Hindawi
Journal of Advanced Transportation
Volume 2017, Article ID 9243039, 8 pages
https://doi.org/10.1155/2017/9243039

Research Article

A Trial-and-Error Method with Autonomous Vehicle-to-Infrastructure Traffic Counts for Cordon-Based Congestion Pricing

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Received 19 April 2017; Accepted 31 May 2017; Published 2 July 2017

Academic Editor: Xiaopeng Li

This study proposes a practical trial-and-error method to solve the optimal toll design problem of cordon-based pricing, where only the traffic counts autonomously collected on the entry links of the pricing cordon are needed. With the fast development and adoption of vehicle-to-infrastructure (V2I) facilities, it is very convenient to autonomously collect these data. Two practical properties of the cordon-based pricing are further considered in this article: the toll charge on each entry of one pricing cordon is identical; the total inbound flow to one cordon should be restricted in order to maintain the traffic conditions within the cordon area. Then, the stochastic user equilibrium (SUE) with asymmetric link travel time functions is used to assess each feasible toll pattern. Based on a variational inequality (VI) model for the optimal toll pattern, this study proposes a theoretically convergent trial-and-error method for the addressed problem, where only traffic counts data are needed. Finally, the proposed method is verified based on a numerical network example.

1. Introduction

Traffic congestion has become one of the key problems for the operation of urban transport systems. Traffic congestion not only is a daily source of frustration for many motorists but also results in environmental issues and social expenses. Congestion causes longer travel times, increased travel time variability, air and noise pollution, and lower productivity in the work force [1–5]. Road pricing has been well recognized as an important tool for the land transport authority to manage the traffic demand, shifting transit ridership [6–8], and eventually ameliorate the traffic congestion; see Small and Verhoef [9], Lawphongpanich et al. [10], and Cheng et al. [11] among many others. A prosperous literature is observed for the theoretical studies of urban congestion pricing. Many studies have focused on determining the optimal toll fares. Two economics concepts, first-best pricing (no restriction on the pricing locations) and second-best pricing (only part of the network is tolled), have been widely discussed in the context of urban congestion pricing. Despite their theoretical soundness, the achievements of many studies on toll determination are not suitable for practical implementations, mainly because they require the accurate data on many network attributes, including travel demand function and link travel time functions as well as network users' value-of-time [12], and it is difficult to obtain these data for the whole transport network. This article thus accounts for the practical implementation of congestion pricing by solving the toll determination problem while obviating these data.

Since the practical implementation of urban congestion pricing is an important focus of this study, we then investigate some existing congestion pricing practices in Singapore since 1975 [13], in London since 2003 [14], and in Stockholm since 2007 [15]. These three well-known practices all adopt cordon-based schemes, where London uses an area licensing charge and the other two cities take entry-based charge. The entry-based charge is known to be more equitable and efficient than area licensing charge [16]. Therefore, this article also takes cordon-based scheme with entry-based charge as a target.

Two practical properties of the cordon-based pricing are further considered in this article. Firstly, when cordon-based pricing is implemented in practice, the network authority is concerned more about the traffic conditions within the cordon area; for instance, when the congestion pricing was first established in Singapore in 1975, Singapore Land Transport Authority's original target of implementing this scheme is to reduce the total traffic volume to the cordon area by 25% to 35% [13]. This article thus further takes into consideration this point; namely, the total inbound traffic volume of one cordon should be restricted by a predetermined threshold. Secondly, the toll charge on each entry to one cordon should be identical, in view of the convenience for recognition by the drivers and for administration by the authority; for example, toll charge on each link is the same at the Orchard Cordon and Bugis-Marina Centre of Electronic Road Pricing (ERP) system in Singapore.

To sum up, this article aims to propose a method for the toll determination of such a cordon-based pricing, where massive data is not needed. Traffic flow data is of crucial value for the studies of urban transport system, for example, the studies on traffic signaling [17, 18] as well as traffic safety studies [19]. With the fast development of intelligent transportation systems and the Internet of things, it is very convenient to collect the number of vehicles in any particular link. Hence, the proposed trial-and-error method only needs the autonomous traffic counts data on the tolled links, which can be easily gathered based on the vehicle-to-infrastructure (V2I) communications.

2. Literature Review

The concept of obviating network attributes data and instead using traffic survey data to adjust the toll fares until achieving the optimal tolls is termed as "trial-and-error" method in the literature. Based on the seminal works by Downs [20] and Vickrey [21], Li [22] first proposed a trial-and-error method for marginal cost pricing based on a simple two-link example, where the Origin-Destination (OD) demand function is avoided. This work by Li has been well recognized in the literature, and only recently Wang and Yang [23] pointed out that under some extreme conditions (when the demand function is too flat) this solution method could not converge, and a bisection-based trial-and-error method was then developed to remedy this problem. Such a concept of trial-and-error is also of considerable importance for other sorts of transportation networks [24, 25].

Yang et al. [26] proposed a trial-and-error method for the first-best pricing in a general transport network assuming deterministic user equilibrium (DUE) circumstances, which was then extended to the logit-based stochastic user equilibrium (SUE) case by Zhao and Kockelman [27]. As an extension of Yang et al. [26], Han and Yang [28] have proposed a more efficient trial-and-error method for the first-best pricing using advanced step sizes. In the context of second-best pricing, Yang et al. [29] then proposed a convergent trial-and-error mechanism for the optimal toll pattern, which is recently extended by Ye et al. [30] by considering the day-to-day dynamics of network flows.

The aforementioned trial-and-error methods obviate the origin-destination (OD) demand function, yet they still rely on the data of link travel time functions and drivers' value-of-time, which are not all readily available in practice. Based on the concept of traffic-restraint pricing (the objective is to restrict traffic volumes on some links to a desirable target level), Meng et al. [31] have proposed a trial-and-error method that only requires the traffic counts on certain links. This traffic count data could be efficiently and accurately obtained in practice from the toll booths or detection loops. Thus, it is an engineering-based method suitable for practical use. Yang et al. [32] then extended this work by considering elastic demand and asymmetric link flow interactions. In the context of general SUE, Meng and Liu [12] built a trial-and-error method based on the gradient projection algorithm, where the convergence can be guaranteed under mild conditions. Zhou et al. [33] developed a new trial-and-error method for the first-best congestion pricing solution with restriction to link capacity constraints. Recently, Xu et al. [34] have developed a trial-and-error method for the traffic-restraint congestion pricing scheme in a network with day-to-day flow dynamics.

As to the cordon-based pricing, these existing methods in the literature, however, are not valid to use, especially considering the two practical properties of cordon-based pricing addressed in this article, namely, (a) the total inbound flow to one cordon should be restricted and (b) toll charge on each link to one cordon should be identical. Considering the wide practicality of cordon-based pricing in reality, an extension of the previous trial-and-error methods for cordon-based pricing is of considerable significance.

Assuming the users' route choice behavior follows the General SUE principle; this article proposed a trial-and-error method based on the variational inequality (VI) model built by Liu et al. [35] for the side-constrained SUE problem. The total inbound traffic flow to each cordon is limited by a predetermined threshold value, which is taken as a side constraint to the SUE traffic assignment problem. It is shown that the optimal toll pattern equals the optimal Lagrangian multiplier of the side constraints.

The remainder of this article is organized as follows. The next section describes the connection between the optimal toll pattern and side constraints to the cordons. Then, a convergent trial-and-error method for the optimal toll pattern is developed, which is numerically verified by a network example. Finally, the conclusions and future works are presented.

3. Problem Description and Mathematical Model

3.1. Assumptions and Notations. Let G denote a strongly connected network; the attributes of this network are represented by the following notations:

N: set of nodes

A: set of links; $|A|$ is the cardinality of set A

W: set of OD pairs; $|W|$ is the cardinality of W

q_w: travel demand between OD pair $w \in W$

$\mathbf{q}$: column vector of all the OD travel demands; $\mathbf{q} = (q_w, w \in W)^{\mathrm{T}}$

R_w: set of all the paths between OD pair $w \in W$, and $|R_w|$ is the cardinality of R_w

δ_{ak}^w: $\delta_{ak}^w = 1$ if path $k \in R_w$ between OD pair $w \in W$ traverses link $a \in A$; $\delta_{ak}^w = 0$ otherwise

Δ_w: link/path incidence matrix associated with OD pair $w \in W$; namely, $\Delta_w = (\delta_{ak}^w,\ a \in A,\ k \in R_w)$

Δ: link/path incidence matrix for the entire network; $\Delta = (\Delta_w,\ w \in W)$

f_{wk}: traffic flow on path $k \in R_w$ between OD pair $w \in W$

$\mathbf{f}_w$: column vector of traffic flows on all the paths between OD pair $w \in W$; namely, $\mathbf{f}_w = (f_{wk},\ k \in R_w)^{\mathrm{T}}$

$\mathbf{f}$: column vector of traffic flow on all the paths in the network; that is, $\mathbf{f} = (\mathbf{f}_w,\ w \in W)^{\mathrm{T}}$

Λ: OD pair/path incidence matrix, $\Lambda = (\delta_{wk},\ w \in W,\ k \in R_w)$, where δ_{wk} equals 1 if path $r \in R_w$ and 0, otherwise

v_a: traffic flow on link $a \in A$

$\mathbf{v}$: column vector of all the link traffic flows, $\mathbf{v} = (v_a,\ a \in A)^{\mathrm{T}}$

$t_a(\mathbf{v})$: asymmetric link travel time function on link $a \in A$, and it is a nonnegative, monotonically increasing, and continuously differentiable function of the link flow vector $\mathbf{v}$

$c_{wk}(\mathbf{v})$: travel time on path $k \in R_w$, and $c_{wk}(\mathbf{v}) = \sum_{a \in A} t_a(\mathbf{v})\delta_{ak}^w$

$\mathbf{c}_w(\mathbf{v})$: column vector of travel times on all the paths connecting OD pair $w \in W$.

According to the flow conservation law, path flows, link flows, and OD travel demands should fulfill the following equations:

$$\begin{aligned} \mathbf{v} &= \Delta \mathbf{f}, \\ \mathbf{q} &= \Lambda \mathbf{f}, \\ \mathbf{f} &\geq 0. \end{aligned} \tag{1}$$

Equations (1) define the feasible set for path flows and link flows, denoted by Ω_f and Ω_v, respectively.

Assume there are totally I cordons in the network, and all the entry links to cordon i, $i = 1, 2, \ldots, I$, are charging with the same toll fare denoted by τ_i. The toll charges of all the cordons are grouped into vector $\boldsymbol{\tau} = (\tau_i,\ i = 1, 2, \ldots I)^{\mathrm{T}}$. Let $\overline{A}_i$ denote the set of all the entry links to cordon i, $i = 1, 2, \ldots, I$, and all the tolled links are then grouped into set $\overline{A}$. Denoted by τ_a, the toll charge on link a equals τ_i if $a \in \overline{A}_i$ and $\tau_a = 0$ if link a is not an entry link to any cordon. Different toll pattern $\boldsymbol{\tau}$ will affect the drivers' route choice and thus give rise to different equilibrium flows. Let $v_a(\boldsymbol{\tau})$ denote the equilibrium link flow on link $a \in A$ in terms of the following generalized link travel time functions:

$$T_a(\mathbf{v}, \boldsymbol{\tau}) = t_a(\mathbf{v}) + \frac{\tau_a}{\alpha}, \quad a \in A, \tag{2}$$

where α is the drivers' value-of-time. It should be noted that this value-of-time is actually not needed when the trial-and-error method proposed below is used in practice.

As mentioned before, the optimal toll pattern $\boldsymbol{\tau}^*$ is one that can restrict the inbound flow to each cordon i to a predetermined threshold H_i, namely,

$$\sum_{a \in \overline{A}_i} v_a(\boldsymbol{\tau}^*) \leq H_i, \quad i = 1, 2, \ldots, I. \tag{3}$$

To further reflect the equity issue, the toll charge should be zero if the total inbound flows are strictly less than the threshold value; that is,

$$\tau_i^* \times \left(\sum_{a \in \overline{A}_i} v_a(\boldsymbol{\tau}^*) - H_i \right) = 0, \quad i = 1, 2, \ldots, I. \tag{4}$$

In addition, the toll charge of each cordon τ_i^* should be nonnegative,

$$\tau_i^* \geq 0, \quad i = 1, 2, \ldots, I. \tag{5}$$

Herein, (3), (4), and (5) define the mathematical conditions for the optimal toll pattern $\boldsymbol{\tau}^*$. The objective of this article then becomes to find a trial-and-error method for such an optimal toll pattern based on only traffic count data, which can be easily gathered based on the V2I facilities on the road side.

3.2. Mathematical Model. We first discuss the mathematical model for the traffic assignment problem, namely, how to get the equilibrium link flows $v_a(\boldsymbol{\tau})$, $a \in A$ in terms of a given toll pattern $\boldsymbol{\tau} = (\tau_i,\ i = 1, 2, \ldots I)^{\mathrm{T}}$. In view of its better representativeness, the general SUE is adopted to depict the drivers route choice behavior. Hence, with given toll charge pattern $\boldsymbol{\tau} = (\tau_i,\ i = 1, 2, \ldots I)^{\mathrm{T}}$, the drivers make their route choice decisions based on the perceived path travel time

$$C_{wk}(\mathbf{v}, \boldsymbol{\tau}) = \sum_{a \in A} T_a(\mathbf{v}, \boldsymbol{\tau}) \delta_{ak}^w + \zeta_{wk}, \quad k \in R_w,\ w \in W, \tag{6}$$

where the random variable ζ_{wk} is assumed to cover the perception error of all the drivers across the whole population on the travel time of path $k \in R_w$. In the literature, ζ_{wk} is usually assumed to follow Gumbel distribution (Logit-based SUE) or normal distribution (probit-based SUE); see Liu et al. [36, 37].

For the general SUE problem with asymmetric link travel time functions, the following fixed point model proposed by Daganzo [38] can be used to formulate the traffic assignment problem

$$v_a = \sum_{w \in W} \left[q_w \times \sum_{k \in R_w} p_{wk}(\mathbf{v}, \boldsymbol{\tau}) \delta_{ak}^w \right], \quad a \in A, \tag{7}$$

where $p_{wk}(\mathbf{v}, \boldsymbol{\tau})$ is the path choice probability equal to the probability that k is perceived as the shortest path among all the paths in set R_w. Equation (7) can be solved by the convergent Method of Successive Average [39].

We proceed to build a mathematical model for the optimal toll pattern $\boldsymbol{\tau}^*$ that can fulfill (3), (4), and (5). In fact, since the feasible set of toll pattern $\boldsymbol{\tau}$ is the whole positive orthant, the nonlinear complementarity conditions (3), (4), and (5) are equivalent to the following variational inequality (VI) problem [40]:

$$\boldsymbol{\Phi}\left(\boldsymbol{\tau}^*\right)\left(\boldsymbol{\tau}-\boldsymbol{\tau}^*\right) \geq 0, \quad \boldsymbol{\tau} \in \Omega, \tag{8}$$

where $\Omega = \{\boldsymbol{\tau} \mid \tau_i \geq 0,\ i = 1, 2, \ldots I\}$ is the feasible set of $\boldsymbol{\tau}$ and the VI function $\boldsymbol{\Phi}(\boldsymbol{\tau})$ is defined as

$$\begin{aligned}\boldsymbol{\Phi}(\boldsymbol{\tau}) &= \left(\Phi_i(\boldsymbol{\tau}),\ i = 1, 2, \ldots, I\right)^{\mathrm{T}} \\ &= \left(H_i - \sum_{a \in \overline{A}_i} v_a(\boldsymbol{\tau}),\ i = 1, 2, \ldots, I\right)^{\mathrm{T}} : \\ &\qquad \Omega \longrightarrow \mathfrak{R}^I.\end{aligned} \tag{9}$$

Hence, solving the VI model gives the optimal toll pattern $\boldsymbol{\tau}^*$ that can satisfy the mathematical conditions (3), (4), and (5). The monotonicity of this type of VI model in the context of cordon-based pricing has been rigorously proven by Liu et al. [35], which guarantees the convergence of many projection-based solution algorithms. It is shown in the following section that a trial-and-error mechanism can be built based on the solution algorithm for the VI model (8).

4. Trial-and-Error Method

Many projection-type methods can be observed in the literature for solving the VI models proposed for transport problems, for example, the works by Nagurney [41], Meng and Liu [39], and Zhou et al. [42]. In this section, we adopt the self-adaptive predictor-corrector (PC) method proposed by He and Liao [43], which is an extension of the PC method with advanced self-adaptive step sizes. We see that this method can converge linearly, which is efficient for practical implementations. The self-adaptive PC method takes the calculation of projection operations of a toll charge pattern as a subroutine. In view of the fact that the feasible set of toll vector here is the nonnegative orthant, calculating the projection operation is very efficient and convenient; for example, for a vector $\boldsymbol{\tau}'$, its projection to the feasible set Ω of the toll patterns, denoted by $P_\Omega[\boldsymbol{\tau}']$, equals

$$P_\Omega\left[\boldsymbol{\tau}'\right] = \left(\max\left(0, \tau_i'\right),\ i = 1, 2, \ldots, I\right)^{\mathrm{T}}. \tag{10}$$

Accordingly, model (8) can be efficiently solved using the self-adaptive PC method. The main steps of the self-adaptive PC method are then summarized as follows.

Step 0 (initialization). Choose an initial vector $\boldsymbol{\tau}^{(1)} = (\tau_a^{(1)} = 0,\ a \in \overline{A})^{\mathrm{T}}$, three constants $0 < \kappa_2 < \kappa_1 < 1$, $\gamma \in (0, 2)$, and initial step size $\eta^{(1)} > 0$. Let the number of iterations $n = 1$.

Step 1 (prediction process).

Step 1.1. Implement the toll pattern $\boldsymbol{\tau}^{(n)}$ in the network and then observe the traffic flows on the entry links to each cordon, which are denoted by $v_a(\boldsymbol{\tau}^{(n)})$, $a \in \overline{A}_i$, $i = 1, 2, \ldots I$, and then compute $\boldsymbol{\Phi}(\boldsymbol{\tau}^{(n)}) = (H_i - \sum_{a \in \overline{A}_i} v_a(\boldsymbol{\tau}^{(n)}),\ i = 1, 2, \ldots, I)^{\mathrm{T}}$.

Step 1.2. Find auxiliary vector $\overline{\boldsymbol{\tau}}^{(n)}$ by the projection:

$$\overline{\boldsymbol{\tau}}^{(n)} = P_\Omega\left[\boldsymbol{\tau}^{(n)} - \eta^{(n)} \boldsymbol{\Phi}\left(\boldsymbol{\tau}^{(n)}\right)\right]. \tag{11}$$

Step 1.3. Consequently, levy the toll pattern $\overline{\boldsymbol{\tau}}^{(n)}$ in the network and then count the corresponding traffic flows on the entry links, denoted by $v_a(\overline{\boldsymbol{\tau}}^{(n)})$, $a \in \overline{A}_i$, $i = 1, 2, \ldots I$, which is used to calculate $\boldsymbol{\Phi}(\overline{\boldsymbol{\tau}}^{(n)}) = (H_i - \sum_{a \in \overline{A}_i} v_a(\overline{\boldsymbol{\tau}}^{(n)}),\ i = 1, 2, \ldots, I)^{\mathrm{T}}$.

Step 1.4. Calculate ratio $r^{(n)}$ by

$$r^{(n)} = \eta^{(n)} \left\|\boldsymbol{\Phi}\left(\boldsymbol{\tau}^{(n)}\right) - \boldsymbol{\Phi}\left(\overline{\boldsymbol{\tau}}^{(n)}\right)\right\|_2 / \left\|\boldsymbol{\tau}^{(n)} - \overline{\boldsymbol{\tau}}^{(n)}\right\|_2. \tag{12}$$

If $r^{(n)} \leq \kappa_1$, go to Step 2; otherwise reduce the step size according to

$$\eta^{(n)} = \frac{2}{3} \eta^{(n)} \min\left\{1, \frac{1}{r^{(n)}}\right\}, \tag{13}$$

and go to Step 1.2.

Step 2 (correction process). Based on $\boldsymbol{\tau}^{(n)}$, $\overline{\boldsymbol{\tau}}^{(n)}$, and $\eta^{(n)}$, calculate a step size $\pi^{(n)}$ for correction and then get an updated vector $\boldsymbol{\tau}^{(n+1)}$.

Step 2.1. Calculate another step size $\pi^{(n)}$ as per the formula

$$\pi^{(n)} = \gamma \times \eta^{(n)} \times \frac{\sum_{i=1}^{I} \left[\left(\tau_i^{(n)} - \overline{\tau}_i^{(n)}\right) \times h_i^{(n)}\right]}{\sum_{i=1}^{I} \left(h_i^{(n)}\right)^2}, \tag{14}$$

where

$$h_i^{(n)} = \left(\tau_i^{(n)} - \overline{\tau}_i^{(n)}\right) - \eta^{(n)}\left(\Phi_i\left(\boldsymbol{\tau}^{(n)}\right) - \Phi_i\left(\overline{\boldsymbol{\tau}}^{(n)}\right)\right), \quad i = 1, 2, \ldots, I. \tag{15}$$

Step 2.2. Update the vector $\boldsymbol{\tau}^{(n+1)}$ by this projection:

$$\boldsymbol{\tau}^{(n+1)} = P_\Omega\left[\boldsymbol{\tau}^{(n)} - \pi^{(n)} \boldsymbol{\Phi}\left(\overline{\boldsymbol{\tau}}^{(n)}\right)\right]. \tag{16}$$

Step 3. Slightly enlarge step size $\eta^{(n)}$ according to the following scheme:

$$\eta^{(n+1)} = \frac{3}{2}\eta^{(n)} \text{ if } \eta^{(n)} \left\|\Phi\left(\boldsymbol{\tau}^{(n)}\right) - \Phi\left(\overline{\boldsymbol{\tau}}^{(n)}\right)\right\|_2 / \left\|\boldsymbol{\tau}^{(n)} - \overline{\boldsymbol{\tau}}^{(n)}\right\|_2 \leq \kappa_2. \quad (17)$$

Step 4 (stop check). If

$$\left\|\boldsymbol{\tau}^{(n)} - \overline{\boldsymbol{\tau}}^{(n)}\right\| \leq \varepsilon, \quad (18)$$

where ε is a predetermined positive tolerance, then stop; otherwise, let $n = n + 1$ and go to Step 1.

The output of PC method gives the optimal toll pattern $\boldsymbol{\tau}^*$, and it can be seen from the solution procedure that only traffic count data autonomously collected on entry links in Steps 1.1 and 1.3 are needed in practice. Thus, this method can avoid gathering the data of OD demand, link travel time functions, or users' VOT that are extremely difficult to be accurately collected.

5. Numerical Example

A numerical example is employed in this section to validate the proposed model and effectiveness of the self-adaptive PC method. As shown in Figure 1, this example contains 7 nodes, 11 links, and one pricing cordon. The cordon area is defined by its vertex nodes 1, 4, 5, and 7. The three entry links 5, 6, and 7 to this cordon are charging. Denoted by $\boldsymbol{\tau}^*$, the optimal toll charges on the entry links are identical.

There are four OD pairs on the network, and their travel demands are tabulated in Table 1.

The travel time function on each link follows the BRP type function:

$$t_a(v_a) = t_a^0 \left(1.0 + 0.15\left(\frac{v_a}{C_a}\right)^4\right), \quad a \in A, \quad (19)$$

where t_a^0 denotes the free flow travel time and C_a is the capacity of each link. The effects of merging links are taken into consideration; for instance, the flows on link 1 and link 7 are merging to link 3; thus traffic flows on link 1 would also influence the travel time on link 7. There are two pairs of merging links on the network: links 1 and 7, as well as links 2 and 6. Thus, for these two pairs of links, their travel time functions are taken as

$$t_a(\mathbf{v}) = t_a^0 \left(1 + 0.15 \times \left(\frac{v_a + 0.5\widehat{v}_{\widehat{a}}}{1.5C_a}\right)^4\right), \quad a \in A, \quad (20)$$

where $\widehat{v}_{\widehat{a}}$ denotes the flow on the paired link of link $a \in A$. For example, for link 1, $\widehat{v}_{\widehat{a}}$ should be the flow on link 7. This type of functions has given rise to the asymmetric link travel time functions. The specific values of t_a^0 and C_a on each link are provided in Table 2.

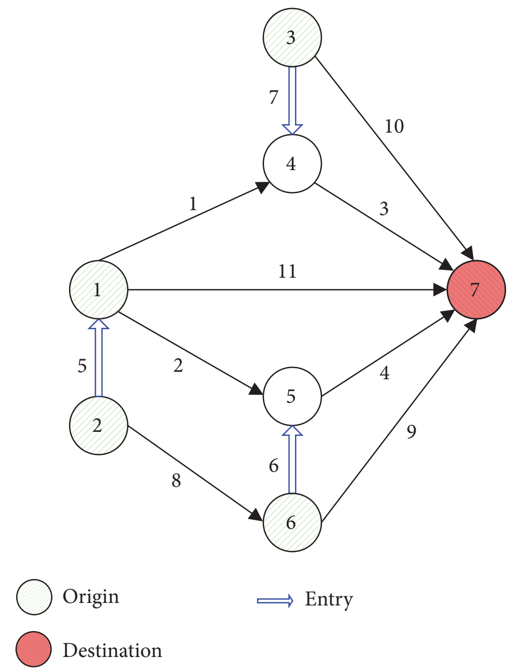

Figure 1: Network structure for the numerical experiment.

Table 1: OD demand.

OD pair	Travel demand (vehicle/hour)
$1 \rightarrow 7$	6000
$2 \rightarrow 7$	5000
$3 \rightarrow 7$	5000
$6 \rightarrow 7$	4000

Table 2: Parameters in link travel time functions.

Link number a	Free flow travel time (seconds) t_a^0	Capacity (vehicles/hour) C_a
1	60	4000
2	50	4000
3	60	4000
4	70	4000
5	60	2000
6	10	2000
7	50	3000
8	100	3000
9	110	4000
10	110	4000
11	150	4000

5.1. Probit-Based SUE Framework. For any given toll charge pattern, its corresponding link flows on the entry links are needed by the autonomously collected traffic counts. In this numerical example, we solve a probit-based SUE problem and

Table 3: Three scenarios of threshold and the optimal toll charges.

	Scenario 1	Scenario 2	Scenario 3
Threshold H	6000.0	5000.0	4000.0
Toll price (cents)	12.4	32.4	59.5
Total inbound flow	5999.9	5000.0	4000.3

use the equilibrium link flows on the entry links to estimate these traffic count data. It should be noted that we assume that the network users' travel behavior follows the probit-based SUE here, because of its better representativeness to the practical conditions, while the proposed methodology is also suitable in terms of other types of SUE models. We also assume that the drivers' value-of-time is 1.0 cent/second. It should be noted that, in practice, the value-of-time is not needed in the trial-and-error method.

The MSA is utilized to solve the probit-based SUE problem. It is well recognized that there is no close form for the path choice probability of probit-based SUE problem, thus the stochastic network loading is approximated by Monte Carlo simulation, as used in Liu and Meng [44], where an accurate approximation can be obtained. To further guarantee the accuracy, the number of simulations in this section is taken as 1000. In each run of the Monte Carlo simulation, there are three tasks: (1) the normally distributed perception errors are first sampled using pseudorandom numbers, (2) then the shortest path between each OD pair is searched, and (3) finally all the OD demands are assigned to the shortest path. Evidently, most of the computation efforts are allocated on solving the shortest path problem.

It should be pointed out that since the travel time perception error ζ_{wk} is defined on paths, thus a direct sampling of the perception errors requires path enumeration. In order to avoid the path enumeration, a link-based interpretation is adopted for ζ_{wk} (see, e.g., Liu and Meng [44]); that is, a perception error is defined on each link, denoted by ξ_a; thus the users' perceived generalized link travel time equals

$$\overline{T}_a = T_a + \xi_a, \tag{21}$$

where T_a is the generalized travel time defined by (2). The link travel time perception error ξ_a is assumed to be normally distributed with zero mean and constant variance.

5.2. Computation Results on Three Scenarios. As claimed in the beginning of Introduction, the objective of cordon-based pricing is to maintain the traffic conditions within cordon area. This objective can be realized by restricting the number of inbound vehicles to the cordon area to a predetermined threshold value. In order to see the sensitivity of optimal toll charge pattern to the threshold, three scenarios are tested, where the threshold values are given in the second row of Table 3.

The self-adaptive PC method is then used to solve the optimal toll charge pattern for each scenario. The stop criteria in (18) is taken as 1×10^{-4}, and the parameters used are $\kappa_1 = 0.9$, $\kappa_2 = 0.1$, $\gamma = 1.8$, and $\eta^{(0)} = 1.0$.

The output optimal toll charge value is provided in the third row of Table 3. As illustrated in the fourth row of Table 3, the total inbound flow (total flow on entry links 5, 6, and 7) to the cordon area of each scenario is equal to the threshold value, implying that the mathematical conditions (3), (4), and (5) can be fulfilled. It can be seen that from scenarios 1 to 3, as the threshold is getting more restrictive, implying a better traffic condition, the toll price becomes larger. This phenomenon means that the drivers need to pay more to achieve and maintain a better traffic condition.

6. Conclusions

This article addressed the trial-and-error method for cordon-based pricing, with the aim of avoiding gathering massive network data including OD demand functions, link travel time functions, and value-of-time. To better reflect the practical conditions, two properties of cordon-based pricing are further considered, including restricting the total inbound flow to the cordon to a predetermined threshold and assuming the tolls on all the entry links of one cordon to be identical.

Assuming the drivers' route choice behavior follows general SUE with asymmetric link travel time functions, a variational inequality (VI) model was used to formulate the optimal toll pattern. Then, based on this VI model, a self-adaptive predictor-corrector (PC) method was then employed, which gives the optimal toll pattern. It can be seen that this PC method provides a mechanism for the trial-and-error method, and only traffic count data on the entry links of each cordon are needed, which is quite convenient for practical use and the accuracy of these data could also be guaranteed. The numerical example showed that the resultant toll pattern can successfully limit the total inbound flow to any predetermined threshold value.

Disclosure

Zhiyuan Liu and Yong Zhang are co-first authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This study is supported by the Projects of International Cooperation and Exchange (no. 51561135003), Key Projects (no. 51638004), and Youth Program (no. 71501038) of the National Natural Science Foundation of China.

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