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A general iterative approach for the system-level joint optimization of pavement maintenance, rehabilitation, and reconstruction planning

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Abstract

We formulate a general bottom-up model for the joint optimization of maintenance, rehabilitation, and reconstruction (MR&R) schedules for a system of heterogeneous pavement segments under budget constraints. The objective is to minimize the total costs incurred to both the highway users and the pavement management agency. We propose a Lagrange multiplier approach together with derivative-free quasi-Newton algorithms to solve the problem for two scenarios: i) with a combined budget constraint for all the treatments; and ii) with one budget constraint for each treatment. The system-level solution approach has the following merits: i) it can be applied to problems with any forms of segment-level models for user and agency costs, deterioration process, and treatment effectiveness, given that the solution to the segment-level problem is available; ii) under the combined budget constraint, it ensures that the optimality gap of the system-level solution is bounded by a term that depends upon the optimality gap of the segment-level solutions; and iii) it exhibits linear complexity with the number of segments.

At the segment level, a new maintenance effectiveness model fitted on empirical data is proposed and incorporated into the MR&R optimization program. A greedy heuristic algorithm is developed, which greatly reduces the computation time without compromising the solution quality. Combining the system- and segment-level models and solution algorithms, we examine a batch of numerical cases. The results show considerable cost savings from the incorporation of maintenance, and from jointly optimizing the use of a combined agency budget. A number of managerial insights stemmed from the numerical case studies are discussed, which can help highway agencies formulate more cost-efficient MR&R plans and budget allocation.

Keywords: System-level MR&R planning; Budget constraints; Preventive maintenance model; Lagrange multiplier; Quasi-Newton methods.

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1. Introduction

1.1. Background

Surface roads constitute the world's largest transportation infrastructure network. For example, the United States alone has over 4 million miles of roads, which served over 3 trillion vehicle-miles in the year of 2015 (CBO, 2016; ASCE, 2017). The constantly increasing vehicle mileage creates ever-growing pavement deterioration and aging, and the deteriorated pavements in turn incur higher costs for vehicle repair, traffic congestion, and extra fuel consumption and emission, among others. This imposes a great challenge for highway agencies to optimally plan MR&R activities for the road pavements, especially given that a large portion of the pavements are already in poor conditions, and that the available budget rises consistently slower than the MR&R costs needed (ASCE, 2017).

Conventionally, a highway agency's long-term planning decision considered only rehabilitation and reconstruction activities. However, in recent decades many studies have reported the sizable effects of preventive maintenance activities (e.g. chip seal, microsurfacing) on slowing down the pavement's deterioration and extending its service life (Chong, 1989; Ponniah and Kennepohl, 1996; Labi and Sinha, 2003; Mamlouk and Dosa, 2014). These cheap maintenance treatments are particularly attractive for highway agencies under budget pressure. However, most highway agencies do not have well-established preventive maintenance planning mechanism (Peshkin et al., 2004). Hence, an optimization model for the joint planning of not only the rehabilitation and reconstruction activities, but also the preventive maintenance activities, is highly desired. Unfortunately such a model is missing in the literature to the best of our knowledge. We next examine the strength and deficiency of existing studies in the realm of MR&R planning optimization.

1.2. Literature review

Studies in this realm commenced by optimizing the rehabilitation planning of a single segment (Friesz and Fernandez, 1979; Fernandez and Friesz, 1981; Markow and Balta, 1985). A variety of segment-level optimization models have thenceforth been developed, which are characterized by the pavement deterioration process (memoryless or history-dependent), the number of treatments, and whether the time and/or pavement states are modeled by discrete or continuous variables. Table 1 summarizes the modeling features and solution approaches of select segment-level studies. Of note is that the table shows a general trend of evolution from simpler models (with memoryless deterioration process, single treatment, and discrete variables) to more complicated but realistic ones (with history-dependent deterioration process, multiple treatments, and continuous variables). This is partly thanks to the development of more sophisticated approaches for seeking global optimal solutions, e.g. calculus of variation (Ouyang and Madanat, 2006; Lee and Madanat, 2014b). The most complicated (and realistic) segment-level model so far seems to be Lee and Madanat (2014a), which optimized the planning of all the three treatments (maintenance, rehabilitation and reconstruction) with a history-dependent deterioration process. However, the solution relied on the technique of approximate dynamic programming, which requires high computation time and thus may not be suitable for large-scale systems of pavements. Another finding is that the solution approaches in Table 1 are usually problem-specific. This means a solution approach cannot be applied directly to solve a different version of the segment-level optimization model. Finally, the maintenance effectiveness models used in segment-level MR&R optimization are unrealistic. For example, the maintenance model used by Gu et al. (2012) and Lee and Madanat (2014a, b) was hypothesized with ungrounded parameter values. As a result, the optimal MR&R plan obtained by Lee and Madanat (2014a) showed that greater deterioration rate reduction could occur when maintenance was applied to a pavement near the end of its lifecycle (see Fig. 4a of the cited work), which contradicts with the common understanding in practice.

On the other hand, a highway agency often manages hundreds of pavement segments or more. Thus they are more interested in models that can jointly optimize for a system of pavement segments under certain budget constraints, which can be incorporated into their pavement management systems. However, the system-level problems are by nature more complicated than the segment-level ones. This is why a smaller number of studies were found in this category, including some works that relied on the highly idealized "top-down" approaches (Kuhn and Madanat, 2005; Durango-Cohen and Sarutipand, 2007). Those top-down models assumed homogeneous pavement segments in a system, and are thus unrealistic and unsuitable for real-world implementation.

The more realistic, "bottom-up" approaches that appreciate the heterogeneity in pavement segments have also been applied to system-level MR&R planning optimization. A number of select bottom-up studies are summarized in Table 2. Despite their contributions, limitations also exist, which are highlighted below:

i) Many studies relied on metaheuristic methods (e.g. genetic algorithm and tabu search) to solve the complicated optimization models. Metaheuristic methods are known to be unable to guarantee the global optimality of the solution (Blum and Roli, 2003). Moreover, in those cited studies it is often unable to assess how close the heuristic is to the global optimum. Other works sought to optimizing Lagrangian and Lagrangian dual functions of their original problems (Sathaye and Madanat, 2011; 2012; Lee et al., 2016; Lee and Madanat, 2017). However, the effectiveness of their solution approaches is contingent on the convexity of the problem formulation. Unfortunately, the convexity is not always guaranteed, given the fact that the empirical models for pavement deterioration and treatment effectiveness may vary from case to case.

ii) For most of the studies cited in Table 2, their solution approaches are highly dependent upon the segment-level empirical models ; i.e., they cannot be directly applied to another system-level problem with different segment-level models. This is undesirable since there are many variants of segment-level models (see again Table 1), and new empirical models may arise in the future to replace the present ones. The only exceptions are Lee et al. (2016) and Lee and Madanat (2017), where the system-level problems were solved by Lagrange methods integrated with dynamic programming. Although in principle their approaches can be extended to solve system-level problems with any discrete-time segment-level models, they might not be applicable to large-scale systems because the dynamic programming method is computationally inefficient.

iii) Most studies only optimized one or two treatments (jointly), possibly because their approaches are insufficient to find optimal solutions within acceptable computation time when more treatments are considered. Note that incorporating preventive maintenance into the optimal MR&R planning would add much to the complexity of the problem. To the authors' knowledge, Chu and Chen (2012) and Lee and Madanat (2017) are the only two that considered three treatments (maintenance, rehabilitation, and reconstruction) in their system-level optimization. However, both models are accompanied with substantial simplifications. Chu and Chen (2012) assumed that the agency can only choose from a limited number of actions, and they searched for threshold-based MR&R policies only, which has been proved to be suboptimal at the segment level when a history-dependent deterioration process is used (Lee and Madanat, 2014b). Lee and Madanat (2017) used the hypothetical maintenance model proposed by Gu et al. (2012), and assumed that a constant maintenance effectiveness is applied to all the pavement segments and over the entire planning horizon, so that the dimensionality of the solution space is much reduced.

1.3. The research question and rundown of the paper

Given the research gap in the literature revealed above, in this paper we will develop a computationally efficient and not problem-specific approach to find globally-optimal or near-optimal MR&R policies for large-scale pavement systems. To this end, we first propose a general formulation of the system-level problem that is independent of any specific segment-level models. Two scenarios are considered in the formulation: i) where a combined budget constraint is applied to all the MR&R treatments; and ii) where each treatment is subject to a separate budget constraint. A general solution approach is then developed to decompose the system-level problem into a number of segment-level subproblems. This is done by relaxing the budget constraint(s) via Lagrange multiplier(s). The optimization program is then converted to a bi-level one where the lower level is the segment-level subproblems which are solved by model-specific algorithms, and the upper level is to find the value(s) of the Lagrange multiplier(s). We show for the combined-budget-constraint scenario that global optimality is retained at the system level via certain derivative-free iterative methods; i.e., if the segment-level subproblems are solved at or near optimality, then the global optimality or near-optimality of the system-level problem is guaranteed. Note that this is true regardless of whether the original problem is convex or not. Also note that the system-level approach can be applied regardless of the form of segment-level models.

Table 1. Select studies on segment-level optimization of MR&R planning

Study	Deterioration process	Number of treatments	Discrete/Continuous time or pavement state	Solution approach
Golabi et al. (1982)	memoryless	1	discrete	linear programming
Carnahan et al. (1987)	memoryless	1	discrete	dynamic programming
Fwa et al. (1994)	memoryless	1	discrete	genetic algorithm
Durango-Cohen (2007)	memoryless	1	hybrid	dynamic programming
Friesz and Fernandez (1979)	memoryless	1	continuous	optimal control
Fernandez and Friesz (1981)	memoryless	1	continuous	optimal control
Tsunokawa and Schofer (1994)	memoryless	1	continuous	optimal control with trend curve approximation
Li and Madanat (2002)	memoryless	1	continuous	using the memoryless property
Ouyang and Madanat (2006)	memoryless	1	continuous	calculus of variation
Madanat (1993)	memoryless	3	discrete	dynamic programming
Madanat and Ben-Akiva (1994)	memoryless	3	discrete	dynamic programming
Gu et al. (2012)	memoryless	2	continuous	numerical method based on optimal conditions from Ouyang and Madanat (2006)
Rashid and Tsunokawa (2012)	memoryless	3	continuous	optimal control with trend curve approximation
Tsunokawa and Ul-Isalm (2002)	history-dependent	1	discrete	exhaustive search
Tsunokawa et al. (2006)	history-dependent	1	discrete	gradient search
Deshpande et al. (2010)	history-dependent	1	discrete	multi-objective genetic algorithm
Bai et al. (2015)	history-dependent	1	hybrid	dynamic programming
Miyamoto et al. (2000)	history-dependent	2	discrete	genetic algorithm
Lee and Madanat (2014a)	history-dependent	3	hybrid	dynamic programming
Lee and Madanat (2014b)	history-dependent	3	continuous	calculus of variation

We propose a segment-level model that incorporates the history-dependent deterioration process and all the three types of treatments (preventive maintenance, rehabilitation, and reconstruction). A realistic maintenance effectiveness model is developed using the recent empirical data reported in the literature (Mamlouk and Dosa, 2014) to replace the hypothetical one that was used previously, and the new model produces reasonable results in the optimal MR&R plans. We also propose a greedy heuristic algorithm that reduces the computation time by 97% without compromising the solution quality (as compared against the dynamic programming approach used in the literature). The segment-level model and the solution algorithm are integrated with the general system-level approach to obtain optimal MR&R policies for pavement systems.

The models and solution approach are tested through a large number of numerical experiments. The results unveil many useful insights regarding how budget and other key operating parameters affect the optimal system-level MR&R policy. The numerical experiments also manifest the computational efficiency of our solution approach. In particular, the computation time increases linearly with the size of the pavement system.

Table 2. Select studies on bottom-up system-level optimization of MR&R planning

Study	Deterioration process	Number of treatments	Discrete/Continuous time or pavement state	Solution approach
Chan et al. (1994)	memoryless	1	discrete	genetic algorithm
Ouyang and Madanat (2004)	memoryless	1	hybrid	branch and bound; greedy heuristic
Ouyang (2007)	memoryless	1	hybrid	approximate dynamic programming
Hajibabai et al. (2014)	memoryless	1	hybrid	Lagrange multiplier method
Sathaye and Madanat (2011)	memoryless	1	continuous	Lagrange multiplier method
Sathaye and Madanat (2012)	memoryless	1	continuous	Lagrange dual method
Fwa et al. (1996)	memoryless	2	discrete	genetic algorithm
Chu and Chen (2012)	history-dependent	3	hybrid	tabu search
Lee and Madanat (2015)	history-dependent	2	hybrid	genetic algorithm
Lee et al. (2016)	history-dependent	2	hybrid	Lagrange dual method
Lee and Madanat (2017)	history-dependent	3	hybrid	Lagrange multiplier method

The rest of the paper is organized as follows: Section 2 presents the general formulation of the system-level problem (i.e., the upper-level problem) and a general solution approach, which incorporates the segment-level (i.e. the lower-level) model and solution to be described in Section 3; numerical case studies are furnished in Section 4; the insights, limitations, and future extensions of this paper are discussed in Section 5.

2. General formulation and solution approach for the system-level optimization of MR&R planning

A general formulation of the system-level MR&R planning problem, regardless of the segment-level models, is presented in Section 2.1. A derivative-free iterative solution approach built upon the Lagrange multiplier method is described in Section 2.2. The description of the solution approach assumes that the solution of the segment-level problem (an example of which will be presented in Section 3) is ready for use.

2.1. A general formulation

The objective of the problem is to minimize the sum of the discounted user and agency costs, $\sum_{k=1}^{K} Z_k$, for all the pavement segments $k \in \{1,2,\dots,K\}$ over a given planning horizon T ($T = \infty$ denotes an infinite-horizon problem), as shown in (1a) below. For each segment k, Z_k is a function of a vector of state variables (e.g. roughness level and age), denoted by $\boldsymbol{q}_k$, and a vector of management decision variables (e.g., timing and intensities of MR&R activities), $\boldsymbol{x}_k$. Note that the elements of $\boldsymbol{q}_k$ and $\boldsymbol{x}_k$ can be discrete or continuous functions of time. The Z_k consists of the costs incurred to the users, C_k^U, and to the management agency, $\sum_{p=1}^{P} C_{kp}$, where $p \in \{1,\dots,P\}$ is the index of a treatment to be planned (i.e., maintenance, rehabilitation, and reconstruction).

Segment-specific constraints are divided into two classes: equality constraints (1b) and inequality constraints (1c), where $\boldsymbol{\Phi}_k$ and $\boldsymbol{\Psi}_k$ are again vectors of discrete or continuous functions of time. These constraints specify the pavements' initial conditions, how each pavement's state evolves over time (i.e. the deterioration process), and how each treatment may change the pavement's state, depending on the type, time and intensity of the treatment (i.e. the treatment effectiveness models). Finally, we present two versions of budget constraints in (1d-e): i) a combined budget that applies to the sum of agency costs for all the treatments across all segments, and ii) a number of separate budgets that each applies to a specific treatment. The B and B_p denote the annual combined budget and separate budget for treatment p, respectively; and r is the annual discount factor. Note here that we assume the budget can be transferred across years over the entire planning horizon so that the number of budget constraints is small (1 for the combined-budget-constraint case and 3 for the separate-budget-constraint case). Similar assumptions have been adopted by a number of previous studies (e.g. Sathaye and Madanat, 2011; 2012). Our system-level approach, however, can be applied to a more general case where the money can be transferred only within a given budget period (e.g., of 5-10 years). Also note that the total budgets in (1d) and (1e) are discounted to the present. We This is done in the interest of simplicity for model formulation and segment-level solution procedure, because now the agency cost terms in the Lagrangian (after relaxing the budget constraints) can be simply combined, as we shall see momentarily.

$$\min \sum_{k=1}^{K} Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = \sum_{k=1}^{K}\left(C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + \sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k)\right) \tag{1a}$$

$$\text{subject to: } \boldsymbol{\Phi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = 0, \text{for } k = 1,\dots,K \tag{1b}$$

$$\boldsymbol{\Psi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) \le 0, \text{for } k = 1,\dots,K \tag{1c}$$

$$\text{combined budget: } \sum_{k=1}^{K}\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) \le \frac{1-e^{-rT}}{r} B \tag{1d}$$

or,

$$\text{separate budgets: } \sum_{k=1}^{K} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) \le \frac{1-e^{-rT}}{r} B_p \text{ for } p = 1,\cdots,P \tag{1e}$$

We next present an iterative approach for solving the above mathematical program.

2.2. *An iterative approach using Lagrange multipliers*

Corresponding to the above system-level formulation, the general formulation of the segment-level problems are given in (2a-c). In the following discussion of this section, we assume that the solution to this segment-level problem has been developed a priori. This segment-level solution will be used as a building block in our proposed approach.

For each $k = 1, \dots, K$

$$\min \; Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + \textstyle\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) \tag{2a}$$

$$\text{subject to: } \boldsymbol{\Phi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = 0 \tag{2b}$$

$$\boldsymbol{\Psi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) \le 0 \tag{2c}$$

To be accurate, we describe the solution approaches for the problems with the combined budget constraint (Section 2.2.1) and separate budget constraints (Section 2.2.2) one by one. However, they follow the same logic: first, the system-level problem is decomposed into K segment-level subproblems, each having the form of (2a-c); and second, built upon the solutions to the segment-level subproblems, a gradient-free iterative algorithm is used to solve the system-level optimization problem.

2.2.1. *Combined-budget-constraint problem*

We introduce a Lagrange multiplier, λ, to relax the combined budget constraint (1d). The relaxed optimization is presented as follows:

$$\min \; L(\boldsymbol{q}, \boldsymbol{x}, \lambda) = \textstyle\sum_{k=1}^{K} Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) + \lambda\left(\sum_{k=1}^{K}\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) - \frac{B}{r}(1 - e^{-rT})\right) =$$

$$\textstyle\sum_{k=1}^{K} H_k(\boldsymbol{q}_k, \boldsymbol{x}_k, \lambda) - \lambda\frac{B}{r}(1 - e^{-rT}) \tag{3a}$$

$$\text{subject to: } \boldsymbol{\Phi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = 0, \text{ for } k = 1, \dots, K \tag{3b}$$

$$\boldsymbol{\Psi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) \le 0, \text{ for } k = 1, \dots, K \tag{3c}$$

$$\lambda \cdot V(\lambda) = \lambda \cdot \left(\textstyle\sum_{k=1}^{K}\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) - \frac{B}{r}(1 - e^{-rT})\right) = 0, \lambda \ge 0, V(\lambda) \le 0 \tag{3d}$$

where L is the partial Lagrange function, and $H_k(\boldsymbol{q}_k, \boldsymbol{x}_k, \lambda) \equiv Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) + \lambda \sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k)$. Constraint (3d) is the complementary slackness condition of optimality: $\lambda > 0$ when the budget constraint is binding, and $\lambda = 0$ otherwise. One can easily verify that the optimal solution of (3a-d) is always optimal to (1a-d); i.e., the relaxed program (3a-d) constructs a sufficient condition for the optimality of (1a-d).

Without constraint (3d), the remaining mathematical program (3a-c) can be decomposed by segment number k as follows:

For each $k = 1, \dots, K$,

min $H_k(\boldsymbol{q}_k, \boldsymbol{x}_k, \lambda) = Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) + \lambda \sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k)$

$= C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + (1+\lambda) \sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k)$

$$= C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + \sum_{p=1}^{P} \bar{C}_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k, \lambda) \quad (4a)$$

$$\text{subject to: } \boldsymbol{\Phi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = 0 \quad (4b)$$

$$\boldsymbol{\Psi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) \leq 0 \quad (4c)$$

where $\bar{C}_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k, \lambda) = (1+\lambda) C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k)$ can be considered as a "weighted" agency cost for treatment p applied to segment k (where the weight is $1+\lambda$). Note that for a given λ, H_k has the same form as Z_k save for only a different weight for agency costs. Thus the solution to the segment-level problem (2a-c) can be readily applied to (4a-c) for each k with a given λ. Also note that if the global optimality of segment-level solutions is guaranteed, then the global optimality of the system-level problem is attained if a λ is found to satisfy the complementary slackness condition (3d). Further, the following lemma ensures that if the segment-level solution is near-optimal (i.e., its relative cost gap from the optimal solution is bounded by a small fraction), then the resulting system-level solution is also near-optimal.

Lemma 1. For a given λ, suppose $\boldsymbol{x}_k^*(\lambda)$ is the exact optimal solution to the subproblem of segment k $(k = 1,2,\cdots,K)$, and $\boldsymbol{x}_k^H(\lambda)$ is a heuristic solution that satisfies:

$$\begin{cases} \left|C_k\left(\boldsymbol{x}_k^*(\lambda)\right) - C_k\left(\boldsymbol{x}_k^H(\lambda)\right)\right| \leq \delta_1 \\ \left|Z_k\left(\boldsymbol{x}_k^*(\lambda)\right) - Z_k\left(\boldsymbol{x}_k^H(\lambda)\right)\right| \leq \delta_2 \end{cases}, \forall k = 1,2,\cdots,K, \lambda \geq 0 \quad (5)$$

where $C_k(\boldsymbol{x}_k) = \sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k)$. Further assume λ^* and λ^H are the Lagrange multiplier values when the exact and the heuristic solutions are used, respectively; i.e.,

$$\lambda^* \cdot \left(\sum_{k=1}^{K} C_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) - B\right) = 0 \quad (6a)$$

$$\lambda^H \cdot \left(\sum_{k=1}^{K} C_k\left(\boldsymbol{x}_k^H(\lambda^H)\right) - B\right) = 0 \quad (6b)$$

Then we have:

$$\left|\sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) - \sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right| \leq K \cdot (\max\{\lambda^*, \lambda^H\}\, \delta_1 + \delta_2) \quad (7)$$

A sketched proof of Lemma 1 is furnished in Appendix A. Note (7) specifies that the percentage cost gap of the system-level problem is in the same magnitude of the percentage cost gaps of the segment-level heuristics, given that λ^* and λ^H are small.[1]

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[1] In our numerical case studies, λ^H is always less than 3. The λ^* is usually comparable to λ^H since $|V(\lambda^H) - V(\lambda^*)| \leq K\delta_1$. Exception may arise only when B is near the maximum annual budget needed, where $V(\lambda)$ becomes flat.

Finally, the following lemma ensures that as long as such a λ exists, we are always able to find it via a properly designed quasi-Newton algorithm. The proof of this lemma is furnished in Appendix B.

Lemma 2. $V(\lambda)$ is a (strictly) decreasing function of λ if each segment-level problem furnishes a unique optimal solution (which is usually true).

An immediate corollary of this lemma is that there exists a unique solution of λ to (3d) (as long as program (3a-d) is feasible), and this solution can be attained by a number of iterative methods, including Newton's and quasi-Newton methods (which presumably converge much faster than the methods of bisection, golden-section, etc.). Since the calculation of derivatives is often difficult and computationally inefficient due to the complicated mathematical forms of MR&R cost and effectiveness models, we next present an algorithm using a derivative-free method (termed the "modified secant method"). In the following algorithm, δ denotes the tolerance level that is sufficiently small to guarantee the algorithm converges. The convergence of the algorithm is proved in Appendix C.

Algorithm 1:

Step 1. Set $\lambda = \lambda^0 = 0$; solve the segment-level subproblems (4a-c) for each k. Evaluate $V(\lambda^0)$. If $V(\lambda^0) \le 0$, end; otherwise go to *Step 2*.

Step 2. Select another initial value $\lambda = \lambda^1 > 0$, solve (4a-c) for each k and evaluate $V(\lambda^1)$. If $|V(\lambda^1)| < \delta$, end; otherwise set $n = 1$ and go to *Step 3*.

Step 3. Set $\lambda = \lambda^{n+1} = \lambda^n - V(\lambda^n)\frac{\lambda^n - \lambda^{n-1}}{V(\lambda^n) - V(\lambda^{n-1})}$. Solve (4a-c) for each k and evaluate $V(\lambda^{n+1})$. If $|V(\lambda^{n+1})| < \delta$, end; otherwise, go to *Step 4*.

Step 4. If $V(\lambda^n) \cdot V(\lambda^{n+1}) > 0$ and $V(\lambda^{n-1}) \cdot V(\lambda^{n+1}) < 0$, set $\lambda^n = \lambda^{n-1}$. Set $n = n + 1$ and repeat *Step 3*.

2.2.2. Separate-budget-constraint problem

Similarly, we use a Lagrange multiplier, λ_p, to relax each of the P budget constraints in (1e). The Lagrange function becomes:

$$\min\ L(\boldsymbol{q}, \boldsymbol{x}, \lambda) = \sum_{k=1}^{K} Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) + \sum_{p=1}^{P} \lambda_p \left(\sum_{k=1}^{K} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) - \frac{B_p}{r}(1 - e^{-rT})\right) =$$

$$\sum_{k=1}^{K} H_k(\boldsymbol{q}_k, \boldsymbol{x}_k, \boldsymbol{\lambda}) - \sum_{p=1}^{P} \lambda_p \frac{B_p}{r}(1 - e^{-rT}) \tag{8}$$

where $H_k(\boldsymbol{q}_k, \boldsymbol{x}_k, \boldsymbol{\lambda}) \equiv C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + \sum_{p=1}^{P}(1 + \lambda_p) C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k)$, and $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_P]^T$. The corresponding segment-level problem can be written as follows:

For each $k = 1, \dots, K$

$$\min \ H_k(\boldsymbol{q}_k, \boldsymbol{x}_k, \boldsymbol{\lambda}) = C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + \textstyle\sum_{p=1}^{P}(1+\lambda_p)C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) \quad (9a)$$

$$\text{subject to: } \boldsymbol{\Phi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = 0 \quad (9b)$$

$$\boldsymbol{\Psi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) \le 0 \quad (9c)$$

The complementary slackness conditions are:

$$\text{For } p = 1, \cdots, P, \ \lambda_p V_p(\boldsymbol{\lambda}) = \lambda_p \left(\textstyle\sum_{k=1}^{K} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) - \frac{B_p}{r}(1-e^{-rT})\right) = 0, \lambda_p \ge 0, V_p(\boldsymbol{\lambda}) \le 0 \quad (10)$$

Similar to the combined-budget-constraint problem, the optimal solution of the relaxed program above is also optimal to the original problem under separate budget constraints. Here we propose a modified Broyden's method to formulate the following algorithm for solving the relaxed program.[2]

Algorithm 2:

Step 1. Set $\boldsymbol{\lambda} = \boldsymbol{\lambda}^0 \equiv [\lambda_1^0, \lambda_2^0, \dots, \lambda_P^0]^T = \boldsymbol{0} \equiv [0,0,\dots,0]^T$; solve the segment-level subproblems (9a-c) for each k. Evaluate $\boldsymbol{V}(\boldsymbol{\lambda}^0) = [V_1, \dots, V_P]^T$. If $\boldsymbol{V}(\boldsymbol{\lambda}^0) \le \boldsymbol{0}$, end; otherwise go to *Step 2*.

Step 2. Calculate the initial $P \times P$ Jacobian matrix J^0. For each $p = 1, \dots, P$, define $\boldsymbol{\lambda}^{p,0}$ as a P-dimensional vector whose p-th element is a small positive number δ_p and all the other elements are 0. The J^0 is calculated by setting its element on the i-th row and the j-th column as: $J_{i,j}^0 = \frac{V_i(\boldsymbol{\lambda}^{j,0}) - V_i(\boldsymbol{0})}{\delta_j}$.

Step 3. Set $\boldsymbol{\lambda}^1 = \boldsymbol{\lambda}^0 - (J^0)^{-1}\boldsymbol{V}(\boldsymbol{\lambda}^0)$. End the search if $\boldsymbol{V}(\boldsymbol{\lambda}^1)$ satisfies the complementary slackness conditions (10); i.e., for each $p = 1, \dots, P$, $V_p(\boldsymbol{\lambda}^1) \le 0$ if $\lambda_p^1 = 0$, and $\left|V_p(\boldsymbol{\lambda}^1)\right| < \delta$ if $\lambda_p^1 > 0$. Otherwise set $n = 1$ and go to *Step 4*.

Step 4. Set $(J^n)^{-1} = (J^{n-1})^{-1} + \frac{(\boldsymbol{\lambda}^n - \boldsymbol{\lambda}^{n-1}) - (J^{n-1})^{-1} * (\boldsymbol{V}(\boldsymbol{\lambda}^n) - \boldsymbol{V}(\boldsymbol{\lambda}^{n-1}))}{(\boldsymbol{\lambda}^n - \boldsymbol{\lambda}^{n-1})^T * (J^{n-1})^{-1} * (\boldsymbol{V}(\boldsymbol{\lambda}^n) - \boldsymbol{V}(\boldsymbol{\lambda}^{n-1}))} * (\boldsymbol{\lambda}^n - \boldsymbol{\lambda}^{n-1})^T * (J^{n-1})^{-1}$ and $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{n+1} = \boldsymbol{\lambda}^n - (J^n)^{-1}\boldsymbol{V}(\boldsymbol{\lambda}^n)$. End the search if $\boldsymbol{V}(\boldsymbol{\lambda}^{n+1})$ satisfies the complementary slackness conditions (10). Otherwise go to *Step 5*.

Step 5. Define vector operator $\otimes$ as $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \otimes \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_1 b_1 \\ a_2 b_2 \\ a_3 b_3 \end{bmatrix}$ for any $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. If the number of

[2] The original Broyden's method is the multivariate version of the secant method; see an introduction in Jorge and Stephen (2006). One can also show that the relaxed program has a unique optimum given that each segment-level problem has a unique optimal solution (similar to Lemma 2). However, unlike the combined-budget-constraint case, Algorithm 2 cannot guarantee the global convergence to the optimum. Nevertheless, we believe this algorithm does converge to the global minimum in the numerical experiments presented in Section 4.4, since the solutions under the combined budget constraint (which are guaranteed to be optimal) are consistent with the corresponding solutions under the separate budget constraints.

negative elements in vector $V(\lambda^{n-1})\otimes V(\lambda^{n+1})$ is larger than that in $V(\lambda^{n})\otimes V(\lambda^{n+1})$, set $\lambda^n = \lambda^{n-1}$. Set $n = n+1$ and return to *Step 4*.

3. Segment-level MR&R models and solution approaches

This section presents the formulation and solution approaches of the segment-level subproblem (i.e. a special case of (2a-c)) that jointly optimizes all the three treatments, i.e. preventive maintenance (chip seal), rehabilitation and reconstruction. While the framework in Section 2 applies to almost all segment level subproblems, to stay focused, we present here only a segment-level formulation that is discrete in time but continuous in the pavement condition (i.e., the roughness index) for an infinite planning horizon. Most of the problem formulation, except for the maintenance model, is similar to the one presented in Lee and Madanat (2014a), and is presented in Section 3.1. We develop an efficient greedy heuristic, and compare its solution quality and computation efficiency against a benchmark dynamic programming algorithm (Section 3.2).

3.1. General formulation

The state variables are $\boldsymbol{q}_k = (q_{kt}|t = 0,1,2,\cdots) = (s_k(t), h_{kt}|t = 0,1,2,\cdots)$, where $s_k(t)$ and h_{kt} are the pavement roughness index and the pavement's age (number of years since the last reconstruction), respectively, for segment k in year t. The decision variables are $\boldsymbol{x}_k = \left(v_{kt}, \omega_{kt}, x_{kt,1}, x_{kt,2}, x_{kt,3}\middle| t = 0,1,2,\cdots\right)$, where the binary variable $x_{kt,p}$ $(p = 1,2,3)$ is equal to 1 if a maintenance (corresponding to $p = 1$), rehabilitation $(p = 2)$ or reconstruction $(p = 3)$ activity is executed in year t for segment k, respectively, and 0 otherwise; v_{kt} and ω_{kt} represent the maintenance and rehabilitation intensities in year t for segment k, respectively. The full formulation is presented as follows:

$$\min Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + \textstyle\sum_{p=1}^{3} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) \tag{11a}$$

subject to:
$$C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) = \textstyle\sum_{t=0}^{\infty} \int_t^{t+1} l_k\left(c_k^1 s_k(u) + c_k^2\right) e^{-ru} du \tag{11b}$$

$$C_{k,1}(\boldsymbol{q}_k, \boldsymbol{x}_k) = \textstyle\sum_{t=0}^{\infty} x_{kt,1} \left(\gamma_k^1 v_{kt} + \gamma_k^2\right) e^{-rt} \tag{11c}$$

$$C_{k,2}(\boldsymbol{q}_k, \boldsymbol{x}_k) = \textstyle\sum_{t=0}^{\infty} x_{kt,2} \left(m_k^1 \omega_{kt} + m_k^2\right) e^{-rt} \tag{11d}$$

$$C_{k,3}(\boldsymbol{q}_k, \boldsymbol{x}_k) = \textstyle\sum_{t=0}^{\infty} x_{kt,3} \left(z_k^1 + z_k^2 l_k\right) e^{-rt} \tag{11e}$$

$$\bar{b}_k - b_{kt} = x_{kt,1} E_k\left(v_{kt}, s_k(t)\right), \forall t \tag{11f}$$

$$E_k\left(v_{kt}, s_k(t)\right) = \frac{\alpha_k v_{kt}}{\left(s_k(t)\right)^{\beta_k}} \tag{11g}$$

$$0 \le v_{kt} \le D_{kt} = \min\left\{\bar{v}_k, \frac{(\bar{b}_k - b_k^L)\left(s_k(t)\right)^{\beta_k}}{\alpha_k}\right\}, \forall t \tag{11h}$$

$$s_k(t) - s_k^+(t) = x_{kt,2} G_k\left(\omega_{kt}, s_k(t)\right) + x_{kt,3}\left(s_k(t) - s_k^{new}\right), \forall t \tag{11i}$$

$$G_k\left(\omega_{kt}, s_k(t)\right) = \frac{g_k^1}{g_k^2 s_k(t) + g_k^3} \omega_{kt} \tag{11j}$$

$$0 \le \omega_{kt} \le R_{kt} = \left(\frac{g_k^2}{g_k^1} + \frac{g_k^3}{g_k^1 s_k(t)}\right) \max\left(0, \min\{s_k(t) - s_k^L, g_k^1 s_k(t)\}\right), \forall t \tag{11k}$$

$$s_k(u) = F_k(s_k^+(t), u - t, h_{kt}^+, b_{kt}), \forall u \in (t, t+1], \forall t \tag{11l}$$

$$F_k(s_k^+(t), u - t, h_{kt}^+, b_{kt}) = s_k^+(t) e^{b_{kt}(u-t)} + f_k l_k (u - t) e^{b_{kt}(h_{kt}^+ + u - t)} \tag{11m}$$

$$\textstyle\sum_{p=1}^{3} x_{kt,p} \le 1, \forall t \tag{11n}$$

$$h_{kt}^+ = h_{kt}(1 - x_{kt,3}), \forall t \tag{11o}$$

$$s_k^{new} \le s_k(t) \le s_k^{max}, \forall t \tag{11p}$$

$$T_k^{min} x_{kt,3} \le h_{kt} x_{kt,3} \le T_k^{max} x_{kt,3}, \ \forall t \tag{11q}$$

$$q_{k0} = (s_k(0), h_{k0}) \tag{11r}$$

The models for the user cost C_k^U, maintenance cost $C_{k,1}$, rehabilitation cost $C_{k,2}$ and reconstruction cost $C_{k,3}$ are described in (11b-e), respectively, where l_k is the annual traffic loading on segment k, which is assumed to be constant (Sathaye and Madanat, 2011; 2012; Lee and Madanat, 2017); and c_k^1, c_k^2, γ_k^1, γ_k^2, m_k^1, m_k^2, z_k^1 and z_k^2 are (non-negative) cost coefficients.

Of note is that the maintenance cost model (11c) is for chip seal only, which is one of the most commonly used preventive maintenance activities (Labi and Sinha, 2003). Here the maintenance intensity variable v_{kt} is defined as the average least dimension (ALD) of chip seal in year t for segment k. The non-negative cost coefficients γ_k^1 and γ_k^2 depend upon the oil price, geographical location of the pavement, labor cost, etc. Our maintenance effectiveness model is shown by (11f-g). The model is built upon the following two facts: i) the pavement roughness before and after the chip seal is approximately the same but the deterioration rate diminishes, which is consistent with the findings of Mamlouk and Dosa (2014) among others; and ii) the reduction in deterioration rate is a non-increasing function of the pavement roughness level (see Table 2 and Figures 4-7 of Mamlouk and Dosa, 2014). The latter means maintenance (e.g. chip seal) is less effective when being applied to a pavement in worse condition. Note that a previous maintenance cost and effectiveness model (Gu et al., 2012; Lee and Madanat, 2014a, b; Lee and Madanat, 2017) resulted in predictions that were at odds with this simple and obvious fact. For example, Lee and Madanat (2014a) predicted a non-monotonic trend between deterioration rate reduction and the pavement's roughness level (see Fig. 4a of that paper), where a larger deterioration rate reduction may occur when the roughness level is higher. This mistake has been corrected by using our maintenance model. The $\bar{b}_k$ and b_{kt} in (11f) are the deterioration rates before and after applying chip seal. The mathematical form of (11g) is selected to fit the real test data of chip seal from Mamlouk and Dosa (2014), where parameters $\alpha_k > 0$, $\beta_k \ge 1$. In addition, there is a technical upper bound for the ALD, $\bar{v}_k$, and the deterioration rate has a lower bound b_k^L, at which any additional maintenance has no effect. Thus, the effective maintenance intensity is bounded by D_{kt}, which is defined in (11h).

Other parts of the segment-level formulation are borrowed from previous studies, mostly from

Ouyang and Madanat (2004; 2006) and Lee and Madanat (2014a). Constraints (11i) specify the roughness index reduction caused by a rehabilitation or reconstruction activity, where function G_k represents the rehabilitation effectiveness as defined in (11j); $s_k(t)$ and $s_k^+(t)$ denote the roughness indices right before and after the activity, respectively; s_k^{new} is the roughness index immediately after a reconstruction; and g_k^1, g_k^2, and g_k^3 are coefficients. Constraints (11k) stipulate the upper bound, R_{kt}, for the rehabilitation intensity, where s_k^L is the best possible roughness level after a rehabilitation. Constraints (11l) indicate how the pavement state is updated at moment $u \in (t, t+1]$, where F_k is a history-dependent deterioration model shown in (11m); and h_{kt}^+ is the pavement age after the activity. Constraints (11n) ensure that at most one activity is performed every year. Constraints (11o) reset the pavement age to 0 after a reconstruction. Constraints (11p-q) specify the upper and lower bounds of the roughness level and the pavement's lifecycle duration. Constraint (11r) defines the initial pavement state.

3.2. Solution method

We first decompose the infinite-horizon optimization problem (11a-r) into two finite-horizon subproblems (Section 3.2.1). Each subproblem has fewer decision variables and is thus easier to solve. We present in Section 3.2.2 two algorithms to solve the subproblems: a dynamic programming algorithm similar to the one used by Lee and Madanat (2014a) and a greedy heuristic. The heuristic can achieve the same solution quality as the dynamic programming approach with only a small fraction of the computation time, as is validated later in Section 4.2.

3.2.1. Problem decomposition

With the augmented state $q_{kt} = (s_k(t), h_{kt})$, the infinite horizon problem still follows a Markov Decision Process (Li and Madanat, 2002; Lee and Madanat, 2015); i.e., the optimal MR&R decisions from year t onwards (and the future pavement states) depend only on the present state q_{kt}. Based on the Principle of Optimality (Bellman, 1957), the optimal roughness trajectory after the first reconstruction enters a periodic steady state, since every reconstruction will reset the pavement to $(s_k^{new}, 0)$. The steady-state solution is thus characterized by a fixed lifecycle duration denoted by T_k. The period prior to the first reconstruction is termed as the transient period, which will be optimized separately. Therefore, the objective function (11a) is reformulated as follows:

$$\min Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = Z_k^T(\boldsymbol{q}_k, \boldsymbol{x}_k) + \frac{e^{-rt_k^T}}{1-e^{-rT_k}} Z_k^S(\boldsymbol{q}_k, \boldsymbol{x}_k) \tag{12}$$

where Z_k^T is the discounted cost for the transient period, and Z_k^S is the cost for one steady-state cycle (with a reconstruction activity at the beginning) discounted to the beginning of the cycle, and t_k^T is the time of the first reconstruction. The Z_k^T and Z_k^S are given by the following equations.

$$Z_k^T(\boldsymbol{q}_k, \boldsymbol{x}_k) = \sum_{t=0}^{t_k^T-1} \left(\int_t^{t+1} l_k \left(c_k^1 s_k(u) + c_k^2 \right) e^{-ru} du + x_{kt,1} \left(\gamma_k^1 v_{kt} + \gamma_k^2 \right) e^{-rt} + x_{kt,2} (m_k^1 \omega_{kt} + \right.$$

$m_k^2)e^{-rt}\Big)$ (13)

$$Z_k^S(\boldsymbol{q}_k,\boldsymbol{x}_k)=\sum_{\tau=0}^{T_k-1}\Big(\int_\tau^{\tau+1} l_k\big(c_k^1 s_k(u)+c_k^2\big)\,e^{-ru}du+x_{k\tau,1}\big(\gamma_k^1 v_{k\tau}+\gamma_k^2\big)e^{-r\tau}+x_{k\tau,2}(m_k^1\omega_{k\tau}+$$

$$m_k^2)e^{-r\tau}\Big)+z_k^1+z_k^2 l_k \quad (14)$$

In equation (14) we use τ to denote the "age" in a steady-state lifecycle (counted from 0 when the last reconstruction is performed), and $q_{k0}=(s_k^{new},0)$.

Note that Z_k^S is independent of the transient period duration t_k^T and the MR&R schedule during that period. We can thus decompose this problem into two subproblems: the first subproblem for optimizing $\frac{Z_k^S}{1-e^{-rT_k}}$, and the second for optimizing Z_k given the optimal solution of the first one. Further note that Lee and Madanat (2014a) proved ω_{kt} is either 0 or R_{kt} at optimality. One can easily verify by applying Lee and Madanat's method that the same optimality condition is true for our model. Thus, ω_{kt} can be eliminated from the list of decision variables. Now the two subproblems are summarized as follows:

Subproblem 1: Minimize $\frac{Z_k^S}{1-e^{-rT_k}}$ subject to constraints (11f-r) with decision variables T_k and $v_{k\tau}, x_{k\tau,1}, x_{k\tau,2}$ $(\tau=0,1,2,\dots,T_k-1)$.

Subproblem 2: Minimize $Z_k=Z_k^T+e^{-rt_k^T}\left(\frac{Z_k^S}{1-e^{-rT_k}}\right)^*$ subject to constraints (11f-r) with decision variables t_k^T and $v_{kt}, x_{kt,1}, x_{kt,2}$ $\big(t=0,1,2,\dots,t_k^T-1\big)$, where $\left(\frac{Z_k^S}{1-e^{-rT_k}}\right)^*$ is the optimal value found in subproblem 1.

3.2.2. Algorithms for the subproblems

we first formulate a dynamic programming algorithm, which is modified from the one developed by Lee and Madanat (2014a, 2015). The algorithm is relegated to Appendix D in the interest of brevity. To apply the algorithm, we discretize both the maintenance intensity $v_{k\tau}$ and the pavement roughness level $s_k(\tau)$ into $d+1$ and $N+1$ points, respectively, where d and N are integers. As d and N approach to infinity, the dynamic programming solution converges to the global optimum. Thus, solutions of the dynamic programming approach can be used as benchmarks for verifying the solution quality of a much faster greedy heuristic. We next describe the details of this heuristic algorithm.

The heuristic is based upon the assumption that preventive maintenance is much cheaper than rehabilitation, which is true for most prevailing preventive maintenance treatments including chip seal

(Labi and Sinha, 2003). Thus, we start by seeking solutions where maintenance is performed more frequently, while rehabilitation is adopted only when that becomes a must. For the same reason, we also postulate that a maintenance activity is always executed with the maximum intensity $D_{k\tau}$. (This postulation was verified by our extensive numerical tests.) To further avoid solutions with high frequency of rehabilitation, we specify a lower bound of roughness level, W_k, below which rehabilitation should not be executed. Different values of W_k were used in the algorithm to balance off the solution quality and the computational efficiency. The algorithm for subproblem 1 is presented as follows:

Algorithm 3:

For each W_k, do the following and record the least-cost solution:

Step 1. Initialize $\tau = 1, cost_2 = \infty$.

Step 2. If $\tau < T_k^{max}$, find the action in year τ from the action set: $\{$Do-nothing $(x_{k\tau,1} = x_{k\tau,2} = 0)$, Maintenance $(x_{k\tau,1} = 1, x_{k\tau,2} = 0)$, Rehabilitation $(x_{k\tau,1} = 0, x_{k\tau,2} = 1)\}$, which minimizes the objective function $\frac{Z_k^S}{1-e^{-rT_k}}$ for the MR&R plan generated by the following *Steps 2.1-2.3*. Record the minimum objective value as $cost_1$:

Step 2.1. Keep the recorded MR&R plan before year τ and execute the selected action in year τ.

Step 2.2. For each year $y > \tau$, execute a maintenance with the maximum intensity D_{ky}; and if $s_k(y+1) > s_k^{max}$, replace this maintenance in year y by a rehabilitation.

Step 2.3. Among year T_k^{max} and all those years of rehabilitation between T_k^{min} and T_k^{max}, find the year in which a reconstruction minimizes the objective function $\frac{Z_k^S}{1-e^{-rT_k}}$.

The selected action of year τ should also satisfy the following conditions: $s_k(\tau) > W_k$ if the selected action is rehabilitation; and $s_k(\tau+1) < s_k^{max}$ if the selected action is executed in year τ.

Step 3. If $T_k^{min} \le \tau \le T_k^{max}$, calculate the objective function $\frac{Z_k^S}{1-e^{-rT_k}}$ associated with the following MR&R plan: keep the recorded plan before year τ and execute reconstruction in year τ. Set $cost_2 = \frac{Z_k^S}{1-e^{-rT_k}}$.

Step 4. If $\tau = T_k^{max}$ or $cost_2 < cost_1$, record the reconstruction in year τ, end; otherwise, set $\tau = \tau + 1$ and go to *Step 2*.

Only minor changes are made to the above algorithm when it is applied to subproblem 2.

Specifically, τ is initialized by 0 instead of 1; the objective function is changed to $Z_k = Z_k^T + e^{-rt_k^T}\left(\frac{Z_k^S}{1-e^{-rT_k}}\right)^*$; and finally, the time range for reconstruction is replaced by $\left[T_k^{min'}, T_k^{max'}\right]$, where $T_k^{min'} = \max\{0, T_k^{min} - h_{k0}\}$ and $T_k^{max'} = \max\{0, T_k^{max} - h_{k0}\}$.

4. Numerical case studies

Most of the numerical experiments presented in this section are for a pavement system with 100 heterogeneous segments. Although our approach is able to optimize for pavement systems that are 10 times larger within reasonable computation time (see Section 4.5), we choose this medium-size system for analysis simply because it is easier to run for a large batch of numerical experiments with various parameter values. We are thus able to discuss the general findings and insights unveiled by these results. Section 4.1 describes the parameter values. Section 4.2 examines the solution quality and computation efficiency of the segment-level heuristic. The system-level case studies under the combined and separate budget constraints are discussed in Sections 4.3 and 4.4, respectively. The computational efficiency of our solution method is examined in Section 4.5. All the numerical instances presented in this section are carried out via Matlab R2014a on a PC with Inter® Xeon® 3.60GHz CPU, 32.0GB RAM, and Windows 10 Pro 64-bit.

4.1. Parameter values

Most parameter values used in our numerical cases are summarized in Table 3. The cost parameters γ_k^1, γ_k^2 are derived from the empirical cost model of chip seal in Labi and Sinha (2003); the parameters for the chip seal effectiveness model (α_k, β_k) are obtained by fitting the model to the data in Table 2 of Mamlouk and Dosa (2014)[3]; the other parameter values are borrowed from Lee and Madanat (2014a). To account for heterogeneous segments, we specify that the initial pavement states, traffic loading, and some cost coefficients follow certain uniform distributions, which are denoted by the form of $U[a, b]$ in the table.

4.2. Validation of the segment-level greedy heuristic

To verify the quality of the segment-level greedy heuristic, we test 216 numerical cases with varying values of λ_p $(p = 1,2,3)$, q_{k0}, l_k, and r: $\lambda_p \in \{0,4\}$, $q_{k0} = (s_k(0), h_{k0}) \in \{(1,3), (2,8), (4,15)\}$, $l_k \in \{0.5, 0.8, 1.2\}$, $r \in \{0.05, 0.07, 0.1\}$. Note that the agency cost of treatment p in the objective function is multiplied by the weight $1 + \lambda_p$. The other parameters take values as in Table 3.

[3] Table 2 of Mamlouk and Dosa (2014) contains a number of (deterioration) performance models under various conditions. For each of them, the linear coefficient is the initial roughness and the exponent indicates the deterioration rate. To regress α_k and β_k, we also assume that the ALD used in their experiments takes a typical value in practice (7.18 mm); see Transit NZ, RCA, and Roading NZ (2005).

Table 3. Parameter values

Parameter	Value	Unit	Parameter	Value	Unit
c_k^1	$U[20500,22500]$	\$/IRI/km/lane/ million ESAL	$\bar{b}_k$	0.04	-
c_k^2	0	-	b_k^L	0.025	-
m_k^1	$U[10000,12000]$	\$/mm/km/lane	z_k^1	900000	\$/km/lane
m_k^2	$U[140000, 170000]$	\$/km/lane	z_k^2	917000	\$·year/km/ million ESAL
g_k^1	0.66	-	s_k^L	0.8	IRI
g_k^2	7.15	mm/IRI	s_k^{new}	0.75	IRI
g_k^3	18.3	mm	s_k^{max}	6	IRI
γ_k^1	130	\$/mm/lane/km	f_k	0.093	IRI·lane·year/ million ESAL
γ_k^2	300	\$/lane/km	T_k^{min}	20	year
α_k	0.002	-	T_k^{max}	60	year
β_k	1.483	-	s_k^0	$U[1,3]$	IRI
$\bar{v}_k$	14	mm	l_k	$U[0.4,0.9]$	million ESAL/year/lane
r	0.07	-			

We compare the greedy heuristic against two instances of the dynamic programming algorithm: where $N = d = 3$ (denoted as DP1), and where $N = 300,\ d = 5$ (denoted as DP2). The solutions generated from DP2 is treated as the global optima because no meaningful improvement of the solutions was observed by further increasing N or d. The runtimes and the cost gaps of the heuristic and the DP algorithms are summarized in Table 4, where the cost gaps are defined as:

$$\frac{\text{cost of the greedy heuristic or DP1} - \text{cost of DP2}}{\text{cost of DP2}}$$

Both the averages and the maxima of all the 216 cases are presented.

The tabulated values confirm that our heuristic algorithm produces solutions that are very close to the global optima. Note that the average gap in the total cost is only 0.37%. Comparison between the greedy heuristic and DP1 shows that both algorithms furnished solutions of similar quality, but our heuristic took much shorter (about 97% less) runtimes. We will thus use the greedy heuristic in the following sections for the purpose of computation efficiency. Recall that our system-level approach preserves the solution quality as long as the segment-level subproblems are solved near the optimality.

Table 4. Runtimes and cost gaps for the greedy heuristic and the dynamic programming algorithms

	Greedy heuristic	DP1	DP2
Average runtime (second)	1.20	49.21	1439.31
Maximum runtime (second)	1.47	73.43	1981.32
Average total cost gap	0.37%	0.41%	-
Maximum total cost gap	3.56%	2.05%	-
Average maintenance cost gap	0.29%	0.28%	-
Maximum maintenance cost gap	4.17%	3.83%	-
Average rehabilitation cost gap	0.36%	0.52%	-
Maximum rehabilitation cost gap	4.35%	2.69%	-
Average reconstruction cost gap	0.42%	0.40%	-
Maximum reconstruction cost gap	3.98%	2.46%	-

4.3. Under the combined budget constraint

First, we randomly generate a 100-segment pavement system, and optimize the total discounted cost for a range of combined annual budget: $B \in [4 \times 10^6, 5 \times 10^6]$ \$/year. The optimal total discounted cost and the cost components are plotted against B as the solid curves in Fig. 1. These curves start from $B = 4.02 \times 10^6$ \$/year on the left because this value of B represents the minimum budget required to find a feasible MR&R plan. This minimum required budget can be calculated by optimizing the decomposed problems (4a-c) with a sufficiently large λ. The figure shows that the optimal total cost (the solid curve with dot markers) decreases as B grows, until it reaches a threshold of 4.61×10^6 \$/year as marked by the arrow. This threshold represents the maximum budget needed for the pavement system; i.e., any additional budget would be redundant, and the optimal total cost would stay the same (11.73×10^7\$).

The figure also shows that the user cost (the triangle-marked solid curve) also decreases as B increases, which is as expected. Meanwhile, the rehabilitation cost (the "x"-marked solid curve) generally diminishes, while the reconstruction cost (the square-marked solid curve) increases with B. This means with more budget to spend, the agency should apply more reconstruction but less rehabilitation to reduce the user cost. On the other hand, when the budget is highly limited, more rehabilitation activities should be performed to extend the pavements' service life. The maintenance cost (the diamond-marked curve near the bottom of the figure) is much lower than the other cost components, and is insensitive to B. This is because there is no incentive to trade off the maintenance activities: they are very cheap, but have considerable effects on the pavements.

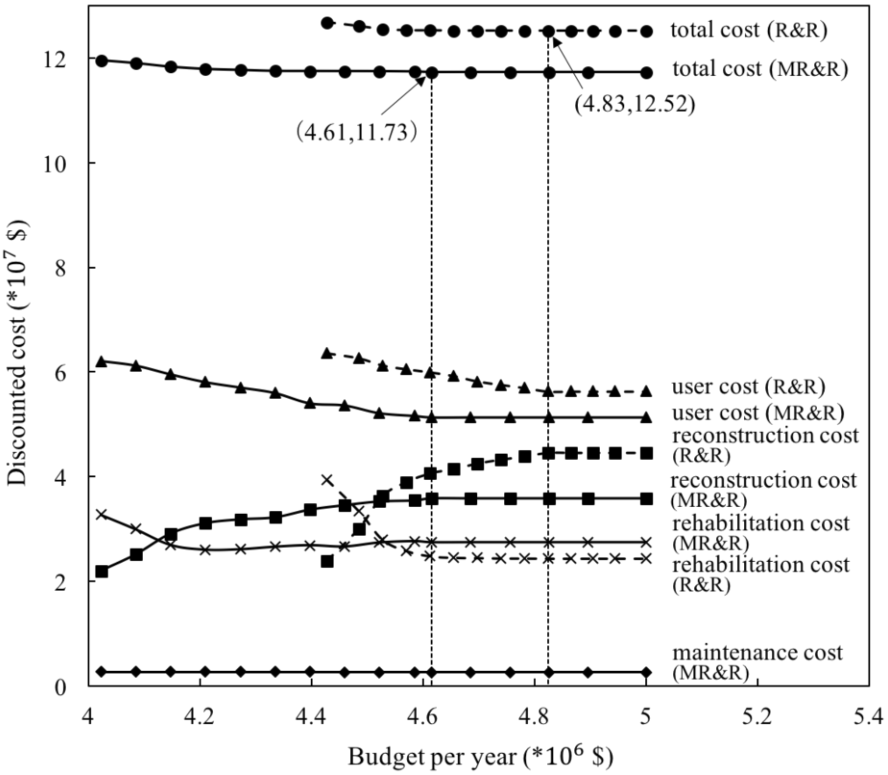
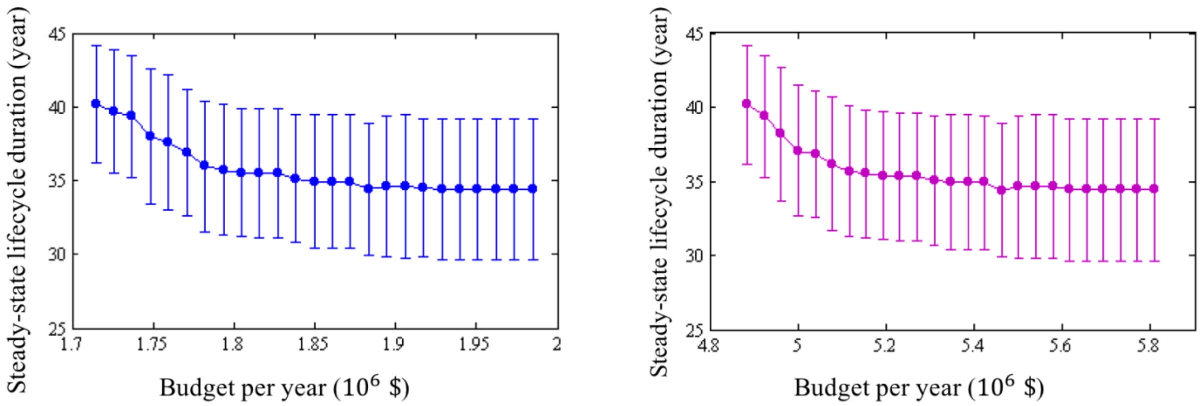


Fig. 1. Effects of the combined agency budget on the system-level optimal costs

To examine how adding maintenance affects the optimal MR&R plan, we compare the above total cost and cost components against those for the optimal R&R plans (i.e., without maintenance). The R&R costs are plotted as the dashed curves in Fig. 1. Comparison reveals a total cost saving of 6.3~7.5% from applying maintenance for $B \in [4.43 \times 10^6, 5 \times 10^6]$ \$/year. The minimum annual budget required is also reduced by 9.3% (from 4.43 to 4.02×10^6 \$/year). Comparison between the cost components reveals that adding maintenance usually results in a lower reconstruction cost but a higher rehabilitation cost. This means maintenance extends the pavements' service life, which in turn entails more rehabilitation activities.

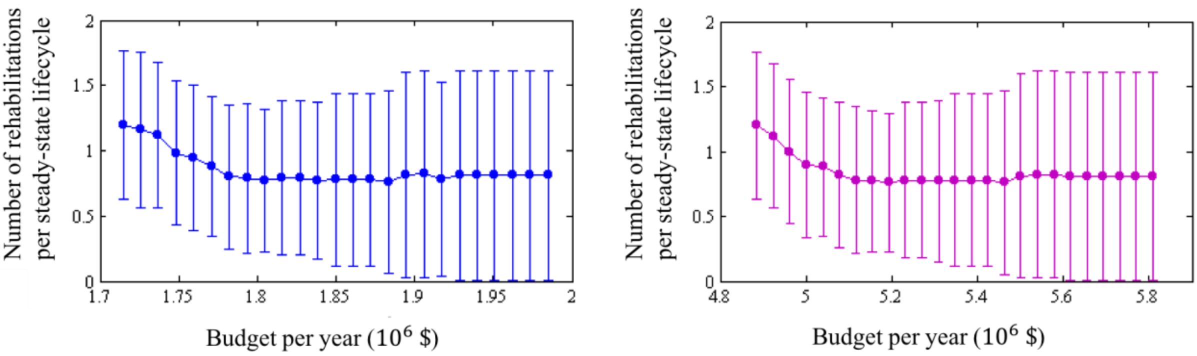
The effects of initial pavement conditions on the optimal MR&R plans for individual segments were often overlooked by previous studies (e.g. Lee and Madanat, 2015). Here we plot against the budget constraint the distributions of i) steady-state lifecycle duration (Fig. 2a and b) and ii) number of rehabilitations per steady-state lifecycle (Fig. 3a and b) of the 100 segments. Fig. 2a and 3a are for a system with good initial conditions ($s_k(0) \sim U[0,1], \forall k$), and Fig. 2b and 3b are for the same system but with poor initial conditions ($s_k(0) \sim U[2,3], \forall k$). In each figure, a black dot indicates the mean value (lifecycle duration or rehabilitation count) of the 100 segments, and an error bar describes the interval of two standard deviations centered at the mean. As expected, both the mean lifecycle duration and the mean rehabilitation count decrease as B increases until the constraint becomes unbinding, which is consistent with the findings from Fig. 1. Smaller standard deviations are observed for smaller B, indicating that a tighter budget tends to "homogenize" the segment-level MR&R plans.

Comparison between Fig. 2a and b unveils that the mean and standard deviation of steady-state lifecycle durations vary along very similar paths as B increases, despite the largely different initial pavement conditions. A high similarity is also observed between Fig. 3a and b for the distribution of rehabilitation counts per steady-state lifecycle. This means the steady-state MR&R plans of individual segments are *almost* independent of their initial conditions. Scrutinization of the numerical results shows that most of the pavement segments have nearly (but not exactly) the same steady-state MR&R plans between the two cases. However, the optimal MR&R plans during the transient periods are significantly affected by the initial pavement conditions; see the large differences between the distributions of the transient period durations (Fig. 4a and b) and the rehabilitation counts in the transient periods (Fig. 5a and b) for the cases with good and poor initial conditions.



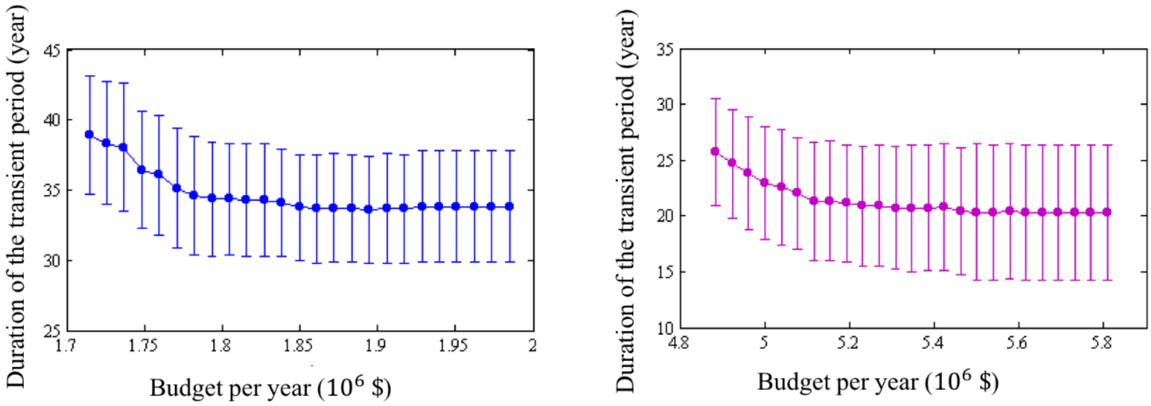
(a) (b)

Fig. 2. Distribution of steady-state lifecycle durations versus the budget constraint: (a) the case with good initial conditions; (b) the case with poor initial conditions



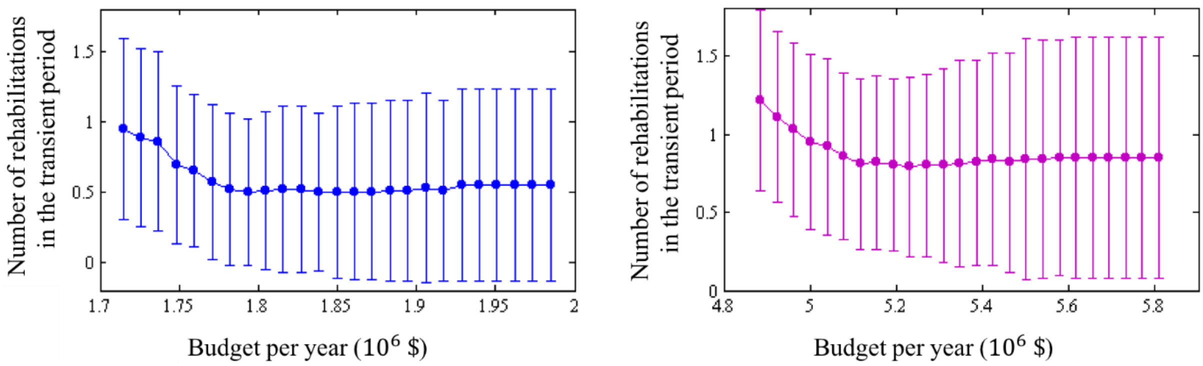
(a) (b)

Fig. 3. Distribution of rehabilitation counts per steady-state lifecycle versus the budget constraint: (a) good initial conditions; (b) poor initial conditions



(a) (b)

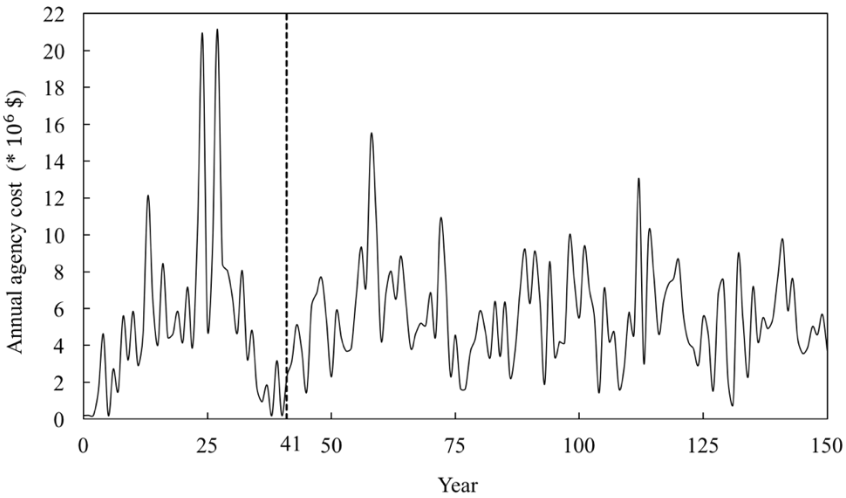
Fig. 4. Distribution of the first lifecycle's duration versus the budget constraint: (a) good initial conditions; (b) poor initial conditions



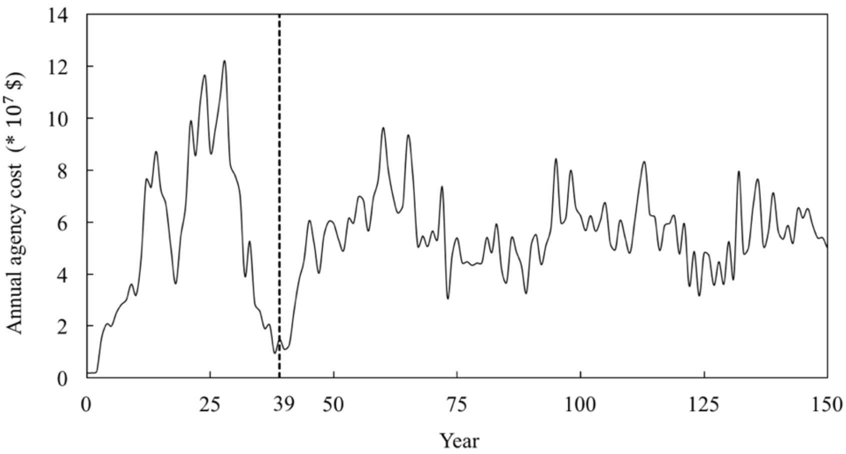
(a) (b)

Fig. 5. Distribution of rehabilitation counts in the first lifecycle versus the budget constraint: (a) good initial conditions; (b) poor initial conditions

One may also wonder how the assumption that the budget can be transferred across the years affects the optimal solutions. A coarse look into this problem can be made by examining the actual annual expenditures for the optimal MR&R plan when budget is allowed to transfer across years. To examine if the optimal annual agency cost follows any periodic pattern, we plot that for a long planning horizon of 150 years for a 100-segment system with $B = 4.42 \times 10^6$ \$/year (Fig. 6a) and a 1000-segment system with $B = 4.38 \times 10^7$ \$/year (Fig. 6b). Both figures show large variations in the annual agency expenditures. The variation is especially large for a smaller-sized system (see Fig. 6a), and during the transient period of the pavement system (i.e. before the dashed vertical line in both figures, which marks the time when the last segment enters a steady state). Also, no periodic pattern is observed in either figure. This is also as expected because each segment has a different lifecycle duration. The above findings imply that, if a constant, non-transferable budget is set in each year, the optimal MR&R plan would be very different, and the optimal cost would likely be much larger than what we obtain in this paper. In reality, an agency may have the freedom to decide how to allocate the budget over a short planning period, e.g. 5 years (Lee and Madanat, 2015). However, this would also be suboptimal as is revealed by Fig. 6a and b. (Interestingly, this result is at odds with the claim made by Lee and Madanat, 2015.) The agencies are suggested that they also seek to borrow and lend money across the planning periods (e.g., via some financial tools).



(a)



(b)

Fig. 6. Annual agency costs under optimal MR&R plans with budget transfers allowed: (a) a system of 100 segments; (b) a system of 1000 segments

4.4. Under the separate budget constraints

In reality, an agency often manages separate budgets for different treatments (Lee and Madanat, 2015). In this section we revisit how this suboptimal practice affects the performance of the optimal MR&R plan. We examine the same 100-segment pavement system analyzed in Fig. 1, but now under the separate budget constraints. For the clarity of illustration, we here present the results of a reduced problem with two budget constraints only: one for reconstruction and the other for maintenance and rehabilitation combined.[4] Fig. 7a plots a contour map of the optimal total cost for the annual reconstruction budget ranging in $[0, 5.5 \times 10^6]$ \$/year, and the annual maintenance and rehabilitation budget in $[0, 5 \times 10^6]$ \$/year. Each thin, solid curve in the figure represents a contour line with the total discounted cost marked on the curve (in the unit of 10^7 \$). Examination of this figure unveils interesting findings that complement those in the literature.

First of all, no contour line is present in the region in the lower-left part of Fig. 7a (labeled "***INF***"), because the MR&R optimization problem is infeasible in this region due to insufficient budgets. Note that the area on the left side of the vertical dashed line at 0.36×10^6 \$/year (the minimum reconstruction budget associated with $T_k^{max}, \forall k$) all belongs to region ***INF***, regardless of the maintenance and rehabilitation budget. On the other hand, no contour line exists in the rectangular region in the upper-right corner of Fig. 7a (labeled "***A***"), because in this region both budget constraints are unbinding and the optimal total cost remains constant at 11.73×10^7 \$. Note the bottom-left corner of region ***A*** indicates the maximum budgets needed: 2.51×10^6 \$/year for reconstruction and 2.10×10^6 \$/year for maintenance and rehabilitation combined.

The remaining part of the figure is divided into three regions: ***B***, ***C***, and ***D***, as demarcated by the thick solid lines in the figure. Region ***B*** refers to the set of cases where the reconstruction budget constraint is unbinding and the maintenance and rehabilitation budget constraint is binding. Hence the contours in this region are horizontal lines. Region ***C***, on the other hand, is where the maintenance and rehabilitation budget constraint is unbinding but the reconstruction one is binding. Finally, region ***D*** is where both budget constraints are binding. Note that each contour line that extends from the top-left to the bottom-right corner of the figure is tangent to a line with slope -1, and the tangent point indicates the optimal solution under the combined budget constraint. Some of these combined-budget-constraint solutions are shown as black dots on the contour lines of 11.90, 11.80, and 11.75×10^7 \$. The lower boundary of ***D*** is also tangent to a line with slope -1 (the dashed line shown in Fig. 7a); this dashed line specifies the minimum budget required for the combined budget scenario (4.02×10^6), which is consistent with Fig. 1. This is also intuitive: if a feasible MR&R plan is found for a given pair of separate budget constraints, then the corresponding problem when all the budgets are combined is also feasible.

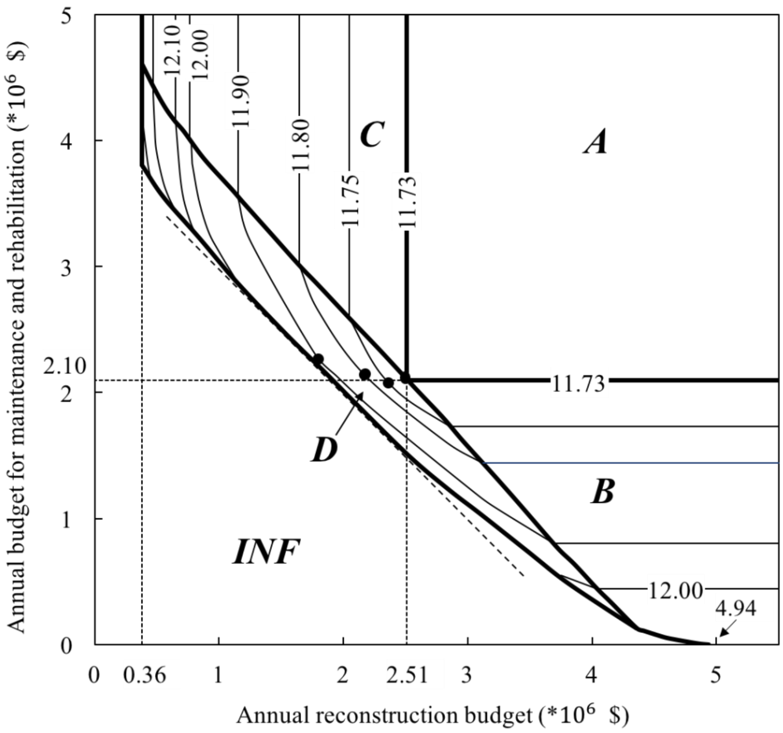
Fig. 7b shows the contour map of the percentage of cost savings created by optimizing for a combined budget for all the three treatments. The figure shows a cost saving of up to 4% when the

[4] We choose to present the results of this reduced problem simply for the sake of clarity. Note now the effects of the two budget constraints can be clearly illustrated by two-dimensional contour maps (like Fig. 7a-d). A three-budget-constraint problem can also be solved by our approach, but the effects of the three budget constraints cannot be presented in a similar way in the paper. The analysis of the reduced problem does not compromise our findings since the maintenance cost is always small and easy to accommodate; see again Fig. 1.

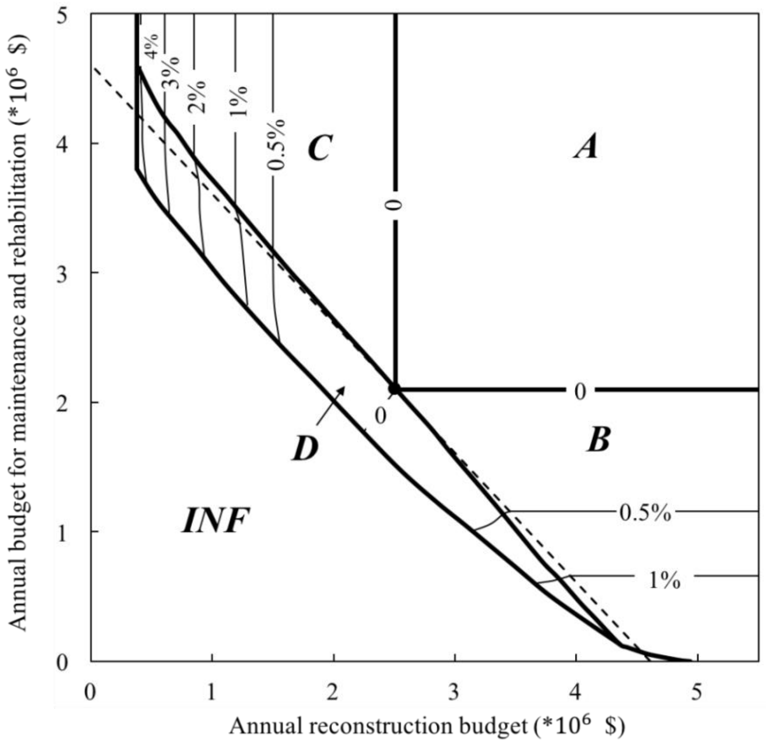
reconstruction budget is highly limited. On the other hand, if only the maintenance and rehabilitation budget is limited, the cost saving is below 2%. The dashed line with slope -1 indicates the maximum required combined budget, and the contour lines above this dashed line should overlap with the contours of the optimal total cost shown in Fig. 7a.

To further illustrate the effectiveness of maintenance, Fig. 7c compares the five solution regions defined above (***A***, ***B***, ***C***, ***D***, and ***INF***) against those for the optimal R&R plan (i.e. without preventive maintenance). The solution regions for the R&R case are demarcated by thick, dashed lines in the figure. The figure shows that when preventive maintenance is included, region ***D*** expands and moves downward, while region ***INF*** diminishes. This means including maintenance can largely reduce the budget needed to keep the pavements workable.

Finally, the percentage of cost saving between MR&R and R&R is plotted in Fig. 7d. It shows that including maintenance can bring an over 5% reduction in the optimal total cost for most of the cases. Highest cost savings (almost 8%) are achieved when the reconstruction budget is most limited, because maintenance can extend the pavements' lifecycles and thus reduce the need for reconstruction.

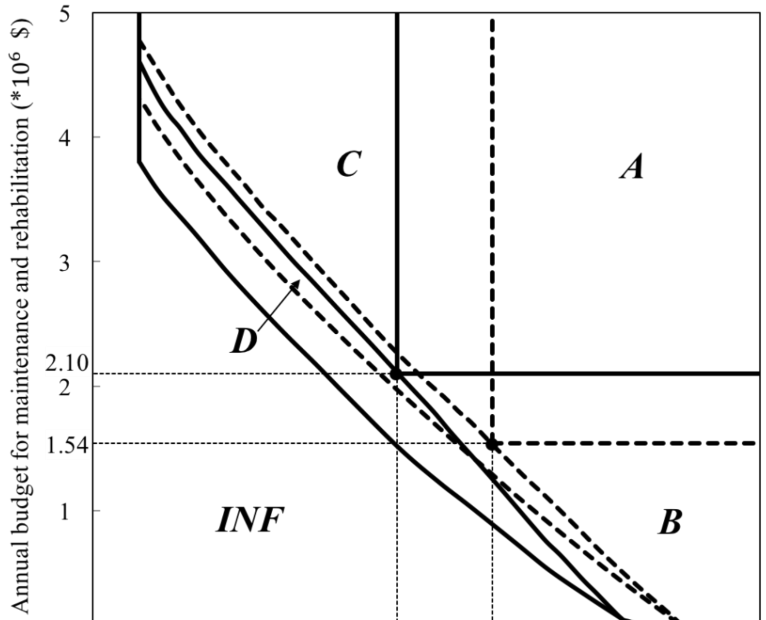


(a)



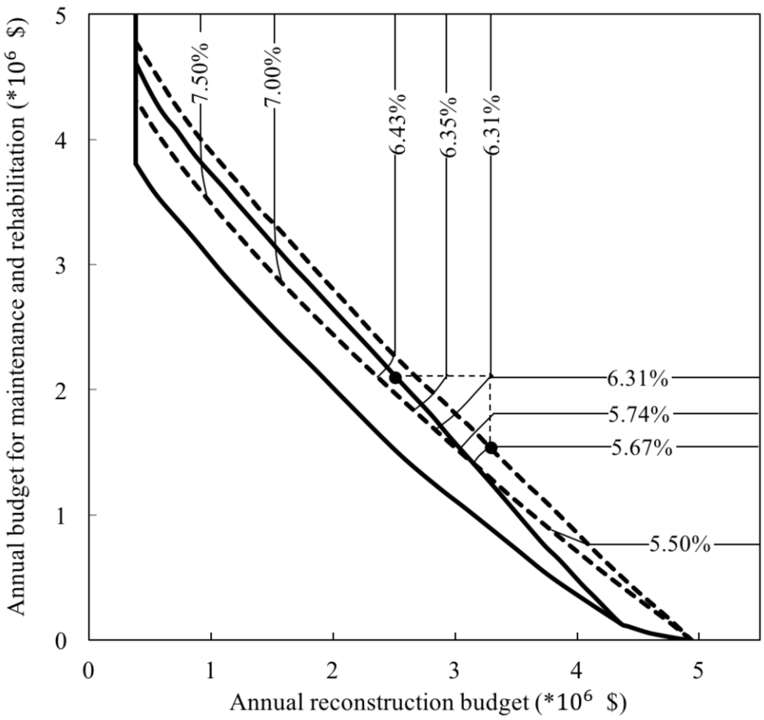
Annual budget for maintenance and rehabilitation (*10^6 $)
Annual reconstruction budget (*10^6 $)
4%
3%
2%
1%
0.5%
C
A
0
0
0
B
D
INF
0.5%
1%

(b)



Annual budget for maintenance and rehabilitation (*10^6 $)
Annual reconstruction budget (*10^6 $)
C
A
D
2.10
1.54
INF
B
2.51
3.29

(c)



(d)

Fig. 7. Solutions under separate budget constraints: (a) contours of optimal total cost and the solution regions; (b) percentage of cost savings from optimally allocating a combined budget; (c) comparison of the solution regions with and without maintenance; (d) percentage of cost savings from adding maintenance.

One may note that the solution regions shown in Fig. 7a-d are different from those presented by Lee and Madanat (2015). Specifically, in Lee and Madanat the right boundary of region ***INF*** is a vertical line, and the lower boundary of region ***D*** is the horizontal axis; see Fig. 4 in the cited paper. The difference is due to the different input parameters used in our case studies. For more details, please refer to Appendix E, which presents all the possible patterns of the solution regions that may arise from real-life pavement systems.

4.5. Computational efficiency

The solid dots in Fig. 8 present the computation times of 110 randomly generated numerical instances under the combined budget constraint against the number of pavement segments (ranging from 50 to 1000). These dots exhibit a clear linear relationship between the computational time and the size of the problem. Similar linear relationship is found for the cases under separate budget constraints. This is because the number of iterations needed for the Lagrange multiplier(s) λ (or λ_p) to converge is uncorrelated with the size of the system. With our selected error tolerance level (1% of the budget), this number of iterations is usually 4-5 under the combined budget constraint, and 20-30 under three separate budget constraints. Note too that a 1000-segment system takes about 1.5 hours to solve under the combined budget constraint, and about 8-10 hours under three separate budget constraints. The runtime is very reasonable for real-world implementation.

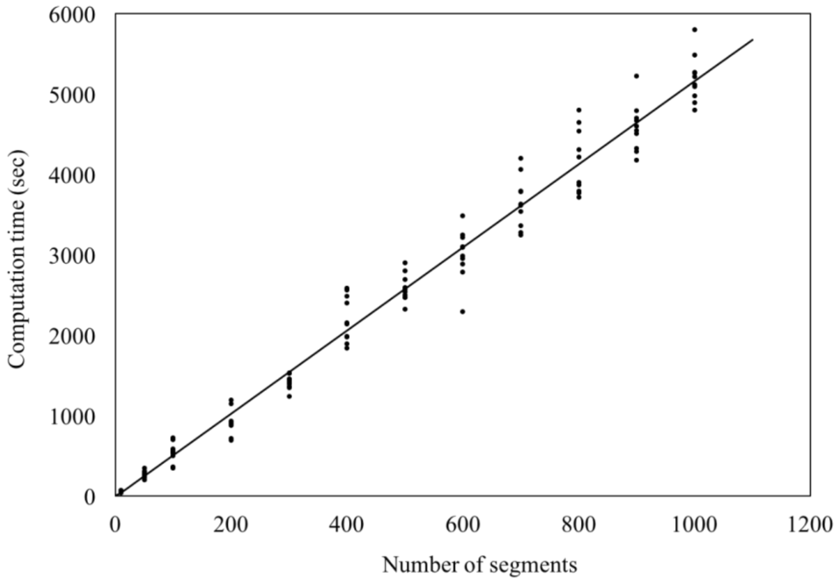


Fig. 8. Computation times for the numerical instances under the combined budget constraint

In comparison, the GA algorithm developed by Lee and Madanat (2015) for solving the joint R&R optimization (i.e. without maintenance) seems to exhibit a polynomial complexity (see Fig. 6 of the cited work); i.e. the computation time increases much faster than the linear trend. Thus our approach is more computationally efficient than the GA algorithm especially for larger-scale systems.

5. Conclusions

We formulate a general mathematical model for the joint optimization of MR&R planning for a system of heterogeneous pavement segments under budget constraints. We propose a Lagrange multiplier approach combined with derivative-free quasi-Newton methods to solve the system-level program. The approach relaxes the budget constraints and decomposes the system-level problem into multiple segment-level subproblems, whose solutions can be more easily derived. Hence, our approach can be applied to segment-level models that take any specific forms. Our work has extended the literature in the realm of pavement MR&R optimization in multiple aspects. We are, to our best knowledge, the first to formulate and solve the system-level MR&R optimization problem that incorporates preventive maintenance activities modeled by a more realistic formulation fitted on the real data. The inclusion of maintenance adds another dimension to the solution space, as compared to the previous system-level studies (e.g. Lee and Madanat, 2015). Despite the added complexity, however, the problem is solved within only moderate runtimes, thanks in part to the derivative-free quasi-Newton methods used to search for the optima, and to the efficient segment-level heuristic. More importantly, the runtime increases linearly with the number of segments in a system, which ensures the applicability of our solution approach to large-scale systems. Further, the computational efficiency is achieved without compromising the solution quality. Particularly for the problem under the combined budget constraint, our approach guarantees the global optimality or near-optimality at the system level as long as the segment-level subproblems are solved at or near the optima. Note that high-quality solutions are always preferred because even one more percent of cost reduction would mean a saving of millions of dollars.

Our numerical case studies reveal a number of useful findings. For example, the results show that by optimally allocating a combined agency budget among the treatments, the minimum total cost can be reduced by up to 4% (see again Fig. 7b). Incorporating maintenance in the optimal MR&R planning will result in a total cost saving of over 6% (see Fig. 7d), and more importantly, it can significantly lower the minimum budget required to keep the pavement system workable (by over 9% in our numerical cases; see Fig. 1). Highway agencies can obtain the optimal allocation of the budget for each treatment, and the minimum total budget required from our model, which are very useful for them to prepare for future budget proposals. Managerial insights are also unveiled, including: i) that the agency should perform fewer reconstructions but more rehabilitations when the budget is more limited; ii) that incorporating maintenance will reduce the need for reconstruction but increase the rehabilitation frequency; and iii) that the pavements' initial conditions have a significant effect on the minimum budget required and the transient periods of the optimal MR&R plans, but have almost no effect on the steady-state periods of the MR&R plans. These insights are helpful for agencies to plan for future pavement management activities.

To be sure, our work still has limitations. The limitations first exist in the present segment-level models. These include: i) the present models fail to account for the influence of a number of factors including the environmental conditions and the dynamic traffic loading; ii) the cost and effectiveness models for a variety of other preventive maintenance treatments (e.g. fog seal and microsurfacing) are not included; and iii) the roughness index is not a perfect indicator of pavement conditions. However, our system-level approach can still be applied to more complicated scenarios that address these practical concerns, should more realistic segment-level models be made available. Work in this regard is underway.

Also, the effects of some key parameters on the optimal MR&R plan are not fully examined due to the limited space of the present paper. Of particular note is that the maximum allowable roughness index, s_k^{max} (which defines the worst acceptable condition for a pavement), has a significant impact on the cost savings stemmed from incorporating maintenance and from optimally allocating budget among the treatments. Larger s_k^{max} (i.e. higher tolerance for poor pavements) would result in more savings.

Potential extensions of our work also include modeling the uncertainties in the deterioration process and MR&R effectiveness, addressing more budget constraints for every period without transferring between the periods and accounting for other operational and practical constraints like the greenhouse gas emissions, network connectivity, time-varying traffic loading etc. These extensions would require not only a revised formulation of the problem, but also more efficient search algorithms to ensure convergence within reasonable runtimes. Some of these extensions are currently under investigation too.

Acknowledgements

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Appendix A. Proof sketch of Lemma 1

First note that the case of $\lambda^* = 0$ is trivial. In the following proof we assume that $\lambda^* \neq 0$ and $\lambda^H \neq 0$ (note it is unlikely that $\lambda^* \neq 0$ and $\lambda^H = 0$ when δ_1 and δ_2 are both small). So from (6a-b), we have:

$$\sum_{k=1}^{K} C_k\big(\boldsymbol{x}_k^*(\lambda^*)\big) = \sum_{k=1}^{K} C_k\left(\boldsymbol{x}_k^H(\lambda^H)\right) = B \tag{A1}$$

Then,

$$\left|\sum_{k=1}^{K}\left(C_k\big(\boldsymbol{x}_k^*(\lambda^H)\big) - C_k\big(\boldsymbol{x}_k^*(\lambda^*)\big)\right)\right| = \left|\sum_{k=1}^{K}\left(C_k\big(\boldsymbol{x}_k^*(\lambda^H)\big) - C_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right)\right| \leq$$

$$\sum_{k=1}^{K}\left|C_k\big(\boldsymbol{x}_k^*(\lambda^H)\big) - C_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right| \leq K \cdot \delta_1 \tag{A2}$$

On the other hand, since $x_k^*(\lambda)$ minimizes $Z_k(x_k) + \lambda C_k(x_k)$, we have:

$$Z_k\big(\boldsymbol{x}_k^*(\lambda^*)\big) + \lambda^* \cdot C_k\big(\boldsymbol{x}_k^*(\lambda^*)\big) \leq Z_k\big(\boldsymbol{x}_k^*(\lambda^H)\big) + \lambda^* \cdot C_k\big(\boldsymbol{x}_k^*(\lambda^H)\big) \tag{A3}$$

and,

$$Z_k\big(\boldsymbol{x}_k^*(\lambda^H)\big) + \lambda^H \cdot C_k\big(\boldsymbol{x}_k^*(\lambda^H)\big) \leq Z_k\big(\boldsymbol{x}_k^*(\lambda^*)\big) + \lambda^H \cdot C_k\big(\boldsymbol{x}_k^*(\lambda^*)\big) \tag{A4}$$

The (A3) and (A4) can be combined into:

$$\lambda^* \cdot \left(C_k\big(\boldsymbol{x}_k^*(\lambda^*)\big) - C_k\big(\boldsymbol{x}_k^*(\lambda^H)\big)\right) \leq Z_k\big(\boldsymbol{x}_k^*(\lambda^H)\big) - Z_k\big(\boldsymbol{x}_k^*(\lambda^*)\big) \leq \lambda^H \cdot \left(C_k\big(\boldsymbol{x}_k^*(\lambda^*)\big) - C_k\big(\boldsymbol{x}_k^*(\lambda^H)\big)\right) \tag{A5}$$

Hence,

$$\left|\sum_{k=1}^{K}\left(Z_k\big(\boldsymbol{x}_k^*(\lambda^*)\big) - Z_k\big(\boldsymbol{x}_k^*(\lambda^H)\big)\right)\right| \leq \max\{\lambda^*, \lambda^H\} \cdot \left|\sum_{k=1}^{K}\left(C_k\big(\boldsymbol{x}_k^*(\lambda^*)\big) - C_k\big(\boldsymbol{x}_k^*(\lambda^H)\big)\right)\right| \leq$$

$$\max\{\lambda^*, \lambda^H\} \cdot K\delta_1 \tag{A6}$$

Now we have:

$\left|\sum_{k=1}^{K} Z_k(\boldsymbol{x}_k^*(\lambda^*)) - \sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right| \leq \left|\sum_{k=1}^{K} Z_k(\boldsymbol{x}_k^*(\lambda^*)) - \sum_{k=1}^{K} Z_k(\boldsymbol{x}_k^*(\lambda^H))\right| +$

$\left|\sum_{k=1}^{K} Z_k(\boldsymbol{x}_k^*(\lambda^H)) - \sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right| \leq \max\{\lambda^*, \lambda^H\} \cdot K\delta_1 + \sum_{k=1}^{K} \left|Z_k(\boldsymbol{x}_k^*(\lambda^H)) -\right.$

$\left. Z_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right| \leq K \cdot (\max\{\lambda^*, \lambda^H\}\, \delta_1 + \delta_2)$ (A7)

Note in a real pavement system that $C_k(\boldsymbol{x}_k^*(\lambda^H)) - C_k(\boldsymbol{x}_k^*(\lambda^*))$ can be either positive or negative for any k; and the positive and negative components of the sum $\sum_{k=1}^{K}\left(C_k(\boldsymbol{x}_k^*(\lambda^H)) - C_k(\boldsymbol{x}_k^*(\lambda^*))\right)$ will cancel out. Thus inequality (A2) may be a very weak one, and as a result, (A7) may be weak too. ■

Appendix B. Proof of Lemma 2

We prove Lemma 2 by contradiction. Suppose there exists $\lambda_1 > \lambda_2 \geq 0$, such that $V(\lambda_1) \geq V(\lambda_2)$. We denote $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$ as the solutions associated with λ_1 and λ_2, respectively; i.e., for each $k = 1,2, \ldots, K$, $\boldsymbol{x}_k^1$ is the unique minimizer of $H_k(\boldsymbol{x}_k, \lambda_1) \equiv C_k^U(\boldsymbol{x}_k) + (1 + \lambda_1)C_k(\boldsymbol{x}_k)$, and $\boldsymbol{x}_k^2$ is the unique minimizer of $H_k(\boldsymbol{x}_k, \lambda_2) \equiv C_k^U(\boldsymbol{x}_k) + (1 + \lambda_2)C_k(\boldsymbol{x}_k)$.

We then have:

$0 > \sum_{k=1}^{K}[H_k(\boldsymbol{x}_k^1, \lambda_1) - H_k(\boldsymbol{x}_k^2, \lambda_1)]$
$= \sum_{k=1}^{K}\{[C_k^U(\boldsymbol{x}_k^1) + (1 + \lambda_1)C_k(\boldsymbol{x}_k^1)] - [C_k^U(\boldsymbol{x}_k^2) + (1 + \lambda_1)C_k(x_k^2)]\}$
$= \sum_{k=1}^{K}[C_k^U(\boldsymbol{x}_k^1) - C_k^U(\boldsymbol{x}_k^2)] + (1 + \lambda_1)(V(\lambda_1) - V(\lambda_2))$
$\geq \sum_{k=1}^{K}[C_k^U(\boldsymbol{x}_k^1) - C_k^U(\boldsymbol{x}_k^2)] + (1 + \lambda_2)(V(\lambda_1) - V(\lambda_2))$
$= \sum_{k=1}^{K}[H_k(\boldsymbol{x}_k^1, \lambda_2) - H_k(\boldsymbol{x}_k^2, \lambda_2)] > 0$
Contradiction! ■

Note if $\boldsymbol{x}_k^1$ and $\boldsymbol{x}_k^2$ are not unique minimizers, then the above equation may hold without creating contradiction, but only when $H_k(\boldsymbol{x}_k^1, \lambda_1) = H_k(\boldsymbol{x}_k^2, \lambda_1)$ and $H_k(\boldsymbol{x}_k^1, \lambda_2) = H_k(\boldsymbol{x}_k^2, \lambda_2)$ for all k (i.e., $\boldsymbol{x}_k^1$ and $\boldsymbol{x}_k^2$ are dual optima under both λ_1 and λ_2), and $V(\lambda_1) = V(\lambda_2)$. Further note that the optimal $C_k(\boldsymbol{x}_k)$ should be a non-increasing function of λ, we have $C_k(\boldsymbol{x}_k^1) = C_k(\boldsymbol{x}_k^2)$ and $C_k^U(\boldsymbol{x}_k^1) = C_k^U(\boldsymbol{x}_k^2)$ for all k. The above conditions are too strict to be satisfied by realistic segment-level models. Thus, we reckon Lemma 2 as a general result.

Appendix C. Proof sketch of the convergence of Algorithm 1

We assume that $V(\lambda)$ is continuously differentiable everywhere and the unique root of $V(\lambda)$ is λ^* (since $V(\lambda)$ is a decreasing function of λ). We prove the convergence of Algorithm 1 by contradiction. Suppose the stop criterion cannot be attained as n increases. Initially, we have $\lambda^0 < \lambda^1$ and $V(\lambda^0) > 0$. According to Lemma 2, we have $V(\lambda^1) < V(\lambda^0)$. One of the following two cases will occur.

Case 1: $V(\lambda^n) > 0$ for all $n \geq 1$

In this case $\lambda^{n+1} = \lambda^n - V(\lambda^n)\frac{\lambda^n - \lambda^{n-1}}{V(\lambda^n) - V(\lambda^{n-1})} > \lambda^n$ for all the n; i.e., the sequence $\{\lambda^n\}$ is strictly increasing. Thus $\{\lambda^n\}$ should be bounded above, because otherwise $V(\lambda^n)$ would be 0 or negative for sufficiently large n. That means $\{\lambda^n\}$ has a supremum: $\tilde{\lambda} = \sup\{\lambda^n\}$. Let $V(\tilde{\lambda}) = \kappa$ as shown in Fig. C1a. We have:

$$\lim_{n\to\infty} V(\lambda^n) = \kappa > 0 \tag{C1}$$

$$\lim_{n\to\infty} \lambda^n = \tilde{\lambda} \tag{C2}$$

However,

$$\tilde{\lambda} = \lim_{n\to\infty} \lambda^{n+1} = \lim_{n\to\infty} \left(\lambda^n - V(\lambda^n)\frac{\lambda^n - \lambda^{n-1}}{V(\lambda^n) - V(\lambda^{n-1})}\right) = \tilde{\lambda} - \kappa \cdot V'(\tilde{\lambda}) > \tilde{\lambda} \tag{C3}$$

Contradiction!

Case 2: $V(\lambda^n) < 0$ for some $n \geq 1$.

According to Algorithm 1, we have $V(\lambda^{n-1}) \cdot V(\lambda^n) < 0$ for all $n \geq n'$, where $n' = \min\{n | V(\lambda^n) < 0\}$; i.e., $\{\lambda^n\}$ oscillates on both sides of λ^*. Then there must be infinite number of λ^n's on at least one side of λ^*. There are two subcases:

(1) Infinite number of λ^n's occur only on one side of λ^*. The contradiction can be shown using the same method presented in Case 1.

(2) Infinite numbers of λ^n's occur on both sides of λ^*. We denote $\{\lambda_L^n\}$ as the λ^n's on the left side of λ^* and $\{\lambda_R^n\}$ as those on the right side. We define $\tilde{\lambda}_L = \sup\{\lambda_L^n\}$ and $\tilde{\lambda}_R = \inf\{\lambda_R^n\}$ as shown in Fig. C1b, where $V(\tilde{\lambda}_L) = \kappa_L$ and $V(\tilde{\lambda}_R) = \kappa_R$. We have:

$$\lim_{n\to\infty} V(\lambda_L^n) = \kappa_L > 0 \tag{C4}$$

$$\lim_{n\to\infty} \lambda_L^n = \tilde{\lambda}_L \tag{C5}$$

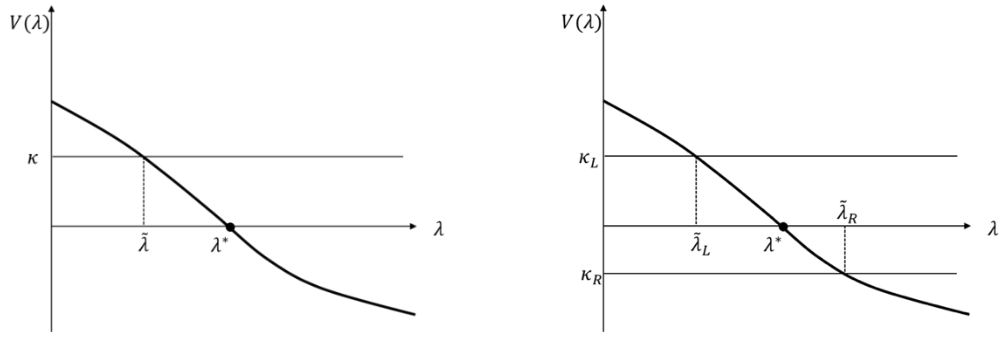
$$\lim_{n\to\infty} V(\lambda_R^n) = \kappa_R < 0 \tag{C6}$$

$$\lim_{n\to\infty} \lambda_R^n = \tilde{\lambda}_R \tag{C7}$$

And,

$$\lim_{n\to\infty} \lambda^{n+1} = \lim_{n\to\infty} (\lambda^n - V(\lambda^n)\frac{\lambda^n - \lambda^{n-1}}{V(\lambda^n) - V(\lambda^{n-1})}) \in (\tilde{\lambda}_L, \tilde{\lambda}_R) \tag{C8}$$

Contradiction! ■



(a) (b)

Fig. C1 Illustrations of the two cases of contradiction: (a) case 1; (b) case 2.

Appendix D. The dynamic programming approach to the segment-level problem

We first reproduced the dynamic programming method used by Lee and Madanat (2014a, 2015) with only minor modifications to solve subproblem 1. To this end, we assume that the maintenance intensity $v_{k\tau}$ and the pavement roughness level $s_k(\tau)$ take values form predefined discrete sets, i.e., $v_{k\tau} \in \left\{0, \frac{1}{d}D_{k\tau}, \frac{2}{d}D_{k\tau}, \cdots, D_{k\tau}\right\}$ for $\tau \in \{1, \cdots, T_k\}$, and $s_k(\tau) \in M_{k\tau} = \left\{s_k^{new}, s_k^{new} + \frac{\bar{s}_k(\tau)-s_k^{new}}{N}, s_k^{new} + 2\frac{\bar{s}_k(\tau)-s_k^{new}}{N}, \cdots, \bar{s}_k(\tau)\right\}$ for $\tau \in \{0, \cdots, T_k\}$, where $\bar{s}_k(\tau)$ is the maximum allowed roughness for segment k in year τ; i.e., if $s_k(\tau) > \bar{s}_k(\tau)$, the roughness level would exceed s_k^{max} in the following year $\tau + 1$. There are $d + 2$ decision options in each year $\tau \in \{1, \cdots, T_k\}$: do nothing; rehabilitation only; and maintenance only with intensity $\frac{1}{d}D_{k\tau}, \frac{2}{d}D_{k\tau}, \cdots, D_{k\tau}$, respectively. Let $Y_k(q_{k\tau})$ denote the cost-to-go in year τ (i.e. the minimum total discounted cost from year τ to T_k), the algorithm is described as follows:

Step 1. For each $T_k \in \{T_k^{min}, \cdots, T_k^{max}\}$, set the boundary condition as $Y_k(q_{k,T_k}) = 0,\ \forall\, q_{k,T_k}$. For each year $\tau = T_k - 1, T_k - 2, \cdots, 0$, $s_k(\tau) \in M_{k\tau}$ and $h_{k\tau} = h_{k0} + \tau$, we generate $Y_k(q_{k\tau})$ in the backward direction by the Bellman equation:

$$Y_k(q_{k\tau}) = \min_{x_{k\tau,1}, x_{k\tau,2}, v_{k\tau}} \left\{\int_{\tau}^{\tau+1} l_k\left(c_k^1 s_k(u) + c_k^2\right) e^{-ru} du + x_{k\tau,1}\left(\gamma_k^1 v_{k\tau} + \gamma_k^2\right) e^{-r\tau} + x_{k\tau,2}\left(m_k^1 R_{k\tau} + m_k^2\right) e^{-r\tau} + Y_k(q_{k,\tau+1})\right\} \quad \text{(D1)}$$

where

$$q_{k,\tau+1} = \{s_k(\tau+1), h_{k,\tau+1}\} = \left\{F_k\left(s_k(\tau) - x_{k\tau,2} G_k\left(R_{k\tau}, s_k(\tau)\right), 1, h_{k\tau}, \bar{b}_k - x_{k\tau,1} E_k(v_{k\tau}, s_k(\tau))\right), h_{k\tau} + 1\right\} \quad \text{(D2)}$$

$$Y_k(q_{k,\tau+1}) = \frac{s_k - s_k(\tau+1)}{s_k - s_k'} Y_k(s_k', h_{k,\tau+1}) + \frac{s_k(\tau+1) - s_k'}{s_k - s_k'} Y_k(s_k, h_{k,\tau+1}) \quad \text{(D3)}$$

s_k' and s_k are the two consecutive roughness indices in $M_{k,\tau+1}$ that satisfy $s_k' \leq s_k(\tau+1) \leq s_k$.

Step 2. For each year $\tau = 0,1,\cdots,T_k - 1$, record the optimal decision in the forward direction:

$$(x^*_{k\tau,1}, x^*_{k\tau,2}, v^*_{k\tau}) = \underset{x_{k\tau,1}, x_{k\tau,2}, v_{k\tau}}{\operatorname{argmin}} \left\{ \int_\tau^{\tau+1} l_k(c_k^1 s_k(u) + c_k^2)\, e^{-ru} du + x_{k\tau,1}(\gamma_k^1 v_{k\tau} + \gamma_k^2) e^{-r\tau} + x_{k\tau,2}(m_k^1 R_{k\tau} + m_k^2) e^{-r\tau} + Y_k(q_{k,\tau+1}) \right\} \quad \text{(D4)}$$

where $q_{k0} = \{s_k(0), h_{k0}\}$.

Step 3. Find the T_k that minimizes $\frac{Z_k^S}{1-e^{-rT_k}}$.

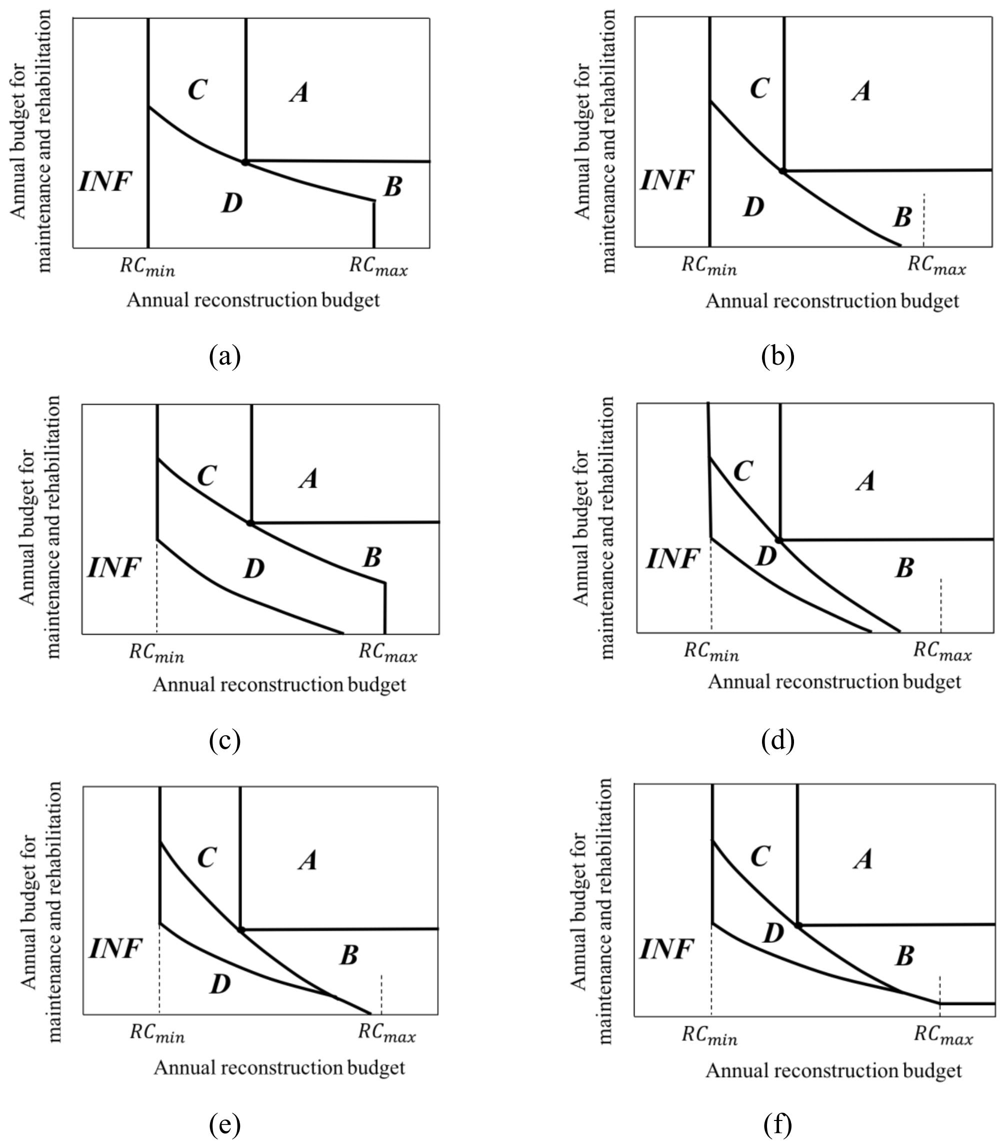
In the first step, we apply the Bellman equation (D1) recursively to generate $Y_k(q_{k\tau})$ for all $\tau \in \{0,\cdots,T_k - 1\}, s_k(\tau) \in M_{k\tau}$ and $h_{k\tau} = h_{k0} + \tau$. The pavement state in year $\tau + 1$ is calculated by equation (D2). The cost-to-go $Y_k(q_{k,\tau+1})$ is approximately by linear interpolation between $Y_k(s_k', h_{k,\tau+1})$ and $Y_k(s_k, h_{k,\tau+1})$; see equation (D3). The optimal decision $(x^*_{k\tau,1}, x^*_{k\tau,2}, v^*_{k\tau})$ that minimizes $Y_k(q_{k\tau})$ in each year τ is obtained and recorded in step 2 with the initial state q_{k0}.

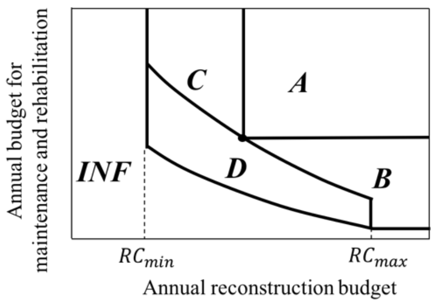
To solve subproblem 2, we make the following changes to the above algorithm: i) in year 0 there are $d + 2$ decisions as in other years; and ii) T_k is replaced by t_k^T, whose range is $[T_k^{min'}, T_k^{max'}]$, where $T_k^{min'} = \max\{0, T_k^{min} - h_{k0}\}$ and $T_k^{max'} = \max\{0, T_k^{max} - h_{k0}\}$. Finally, we choose the t_k^T that minimizes equation (12).

Appendix E. General patterns for the solution regions as defined in Fig. 7a-d

Fig. E1 presents all the seven possible patterns of the solution regions. The result in Lee and Madanat belongs to the pattern shown by Fig. E1a, where RC_{min} and RC_{max} denote the minimum and maximum reconstruction costs that are required when the lifecycle duration is T_k^{max} and T_k^{min}, respectively. (Note that this is the only pattern described in Lee and Madanat, 2015.) This pattern occurs if: i) a feasible solution exists when no maintenance or rehabilitation is applied, and only the minimum reconstruction is executed; and ii) the maximum reconstruction budget RC_{max} will be binding when no maintenance or rehabilitation is applied. If only condition ii) is false, i.e., RC_{max} is unbinding even if no maintenance or rehabilitation is applied, then the upper boundary of region ***D*** would hit the horizontal axis before crossing the vertical line at RC_{max}. This will render the pattern shown by Fig. E1b.

On the other hand, if the above condition i) is false, then the lower boundary of region ***D*** will decline as the reconstruction budget increases. This oblique lower boundary may end by: I) hitting the horizontal axis (before reaching RC_{max}); II) crossing the upper boundary of ***D*** (before reaching RC_{max}); and III) crossing the vertical line at RC_{max}. Case I) can be further divided into two patterns: when the upper boundary of ***D*** ends at the vertical line at RC_{max} (Fig. E1c), and when that boundary also ends at the horizontal axis (Fig. E1d). In case II), the two boundaries of region ***D*** merge to a single line which is decreasing as reconstruction budget increases. This line will cross the horizontal axis (Fig. E1e) or the vertical line at RC_{max} (Fig. E1f). Finally, case III) will render the patterns described by Fig. E1g. Note any interface that appears on the right of RC_{max} has to be horizontal. The results shown in Fig. 7a-d belong to the pattern in Fig. E1e.





(g)

Fig. E1. Patterns for the solution regions ***A***, ***B***, ***C***, ***D***, and ***INF***

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