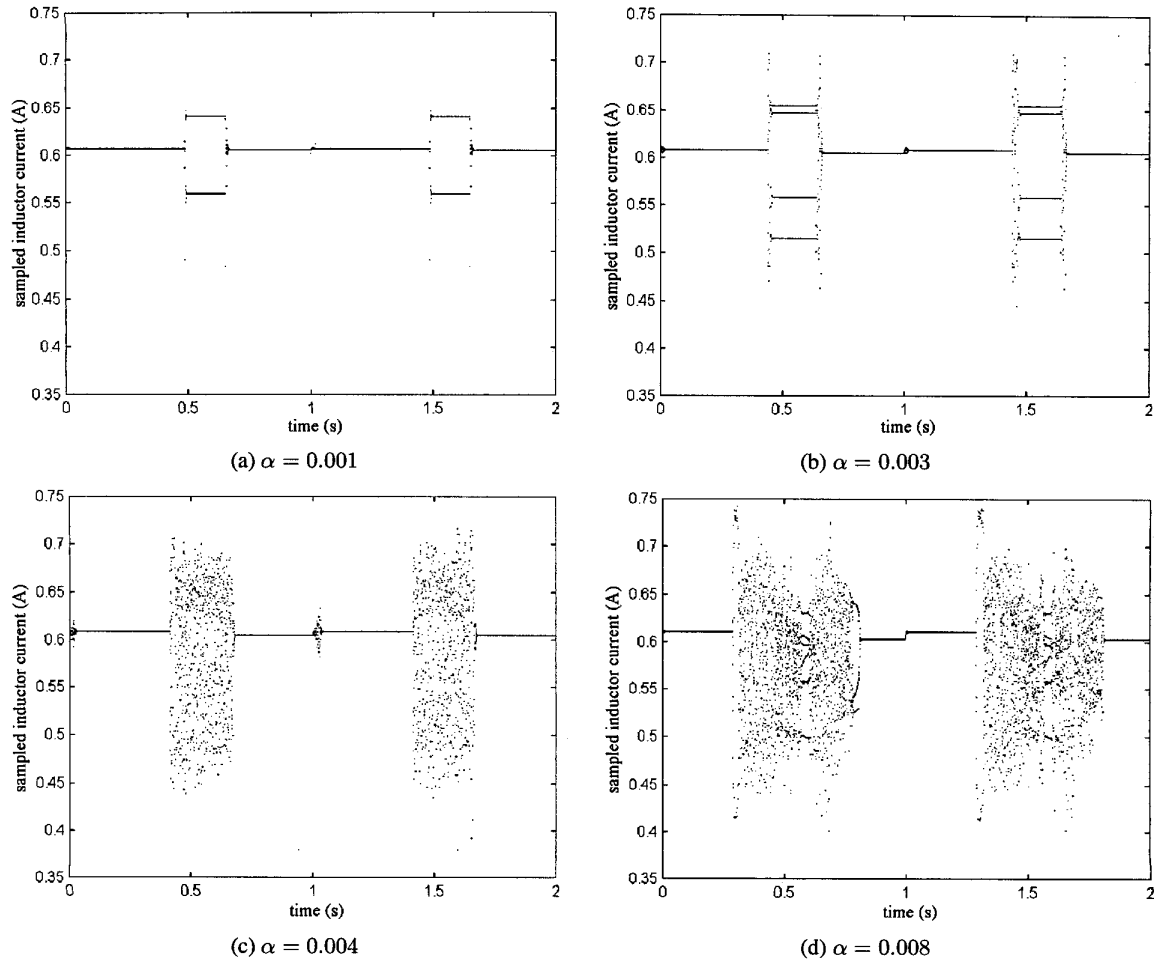
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Fig. 4: Sampled inductor current waveforms (time-bifurcation diagrams) for buck converter with unintended coupling of rectangular intruding source for different coupling strengths. (a)–(b) "intermittent" subharmonics; (c)–(d) "intermittent" chaos.

ular operation.

- When the intruding signal strength is strong, the power supply experiences chaotic operation intermittently with subharmonic operation and regular operation.

- The intermittent period is equal to the $1/|f_o - f_s|$. Thus, if the intruding signal frequency is very close to the switching frequency of the power supply, the intermittency is long.

B. Rectangular Pulse Intruding Source

We now consider another type of spurious signal which is likely to be generated from a PWM generator of another power supply and coupled to the power supply under study through unintended paths. This situation can arise in modular power supplies with individual PWM generators. For the purpose of analysis, we assume that the intruding source v_s is given by

$$v_s = \alpha V_{\text{ref}} \, \text{rect}(f_s t) \qquad (4)$$

where

$$\text{rect}(x) = \begin{cases} +1 & \text{if } 0 \leq x < 0.5 \\ -1 & \text{if } 0.5 \leq x < 1 \\ \text{rect}(x-1) & \text{if } x \geq 1. \end{cases} \qquad (5)$$

Effects similar to the case of sinusoidal intruding signals have been found from simulations. Some time-bifurcation diagrams are shown in Fig. 4.

V. Experimental Observations

We have built a circuit prototype to emulate the proposed coupling circuit. The circuit parameters are the same as those used in the simulations. Sinusoidal as well as rectangular intruding signals have been used in the experiment. By varying the intruding signal strength, we observe "intermittent" subharmonics and chaos similar to those observed from simulations. Figure 5 shows a few time-bifurcation diagrams obtained experimentally. The general phenomenon observed from the simulations is confirmed.

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(a) sinewave intruder $\alpha = 0.0012$

(b) rectangular wave intruder $\alpha = 0.0007$

(c) sinewave intruder $\alpha = 0.0036$

(d) rectangular wave intruder $\alpha = 0.0036$

(e) sinewave intruder $\alpha = 0.004$

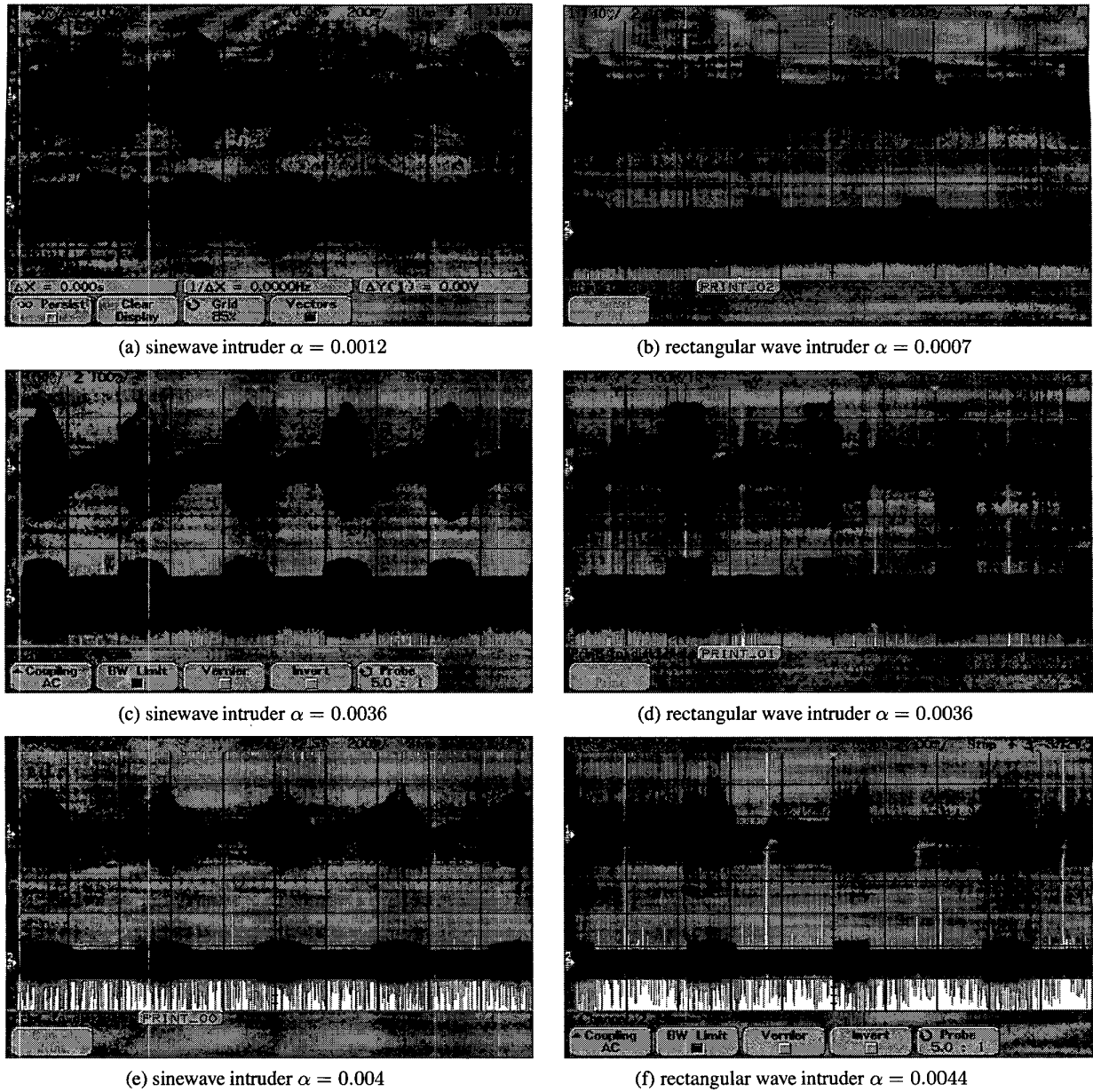(f) rectangular wave intruder $\alpha = 0.0044$

Fig. 5: Measured time-bifurcation diagrams for buck converter with coupling of intruding source for different coupling strengths. (a)–(b) Intermittent period-2 subharmonics (upper trace: output voltage, 90 mV/div for (a) and 140 mV/div for (b); lower trace: inductor current, 100 mA/div); (c)–(d) intermittent period-2 and period-4 subharmonics (upper trace: output voltage, 150 mV/div for (c) and 140 mV/div for (d); lower trace: inductor current, 100 mA/div); (e)–(f) "intermittent" chaos (upper trace: output voltage, 250 mV/div; lower trace: inductor current, 200 mA/div).

VI. PARAMETERS AFFECTING THE OCCURRENCE OF "INTERMITTENT" CHAOS

So far, we have only considered a particular set of parameters and the general phenomena of "intermittent" subharmonics and chaos in a switching converter. Further simulations with different sets of parameters are needed to determine whether α can be smaller or bigger in order to drive the converter into "intermittent" chaos.

The thresholds of the coupling signal strength for intermit-

tent subharmonics and chaos are summarized in Tables 1 and 2. Graphical presentations are also shown in Fig. 6, where the upper surface gives the thresholds for "intermittent" chaos and the lower surface gives the thresholds for "intermittent" subharmonics. It should be noted that these data are obtained for a particular converter operating with a specific set of circuit parameters, and they are useful for inspecting the general trend rather than providing absolute design data. Nonetheless, from these data, it is clear that if the feedback gain is higher (relatively) or the input voltage is higher (for the same

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Feedback gain	$E = 19\,\text{V}$	$E = 20\,\text{V}$	$E = 21\,\text{V}$	$E = 22\,\text{V}$	$E = 23\,\text{V}$	$E = 24\,\text{V}$
6.0	0.0045	0.0043	0.0041	0.0039	0.0036	0.0033
6.4	0.0038	0.0037	0.0035	0.0032	0.0030	0.0027
6.8	0.0033	0.0031	0.0029	0.0025	0.0024	0.0021
7.2	0.0028	0.0026	0.0024	0.0021	0.0019	0.0016
7.6	0.0023	0.0021	0.0019	0.0017	0.0014	0.0011
8.0	0.0019	0.0017	0.0015	0.0013	0.0010	0.0007
8.4	0.0015	0.0014	0.0011	0.0009	0.0005	0.0003

Table 1: Threshold values of α at which "intermittent" subharmonics occur

Feedback gain	$E = 19\,\text{V}$	$E = 20\,\text{V}$	$E = 21\,\text{V}$	$E = 22\,\text{V}$	$E = 23\,\text{V}$	$E = 24\,\text{V}$
6.0	0.0076	0.0076	0.0075	0.0074	0.0072	0.0070
6.4	0.0068	0.0068	0.0067	0.0066	0.0065	0.0063
6.8	0.0062	0.0062	0.0061	0.0060	0.0058	0.0056
7.2	0.0056	0.0056	0.0055	0.0053	0.0051	0.0049
7.6	0.0050	0.0050	0.0049	0.0048	0.0046	0.0043
8.0	0.0045	0.0045	0.0044	0.0043	0.0041	0.0038
8.4	0.0041	0.0041	0.0040	0.0039	0.0036	0.0034

Table 2: Threshold values of α at which "intermittent" chaos occur
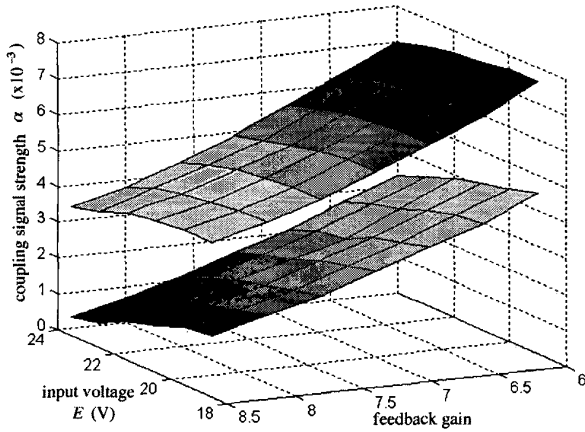

Fig. 6: Graphical representation of the thresholds of coupling signal strength for "intermittent" chaos (upper surface) and "intermittent" subharmonics (lower surface).

harmonics can be explained in terms of coupling of spurious signals. This paper shows that the signal strength and frequency of the intruding signal are vital parameters that affect the type of intermittent behavior and the period of intermittency. Finally, it should be pointed out that our study in this paper is based on a simple voltage-feedback buck converter. Since our purpose is to illustrate the effect the spurious signal coupling, we retain only the essential part of the feedback circuit and the main power stage. The same analysis can be used to study the intermittent chaotic and subharmonic operations in other types of converters. At the time of writing, current-programmed converters have been found to be equally affected by spurious signals which can be coupled into the converter through unintended channels.

output voltage), the converter is more vulnerable to attack by spurious signal coupling, i.e., smaller α suffices to cause "intermittent" subharmonics and chaos. This observation can be easily understood because the converter concerned is closer to the operating boundary with a higher feedback gain or a lower input voltage, and a relatively smaller α will push it to subharmonics and chaos.

VII. CONCLUSIONS

Power electronics are nonlinear systems which have been shown to exhibit a variety of complex behavior [5, 6, 7]. In this paper we attempt to rationalize a commonly observed but rarely explained phenomenon in power supply design. By using an appropriate circuit model, "intermittent" chaos and sub-

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