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TRANSPORTATION NETWORK REDUNDANCY:

COMPLEMENTARY MEASURES AND COMPUTATIONAL METHODS

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ABSTRACT

Redundancy is vital for transportation networks to provide utility to users during disastrous events. In this paper, we develop two network-based measures for systematically characterizing the redundancy of transportation networks: travel alternative diversity and network spare capacity. Specifically, the *travel alternative diversity dimension* is to evaluate the existence of multiple modes and effective routes available for *travelers* or the number of effective connections between a specific origin-destination pair. The *network spare capacity dimension* is to quantify the *network-wide* residual capacity with an explicit consideration of travelers' mode and route choice behaviors as well as congestion effect. They can address two fundamental questions in the pre-disaster transportation system evaluation and planning, i.e., "*how many effective redundant alternatives are there for travelers in the normal or disruptive event*?" and "*how much redundant capacity does the network have*?" To implement the two measures in practice, computational methods are provided to evaluate the network redundancy. Numerical examples are also presented to demonstrate the features of the two redundancy measures as well as the applicability of the computational methods. The analysis results reveal that the two measures have different characterizations on network redundancy from different perspectives, and they can complement each other by providing meaningful information to both travelers and planners.

Keywords: redundancy; travel alternative diversity; network spare capacity

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1 INTRODUCTION

1.1 Research Subject and Motivation

Transportation networks are not only vital in providing accessibility and promoting the safe and efficient movement of people and goods, but also central to the functioning of modern society to support our daily activities and maintain relations in business, social, and family settings. Yet they are vulnerable to disruptions whether planned or unplanned. Recent natural and man-made events such as earthquakes in China, Japan, New Zealand, Nepal, and Afghanistan/Pakistan, hurricanes/typhoons in the United States, Philippines, and Hong Kong have repeatedly emphasized the importance of transportation networks, and the need for government agencies and communities to strengthen transportation networks more *resilient* to planned and unplanned disruptions. For example, the United States Department of Transportation (USDOT) has considered resiliency into the National Transportation Recovery Strategy (USDOT, 2009). The overall goal of this strategy is to enhance the recovery process of transportation networks under disruptions and to increase the resiliency of various infrastructures in the community. Recently, various conceptual and/or computational frameworks have been proposed to analyze *resiliency* (e.g., Chang and Nojima (2001), Victoria Transport Policy Institute (2005), Tierney and Bruneau (2007), Heaslip *et al*. (2010), Croope and McNeil (2011), Urena *et al*. (2011), and Omer *et al*. (2013) for a general transportation network resiliency evaluation framework, Caplice *et al*. (2008), Ortiz *et al*. (2009), Ta *et al*. (2009), Adams and Toledo-Durán (2011) and Miller-Hooks *et al*. (2012) for a freight system resiliency evaluation framework, Faturechi *et al*. (2014) for an airport's runway and taxiway network, and Faturechi and Miller-Hooks (2014, 2015) for a general civil/transportation infrastructure system). Faturechi and Miller-Hooks (2015) provided a comprehensive review on seven common performance measures of transportation infrastructure systems in disasters, including risk, vulnerability, reliability, robustness, flexibility, survivability, and *resiliency*.

The Multidisciplinary Center for Earthquake Engineering (MCEER) provided the four “Rs” concept to characterize *resiliency*: robustness, redundancy, resourcefulness, and rapidity (Bruneau *et al*., 2003). The first two Rs (robustness and redundancy) are mainly related to the pre-disaster planning state, while the last two Rs (resourcefulness and rapidity) are related to the post-disaster recovery and mitigation. The former two Rs apply directly to the transportation infrastructure, network design, and mode options, while the latter two Rs pertain to the transportation system's operating entities. *Redundancy* was defined as "the extent to which elements, systems, or other units of analysis exist that are substitutable, i.e., capable of satisfying functional requirements in the event of disruption, degradation, or loss of function". The Webster/Merriam Dictionary (2012) gives a general definition of redundancy (or state of redundant) as: i) *exceeding what is necessary or normal,* or ii) *serving as a duplicate for preventing failure of an entire system upon failure of a single component*. Faturechi and Miller-Hooks (2014) provided an infrastructure protection framework based on concepts used in describing a system's innate capability (i.e., coping capacity) to endure disruptions, and considering pre- and post-event actions to mitigate the impact of disaster events and increase inherent system qualities of resistance and excess (including expansion, retrofit, resource availability and response activities). Among others, the coping capacity characteristics include the ability to withstand stress, i.e., resistance, and/or excess in terms of redundancies and underutilized capacity; expansion includes pre-event actions to enhance network performance by increasing connectivity (e.g., adding redundancy) or capacity. Also, redundancy has been widely studied and applied in many domains, such as reliability engineering (O'Connor, 2010), communication (Wheeler and O'Kelly, 1999), water distribution system (Kalungi and Tanyimboh, 2003), and supply chain and logistics (Sheffi and Rice, 2005), etc.

In transportation, some researchers have introduced various measures for assessing the resiliency of transportation networks, and redundancy is one of those measures. For example, Berdica (2002) developed a qualitative framework and basic concepts for vulnerability as well as many neighboring concepts such as resiliency and redundancy. According to Berdica (2002), redundancy is the existence of numerous optional routes/means of transport between origins and destinations that can result in less serious consequences in case of a disturbance in some part of the system. In the event of disasters, redundancy not only provides alternatives to travelers to minimize the impacts of disruptions, but also improves recovery and redesign strategies by making transportation networks more resilient against disruptions. The Federal Highway Administration (FHWA, 2006) defined redundancy as the ability to utilize backup systems for critical parts of the system that fail. To improve network resiliency, they emphasized that it is extremely important to consider redundancy in the development of a process or plan for emergency response and recovery. One of the pre-disaster planning strategies is to improve network resiliency by adding redundancy to create more alternatives for travelers or by hardening the existing infrastructures to withstand disruptions. Godschalk (2003) and Murray-Tuite (2006) defined redundancy as the number of functionally similar components that can serve the same purpose, thus the system does not fail when one component fails. Also, Goodchild *et al.* (2009) and Transystems (2011) introduced redundancy as one of the properties of freight transportation resiliency, and defined redundancy as the availability of alternative freight routes and/or modes. In Miller-Hooks *et al*. (2012) and Faturechi *et al*. (2014), the innate capability to resist and absorb disruption impacts through redundancies and underutilized capacity, the effects of adaptive post-event actions, and the preparedness decisions of supporting these actions were integrated into the concept of resiliency. Along a different line, Jenelius (2010) proposed the concept of redundancy importance and two measures (i.e., flow-based and impact-based) by considering the importance of links as backup alternative when other links in the network are disrupted. The flow-based measure considers a net traffic flow that is redirected to the backup link and the impact-based measure considers an increased travel time (cost) due to the rerouting effect. However, these two measures only quantify the localized redundancy importance of a transportation network. In other words, they are unable to capture the diversity of alternatives, which is an important property in measuring network redundancy. The diversity of available routes when the primary choice is inoperative needs to be explicitly considered in the redundancy characterization. In summary, despite that there is a growing body of research on *resiliency and also redundancy has been listed as one of the important concepts in characterizing resiliency*, *it is the least study in the context of transportation networks according to the above comprehensive review by Faturechi and Miller-Hooks (2015) on the transportation infrastructure system performance in disasters*. *Few research studies have provided concrete definitions of transportation network redundancy, and even fewer studies have been devoted to quantitative measures and computational methods for assessing the multifaceted characteristics of transportation network redundancy*.

There are a few challenges and practical considerations associated with modeling transportation network redundancy (e.g., flexibility of travel alternatives, congestion effect, practical requirements on an acceptable route substitution, mode and route choice behaviors, etc.). Adding redundancy to create more alternatives for travelers could involve not only routes but also travel modes. In addition, multiple travel modes within the system could increase the redundancy by providing substitutions to maintain transport service if one or more modes are disturbed by disruptions. For example in the 1994 Northridge earthquake in California, the transit system helped to alleviate the initial congestion in the Los Angeles highway network. During the

Interstate freeway reconstruction, transit usage was tripled on rail and bus lines; however, it was reduced to the pre-earthquake level one year after the disruption (Deblasio *et al*., 2003). Hence, the redundancy measure should consider the flexibility of travel alternatives as well as the behavioral response of users in the event of a disruption. However, the alternative diversity alone may not be a sufficient measure of network redundancy as it lacks interactions between transport demand and supply. Capacity is not explicitly considered in the evaluation of travel alternatives (i.e., mode and route). It is hence necessary to include network capacity in measuring transportation network redundancy. Evaluating the network-wide capacity is not trivial. Multiple origin-destination (O-D) pairs exist and the demands between different O-D pairs are not exchangeable or substitutable. The network-wide capacity is not just a simple sum of the individual link capacities. Also, mode and route choice behaviors have to be considered in estimating the multi-modal network capacity. Disruption on an auto link may increase the travel time of auto mode or even change the availability of auto mode. This may further lead to flow shift between modes, changing the multi-modal network capacity.

1.2 Main Contribution of This Paper

The *main purposes* of this paper are twofold: (1) to develop formal definitions for systematically modeling transportation network redundancy from the perspectives of two main decision-making stakeholders in transportation systems (i.e., travelers and planners), and (2) to develop network-based measures and computational methods for evaluating the network-based redundancy measures. Specifically, *travel alternative diversity* and *network spare capacity* are developed as two quantitative measures to capture the considerations of travelers and planners (i.e., the two main decision-making stakeholders in transportation systems). They can address two fundamental questions in the pre-disaster transportation system evaluation and planning, i.e., "*how many effective redundant alternatives are there for travelers in the event of a disruption*?" and "*how much redundant capacity does the network have*?" In the context of a general network resiliency evaluation framework, the proposed measures of network redundancy can be considered as a critical component in assessing network resiliency and also designing a more resilient transportation network against disruptions.

On the one hand, the *travel alternative diversity dimension* is to evaluate the existence of multiple modes and effective routes available for travelers, or the number of effective connections between a specific O-D pair. Travelers might not treat all simple routes as their effective alternatives. Shorter detoured routes with an acceptable travel cost (i.e., not-too-long routes) are more likely to be considered by travelers as a reasonable substitution when the primary or secondary route is not available. Also, as different routes may share the same links or segments in the network, the number of routes may drop significantly when one main link fails to function. The travel alternative diversity is mainly measured based on network topology and structure from travelers' perspective, without considering travelers' mode and route choice behaviors responding to the congestion effect. On the other hand, the *network spare capacity dimension* is to quantify the *network-wide* residual capacity from planners' perspective with an explicit consideration of travelers' mode and route choices as well as congestion effect. These two measures can complement each other (i.e., modeling network topology and travel choice behaviors under congestion) by providing a two-dimensional characterization of transportation network redundancy from the perspective of both travelers and planners.

To implement these two network-based measures in practice, a formal methodology is provided to evaluate the transportation network redundancy. Our developments are based on some ground works in the literature (e.g., the basic network reserve capacity model (Wong and Yang, 1997), and the method of counting the number of efficient routes (Meng *et al*., 2005)), but with important modifications for the needs of modeling network redundancy. As to the *travel alternative diversity dimension*, we explicitly model the requirement of not-too-long routes (i.e., effective routes) by using a user-specified threshold in the measure of travel alternative diversity and extend Meng *et al*. (2005) to achieve this in the computation. The number of effective routes could be further used to evaluate the number of effective routes traversing a particular link and to identify the heavily overlapped links. These important modifications could enhance the assessment realism of route diversity. As to the *network spare capacity dimension*, we explicitly model the travelers' mode substitution (via the Logit model) and route choice (via the C-logit model for capturing route overlapping) as well as the congestion effect based on the basic network reserve capacity model (without considerations of mode substitution, perception error, and route overlapping) by Wong and Yang (1997). The Logit and C-Logit models are used to consistently capture the travelers' mode choice and route choice behaviors under the network equilibrium framework. In summary, this paper addresses three hierarchical and relevant issues: *How to systematically define network redundancy, why the two dimensions, and how to compute the two dimensions*.

The remainder of this paper is organized as follows. Section 2 and Section 3 present the two measures and the evaluation methodology, respectively. Section 4 then provides numerical results to demonstrate the features of the redundancy measures as well as the evaluation methodology. Finally, conclusions are summarized in Section 5.

2 NETWORK-BASED REDUNDANCY MEASURES

In this section, we characterize transportation network redundancy from two perspectives: travel alternative diversity and network spare capacity.

2.1 Travel Alternative Diversity

Travel alternative diversity refers to the existence of multiple modes and effective routes available for travelers, or the number of effective connections between a specific O-D pair. We use K_{rs} to denote the set of available routes connecting a generic O-D pair (r,s), and $|K_{rs}|$ to denote the number of routes between O-D pair (r,s). A route consists of a set of links, which are characterized by zero-one variable denoting the state of each link (operating or failed). If there is only one route between O-D pair (r,s), i.e., $|K_{rs}|=1$, the travelers from origin r cannot reach destination s when one or more links on this single route are failed under an earthquake or a severe traffic accident. Note that more available routes correspond to more opportunities of realizing the evacuation trips when encountering disastrous events. Hence, it is vital to provide multiple alternatives, particularly for an important O-D pair with a large amount of commuting trips.

Conceptually, the travel alternative diversity is general. According to the specification of available routes, we may use simple routes, efficient routes (Dial, 1971), or distinct routes (Kurauchi *et al*., 2009). Even within the category of efficient routes, there are also different definitions such as 'always moving further away from the origin and closer to the destination', 'always moving further away from the origin' (Dial, 1971), 'either always moving further away from the origin or always moving closer to the destination' (Tong, 1990), and 'efficient and not-too-long routes' (Leurent, 1997). Note that the specification of route diversity needs to explicitly consider the tradeoff between computational tractability and modeling realism. For example, it is known that there is no polynomial-time algorithm that is able to count the number of different simple routes between an O-D pair (Valiant, 1979; Meng *et al.*, 2005). Also, counting distinct routes with acceptable travel time between each O-D pair is computationally non-trivial due to the bi-level programming structure (Kurauchi *et al*., 2009). On the other hand, counting the efficient routes seems computationally efficient according to the polynomial-time combinatorial algorithm of Meng *et al*. (2005). In view of the computational advantage, we focus on the specification of efficient routes but with two important modifications to enhance the assessment realism of route diversity.

Travelers might not treat all simple or efficient routes as their effective alternatives. Shorter detoured routes with an acceptable travel cost (i.e., not-too-long routes) are more likely to be considered by travelers as a reasonable substitution when the primary or secondary route is not available. In addition, as different routes may share the same links or segments in the network, the number of routes may drop significantly when one main link fails to function. Below we model the above two requirements.

If a route includes only links that make travelers further away from the origin, it is an efficient route (Dial, 1971). Mathematically, all links in an efficient route satisfy

$$l_r\left(\text{head}_a\right) > l_r\left(\text{tail}_a\right), \forall a \in \Gamma_k, \tag{1}$$

where tail_a and head_a are the tail and head of link a; $l_r(\text{tail}_a)$ and $l_r(\text{head}_a)$ are respectively the shortest route cost from origin r to the tail and head of link a; Γ_k is the set of links on route k.

The requirement of *shorter detoured routes with an acceptable cost* (i.e., not-too-long routes) can be implemented in a link manner by requiring every link is reasonable enough relative to the shortest path (Leurent, 1997). Mathematically,

$$\left(1+\tau_r^a\right)\left(l_r\left(\text{head}_a\right)-l_r\left(\text{tail}_a\right)\right) \geq l_a,\ \forall a \in \Gamma_k, \tag{2}$$

where l_a is the cost (e.g., length or free-flow travel time) of link a; τ_r^a is an allowable/acceptable elongation ratio for link a with respect to origin r. τ_r^a may be set to 1.6 for inter-urban studies or between 1.3 and 1.5 for urban studies (Tagliacozzo and Pirzio, 1973; Leurent, 1997). By summing up all links on route k, we have

$$\begin{aligned} l_k = \sum_{a\in\Gamma_k} l_a &\leq \sum_{a\in\Gamma_k}\left(1+\tau_r^a\right)\left(l_r\left(\text{head}_a\right)-l_r\left(\text{tail}_a\right)\right) \\ &\leq \sum_{a\in\Gamma_k}\left(1+\tau_r^{\max}\right)\left(l_r\left(\text{head}_a\right)-l_r\left(\text{tail}_a\right)\right) \\ &= \left(1+\tau_r^{\max}\right)\left(l_r(s)-l_r(r)\right) = \left(1+\tau_r^{\max}\right)\min_p l_p \quad, \end{aligned} \tag{3}$$

where l_k (or l_p) is the cost of route k (or p); $l_r(r)$ and $l_r(s)$ are the shortest costs from origin r to r and to destination s; and $\tau_r^{\max} = \max_{a\in\Gamma_k} \tau_r^a$. One can see that Eq. (2) is at the link level, which circumvents the computationally demanding path enumeration issue. Also, it can ensure that the route cost does not exceed $\left(1+\tau_r^{\max}\right)$ times of the shortest path cost as shown in Eq. (3). For illustration purposes, Fig. 1 provides an example for the not-too-long route. This simple network has one O-D pair, five links (their costs are shown in the figure) and three routes. We look at the lower detoured link. The elongation ratio is set at 1.6. The left-hand side of Eq. (2) is (1+1.6)(l_r(2)-l_r(1))=2.6, which is less than the link cost of 3. Hence, this link is not a reasonable link with respect to origin r.

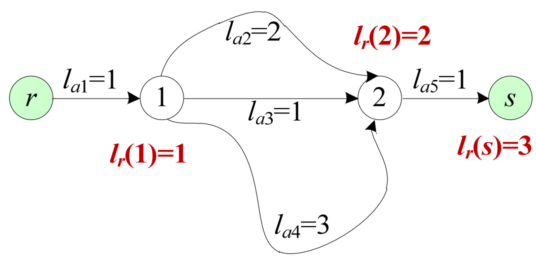

Fig. 1. Illustration of not-too-long route.

The route overlapping issue could be considered by modifying link costs. For example, a link-size factor (Fosgerau *et al*., 2013) or a link-based commonality factor (Russo and Vitetta, 2003) could be added to the link cost to 'penalize' the link shared by multiple routes, and subsequently the check of efficient route in Eq. (1). However, this manner is not intuitive and it is difficult to quantify the impact of link-size factor or link-based commonality factor. In this paper, we use an indirect way to treat this requirement. Specifically, we evaluate the *number of efficient routes from an O-D pair using a particular link* N_a^{rs}, which is also referred to as the link multiplicity (Russo and Vitetta, 2003). This information would also assist in identifying critical links associated with network redundancy. A link used by a large number of efficient routes is obviously an important link, whose disruption will have a significant impact on the network.

Remark 1: Note that the above definition of travel alternative diversity is at an O-D pair level. In other words, we obtain an assessment on the number of effective connections for each O-D pair. For a more aggregate assessment, we could simply add up the O-D travel alternative diversity to different spatial levels (e.g., zonal and network levels) according to different evaluation purposes. However, more travelers within an O-D pair typically need more available routes to disperse travels. To cater for this consideration, the aggregation could explicitly consider the effect of travel demand as weights on route alternative diversity.

2.2 Network Spare Capacity

The travel alternative diversity is assessed using only network topology characteristics. It lacks interactions between transport demand and supply. Capacity is not explicitly considered in the evaluation of travel alternatives (i.e., mode and route). Also, *congestion effect* and *travelers' choice behavior* are two critical characteristics of transportation systems. In order to adequately capture these characteristics, we consider network spare capacity as the second dimension of network redundancy. Network spare capacity can consider the rerouting traffic and stochasticity under disruptions. Evaluating the network-wide capacity is not trivial since it is not just a simple sum of the individual link capacities. Multiple O-D pairs exist and the demands between different O-D pairs are not exchangeable or substitutable. Also, mode and route choice behaviors have to be considered in estimating the multi-modal network capacity. Disruption on an auto link may increase the travel time of auto mode or even change the availability of auto mode. This may further lead to flow shift between modes, changing the multi-modal network capacity.

For the network capacity model, Wong and Yang (1997) proposed the concept of reserve capacity for a signal-controlled road network. It was defined as the largest multiplier μ applied to a given existing O-D demand matrix $\mathbf{q}$ that can be allocated to a network without violating a pre-specified level of service (or maximum flow-to-capacity ratio). The largest value of μ indicates whether the current network has spare capacity or not: $\mu\mathbf{q}$ is the maximum throughput of the network; if μ>1, the current network has a reserve (or spare) capacity amounting to 100(μ-1) percent of $\mathbf{q}$; otherwise, the current network is overloaded by 100(1- μ) percent of $\mathbf{q}$. Yang *et al*. (2000) formulated the network capacity and level of service problem as determining the maximum zonal trip generation subject to combined trip distribution and assignment equilibrium constraints. Gao and Song (2002) extended the reserve capacity model of Wong and Yang (1997) by considering O-D pair-specific demand multipliers. Chen and Kasikitwiwat (2011) and Chen *et al.* (2013) further detailed the network capacity model of Yang *et al*. (2000) as the ultimate and practical network capacity models in assessing the capacity flexibility and capacity reliability of transportation networks.

To cater for the consideration of both mode choice and route choice, we propose a multi-modal network spare capacity model. It quantifies the maximum throughput of a network while considering travelers' mode choice and route choice behaviors as well as the congestion effect. We use the Logit model to capture the travelers' mode choice behavior. As to the route choice behavior, we adopt the C-logit model proposed by Cascetta *et al*. (1996) to account for similarities between overlapping routes by adding a commonality factor (CF) in the systematic utility term. The C-logit model has been used in many applications, such as the path flow estimator for estimating O-D trip table from traffic counts (Bell, 1998), microscopic traffic simulation (e.g., AIMSUM), and network design problem (Yin *et al*., 2009). The popularity is due to its analytical closed-form probability expression, relatively low calibration effort, and sound rational behavior consistent with random utility theory. Zhou *et al.* (2012) developed equivalent mathematical formulations of the C-logit stochastic user equilibrium (SUE) assignment problem, Xu *et al.* (2012) developed path-based algorithms for solving the C-logit SUE problem, while Chen *et al.* (2012) examined the scaling effect and overlapping problem in various logit-based SUE models. By integrating the Logit and C-logit models together, the combined mode and route choice model used in the network spare capacity has a consistent modeling rationale between mode and route choices as well as an explicit consideration of route overlapping.

For simplicity, we consider two modes: road traffic and metro traffic. Finding the multi-modal network spare capacity can be formulated as the following bi-level programming (BLP) problem:

$$\max \mu, \tag{4}$$

s.t.

$$v_a(\mu) \leq \theta_a C_a, \ \forall a \in A, \tag{5}$$

$$q_{rs}^{\text{metro}}(\mu) \leq \bar{q}_{rs}^{\text{metro}}, \ \forall r \in R, s \in S, \tag{6}$$

where A is the set of links in the road network; R and S are the sets of origins and destinations, respectively; θ_a is a parameter denoting the pre-specified maximum flow-to-capacity ratio required on link a; C_a is the capacity of link a; $\bar{q}_{rs}^{\text{metro}}$ is the capacity of the metro line between O-D pair (r,s); $v_a(\mu)$ is the flow on link a and $q_{rs}^{\text{metro}}(\mu)$ is the metro travel demand, which are obtained by solving the lower-level combined mode split and traffic assignment model under a given multiplier μ:

$$
\begin{aligned}
\min Z\left(\mathbf{v}, \mathbf{q}^{\text{metro}}\right) = & \sum_{a \in A} \int_0^{v_a} t_a\left(w\right) dw \\
& + \frac{1}{\theta_1} \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} f_k^{rs} \ln f_k^{rs} + \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} f_k^{rs} CF_k^{rs} \quad , \\
& + \sum_{r \in R} \sum_{s \in S} \int_0^{q_{rs}^{\text{metro}}} \left(\frac{1}{\theta_2} \ln \frac{w}{q_{rs}^{\text{total}} - w} + u_{rs}^{\text{metro}} - \varphi_{rs}^{\text{metro}} \right) dw
\end{aligned} \tag{7}
$$

$$
s.t. \quad q_{rs}^{\text{total}} = \mu \cdot q_{rs}^{0}, \ \forall r \in R, s \in S, \tag{8}
$$

$$
\sum_{k \in K_{rs}} f_k^{rs} + q_{rs}^{\text{metro}} = q_{rs}^{\text{total}}, \ \forall r \in R, s \in S, \tag{9}
$$

$$
v_a = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} f_k^{rs} \delta_{ak}^{rs}, \ \forall a \in A, \tag{10}
$$

$$
f_k^{rs} \geq 0, \ \forall k \in K_{rs}, r \in R, s \in S, \tag{11}
$$

$$
0 < q_{rs}^{\text{metro}} \leq q_{rs}^{\text{total}}, \ \forall r \in R, s \in S, \tag{12}
$$

where t_a is the travel time on link a in the road network; f_k^{rs} is the flow on route k between O-D pair (r,s); CF_k^{rs} is a commonality factor (CF) of route k between O-D pair (r,s); θ_1 and θ_2 are parameters associated with route choice and mode choice; q_{rs}^{total} and q_{rs}^{metro} are the total travel demand and metro travel demand of O-D pair (r,s) corresponding to the network capacity; q_{rs}^{0} is the *current* total travel demand of O-D pair (r,s); u_{rs}^{metro} is the fixed travel cost of metro between O-D pair (r,s); $\varphi_{rs}^{\text{metro}}$ is the exogenous attractiveness of metro between O-D pair (r,s); δ_{ak}^{rs} is the link-route incidence indicator: $\delta_{ak}^{rs}=1$ if link a is on route k between O-D pair (r,s), $\delta_{ak}^{rs}=0$ otherwise; π_{rs} is the Lagrangian multiplier associated with Eq. (9). As to the CF, Cascetta *et al*. (1996) proposed several functional forms, and a typical form is as follows:

$$
CF_k^{rs} = \beta \ln \sum_{l \in K_{rs}} \left(\frac{L_{kl}}{\sqrt{L_k} \sqrt{L_l}} \right)^{\gamma}, \ \forall k \in K_{rs}, r \in R, s \in S, \tag{13}
$$

where L_{kl} is the length of links common to routes k and l, L_k and L_l are the overall lengths of routes k and l, respectively, and β and γ are two parameters. If β is equal to zero, the C-logit model collapses to the traditional Logit model.

In the above formulation, the objective function in Eq. (4) is to maximize the multiplier μ. In essence, it is to maximize the throughput of the multi-modal network, i.e., $\sum_{rs} q_{rs}^{\text{total}} = \mu \cdot \sum_{rs} q_{rs}^{0}$; Eq. (5) is the road link maximum flow-to-capacity ratio constraint or capacity constraint; Eq. (6) is the metro line capacity constraint; Eq. (8) links the current O-D demand and the 'future' O-D demand corresponding to the network capacity; Eq. (9) is the demand conservation constraint; Eq. (10) is a definitional constraint that sums up all route flows that pass through a given link; and Eqs. (11)-(12) are non-negativity constraints on route flows and metro demands. Eqs. (4) and (8) use a uniform multiplier in the network reserve capacity model. The network reserve capacity using a predefined travel pattern can be considered as a conservative estimation since no available information is known about the future growth or decline (stochasticity). By deriving the first-order optimality conditions, we have the following C-Logit model for route choice and binary Logit model for mode choice, respectively.

$$P_k^{rs} = \frac{\exp\left(-\theta_1\left(c_k^{rs} + CF_k^{rs}\right)\right)}{\sum_{l \in K_{rs}} \exp\left(-\theta_1\left(c_l^{rs} + CF_l^{rs}\right)\right)}, \quad \forall k \in K_{rs}, r \in R, s \in S, \tag{14}$$

$$\frac{q_{rs}^{\text{metro}}}{q_{rs}^{\text{total}}} = \frac{1}{1 + \exp\left(-\theta_2\left(\pi_{rs} - u_{rs}^{\text{metro}} + \varphi_{rs}^{\text{metro}}\right)\right)}, \quad \forall r \in R, s \in S. \tag{15}$$

From Eq. (15), a large π_{rs} (i.e., road traffic O-D cost), a small u_{rs}^{metro} (i.e., metro traffic O-D cost) or a large $\varphi_{rs}^{\text{metro}}$ (i.e., metro attractiveness) corresponds to a large choice proportion of metro traffic.

Remark 2: Santos *et al*. (2010) developed the following *weighted link spare capacity* measure to quantify the *network-wide spare capacity*:

$$\sum_{a \in A} \left(C_a - v_a\right)^{\alpha} v_a L_a \Big/ \sum_{a \in A} v_a L_a, \tag{16}$$

where L_a is the length of link a; α is a weighting parameter. When α is larger than 1.0, the spare capacities are large but concentrated on a small number of links; otherwise, the spare capacities are relatively small but more dispersed across the network. Note that the denominator is the total vehicle miles traveled. Thus, this measure is the aggregation of *link* spare capacity (i.e., $C_a - v_a$) weighted by the relative proportion of vehicle miles traveled on this link. This weighing scheme implies that we pay more attention to the spare capacity on long and heavy-flow links. This measure is simple and easy to calculate. However, it only serves as a '*proxy*' or a localized approximation of the network-wide spare capacity. In contrast, the network spare capacity measure adopted in this paper is a global (or network-wide) measure based on an optimization-based approach that can explicitly determine the maximum throughput to address the question of '*how much additional demand can this multi-modal network accommodate?*' This desirable feature enables planners to have a systematic assessment of multi-modal network spare capacity.

3 COMPUTATIONAL METHODS OF NETWORK REDUNDANCY

This section provides the computational methods for evaluating the two network redundancy measures.

3.1 Evaluating Route Diversity

Meng *et al*. (2005) developed a combinatorial algorithm with polynomial-time complexity for counting the number of efficient routes between an O-D pair. This algorithm consists of two parts: 1) constructing a sub-network for each origin r, $G_r=(N_r, A_r)$, and 2) counting the number of efficient routes from origin r to all nodes in the sub-network $G_r=(N_r, A_r)$. The sub-network $G_r=(N_r, A_r)$ is a connected and acyclic network. The concept of *efficient routes* is used in the sub-network construction. In other words, the sub-network only includes the links that are on the efficient routes from this origin. Also, we modify Meng *et al*. (2005) to explicitly consider the requirement of *not-too-long routes*. The procedure of constructing the sub-network $G_r=(N_r, A_r)$ is as follows.

Constructing the sub-network $G_r=(N_r, A_r)$
For each origin r,
    Perform a shortest route algorithm to find the minimum cost from origin r to all
        nodes, $l_r(n)$, $n\neq r$
    For all nodes $n\neq r$
        If $l_r(n)=\infty$
            $N_r=N_r\backslash\{n\}$
    For all links a
        If $l_r(\text{tail}_a) \geq l_r(\text{head}_a)$ or $\left(1+\tau_r^a\right)\left(l_r\left(\text{head}_a\right)-l_r\left(\text{tail}_a\right)\right)<l_a$
            $A_r=A_r\backslash\{a\}$

Note that the last if-then condition is added to satisfy the requirement of *not-too-long routes*. In other words, all links in the sub-network are in an efficient route and also in an effective route with an acceptable elongation ratio relative to the shortest route (i.e., reasonable with respect to origin r).

Counting the number of efficient routes from origin r to all nodes in the sub-network is essentially based on the node adjacent matrix operation. In the following, we present the procedure for counting the different efficient routes for each origin r. For the theoretical proof, please refer to Meng *et al*. (2005).

Counting the number of efficient routes from origin *r* to all nodes

Step 1 Initialization:

$\mathbf{u}=\mathbf{0}(|N_r|, |N_r|)$

For all links $a \in A_r$

$u(\text{tail}_a, \text{head}_a)=1$

Step 2 Matrix Operations:

For all nodes $j \in N_r$

For all nodes $m \in N_r \backslash j$

For all nodes $n \in N_r \backslash j \backslash m$

$u(m, n) := u(m, n) + u(m, j) \times u(j, n)$

Based on the number of efficient routes counted above, we can further evaluate the number of efficient routes using a particular link:

$$N_a^{rs} = u(r, \text{tail}_a) \times u(\text{head}_a, s), \tag{17}$$

where N_a^{rs} is the number of efficient routes between O-D pair (r,s) using link a; $u(r, \text{tail}_a)$ and $u(\text{head}_a, s)$ are the number of efficient routes between node pair (r, tail_a) and between node pair (head_a, s), respectively. If N_a^{rs} is equal to $u(r, s)$, then all efficient routes connecting O-D pair (r, s) need to traverse link a.

3.2 Evaluating Multi-Modal Network Spare Capacity

The multi-modal network spare capacity model is a bi-level programming (BLP) problem. Solving this BLP problem is not a trivial task because evaluating the upper-level objective function (i.e., multiplier μ) requires solving the lower-level subprogram and also considering the capacity constraints in the upper-level subprogram. The main challenge lies in the implicit and nonlinear functions of link flow and metro demand with respect to the multiplier μ in Eqs. (5)-(6). Hence, despite with a simple linear objective function, the upper-level programming has a nonlinear and implicitly defined constraint set. To handle this issue, we can use the first-order Taylor expansion to linearly approximate the implicit link flow function $v_a(\mu)$ and metro demand function $q_{rs}^{\text{metro}}(\mu)$ at the current point $\mu^{(n)}$.

$$v_a(\mu) \approx v_a\left(\mu^{(n)}\right) + \nabla_\mu v_a\left(\mu^{(n)}\right) \cdot \left(\mu - \mu^{(n)}\right), \ \forall a \in A, \tag{18}$$

$$q_{rs}^{\text{metro}}(\mu) \approx q_{rs}^{\text{metro}}\left(\mu^{(n)}\right) + \nabla_\mu q_{rs}^{\text{metro}}\left(\mu^{(n)}\right) \cdot \left(\mu - \mu^{(n)}\right), \ \forall r \in R, s \in S, \tag{19}$$

where $v_a\left(\mu^{(n)}\right)$ and $q_{rs}^{\text{metro}}\left(\mu^{(n)}\right)$ are the link flow and metro demand under multiplier $\mu^{(n)}$, which can be obtained by solving the lower-level programming; $\nabla_\mu v_a\left(\mu^{(n)}\right)$ and $\nabla_\mu q_{rs}^{\text{metro}}\left(\mu^{(n)}\right)$ are the derivatives of link flow and metro demand with respect to multiplier μ, which can be obtained from the sensitivity analysis method. For our case, the Logit-based probability expression for both mode and route choice dimensions ensures that the solution to the lower-level programming is unique. Hence, the standard sensitivity analysis method for nonlinear programming problem can be used directly to derive the sensitivity information. Interested readers may refer to Yang and Chen (2009) and Yang *et al.* (2013) for the detailed derivation. With the above linear approximations, the nonlinear and implicitly defined constraints in Eqs. (5)-(6) can be approximated as

$$v_a\left(\mu^{(n)}\right) + \nabla_\mu v_a\left(\mu^{(n)}\right) \cdot \left(\mu - \mu^{(n)}\right) \le \theta_a C_a, \ \forall a \in A, \tag{20}$$

$$q_{rs}^{\text{metro}}\left(\mu^{(n)}\right) + \nabla_\mu q_{rs}^{\text{metro}}\left(\mu^{(n)}\right) \cdot \left(\mu - \mu^{(n)}\right) \le \bar{q}_{rs}^{\text{metro}}, \ \forall r \in R, s \in S. \tag{21}$$

Note that Eqs. (20)-(21) are linear inequalities with a single solution variable μ. Hence, the upper-level programming becomes a linear programming with a single continuous variable, which is readily solvable. The solution to the above approximated linear programming generates a new solution point $\mu^{(n+1)}$, which will be iteratively used to construct a new linear approximation of Eqs. (5)-(6). Essentially, we solve a sequence of linear approximations to the upper-level nonlinear problem. Below we present the sensitivity analysis-based algorithm for solving the bi-level programming formulated multi-modal network spare capacity.

Estimating multi-modal network spare capacity

Step 1: Determine an appropriate initial value $\mu^{(0)}$, and set n=0.

Step 2: Solve the lower-level combined modal split and traffic assignment model based on $\mu^{(n)}$ and obtain the link flow pattern $\mathbf{v}(\mu^{(n)})$ and metro demand pattern $\mathbf{q}^{\text{metro}}(\mu^{(n)})$.

Step 3: Calculate the derivatives $\nabla_\mu v_a\left(\mu^{(n)}\right)$ and $\nabla_\mu q_{rs}^{\text{metro}}\left(\mu^{(n)}\right)$ using the sensitivity analysis method for the Logit-based combined modal split and traffic assignment problem.

Step 4: Formulate a linear approximation of Eqs. (5)-(6) using the derivative information, and solve the resultant linear programming to obtain a new multiplier $\mu^{(n+1)}$.

Step 5: If $\left|\mu^{(n+1)} - \mu^{(n)}\right| \le \varepsilon$, then terminate, where ε is a predetermined tolerance error; otherwise, let n:=n+1, and go to Step 2.

4 NUMERICAL EXAMPLES

This section provides numerical examples to demonstrate the desirable features of the two network redundancy measures (particularly the necessity of having two dimensions for systematically characterizing transportation network redundancy) and the applicability of the evaluation methodology in a large-scale transportation network.

4.1 Example 1: Simple Network

Example 1 uses a simple network, shown in Fig. 2, to demonstrate the features of the proposed network redundancy measures. This network has six nodes, seven links, two origins, two destinations, and four O-D pairs. The travel demand of O-D pairs (1, 3), (1, 4), (2, 3) and (2, 4) are 40, 10, 10, and 50, respectively. We use the standard BPR (bureau of public road)-type road link performance function:

$$t_a(v_a) = t_a^0\left[1 + 0.15(v_a/C_a)^4\right], \tag{22}$$

where t_a^0 is the free-flow travel time on link a. The free-flow travel time and capacity of the seven road links are also shown in Fig. 2.

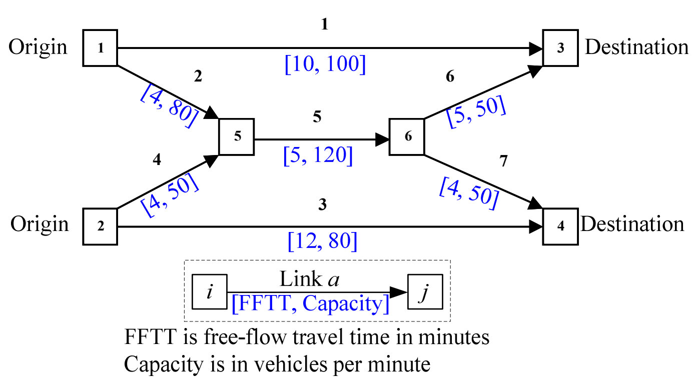

Fig. 2. Network in Example 1.

We consider the following eight scenarios of network reconfiguration or improvement as shown in Table 1.

Table 1. A set of scenarios for the small network.

Scenario	Description
0	The current road network (base case)
1	Construct a new road from node 1 to node 2 (t_a^0=4, C_a=80)
2	Expand the capacity of link 5 by 50%
3	Expand the capacity of link 3 by 50%
4	Construct a new road from node 1 to node 6 (t_a^0=6, C_a=80)
5	Construct a new road from node 2 to node 6 (t_a^0=6, C_a=80)
6	Construct a metro line from origin 1 to destination 3
7	Construct a metro line from origin 2 to destination 4

The purpose of *Scenario 1* is to enhance the connection between the two origins (i.e., 1 and 2) by constructing a local road parallel to the major highway between two communities so that travelers do not need to enter the major highway. *Scenario 2* is designed to expand the capacity of link 5 by 50%. Link 5 seems to be a critical link from a pure network topology perspective. It serves all four O-D pairs, and it is part of the single route that serves O-D pairs (1, 4) and (2, 3). *Scenario 3* serves as a comparison of Scenario 2 by expanding the capacity of link 3 by 50%. In addition, O-D pairs (1, 3) and (2, 4) have a large travel demand volume. In order to enhance their route connections, we construct a new road from node 1 to node 6 and from node 2 to node 6 in *Scenario* 4 and *Scenario* 5, respectively. In addition, *Scenario 6* and *Scenario 7* construct a metro mode to connect O-D pair (1, 3) and O-D pair (2, 4), respectively. For both O-D pairs, the fixed metro travel time deducted by the exogenous attractiveness of metro is equal to 8 equivalent minutes. The capacity of the two metro lines is equal to 150 units. Other parameters are set as follows: θ_1=2 (route choice), θ_2=1 (mode choice), β=1 and γ=1 (commonality factor). The main purpose of Example 1 is to demonstrate why it is necessary to have two dimensions for systematically characterizing the transportation network redundancy. To this end, we use the simplest specification for each dimension: we enumerate all simple routes rather than counting the effective routes in Example 1.The details of the two measures (e.g., efficient route, not-too-long route, etc.) and their evaluation methodologies will be discussed in Example 2.

(1) General Relationship of the Two Dimensions

First of all, we examine how travel alternative diversity and network spare capacity complement each other for network redundancy characterization. The number of travel alternatives (e.g., routes and modes) of the four O-D pairs and the network capacity multiplier under the above eight scenarios are shown in Table 2.

Table 2. Network redundancy performances under different scenarios.

Scenario	Number of alternatives (modes + routes)				Network spare capacity (multiplier)
	O-D (1,3)	O-D (1,4)	O-D (2,3)	O-D (2,4)	
0	2	1	1	2	2.16
1	***3***	***3***	1	2	***2.16***
2	2	1	1	2	2.15 (↓)
3	2	1	1	2	2.51 (↑)
4	**3**	**2**	1	2	2.15 (↓)
5	2	1	**2**	**3**	0.93 (↓)
6	3	1	1	2	2.16
7	2	1	1	3	2.51 (↑)

By comparing Scenario 1 with Scenario 0 (base case), constructing a new road from node 1 to node 2 will increase the number of connections of O-D pairs (1, 3) and (1, 4) from 2 to 3 and from 1 to 3, respectively. However, the network spare capacity multiplier is the same as that in Scenario 0. On the other hand, the comparison among Scenario 2, Scenario 3 and Scenario 0 indicates that expanding these link capacities can only change (decrease in Scenario 2 and increase in Scenario 3) the network spare capacity while keeping the number of connections intact. Scenarios 0, 1, 4, and 6 have similar or even identical network spare capacity values, whereas the numbers of connections of O-D pairs (1, 3) and (1, 4) are obviously different. In addition, Scenario 4 and Scenario 5 change both dimensions simultaneously. However, they increase the number of connections but decrease the network spare capacity. Adding a new road may not always increase the network-wide spare capacity (to be explained later). Similar phenomenon also occurs in Scenario 6 and Scenario 7 of constructing a metro line for the two O-D pairs with large travel demands. It seems that there exists a trade-off between travel alternative diversity and network spare capacity. Using either travel alternative diversity or network spare capacity solely may not be able to capture the full picture of network redundancy under different network reconfiguration or enhancement schemes. However, they can complement each other to provide a two-dimensional transportation network redundancy characterization. This also shows the importance of 'integrating' the two dimensions in order to avoid a biased network redundancy assessment. Therefore, we need to optimize them simultaneously (as a bi-objective problem) in order to design an optimal redundant transportation network.

(2) Travel Alternative Diversity

Secondly, we examine the travel alternative diversity dimension. Note that the basic definition of alternative (mode and route) diversity is at an O-D pair level measuring the number of connections for a specific O-D pair. However, we can aggregate it to different spatial levels according to planners' different evaluation purposes. Below we use the corresponding O-D demands as the weights to aggregate the O-D pair level to zonal level and network level in Table 3. Scenario 4 and Scenario 5 have a symmetric number of connections at the O-D pair level. However, the numbers of connections at the zonal and network levels are different as indicated in Table 3. Particularly, the numbers of routes to destination 3 and destination 4 are the same (i.e., 4) in both scenarios, whereas the aggregated numbers of connections to these two destinations are different. The reason is that the above aggregation explicitly considers the effect of travel demand on route diversity. Typically, O-D pairs with large travel demands need more available routes to disperse the travel demands. In addition, constructing a new road from node 1 to node 6 in Scenario 4 (from node 2 to node 6 in Scenario 5) is quite beneficial for the connections of origin 1 and destination 3 (origin 2 and destination 4). Similar changes also occur in Scenarios 6 and 7 when constructing a new metro line from origin 1 to destination 3 and from origin 2 to destination 4. The direct users of this dimension are travelers. Particularly, the evacuees from a residential zone are eager to know how many choices (routes and/or modes) are available for getting to a particular shelter. In addition, the network planners could use the above aggregation for improving the route diversity of important zones.

Table 3. Travel alternative diversity under different scenarios.

Scenario	O-D (1,3)	O-D (1,4)	O-D (2,3)	O-D (2,4)	O-1	O-2	D-3	D-4	Network
0/2/3	2	1	1	2	1.80	1.83	1.80	1.83	1.82
1	3	3	1	2	3.00	1.83	2.60	2.17	2.36
4	3	2	1	2	2.80	1.83	2.60	2.00	2.27
5	2	1	2	3	1.80	2.83	2.00	2.67	2.36
6	3	1	1	2	2.60	1.83	2.60	1.83	2.18
7	2	1	1	3	1.80	2.67	1.80	2.67	2.27

(3) Network Spare Capacity

Thirdly, we explain why the network spare capacity has different changes under the above scenarios. Recall that the network spare capacity model determines the maximum throughput of the network while considering congestion effect, route choice behavior (via the C-Logit model with route overlapping consideration) and mode choice behavior (via the Logit model). Also, the link capacity constraint is a main barrier of preventing the network capacity increase. Table 4 shows the binding links (i.e., link flow equals capacity) in network capacity evaluation under the above scenarios.

Table 4. Binding links in network capacity evaluation under different scenarios.

Scenario	Binding Links							
	1	2	3	4	5	6	7	8
0	×	×	√	√	×	×	√	×
1	×	×	√	√	×	×	√	×
2	×	×	×	√	×	×	√	×
3	√	×	×	×	×	×	×	×
4	×	×	√	√	×	×	√	×
5	×	×	×	×	×	×	√	×
6	×	×	√	√	×	×	√	×
7	√	×	×	×	×	×	×	×

- *Scenario 1*: In the base scenario, links 3, 4, and 7 are the binding links. These three links remain active in Scenario 1 despite a new road is added from node 1 to node 2. Accordingly, the network spare capacity cannot be further increased. The reason is that: even though it increases the number of connections of origin 1, this new road is seldom used by travelers due to the large route travel cost.
- *Scenarios 2 and 3*: These two scenarios expand the capacity of link 5 and link 3, respectively. Link 5 seems to be a critical link from a pure network topology perspective. However, it is not a critical link in terms of congestion as shown in Table 4. Expanding link 5 in Scenario 2 may divert some travelers from link 3 to links 4-5-7. This diversion will increase the burden on the binding links 4 and 7, leading to a slight decrease of network capacity. Instead, link 3 is actually the most critical binding link in this network (to be further shown in Fig. 3). Scenario 3 considers all three critical binding links by expanding link 3. Hence, it has a substantial increase of network spare capacity.
- *Scenarios 4 and 5*: From a pure network topology standpoint, these two scenarios have a symmetric effect on network redundancy. This is witnessed by the improvement of route diversity. However, they have significantly different network spare capacity values. In Scenario 4, constructing a new road from node 1 to node 6 will divert flows from links 2 and 5 to the new link. However, the remaining binding link 7 blocks the possible throughput increase of O-D pair (1, 4) (i.e., link 2-5-7). In Scenario 5, constructing a new road from node 2 to node 6 has made links 3 and 4 non-binding. However, the traffic diversion from link 3 and links

4-5-7 to the new road will further overwhelm the binding link 7, making the network overloaded by 7% of the existing O-D demand.

- *Scenarios 6 and 7*: In Scenario 6, the new metro line from origin 1 to destination 3 does not relax the three binding links 3, 4, and 7, since it mainly diverts demands of O-D pair (1, 3) from road to metro. Accordingly, the network spare capacity remains unchanged. However, this scenario is still meaningful since it creates an alternative mode besides the road traffic mode, especially when the road network encounters a significant disruption (e.g., bridge collapse). As to Scenario 7, the construction of a new metro line from origin 2 to destination 4 increases both travel alternative diversity and network spare capacity. It diverts demands of O-D pair (2, 4) from road to metro, relaxing all three binding links.

Finally, we continue to examine the role of the three congested critical links (i.e., links 3, 4 and 7) in the network spare capacity. These congested critical links are associated with the network-wide capacity rather than their individual congestion only, since they are vital for network capacity improvement. Fig. 3 shows the network spare capacity under all possible combinations of their link capacity enhancements. One can observe that link 3 is the most critical link for network spare capacity improvement. All cases with capacity enhancement on link 3 (i.e., Cases 2, 5, 6, and 8) have the largest network capacity multiplier value. After expanding link 3, the original binding capacity constraints on the three critical links become inactive due to the flow shift from links 4-5-7 to link 3, and subsequently the network can thus absorb more demands. However, if we only expand link 4 as in Case 3 (or link 7 in Case 4), flows will be diverted from link 3 to links 4-5-7. This flow shift will increase the burden on the binding link 7 (or 4), resulting in a decrease of network spare capacity. Hence, ranking the congested critical links appropriately enables planners to prioritize the candidate capacity enhancement projects more cost-effectively for improving network spare capacity.

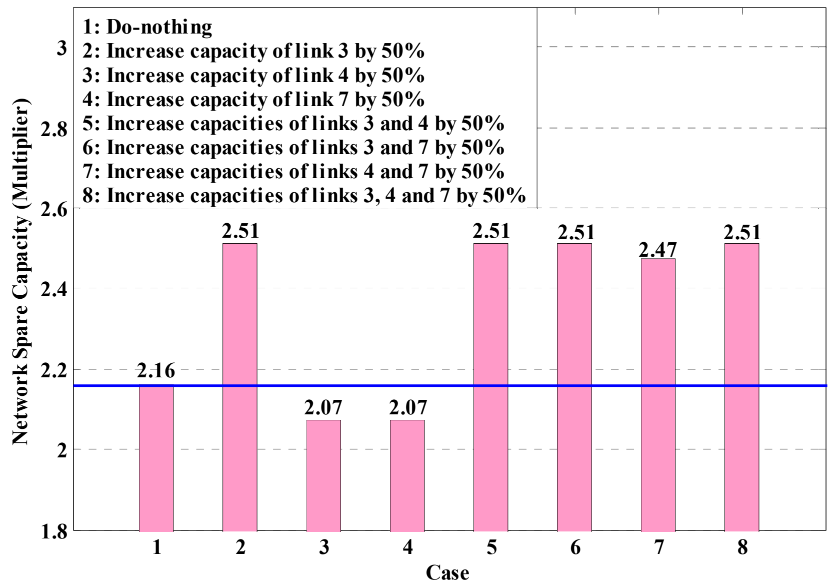

Fig. 3. Network spare capacities under different capacity enhancement schemes.

4.2 Example 2: Winnipeg Network

In this section, we conduct a case study using the Winnipeg network in Manitoba, Canada to demonstrate the applicability of the computational methods. The Winnipeg network, shown in Fig. 4, consists of 154 zones, 1,067 nodes, 2,535 links, and 4,345 O-D pairs. This network includes the perimeter highway serving as a by-pass for the city, and several major arterial roads. There are totally 14 major bridges in the downtown and suburban areas as shown in Fig. 4 (Ryu et al., 2014). They are used as the main crossings of the Red River (eastbound -westbound direction) and the Assiniboine River (northbound-southbound direction). The network structure, O-D trip table, and link performance parameters are from the Emme/2 software (INRO Consultants, 1999). Due to lack of metro data in the Winnipeg network, we only consider the road traffic network. Other parameters are set as follows: θ_1=1.2 (route choice), $\beta = 1$ and $\gamma = 1$ (commonality factor), and $\tau_r^a = 1.4, \forall a \in A, r \in R$ (elongation ratio in not-too-long routes). The computational methods presented in Section 3 are performed to evaluate the travel alternative diversity and network spare capacity.

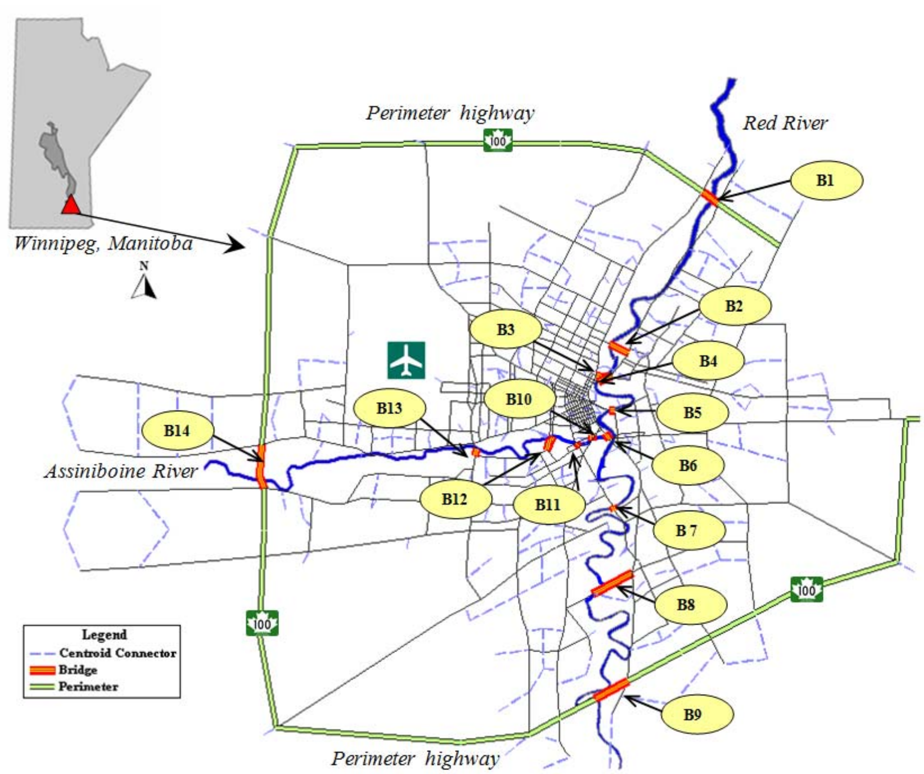

Fig. 4. Winnipeg network and major bridge locations (B1-B14)

(1) Redundancy Assessment of the Current Network

Travel Alternative Diversity: Recall that we define an effective route is not only an efficient route but also a not-too-long route. From Fig. 5, we can see that all O-D pairs have at least one *effective* route and at most 634 *effective* routes. The average number of effective routes per O-D pair is 11.63 and the median is 4 for all O-D pairs. About 62% and 78% of all O-D pairs are connected by at most 5 and 10 effective routes, respectively. As a comparison, Fig. 6 shows the number of *efficient* routes (may be too-long routes relative to the shortest route) for all O-D pairs. One can see that ignoring the requirement of not-too-long routes will significantly overestimate the number of valid connections. Behaviorally, shorter detoured routes with an acceptable travel cost are more likely to be considered by travelers as a reasonable substitution when the primary or secondary route is not available.

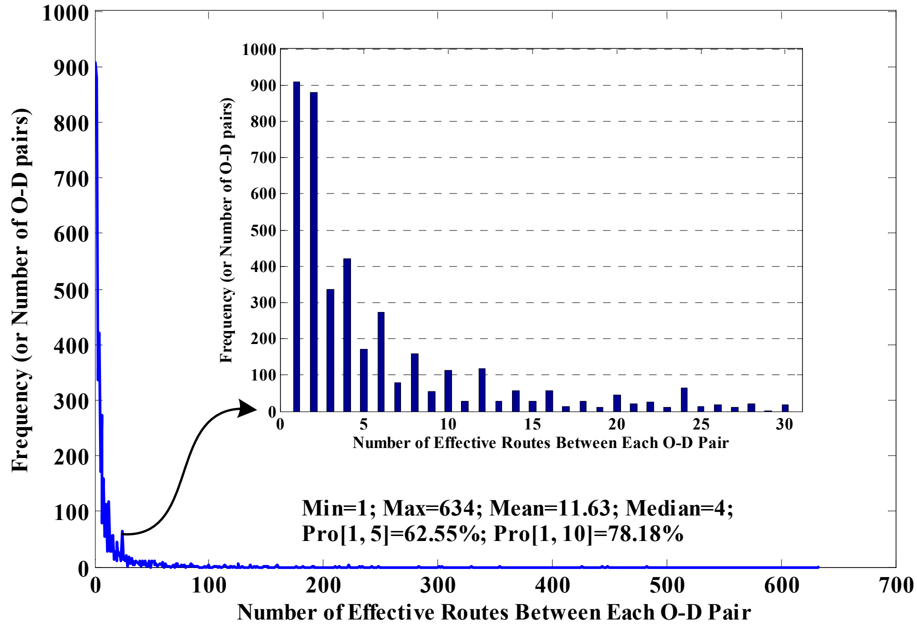

Fig. 5. Number of effective routes (efficient and not-too-long) in Winnipeg network.

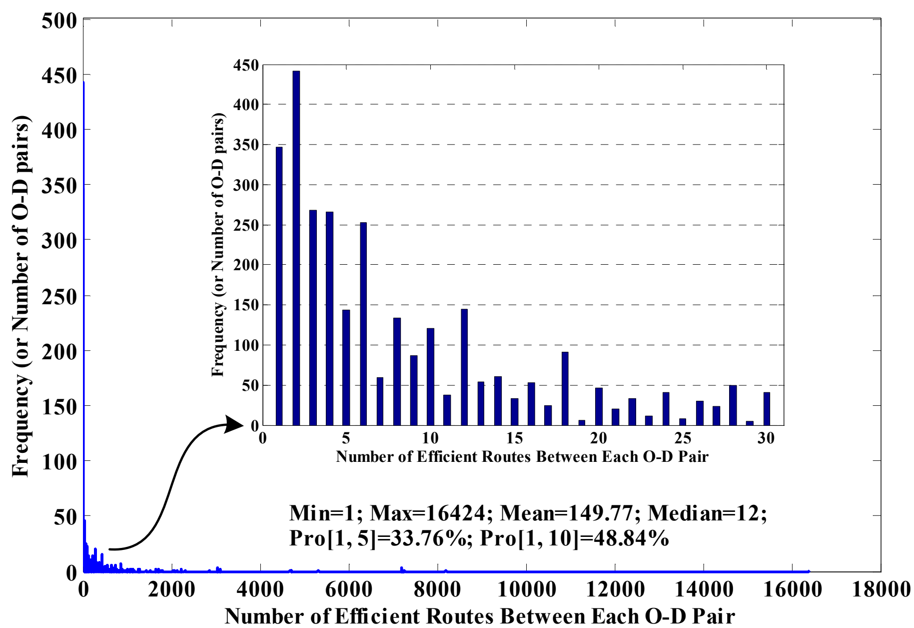

Fig. 6. Number of efficient routes in Winnipeg network.

Network Spare Capacity: As mentioned before, the link capacity constraint is a main barrier of preventing the network capacity improvement. Fig. 7 shows the number of links with V/C>1 under each value of multiplier μ. One can see then at the current demand pattern, the flows of 206 links (i.e., 8% of 2,535 links) exceed their capacities. With the decrease of multiplier μ, the number of links with V/C>1 are reduced quickly. When multiplier μ is equal to 0.37, all links can satisfy the link capacity constraints.

This network appears to have a lot of room for improving the network capacity, at least in this particular scenario. In order to accommodate the current travel demands, planners should improve the network by expanding existing roads, constructing new roads and/or alternative travel modes, or both. Similar to Example 1, congested critical links associated with network capacity should be identified in the planning process.

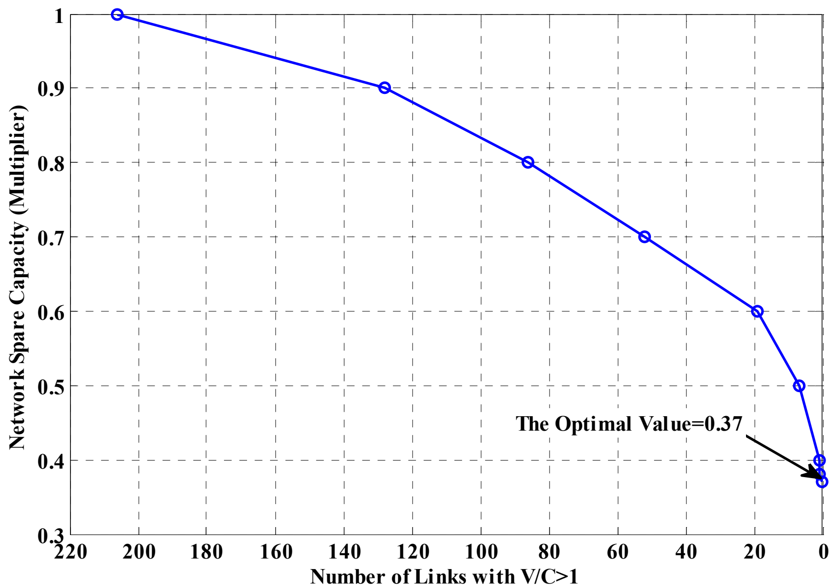

Fig. 7. Network spare capacity of the Winnipeg network.

(2) Redundancy Assessment under Bridge Disruptions

In this section, we assess the network redundancy under various potential disruptions of bridges. For each scenario, we examine the consequence in terms of the two redundancy measures after a bridge disruption. This analysis could assist in the bridge enhancement prioritization and budget allocation, and is useful in assessing and managing bridge safety in practice. Planners could prioritize the candidate retrofit projects for enhancing network redundancy more effectively. Table 5 and Fig. 8 show the average number of effective paths of each O-D pair and the network reserve capacity multiplier under each bridge (bi-directional) disruption. For the average number of effective paths, we do not need to rerun the procedure presented in Section 3.1 for each disruption scenario. Instead, we just need to calculate the number of effective routes between each O-D pair in the pre-disruption network, and then make use of Eq. (17) to calculate the number of effective routes between each O-D pair using each bridge in the pre-disruption network. The number of effective paths in each bridge disrupted network is equal to the number of effective paths in the pre-disruption network deducted by the number of effective routes using that bridge.

As different routes may share the same links or segments in the network, the number of routes may drop significantly when one main bridge fails to function. From Table 5 and Fig. 8, one can see that the disruption of B4 and B6 will significantly reduce the average number of effective paths from 11.66 to 9.47 and 9.77, followed by B10 and B11. The disruptions of other bridges do not have too much impact on route diversity. From the route diversity perspective, Bridges 4 and 6 should be treated as critical bridges that should be strategically protected and enhanced, because 9,524 effective routes of 498 O-D pairs need to use B4, and 8,228 effective routes of 543 O-D pairs need to use B6. Hence, their disruptions will affect many O-D pairs, leading to their critical role in ensuring travelers' route diversity.

As mentioned before, the network reserve capacity is a network-wide aggregated measure of network capacity with an explicit consideration of travelers' route choice adjustment in the process of demand increase. In general, one can imagine that a single-bridge disruption does not have much impact on the network-wide aggregated reserve capacity. Other than B13 and B6, a single-bridge disruption of almost all other bridges does not change the network reserve capacity significantly. The disruptions of B5 and B8 slightly increase the network capacity. This further demonstrates the above characteristics of network reserve capacity measure. A bridge disruption may change the travelers' route choice behaviors, reduce the V/C ratio of some previously binding links, and consequently allows the network to absorb more travel demands. From the network capacity perspective, B13 and B6 should be treated as critical bridges, whose disruptions will reduce the network capacity multiplier from 0.37 to 0.30 and 0.31, respectively. Even though this change seems trivial, in fact they correspond to the dissatisfaction of 7% of the total demand (i.e., 7%×54,459=3,812 trips) due to a single-bridge disruption. To sum up, B4 is the most critical bridge from the perspective of route diversity, followed by B6; B13 is the most critical bridge from the perspective of network reserve capacity, followed by B6. However, when considering the two dimensions simultaneously, B6 appears to be the most critical bridge from both traveler and planner perspectives. Different considerations of the two dimensions lead to different rankings of the critical bridges. Using either dimension solely may not be able to capture the full picture of network redundancy under different bridge disruption scenarios. The ranking of the critical bridges according to their impacts on network redundancy further shows the complementary relationship of the two dimensions of the network redundancy. Overall, the definition, measures and methods of transportation network redundancy assessment are applicable to both normal and failure scenarios.

Table 5. Network redundancy under various bridge disruptions.

Bridge	Average # of effective routes	Network capacity multiplier
B1	11.59	0.37
B2	11.18	0.35
B3	11.47	0.36
B4	***9.47***	0.34
B5	11.20	0.38
B6	***9.77***	***0.31***
B7	11.41	0.37
B8	11.45	0.38
B9	11.64	0.37
B10	10.28	0.36
B11	10.96	0.37
B12	11.54	0.37
B13	11.09	***0.30***
B14	11.64	0.37
Without	11.66	0.37

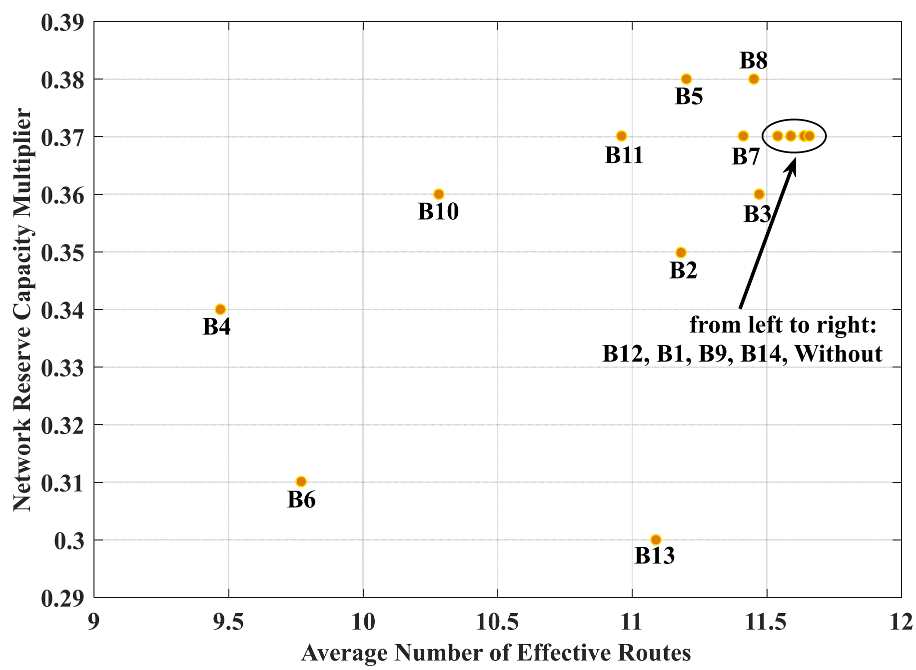

Fig. 8. Network redundancy under various bridge disruptions.

5 Concluding Remarks

This paper developed network-based measures and computational methods to systematically characterize transportation network redundancy, i.e., travel alternative diversity and network spare capacity. Specifically, the travel alternative diversity dimension evaluates the existence of multiple modes and effective routes available for travelers or the number of effective connections between an O-D pair. The network spare capacity dimension quantifies the network-wide residual capacity with an explicit consideration of congestion effect and travelers' route and mode choice behaviors. To implement the two measures in practice, a formal methodology was provided to evaluate the network redundancy.

Two set of numerical examples were provided. Example 1 in the simple network demonstrated the necessity of having the two dimensions together for systematically characterizing transportation network redundancy. Example 2 in the Winnipeg network demonstrated the applicability of the computational methods as well as the importance of considering the requirement of not-too-long routes in the travel alternative diversity measure. The analysis results revealed that the two measures have different characterizations on network redundancy from different perspectives. Using either dimension solely may not be able to capture the full picture of network redundancy under different network reconfiguration/enhancement/failure schemes. They can complement each other by providing meaningful information to travelers as well as assist planners to enhance network redundancy in their infrastructure investment decisions. Adding a new road/metro line or enhancing existing links may not always increase the network capacity. A topologically critical link may not necessarily be a binding link in terms of improving network capacity. A well designed future network with alternative travel modes could significantly increase the network spare capacity to accommodate a substantial demand increase. Multi-modal network redundancy improvement generally involves high capital and long-term investments, and cannot be reversed easily. Therefore, a tailored multi-modal network spare capacity estimation is particularly crucial in the pre-disaster network planning in order to avoid biased and ineffective investment decisions. Particularly, we need to explicitly consider the potential adjustment of travelers' choice behaviors.

Note that vulnerability is another important concept related to resiliency. Network vulnerability has been extensively studied in the literature (e.g., Berdica, 2002; Chen *et al*., 2007, 2012; Taylor, 2012, 2013, 2017; Bell *et al*., 2017). It is particularly applied in the critical link identification, which assesses the potential consequence of disruption scenarios in terms of various performance measures, e.g., travel time/cost/accessibility/network performance/efficiency difference before and after (see, e.g., Nagurney and Qiang, 2009; Xu *et al*., 2017). Redundancy is an excess supply relative to the current travel needs, without assuming/considering unknown disruption/removal scenarios. In fact, the redundancy measures proposed in this paper could also be used as evaluation criteria of network vulnerability, i.e., how many alternatives are remained and how much spare capacity does the network remain if a critical link is disrupted? Recently, Yang *et al.* (2017) applied the route diversity measure to assess the vulnerability of urban rail transit networks in the Beijing metro network.

For future research, we will explore different applications of the redundancy assessment methodology. Also, we will try to integrate the two dimensions to facilitate the redundancy comparison among different cities/regions. In this paper, we considered route overlapping in the C-logit route choice model. Mode similarity could also be considered by a nested logit/weibit model (Kitthamkesorn *et al*., 2016; Kitthamkesorn and Chen, 2017) in a combined modal split and traffic assignment problem. More advanced network capacity models (e.g., Yang *et al*., 2000; Gao and Song, 2002; Chen and Kasikitwiwat, 2011; Chen *et al*., 2013) should be considered in the network redundancy analysis to capture the variations in both travel demand pattern and travel demand volume. To further cater for disaster situations, we need to calibrate link elongation ratios based on stated preference (under hypothetical scenarios) and revealed preference (tracking data under a realistic scenario) studies. In addition, network redundancy improvement belongs to the mixed network design problem (NDP), which involves the continuous capacity expansion for improving the network spare capacity and also the discrete alternative addition for improving both dimensions. The redundant NDP model could be formulated as a bi-objective, bi-level programming problem to improve network resiliency. To solve this complex redundant NDP model, we could use a multi-objective programming approach (e.g., Chen *et al*., 2006) to generate non-dominated network redundancy design schemes, or a goal-programming approach (e.g., Chen and Xu, 2012) to ensure a goal-oriented design for achieving a desired level of network redundancy. We will investigate the mathematical modeling and solution algorithm of this problem in future studies.

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