Admissibility Analysis and Control Synthesis for T–S Fuzzy Descriptor Systems

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***Abstract*—The problem of admissibility analysis and control synthesis for Takagi–Sugeno fuzzy descriptor systems is investigated. First, based on Nonquadratic fuzzy Lyapunov function and fully using the information of fuzzy membership functions, a new relaxed sufficient condition ensuring a fuzzy descriptor system to be admissible (regular, impulse-free, and stable) is proposed, in which it is not necessary to require every fuzzy subsystem to be stable. Second, the other sufficient condition for the admissibility is obtained without the information of time derivatives of fuzzy membership functions. Following the analysis, both parallel and nonparallel distributed compensation controllers are designed, linear matrix inequalities conditions are given to construct the controllers. Finally, some examples are provided to illustrate the main results in this paper less conservative than some earlier related results.**

***Index Terms*—Admissible control, linear matrix inequalities (LMIs), nonparallel distributed compensation (non-PDC) controller, nonquadratic fuzzy Lyapunov function, parallel distributed compensation (PDC) controller, Takagi–Sugeno (T–S) fuzzy descriptor system.**

I. Introduction

Takagi–Sugeno (T–S) fuzzy systems, first proposed by Takagi and Sugeno [1], have attracted great attention because of their effectiveness in approximating a very wide class of nonlinear dynamical systems. Many efficient results for stability analysis and controller synthesis have been proposed. For example, based on a Common Lyapunov function, the stability of T–S fuzzy systems is analyzed in [2]. Since the Common Lyapunov function is independent of fuzzy membership functions, results based on a single Lyapunov function may be conservative. Therefore, fuzzy Lyapunov function [3] and nonquadratic fuzzy Lyapunov function [4], [5] which are of a more general sort than the common quadratic one have been exploited. Based on stability analysis, the problem of stabilization can be considered accordingly. According to [6]–[12], controller design methods for T–S fuzzy systems can be divided into two classes: the parallel distributed compensation (PDC) controller design method and the nonparallel distributed compensation (non-PDC) controller design method. By the PDC method [6]–[9], a nonlinear controller can be blended by using linear feedback gains via fuzzy rules. Therefore, through T–S fuzzy systems and PDC controllers, linear control theory can be further extended over the controller design of complex nonlinear systems. In order to obtain less conservative stabilization results, non-PDC controllers are proposed in [10]–[12].

Descriptor systems provide an effective representation for a wider class of systems in contrast with the standard state-space system representations [14]. Descriptor systems have been extensively studied in the past several decades due to their successful applications in the fields of circuits, economics, and many other fields [13]–[15]. It should be pointed out that the analysis and synthesis for descriptor systems are much more complicated than those for standard state space systems, because for the continuous-time case not only stability but regularity and admissibility have to be addressed as well. A wide class of fuzzy systems in descriptor (differential and algebraic) forms have been studied in [16]. The admissibility of T–S descriptor systems is the important features of systems. The problem of admissibility of descriptor systems has been studied in many literatures. In [17], sufficient conditions are derived for stability and stabilization of a class of extended T–S fuzzy descriptor systems with time-delay. In [18], a novel observer design of nonlinear descriptor systems represented by T–S models has been presented. Fuzzy controllers and fuzzy observers are designed in [19] such that the resultant closed-loop fuzzy descriptor systems with time-delay are both admissible and $(Q, V, R) - \alpha$-dissipative. The problem of robust stability analysis for uncertain discrete extended T–S fuzzy descriptor systems with time-varying norm-bounded parameter uncertainties is considered in [20]. Problems of stability analysis and H_∞ control for a class of fuzzy time-delay descriptor systems described by an extended T–S fuzzy model are investigated in [21]. In [22], the problem of H_∞ fuzzy control for a class of nonlinear time-delay descriptor Markovian jump systems with partially unknown transition rates is investigated. In [23], PDC and fuzzy proportional and derivative state feedback controller are proposed to stabilize T–S extended fuzzy descriptor systems. A delay-dependent guaranteed cost control approach is developed in [24]. In [16]–[24], the problems of admissibility analysis for T–S fuzzy descriptor systems are investigated by means of a Common Lyapunov function. A novel controller design for nonlinear descriptor systems in T–S form has been presented in [25]. In [26], a descriptor system approach to the design of fuzzy control systems using

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fuzzy Lyapunov functions is proposed. In [27], finite-time stability and finite-time boundedness with nonzero initial state for fuzzy descriptor systems are studied. The admissibility conditions of T–S fuzzy descriptor systems via fuzzy Lyapunov functions are derived in [28]. In [29], the problem of H_∞ descriptor fault detection filter design for T–S fuzzy discrete-time systems is studied.

The above existing research for admissibility problem of T–S fuzzy systems need to solve the second-order or higher orders PLMI [34] condition, i.e.,

$$L(\Theta_k) = L_0 + \sum_{i=1}^{r} \theta_i L_i + \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i \theta_j L_{ij} < 0 (>0)$$

for $\sum_{i=1}^{r} \theta_i = 1, 0 \le \theta_i \le 1, 0 \le i \le r$. In order to obtain the linear matrix inequalities (LMIs) conditions, they check the condition L_0, L_i, L_{ij} for every i and j only because of positiveness of θ_j and θ_i. That means only the positiveness of fuzzy membership functions is considered, the other information of fuzzy membership functions is not considered. Therefore, this method significantly increases the conservativeness. By fully using the information of fuzzy membership functions, the problems of admissibility analysis and control synthesis are investigated for T–S fuzzy descriptor systems by means of both PDC and non-PDC controllers in this paper. In our theorem, a new relaxed sufficient condition is established in terms of a Nonquadratic fuzzy Lyapunov function, which guarantees that the system is admissible. In contrast with [26], a complete proof of the admissibility of T–S fuzzy descriptor systems is given. In the proof process, the influence for impulse behavior of descriptor systems taken into consideration. Because of fully using the information of fuzzy membership functions, this theorem does not require that each fuzzy Lyapunov function matrix satisfies $EP_i \ge 0$ and every fuzzy subsystem is stable. The first example is used to demonstrate this feature. Moreover, slack matrices are used to reduce the conservativeness caused by the bounds of time derivatives of fuzzy membership functions. In some cases, the bounds of time derivatives of fuzzy membership functions may be large or the bounds do not exist. Therefore, we present the other new admissibility condition of fuzzy control systems without using those bounds. Both PDC and non-PDC controllers are proposed for guaranteeing the closed-loop system to be admissible. In this paper, based on a matrix inequality, the problem of bilinear matrix inequalities (BMIs) in the PDC design process is overcome. This method is much simpler than the BMI procedures in [3], [28]. In special cases, all of these conditions can be verified by solving strict LMIs.

This paper is organized as follows. The problem formulation and some definitions are introduced in Section II. The main results are developed in Section III. Admissibility for T–S fuzzy descriptor systems is analyzed and then PDC and non-PDC controllers are designed, respectively. Section IV illustrates the advantages of the proposed design approaches over some earlier related results. Finally, Section V concludes this paper.

Notation: Let R and C denote the sets of real and complex numbers, respectively. R^n denotes the n-dimensional real Euclidean space. $\mathrm{R}^{m\times n}$ denotes the set of $m \times n$ matrices with real elements. I is identity matrix with appropriate dimension. For a given matrix P, $P \ge 0$ $(P \le 0)$ means that P is symmetric and positive (negative) semidefinite, $P > 0$ $(P < 0)$ means that P is symmetric and positive (negative) definite. The determinant of P is denoted by $\det(P)$ and rank of P by $\mathrm{rank}(P)$. $\deg(*)$ denotes the degree of a polynomial. $\|X\|$ denotes the norm of a matrix X. For a blockwise symmetric matrix X, offdiagonal blocks are abbreviated with $*$, i.e.,

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} = \begin{bmatrix} X_{11} & * \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix}.$$

X_ξ denotes $\sum_{i=1}^{r} h_i(\xi(t))X_i$.

II. Problem Formulation and Basic Definitions

A fuzzy dynamic model has been proposed by Takagi and Sugeno [1] to represent local linear input/output relations of nonlinear systems. This fuzzy model is described by IF-THEN rules and has been employed to deal with the control design problems of nonlinear systems. Motivated by this, a T–S fuzzy descriptor system is considered in this section, its *i*th fuzzy rule is of the form [16]–[18]:

$\mathbf{R}_i$: if $\xi_1(t)$ is M_{1i}, $\xi_2(t)$ is M_{2i}, ..., and $\xi_p(t)$ is M_{pi}, then

$$\begin{cases} E\dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) + D_i u(t) \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, r$ with r being the number of IF–THEN rules, M_{ji} $(j = 1, 2, \ldots, p)$ is the fuzzy set, $x(t) \in \mathrm{R}^n$ is the state vector, $u(t) \in \mathrm{R}^m$ is the control input, $y(t) \in \mathrm{R}^l$ is the controlled output, E, A_i, B_i, C_i, and D_i are known constant matrices with appropriate dimensions. The matrix E may be singular and it is assumed that $\mathrm{rank}(E) = n_1 \le n$. $\xi_1(t), \xi_2(t), \ldots, \xi_p(t)$ are premise variables which may be functions of the state variables. Let $\xi(t) = [\xi_1(t), \xi_2(t), \ldots, \xi_p(t)]^{\mathrm{T}}$ be a column vector of functions. In this paper, derivative matrices of fuzzy subsystems are the same constant matrix. In the most of previous results, different derivative matrices of fuzzy subsystems are also transformed to the same one.

Taking the weighted average of $E\dot{x}(t)$ as a defuzzification strategy, we may derive the final defuzzified output of the fuzzy model, which is

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t))[A_i x(t) + B_i u(t)] \\ y(t) = \sum_{i=1}^{r} h_i(\xi(t))[C_i x(t) + D_i u(t)] \end{cases} \tag{2}$$

where the fuzzy membership functions

$$h_i(\xi(t)) = \frac{\prod_{j=1}^{p} M_{ji}(\xi_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{p} M_{ji}(\xi_j(t))}.$$

It is obvious that $h_i(\xi(t)) \ge 0$ for all $i = 1, 2, \ldots, r$ and $\sum_{i=1}^{r} h_i(\xi(t)) = 1$.

For convenience, system (2) is denoted as follows:

$$\begin{cases} E\dot{x}(t) = A_\xi x(t) + B_\xi u(t) \\ y(t) = C_\xi x(t) + D_\xi u(t). \end{cases} \tag{3}$$

The admissibility of fuzzy descriptor systems is defined as follows.

Definition II.1 [20], [23], [29]: System (2) is said to be regular if there exists $s \in \mathrm{C}$ satisfying $\det(sE - A_\xi) \neq 0$. System (2) is said to be impulse-free if $\deg(\det(sE - A_\xi) = \mathrm{rank}(E)$. System (2) is said to be stable if all roots of $\det(sE - A_\xi) = 0$ have negative real parts for all $t \in [0, +\infty)$. System (2) is said to be admissible if it is regular, impulse-free, and stable.

Lemma II.1 [28]: Let $A(t) \in \mathrm{R}^{n\times n}$ be a piecewise continuous function matrix, if there exist a norm uniformly bounded function matrix $P(t) \in \mathrm{R}^{n\times n}$ and a scalar $\alpha > 0$ such that

$$P^{\mathrm{T}}(t)A^{\mathrm{T}}(t) + A(t)P(t) < -\alpha I \tag{4}$$

then the following conditions hold:

1) $A(t)$ is invertible for all $t \in [0, +\infty)$.
2) $A^{-1}(t)$ is uniformly bounded for all $t \in [0, +\infty)$.

Lemma II.2 [33]: Given a symmetric matrix $\Xi \in \mathrm{R}^{n\times n}$ and two matrices $\Psi \in \mathrm{R}^{n\times n}$ and $\Upsilon \in \mathrm{R}^{n\times n}$. There exists an *unstructured* matrix Θ such that

$$\Xi + \Upsilon^{\mathrm{T}}\Theta^{\mathrm{T}}\Psi + \Psi^{\mathrm{T}}\Theta\Upsilon < 0$$

if and only if, the following projection inequalities with respect to Θ are satisfied:

$$N_\Psi^{\mathrm{T}}\Xi N_\Psi < 0, N_\Upsilon^{\mathrm{T}}\Xi N_\Upsilon < 0$$

where N_Ψ and N_Υ are matrices whose columns form a basis of the nullspaces of Υ and Ψ, respectively.

The following Lemma is obtained.

Lemma II.3: Given matrix $A \in \mathrm{R}^{n\times n}$ and symmetric matrix $T \in \mathrm{R}^{n\times n}$. There exists an invertible matrix $P \in \mathrm{R}^{n\times n}$ such that

$$T + P^{\mathrm{T}}A^{\mathrm{T}} + AP < 0 \tag{5}$$

if and only if there exist matrices $H \in \mathrm{R}^{n\times n}$ and $L \in \mathrm{R}^{n\times n}$ such that

$$\begin{bmatrix} T + H^{\mathrm{T}}A^{\mathrm{T}} + AH & * \\ P - H + L^{\mathrm{T}}A^{\mathrm{T}} & -L - L^{\mathrm{T}} \end{bmatrix} < 0. \tag{6}$$

Proof: Notice that (5) is equivalent to

$$\begin{bmatrix} I & A \end{bmatrix} \begin{bmatrix} T & P^{\mathrm{T}} \\ P & 0 \end{bmatrix} \begin{bmatrix} I \\ A^{\mathrm{T}} \end{bmatrix} < 0.$$

Similar to the projection inequality in Lemma II.2, (5) can be expressed as a quadratic form with $\Xi = \begin{bmatrix} T & P^{\mathrm{T}} \\ P & 0 \end{bmatrix}$ and $N_\Psi = \begin{bmatrix} I \\ A^{\mathrm{T}} \end{bmatrix}$. Then, setting $N_\Upsilon = 0$, which means $\Upsilon = I$. By Lemma II.2, there exists P such that (5) holds if and only if there exists $\Theta = \begin{bmatrix} H & L \end{bmatrix}$ such that

$$\begin{bmatrix} T & P^{\mathrm{T}} \\ P & 0 \end{bmatrix} + \begin{bmatrix} H^{\mathrm{T}} \\ L^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} A^{\mathrm{T}} & -I \end{bmatrix} + \begin{bmatrix} A \\ -I \end{bmatrix} \begin{bmatrix} H & L \end{bmatrix} < 0.$$

This completes the proof.

Lemma II.4 [30]: For given matrices $\Phi_{ij} \in \mathrm{R}^{n\times n}$, the following inequality

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi)h_j(\xi)\Phi_{ij} < 0 \tag{7}$$

holds if the following inequalities

$$\Phi_{ii} < 0, i = 1, 2, \ldots, r \tag{8}$$

and

$$\frac{2}{r-1}\Phi_{ii} + \Phi_{ij} + \Phi_{ji} < 0, 1 \le i \neq j \le r \tag{9}$$

are satisfied.

Lemma II.5 [33]: For any matrices X, F, and Y of appropriate dimensions, where $F^{\mathrm{T}}F \le I$, and for any scalar $\lambda > 0$ the following inequality:

$$XFY + Y^{\mathrm{T}}F^{\mathrm{T}}X^{\mathrm{T}} \le \lambda XX^{\mathrm{T}} + \lambda^{-1}Y^{\mathrm{T}}Y \tag{10}$$

holds.

Assumption 1: Let $h_k(\xi) \le \alpha_k \le 1$, $h_k(\xi)h_l(\xi) \le \beta_{kl} \le 1$, $\dot{h}_k(\xi) \ge \phi_k$ $(\phi_k \le 0)$, for all $k, l = 1, 2, \ldots, r$, where $\alpha_k, \beta_{ij}, \phi_k$ are scalars.

Remark 1: If the shape of the membership functions is defined by the user, obtaining the α_k and β_{kl} are straightforward. In general case, $\alpha_k = 1, \beta_{kk} = 1, \beta_{kl} = h_k(\xi)h_l(\xi) \le h_k(\xi)(1 - h_k(\xi)) \le 0.25$. Since $\sum_{k=1}^{r}\dot{h}_k(\xi) = 0$, $\dot{h}_k(\xi)$ may be positive or negative. Therefore, in practical applications, the exact value of ϕ_k can be selected to satisfy the assumption $\dot{h}_k(\xi) \ge \phi_k$ $(\phi_k \le 0)$ like [3], [11], [26].

III. Main Result

In this section, we perform admissibility analysis of fuzzy descriptor system (2) and design PDC and non-PDC fuzzy controllers for guaranteeing the closed-loop system to be admissible.

A. Admissibility Analysis

In this section, we give two sufficient conditions for the admissibility of system (2) (when $u(t) = 0$).

Theorem III.1: System (2) is admissible if there exist symmetric matrices $\overline{P}_i$, Z_i, X, $Y \in \mathrm{R}^{n\times n}$, $Q_{ij} > 0\,(i, j = 1, 2, \ldots, r)$ and matrices $Q_i \in \mathrm{R}^{(n-n_1)\times n}\,(i = 1, 2, \ldots, r)$ such that

$$\begin{bmatrix} Y - \sum_{i=1}^{r} Z_i & \frac{1}{2}\alpha_1(Y - \overline{P}_1) & \ldots & \frac{1}{2}\alpha_r(Y - \overline{P}_r) \\ & Z_1 & \ldots & 0 \\ & * & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ & * & \ldots & Z_r \end{bmatrix} > 0 \tag{11}$$

$$EP_i + X \ge 0, r = 1, 2, \ldots, r \tag{12}$$

$$\Theta_{ii} < 0, i = 1, 2, \ldots, r \tag{13}$$

$$\frac{2}{r-1}\Theta_{ii} + \Theta_{ij} + \Theta_{ji} < 0, 1 \le i \neq j \le r \tag{14}$$

where $P_i = \overline{P}_i E^{\mathrm{T}} + SQ_i$, $S \in \mathrm{R}^{n\times(n-n_1)}$ is an arbitrary matrix of full column rank and satisfies $ES = 0$,

$$\begin{aligned}\Theta_{ij} &= A_iP_j + P_j^{\mathrm{T}}A_i^{\mathrm{T}} + P_\Phi + \Omega - Q_{ij}\\ \Omega &= \sum_{k=1}^{r}\sum_{l=1}^{r}\beta_{kl}Q_{kl}\\ P_\Phi &= -\sum_{k=1}^{r}\phi_k(EP_k + X).\end{aligned}$$

Proof: By Lemma II.4, (13) and (14), it can be shown that

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\xi)h_j(\xi)\Theta_{ij}\\ &= P_\Phi + A_\xi P_\xi + P_\xi^{\mathrm{T}}A_\xi^{\mathrm{T}} - \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\xi)h_j(\xi)Q_{ij}\\ &\quad + \sum_{k=1}^{r}\sum_{l=1}^{r}\beta_{kl}Q_{kl} < 0. \qquad (15)\end{aligned}$$

Then, by $h_i(\xi)h_j(\xi) \le \beta_{ij}$ and $Q_{ij} > 0$, it can be seen that

$$P_\Phi + A_\xi P_\xi + P_\xi^{\mathrm{T}}A_\xi^{\mathrm{T}} < 0. \qquad (16)$$

By (12) and $\phi_k \le 0$, we have $P_\Phi \ge 0$. Then,

$$A_\xi P_\xi + P_\xi^{\mathrm{T}}A_\xi^{\mathrm{T}} < 0 \qquad (17)$$

holds.

We first show that the system is regular and impulse-free.

For matrix E satisfies $\mathrm{rank}(E) = n_1$, there exist two nonsingular matrices U and V such that

$$UEV = \begin{bmatrix} I_{n_1} & 0\\ 0 & 0\end{bmatrix}. \qquad (18)$$

Accordingly, write

$$V^{-1}P_iU^{\mathrm{T}} = \begin{bmatrix} P_{1i} & P_{2i}\\ P_{3i} & P_{4i}\end{bmatrix}, UA_iV = \begin{bmatrix} A_{1i} & A_{2i}\\ A_{3i} & A_{4i}\end{bmatrix}. \qquad (19)$$

By $P_i = \overline{P}_iE^{\mathrm{T}} + SQ_i$, one has

$$EP_i = E\overline{P}_iE^{\mathrm{T}} = P_i^{\mathrm{T}}E^{\mathrm{T}}. \qquad (20)$$

Then, pre and postmultiplying (20) by U and U^{T}, respectively, and then using the expressions in (18), (19), it can be shown that $P_{2i} = 0$.

Therefore,

$$V^{-1}P_iU^{\mathrm{T}} = \begin{bmatrix} P_{1i} & 0\\ P_{3i} & P_{4i}\end{bmatrix}.$$

Then, we have

$$V^{-1}P_\xi U^{\mathrm{T}} = \begin{bmatrix} \sum_{i=1}^{r} h_iP_{1i} & 0\\ \sum_{i=1}^{r} h_iP_{2i} & \sum_{i=1}^{r} h_iP_{4i}\end{bmatrix} = \begin{bmatrix} P_{1\xi} & 0\\ P_{2\xi} & P_{4\xi}\end{bmatrix}. \qquad (21)$$

Now, pre and postmultiplying (17) by U and U^{T}, respectively, it can be obtained that

$$\begin{bmatrix} \star & \star\\ \star & P_{4\xi}^{\mathrm{T}}A_{4\xi}^{\mathrm{T}} + A_{4\xi}P_{4\xi}\end{bmatrix} < 0 \qquad (22)$$

where $'\star'$ denotes the blocks which do not affect the proof. It follows that $\| P_{4\xi} \| \le \sum_{i=1}^{r} \| P_{4i} \|$. As a result, $A_{4\xi} = \sum_{i=1}^{r} h_i(\xi)A_{4i}$ is invertible and its inverse matrix is bounded according to Lemma II.1. We have established that the system (2) is regular and impulse-free.

In the following, we prove that the system is stable. Firstly, we prove $\sum_{i=1}^{r} h_i(\xi)\overline{P}_i > 0$.

From $\sum_{i=1}^{r} h_i(\xi) = 1$ and for all symmetric matrix Y, we can obtain

$$\begin{aligned}\sum_{i=1}^{r}h_i(\xi)\overline{P}_i &= Y - \sum_{i=1}^{r}h_i(\xi)Y + \sum_{i=1}^{r}h_i(\xi)\overline{P}_i\\ &= Y - \sum_{i=1}^{r}h_i(\xi)(Y-\overline{P}_i)\\ &= Y - \sum_{i=1}^{r}h_i(\xi)\left[\frac{1}{2}(Y-\overline{P}_i)^{\mathrm{T}} + \frac{1}{2}(Y-\overline{P}_i)\right].\end{aligned}$$

For all nonsingular matrices $W_i \in \mathrm{R}^{n\times n}, i = 1, 2, \ldots, r$, one has

$$\begin{aligned}&h_i(\xi)\left[\frac{1}{2}(Y-\overline{P}_i)^{\mathrm{T}} + \frac{1}{2}(Y-\overline{P}_i)\right]\\ &= \frac{1}{2}(Y-\overline{P}_i)^{\mathrm{T}}W_i^{-1}h_i(\xi)W_i + W_i^{-\mathrm{T}}h_i(\xi)W_i^{\mathrm{T}}\frac{1}{2}(Y-\overline{P}_i).\end{aligned}$$

From $0 \le h_i(\xi) \le \alpha_i$ and Lemma II.5, it can be seen that

$$\begin{aligned}&h_i(\xi)\left[\frac{1}{2}(Y-\overline{P}_i)^{\mathrm{T}} + \frac{1}{2}(Y-\overline{P}_i)\right]\\ &\le \frac{1}{2}(Y-\overline{P}_i)^{\mathrm{T}}W_i^{-1}\alpha_i^2W_i^{-\mathrm{T}}\frac{1}{2}(Y-\overline{P}_i) + W_i^{\mathrm{T}}W_i.\end{aligned}$$

Let, $Z_i = W_i^{\mathrm{T}}W_i$. (Clearly, $Z_i > 0$.) Then

$$\begin{aligned}&h_i(\xi)\left[\frac{1}{2}(Y-\overline{P}_i)^{\mathrm{T}} + \frac{1}{2}(Y-\overline{P}_i)\right]\\ &\le \frac{1}{2}(Y-\overline{P}_i)^{\mathrm{T}}\alpha_i^2Z_i^{-1}\frac{1}{2}(Y-\overline{P}_i) + Z_i.\end{aligned}$$

By (11) and the Schur complement Lemma, we have

$$\begin{aligned}&\sum_{i=1}^{r}h_i(\xi)\overline{P}_i \ge Y\\ &- \sum_{i=1}^{r}\left[\frac{1}{2}(Y-\overline{P}_i)^{\mathrm{T}}\alpha_i^2Z_i^{-1}\frac{1}{2}(Y-\overline{P}_i) + Z_i\right] > 0.\end{aligned}$$

Therefore,

$$\begin{aligned}EP_\xi &= E\sum_{i=1}^{r}h_i(\xi)P_i = E\sum_{i=1}^{r}h_i(\xi)(\overline{P}_iE^{\mathrm{T}} + SQ_i)\\ &= E\sum_{i=1}^{r}h_i(\xi)\overline{P}_iE^{\mathrm{T}} = P_\xi^{\mathrm{T}}E^{\mathrm{T}} \ge 0.\end{aligned}$$

By Lemma II.1 and $A_\xi P_\xi + P_\xi^{\mathrm{T}} A_\xi^{\mathrm{T}} < 0$, one has P_ξ is a nonsingular matrix. Then, pre and postmultiplying $EP_\xi = P_\xi^{\mathrm{T}} E^{\mathrm{T}} \geq 0$ by $P_\xi^{-\mathrm{T}}$ and P_ξ^{-1}, we have $E^{\mathrm{T}} P_\xi^{-1} = P_\xi^{-\mathrm{T}} E \geq 0$.

Second, using this matrix inequality, we take a candidate Nonquadratic fuzzy Lyapunov function of the form

$$V(x(t)) = x^{\mathrm{T}}(t) E^{\mathrm{T}} P_\xi^{-1} x(t). \tag{23}$$

Then,

$$\begin{aligned}\dot{V}(x(t)) = \; & x^{\mathrm{T}}(t) P_\xi^{-\mathrm{T}} E \dot{x}(t) + \dot{x}^{\mathrm{T}}(t) E^{\mathrm{T}} P_\xi^{-1} x(t) \\ & + x^{\mathrm{T}}(t) E^{\mathrm{T}} \frac{dP_\xi^{-1}}{dt} x(t).\end{aligned}$$

Observing that $\frac{dP_\xi^{-1}}{dt} = -P_\xi^{-1} \dot{P}_\xi P_\xi^{-1}$ and by $E^{\mathrm{T}} P_\xi^{-1} = P_\xi^{-\mathrm{T}} E$, we have

$$\begin{aligned}\dot{V}(x(t)) = \; & x^{\mathrm{T}}(t) P_\xi^{-\mathrm{T}} E \dot{x}(t) + \dot{x}^{\mathrm{T}}(t) E^{\mathrm{T}} P_\xi^{-1} x(t) \\ & - x^{\mathrm{T}}(t) E^{\mathrm{T}} P_\xi^{-1} \dot{P}_\xi P_\xi^{-1} x(t) \\ = \; & x^{\mathrm{T}}(t) P_\xi^{-\mathrm{T}} A_\xi x(t) + x^{\mathrm{T}}(t) A_\xi^{\mathrm{T}} P_\xi^{-1} x(t) \\ & - x^{\mathrm{T}}(t) P_\xi^{-\mathrm{T}} E \dot{P}_\xi P_\xi^{-1} x(t) \\ = \; & x^{\mathrm{T}}(t) \left(P_\xi^{-\mathrm{T}} A_\xi + A_\xi^{\mathrm{T}} P_\xi^{-1} - P_\xi^{-\mathrm{T}} E \dot{P}_\xi P_\xi^{-1} \right) x(t).\end{aligned}$$

From $\sum_{k=1}^r h_k(\xi) = 1$, we have $\sum_{k=1}^r \dot{h}_k(\xi) = 0$. Therefore, the following matrix statement with slack symmetry matrix X is proposed:

$$\sum_{k=1}^r \dot{h}_k(\xi) X = 0.$$

Then, by (16), $\dot{h}_k(\xi) \geq \phi_k$ and $EP_i + X \geq 0$, one has

$$\begin{aligned}\Pi = \; & -E\dot{P}_\xi + A_\xi P_\xi + P_\xi^{\mathrm{T}} A_\xi^{\mathrm{T}} \\ = \; & -\sum_{k=1}^r \dot{h}_k(\xi) E P_k + A_\xi P_\xi + P_\xi^{\mathrm{T}} A_\xi^{\mathrm{T}} \\ = \; & -\sum_{k=1}^r \dot{h}_k(\xi)(EP_k + X) + A_\xi P_\xi + P_\xi^{\mathrm{T}} A_\xi^{\mathrm{T}} \\ \leq \; & -\sum_{k=1}^r \phi_k (EP_k + X) + A_\xi P_\xi + P_\xi^{\mathrm{T}} A_\xi^{\mathrm{T}} \\ = \; & P_\Phi + A_\xi P_\xi + P_\xi^{\mathrm{T}} A_\xi^{\mathrm{T}} < 0.\end{aligned}$$

Multiplying Π on the left by $P_\xi^{-\mathrm{T}}$ and right by P_ξ^{-1}, respectively, it can be seen that

$$\begin{aligned}& P_\xi^{-\mathrm{T}} \left(-E\dot{P}_\xi + A_\xi P_\xi + P_\xi^{\mathrm{T}} A_\xi^{\mathrm{T}} \right) P_\xi^{-1} \\ = \; & P_\xi^{-\mathrm{T}} A_\xi + A_\xi^{\mathrm{T}} P_\xi^{-1} - P_\xi^{-\mathrm{T}} E \dot{P}_\xi P_\xi^{-1} < 0.\end{aligned}$$

Therefore, $\dot{V}(x(t)) < 0$, system (2) is stable. This completes the proof.

Remark 2: In [26], the impulsive behavior of T–S fuzzy descriptor systems was not considered fully, which is an important feature of descriptor systems. In Theorem III.1, by making use of the full information of the fuzzy membership functions, we have obtained more precise admissibility condition for T–S fuzzy descriptor systems including the existence of impulse. Meanwhile, we have used the slack matrix X to reduce the conservativeness caused by the derivatives of fuzzy basis functions. Finally, because LMIs (12) are not conventional strict LMIs, the LMI toolbox in MATLAB cannot be used directly to solve these inequalities. However, by choosing $X = EZE^{\mathrm{T}}$ and $\overline{P}_i + Z > 0$, we have $EP_i + X = EP_i + EZE^{\mathrm{T}} = E(\overline{P}_i E^{\mathrm{T}} + SQ_i) + EZE^{\mathrm{T}} = E(\overline{P}_i + Z)E^{\mathrm{T}} \geq 0$. Therefore, LMIs (12) can be converted to strict LMIs.

When $E = I$, T–S fuzzy descriptor system (2) reduces to a standard one. A standard system is always regular and impulse free. The following corollary is obtained.

Corollary III.1: The standard fuzzy system is stable if there exist symmetric matrices $\overline{P}_i, Y, Z, Q_{ij} > 0, Z_i \in \mathrm{R}^{n\times n}$ $(i, j = 1, 2, \ldots, r)$ such that LMIs (11) and

$$\begin{aligned}& \overline{P}_i + Z \geq 0, i = 1, 2, \ldots, r \\ & \Theta_{ii} < 0, i = 1, 2, \ldots, r \\ & \frac{2}{r-1}\Theta_{ii} + \Theta_{ij} + \Theta_{ji} < 0, 1 \leq i < j \leq r\end{aligned}$$

where

$$\begin{aligned}\Theta_{ij} = \; & A_i \overline{P}_j + \overline{P}_j^{\mathrm{T}} A_i^{\mathrm{T}} + \overline{P}_\Phi + \Omega - Q_{ij} \\ \Omega = \; & \sum_{k=1}^r \sum_{l=1}^r \beta_{kl} Q_{kl} \\ \overline{P}_\Phi = \; & -\sum_{k=1}^r \phi_k (\overline{P}_i + Z).\end{aligned}$$

Remark 3: In Theorem III.1 and Corollary III.1, it is not required that every fuzzy Lyapunov function matrix satisfies $EP_i \geq 0$ or $\overline{P}_i > 0$. It is only required that the total fuzzy Lyapunov function matrix meets $E\sum_{i=1}^r h_i(\xi) P_i > 0$ or $\sum_{i=1}^r h_i(\xi) \overline{P}_i \geq 0$. Moreover, the bounds of fuzzy membership functions (α_k, β_{kl}) are used in Theorem III.1 and Corollary III.1. These improvements remove the requirement that every fuzzy subsystem is stable. This is in contrast with the existing results in [3], [26], [28], [31], [32], where it is required that

$$\begin{cases} E^{\mathrm{T}} P_i = P_i^{\mathrm{T}} E \geq 0 \\ A_i^{\mathrm{T}} P_i + P_i^{\mathrm{T}} A_i < 0 \end{cases} \quad \text{or} \quad \begin{cases} P_i > 0 \\ A_i^{\mathrm{T}} P_i + P_i A_i < 0 \end{cases}.$$

That is, every fuzzy subsystem is supposed to be stable.

In Theorem III.1, bounds of time derivatives of fuzzy basis functions are used, which may complicate of the admissibility analysis in some cases. To overcome it, the following theorem is proposed.

Theorem III.2: System (2) is admissible if there exist matrices $Z, Q_i \in \mathrm{R}^{n\times n}$ $(i = 1, 2, \ldots, r)$ and symmetric matrices $Q_{ij} > 0, P \in \mathrm{R}^{n\times n}$ $(i, j = 1, 2, \ldots, r)$ such that

$$EPE^{\mathrm{T}} + E^{\dagger} Z + Z^{\mathrm{T}} E^{\dagger\mathrm{T}} > 0 \tag{24}$$

$$M_{ii} < 0 \quad i = 1, 2, \ldots, r \tag{25}$$

$$\frac{2}{r-1} M_{ii} + M_{ij} + M_{ji} < 0, 1 \leq i \neq j \leq r \tag{26}$$

where

$$\begin{aligned} M_{ij} = & A_i(PE^{\mathrm{T}} + E^{\perp}Q_j) + (PE^{\mathrm{T}} + E^{\perp}Q_j)^{\mathrm{T}} A_i^{\mathrm{T}} \\ & + \Omega - Q_{ij} \\ \Omega = & \sum_{k=1}^{r}\sum_{l=1}^{r}\beta_{kl}Q_{kl} \\ E^{\perp} = & V(I - UEV)U \\ E^{\dagger} = & U^{-1}(I - UEV)U. \end{aligned}$$

Proof: Using Lemma II.4, it can be obtained that

$$\begin{aligned} & \sum_{i=1}^{r}\sum_{j=1}^{r} h_j(\xi)h_i(\xi)M_{ij} \\ = & A_\xi(PE^{\mathrm{T}} + E^{\perp}Q_\xi) + (PE^{\mathrm{T}} + E^{\perp}Q_\xi)^{\mathrm{T}} A_\xi^{\mathrm{T}} \\ & - \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi)h_j(\xi)Q_{ij} + \sum_{k=1}^{r}\sum_{l=1}^{r}\beta_{kl}Q_{kl} < 0. \end{aligned}$$

Then, by $h_i(\xi)h_j(\xi) \le \beta_{ij}$ and $Q_{ij} > 0$, it can be obtained that

$$A_\xi \widehat{P}_\xi + \widehat{P}_\xi^{\mathrm{T}} A_\xi^{\mathrm{T}} < 0 \tag{27}$$

where $\widehat{P}_\xi = PE^{\mathrm{T}} + E^{\perp}Q_\xi$.

Notice that

$$\begin{aligned} & U(EPE^{\mathrm{T}} + E^{\dagger}Z + Z^{\mathrm{T}}E^{\dagger\mathrm{T}})U^{\mathrm{T}} \\ = & UEVV^{-1}PV^{-\mathrm{T}}(UEV)^{\mathrm{T}} + UE^{\dagger}U^{-1}UZU^{\mathrm{T}} \\ & + UZ^{\mathrm{T}}U^{\mathrm{T}}U^{-\mathrm{T}}E^{\dagger\mathrm{T}}U^{\mathrm{T}} \\ = & \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{P}_1 & \star \\ \star & \star \end{bmatrix} \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix} \\ & + \begin{bmatrix} 0 & 0 \\ 0 & I_{n-n_1} \end{bmatrix} \begin{bmatrix} \bar{Z}_1 & \bar{Z}_2 \\ \bar{Z}_3 & \bar{Z}_4 \end{bmatrix} \\ & + \begin{bmatrix} \bar{Z}_1 & \bar{Z}_2 \\ \bar{Z}_3 & \bar{Z}_4 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 0 & 0 \\ 0 & I_{n-n_1} \end{bmatrix} \\ = & \begin{bmatrix} \bar{P}_1 & \bar{Z}_2^{\mathrm{T}} \\ \bar{Z}_2 & \bar{Z}_4 + \bar{Z}_4^{\mathrm{T}} \end{bmatrix} > 0. \end{aligned}$$

We conclude that $\bar{P}_1 > 0$ if (24) holds. Therefore,

$$UEPE^{\mathrm{T}}U^{\mathrm{T}} = \begin{bmatrix} \bar{P}_1 & 0 \\ 0 & 0 \end{bmatrix} \ge 0.$$

Then, by $EE^{\perp} = 0$, we can obtain

$$E\widehat{P}_\xi = E(PE^{\mathrm{T}} + E^{\perp}Q_\xi) = EPE^{\mathrm{T}} = \widehat{P}_\xi^{\mathrm{T}} E^{\mathrm{T}} \ge 0.$$

We take a candidate Nonquadratic fuzzy Lyapunov function as follows:

$$V(x(t)) = x^{\mathrm{T}}(t)E^{\mathrm{T}}\widehat{P}_\xi^{-1}x(t). \tag{28}$$

Meanwhile, by $\sum_{k=1}^{r} \dot{h}_k(\xi) = 0$, we have

$$\begin{aligned} E\dot{\widehat{P}}_\xi = & \sum_{k=1}^{r} \dot{h}_k(\xi)E(PE^{\mathrm{T}} + E^{\perp}Q_\xi) \\ = & \sum_{k=1}^{r} \dot{h}_k(\xi)EPE^{\mathrm{T}} = 0. \end{aligned}$$

The rest of proof is the same as Theorem III.1. This completes the proof.

Remark 4: The distinction between Theorems III.2 and III.1 is that it does not need the bounds of the time derivatives of fuzzy membership functions. In some cases, it is very difficult to obtain these bounds of the time derivatives of fuzzy membership functions. Therefore, Theorem III.2 is more convenient for checking the admissibility without knowledge of the time derivatives of fuzzy membership functions.

B. Fuzzy Controller Design

In this section, the design problem of PDC and non-PDC controllers is considered.

First, let the PDC controller be of the form

R_i: if $\xi_1(t)$ is M_{1i}, $\xi_2(t)$ is $M_{2i}, \ldots$, and $\xi_p(t)$ is M_{pi} then

$$u(t) = K_i x(t), i = 1, 2, \ldots, r \tag{29}$$

where K_i is a gain of the *i*th rule to be determined. Then overall state feedback control law is inferred by

$$\begin{aligned} u(t) = & \sum_{i=1}^{r} \frac{\prod_{j=1}^{p} M_{ji}(\xi_j(t))K_i x(t)}{\sum_{i=1}^{r}\prod_{j=1}^{p} M_{ji}(\xi_j(t))} \\ = & \sum_{i=1}^{r} h_i(\xi(t))K_i x(t) \\ = & K_\xi x(t). \end{aligned} \tag{30}$$

The resultant closed-loop system is as follows:

$$E\dot{x}(t) = (A_\xi + B_\xi K_\xi)x(t). \tag{31}$$

To design PDC controller with corresponding fuzzy Lyapunov function, previous sufficient results are usually in the form of BMIs [3], [28]. The following theorem gives a sufficient condition in the form of LMIs for the closed-loop system to be admissible.

Theorem III.3: System (31) is admissible if there exist a scalar $\alpha > 0$, symmetric matrices $\overline{P}_i, Z_i, Q_{ij} > 0, X, Y \in \mathrm{R}^{n\times n}$ $(i, j = 1, 2, \ldots, r)$, matrices $Q_i \in \mathrm{R}^{(n-n_1)\times n}$, $H \in \mathrm{R}^{n\times n}$ and $L_i \in \mathrm{R}^{m\times n}$ $(i = 1, 2, \ldots, r)$ such that LMIs (11) and (12) and

$$\Gamma_{ii} < 0 \quad i = 1, 2, \ldots, r \tag{32}$$

$$\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, 1 \le i \ne j \le r \tag{33}$$

where

$$\Gamma_{ij} = \begin{bmatrix} P_\Phi + \Psi + \Psi^{\mathrm{T}} + \Omega - Q_{ij} & * \\ P_i + \alpha\Psi^{\mathrm{T}} - H & -\alpha(H + H^{\mathrm{T}}) \end{bmatrix}$$

$$\Psi = A_i H + B_i L_j$$

$$\Omega = \sum_{k=1}^{r}\sum_{l=1}^{r} \beta_{kl} Q_{kl}, P_i = \overline{P}_i E^{\mathrm{T}} + SQ_i$$

$$P_\Phi = -\sum_{k=1}^{r} \phi_k (EP_k + X).$$

In this case, the state feedback gain K_i can be chosen to be $K_i = L_i H^{-1}$.

Proof: If LMIs (32) and (33) hold, by Lemma II.4, it can be obtained that

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi)h_j(\xi)\Gamma_{ij} = \begin{bmatrix} \Delta & * \\ P_\xi + \alpha\Xi^{\mathrm{T}} - H & -\alpha(H + H^{\mathrm{T}}) \end{bmatrix} < 0$$

where

$$\Delta = P_\Phi + \Xi + \Xi^{\mathrm{T}} - \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi)h_j(\xi)Q_{ij} + \Omega$$

$$\Xi = A_\xi H + B_\xi L_\xi.$$

By $h_i(\xi)h_j(\xi) \le \beta_{ij}, Q_{ij} > 0$ and $K_i = L_i H^{-1}$, one has

$$\begin{bmatrix} P_\Phi + \Pi + \Pi^{\mathrm{T}} & * \\ P_\xi + \alpha\Pi^{\mathrm{T}} - H & -\alpha(H + H^{\mathrm{T}}) \end{bmatrix} < 0 \tag{34}$$

where

$$\Pi = [A_\xi + B_\xi K_\xi]H.$$

Using Lemma II.3 with $T = P_\Phi$, $A = A_\xi + B_\xi K_\xi$, $L = \alpha H$ and $P = P_\xi$, one obtains

$$P_\Phi + [A_\xi + B_\xi K_\xi]P_\xi + P_\xi^{\mathrm{T}}[A_\xi + B_\xi K_\xi]^{\mathrm{T}} < 0. \tag{35}$$

According to Theorem III.1, when the LMIs (11), (12), and (35) hold, the resultant closed-loop system (31) is admissible. This completes the proof.

Remark 5: Using Lemma II.3, LMI (34) does not involve the product between the Lyapunov matrix P_ξ and the system dynamic matrix A_ξ in PDC design process, which can make the design PDC controller easier. The remaining problem is how to choose the scalar $\alpha > 0$. This problem can be solved by the following optimal algorithm.

Step 1: Initialization give the initial value $\alpha^0 = 1$.

Step 2: Find a feasible solution $H = H^0$ for the following optimization problem:

$$\min_H \Lambda\left(\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji}\right), \Lambda(\Gamma_{ii})$$

where $\Lambda()$ is maximum eigenvalue of the matrix. If $\Lambda(\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji}) < 0, \Lambda(\Gamma_{ii}) < 0$, stop. Otherwise, go to step 3.

Step 3: Using the matrices H^0 obtained in Step 2, find a feasible solution $\alpha = \alpha^1$ for the following optimization problem:

$$\min_\alpha \Lambda\left(\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji}\right), \Lambda(\Gamma_{ii}).$$

If $\Lambda(\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji}) < 0, \Lambda(\Gamma_{ii}) < 0$, stop. Otherwise, go to step 2.

Iteration termination condition, if $|\Lambda^k(\Gamma_{ii}) - \Lambda^{(k-1)}(\Gamma_{ii})| < e$, where e is a prescribed tolerance, stop.

In what follows, we deign a fuzzy non-PDC controller of the form

$$u(t) = \sum_{i=1}^{r} h_i(\xi)F_i \left(\sum_{i=1}^{r} h_i(\xi)P_i\right)^{-1} x(t) = F_\xi P_\xi^{-1} x(t). \tag{36}$$

The resultant closed-loop system is as follows:

$$E\dot{x}(t) = \left(A_\xi + B_\xi F_\xi P_\xi^{-1}\right) x(t). \tag{37}$$

Theorem III.4: System (37) is admissible if there exist symmetric matrices $\overline{P}_i, Z_i, Q_{ij} > 0, X, Y \in \mathrm{R}^{n\times n}$ $(i, j = 1, 2, \ldots, r)$ and matrices $Q_i \in \mathrm{R}^{(n-n_1)\times n}$, $F_i \in \mathrm{R}^{m\times n}$ $(i = 1, 2, \ldots, r)$ such that LMIs (11) and (12) and

$$\Xi_{ii} < 0 \quad i = 1, 2, \ldots, r \tag{38}$$

$$\frac{2}{r-1}\Xi_{ii} + \Xi_{ij} + \Xi_{ji} < 0, 1 \le i \ne j \le r \tag{39}$$

where

$$\Xi_{ij} = P_\Phi + A_i P_j + B_i F_j + P_j^{\mathrm{T}} A_i^{\mathrm{T}} + F_j^{\mathrm{T}} B_i^{\mathrm{T}} + \Omega - Q_{ij}$$

$$\Omega = \sum_{k=1}^{r}\sum_{l=1}^{r} \beta_{kl} Q_{kl}, P_i = \overline{P}_i E^{\mathrm{T}} + SQ_i$$

$$P_\Phi = -\sum_{k=1}^{r} \phi_k (EP_k + X).$$

Proof: If LMIs (38) and (39) hold, by Lemma II.4, it can be shown that

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi)h_j(\xi)\Xi_{ij} = P_\Phi + A_\xi P_\xi + B_\xi F_\xi + P_\xi^{\mathrm{T}} A_\xi^{\mathrm{T}} + F_\xi^{\mathrm{T}} B_\xi^{\mathrm{T}} + \sum_{k=1}^{r}\sum_{l=1}^{r} \beta_{kl} Q_{kl} - \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi)h_j(\xi)Q_{ij}.$$

By $h_i(\xi)h_j(\xi) \le \beta_{kl}$ and $Q_{ij} > 0$, one has

$$P_\Phi + A_\xi P_\xi + B_\xi F_\xi + P_\xi^{\mathrm{T}} A_\xi^{\mathrm{T}} + F_\xi^{\mathrm{T}} B_\xi^{\mathrm{T}} = P_\Phi + P_\xi^{\mathrm{T}} \left[A_\xi + B_\xi F_\xi P_\xi^{-1}\right]^{\mathrm{T}} + \left[A_\xi + B_\xi F_\xi P_\xi^{-1}\right] P_\xi < 0. \tag{40}$$

Therefore, according to Theorem III.1, it can be seen that system (37) is admissible. This completes the proof.

Remark 6: It should be pointed out that the original bounds of the derivatives of membership functions need to be checked after the completion of the fuzzy controller design process. Similar to [3], these original bounds can be found within the operating domain of the state of consideration. Then, by Theorems III.3 and III.4, the fuzzy controller is obtained and we should check whether the domain of the state of the closed-loop system can be satisfied to the original bounds. If these bounds are relatively difficult to achieve, we can use the following Theorems III.5 and III.6 independent of these bounds to design the fuzzy controller.

Let the other non-PDC fuzzy controller be

$$\begin{aligned} u(t) &= \sum_{i=1}^{r} h_i(\xi) F_i \Big(\sum_{i=1}^{r} h_i(\xi) \widehat{P}_i\Big)^{-1} x(t) \\ &= F_\xi \widehat{P}_\xi^{-1} x(t) \end{aligned} \tag{41}$$

where $\widehat{P}_i = PE^{\mathrm{T}} + SQ_i$. The resultant closed-loop system is as follows:

$$\dot{x}(t) = \left(A_\xi + B_\xi F_\xi \widehat{P}_\xi^{-1}\right) x(t). \tag{42}$$

In view of the above Theorems III.3 and III.4, the admissibility of closed-loop system (31) and (37) have been considered based on Theorem III.1. Similar to the method of Theorems III.3 and III.4, the admissibility of closed-loop system (31) and (42) can also be considered based on Theorem III.2. Then, the following two results can be obtained.

Theorem III.5: System (31) is admissible if there exist a scalar $\alpha > 0$, matrices $Z, Q_i, H \in \mathrm{R}^{n\times n}\,(i = 1, 2, \ldots, r)$, symmetric matrices $Q_{ij} > 0, P \in \mathrm{R}^{n\times n}$ $(i, j = 1, 2, \ldots, r)$ and $L_i \in \mathrm{R}^{m\times n}$ $(i = 1, 2, \ldots, r)$ such that LMI (24) and

$$\begin{aligned} &\overline{M}_{ii} < 0, i = 1, 2, \ldots, r \\ &\frac{2}{r-1}\overline{M}_{ii} + \overline{M}_{ij} + \overline{M}_{ji} < 0, 1 \le i \ne j \le r \end{aligned}$$

where

$$\begin{aligned} \overline{M}_{ij} &= \begin{bmatrix} \Pi + \Pi^{\mathrm{T}} + \Omega - Q_{ij} & * \\ PE^{\mathrm{T}} + E^{\perp}Q_i + \alpha\Pi^{\mathrm{T}} - H & -\alpha(H + H^{\mathrm{T}}) \end{bmatrix} \\ \Pi &= AH + B_i L_j \\ \Omega &= \sum_{k=1}^{r}\sum_{l=1}^{r} \beta_{kl} Q_{kl}. \end{aligned}$$

In this case, K_i can be chosen as $K_i = L_i H^{-1}$.

Theorem III.6: System (42) is admissible if there exist matrices $Z, Q_i \in \mathrm{R}^{n\times n}$ $(i = 1, 2, \ldots, r,)$, symmetric matrices $Q_{ij} > 0, P \in \mathrm{R}^{n\times n}$ $(i, j = 1, 2, \ldots, r)$, and $F_i \in \mathrm{R}^{m\times n}(i = 1, 2, \ldots, r)$ such that LMI (24) and

$$\begin{aligned} &\overline{\Xi}_{ii} < 0 \quad , i = 1, 2, \ldots, r \\ &\frac{2}{r-1}\overline{\Xi}_{ii} + \overline{\Xi}_{ij} + \overline{\Xi}_{ji} < 0, 1 \le i \ne j \le r \end{aligned}$$

where

$$\begin{aligned} \overline{\Xi}_{ij} &= A_i(PE^{\mathrm{T}} + E^{\perp}Q_j) + B_i F_j \\ &\quad + (PE^{\mathrm{T}} + E^{\perp}Q_j)^{\mathrm{T}} A_i^{\mathrm{T}} + F_j^{\mathrm{T}} B_i^{\mathrm{T}} + \Omega - Q_{ij} \\ \Omega &= \sum_{k=1}^{r}\sum_{l=1}^{r} \beta_{kl} Q_{kl}. \end{aligned}$$

IV. Illustrative Examples

In this section, four examples are used to illustrate some of the main results of this paper.

Example 1: Consider the following T–S fuzzy descriptor system:

$$E\dot{x}(t) = \sum_{i=1}^{2} h_i(\xi(t)) A_i x(t)$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -2 & 0.2 & 1 \\ 2 & -5 & 0 \\ 1 & 0 & 1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1 & 0.2 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The membership functions are $h_1(\xi(t)) = (1 - \frac{x_1^2(t)}{2}), h_2(\xi(t)) = \frac{x_1^2(t)}{2}, x_1(t) \in [-1, 1]$.

By computing the characteristic equation $\det(sE - A_2) = (-s - 4)(s - 1) = 0$, we find that one of the eigenvalues is 1. Accordingly, it can be seen that the second fuzzy subsystem is not stable. The admissibility of this system can be checked by Theorem III.1. Specifically, choosing $\alpha_1 = 1, \alpha_2 = 0.5, \beta_{11} = 1, \beta_{12} = 0.25, \beta_{22} = 0.25, \phi_1 = \phi_2 = -1$, it can be verified that the LMIs (11–14) have the feasible solution

$$P_1 = \begin{bmatrix} 31.3242 & 3.2349 & 0 \\ 3.2349 & 25.5349 & 0 \\ -7.9780 & -5.7466 & -53.3097 \end{bmatrix}$$
$$P_2 = \begin{bmatrix} 51.1251 & 11.5460 & 0 \\ 11.5460 & 24.0675 & 0 \\ -37.4635 & -10.6991 & -33.7787 \end{bmatrix}.$$

The system is thus admissible. The initial condition is taken as $x_0 = [0.2, 0.4, 1]^{\mathrm{T}}$, a trajectory of the T–S fuzzy descriptor system is shown in Fig. 1. From Fig. 1, we can find this system is stable. Further, by $\det[sE - (h_1A_1 + h_2A_2)] = (-\frac{1}{5})$ $[s^2 + (40h_1^2 + 50h_1h_2 + 15h_2^2)s + 73h_1^3 + 81h_1^2h_2 - 7h_1h_2^2 - 20h_2^3]$, we can obtain this system is regular and impulse-free. Therefore, this system is admissible.

Remark 7: From Example 1, we find that Theorem III.1 can be used when the fuzzy subsystems is not stable. This is different from the previous results in [16], [23], [26], [28] which require fuzzy subsystems is stable.

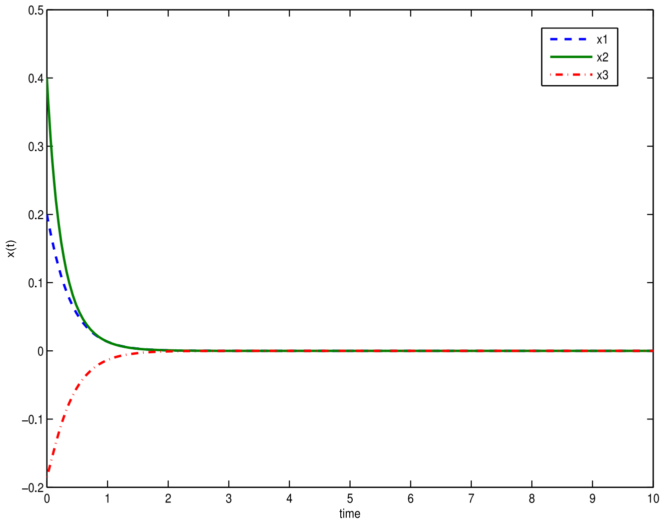

Fig. 1. Trajectories of the T–S fuzzy descriptor system.

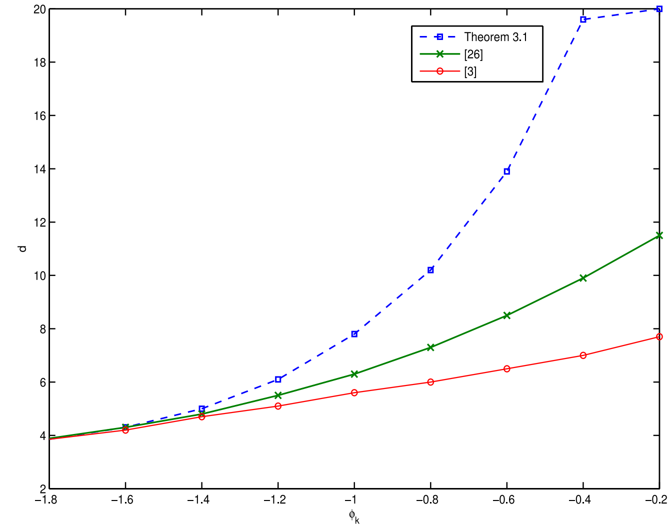

Fig. 2. Maximum values of d guaranteeing feasibility for each ϕ_k.

Example 2: Consider the following nonlinear system [26]:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -2x_1(t) - x_2(t) - f(t)x_1(t) \end{aligned} \tag{43}$$

where the $f(t) \in [0, d]$ is at least first-order differentiable. The nonlinear dynamics of this system can be exactly represented by the following fuzzy model:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(\xi(t))A_i x(t)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -2-d & -1 \end{bmatrix}.$$

The membership functions are $h_1(\xi(t)) = \frac{d-f(t)}{d}$ and $h_2(\xi(t)) = \frac{f(t)}{d}$.

Similar to [26], this system can be rewritten in the descriptor form. Choosing $\alpha_1 = 1, \alpha_2 = 1, \beta_{11} = 1, \beta_{12} = 0.25, \beta_{22} = 1$, the maximum values d of guaranteeing the feasibility of Theorem III.1, results in [3] and [26] for each ϕ_k can be checked. The maximal d values obtained by means of these methods are depicted in Fig. 2. From Fig. 2, it is easy to see that Theorem III.1 guarantees larger feasible area than those existing ones.

Example 3: Consider the following T–S fuzzy system [31]:

$$\dot{x}(t) = \sum_{i=1}^{3} h_i(\xi(t))A_i x(t)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.06 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & a \\ -1.94 & -1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0 & b \\ -0.5 & -1.5 \end{bmatrix}.$$

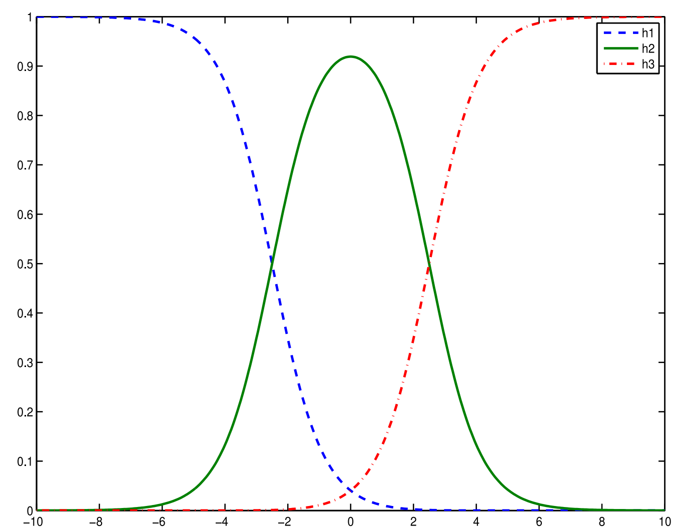

Fig. 3. Membership functions $h_1(\xi)$, $h_2(\xi)$, and $h_3(\xi)$.

The membership functions are

$$h_i(\xi(t)) = \frac{\omega_i(x_1(t))}{\omega_1(x_1(t)) + \omega_2(x_1(t)) + \omega_3(x_1(t))} \; (i = 1, 2, 3),$$

where

$$\begin{aligned} \omega_1(x_1(t)) &= \exp\left(-\frac{1}{2}\left(\frac{x_1(t)+5}{2}\right)^2\right), \\ \omega_2(x_1(t)) &= \exp\left(-\frac{1}{2}\left(\frac{x_1(t)}{2}\right)^2\right), \end{aligned}$$

and $\omega_3(x_1(t)) = \exp(-\frac{1}{2}(\frac{x_1(t)-5}{2})^2)$. Membership functions of this system are depicted in Fig. 3.

Choosing $\alpha_1 = \alpha_3 = 1, \alpha_2 = 0.95, \beta_{11} = \beta_{33} = 1, \beta_{22} = 0.91, \beta_{12} = \beta_{23} = 0.25, \beta_{13} = 0.0025, \phi_1 = \phi_2 = \phi_3 = -0.3$ and for several values of a, the maximum b of the admissible

TABLE I
MAXIMUM b FOR THE STABLE REGION

a=	5	15	30	50	100
[3]	7	12.5	19.5	27.8	46.6
[32]	7.2	13.8	20.7	29	46.3
[31]	7.2	14.3	22.4	31.5	50.3
Corollary III.1	8.8	17.6	32.2	44.4	63.2

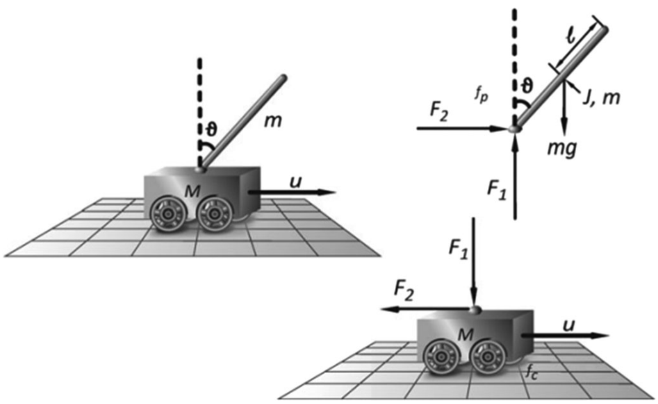

Fig. 4. Model of the cart and inverted pendulum and the Free Body diagrams.

region has been checked and compared in Table I. It is easy to see that Corollary III.1 provides less conservative results than those existing ones.

Example 4 [27]: Consider the inverted pendulum model described as follows:

$$\begin{cases} J\ddot{\theta}(t) = F_1(r_x(t) - r(t)) - F_2 r_y(t) - f_p\dot{\theta}(t) \\ M\ddot{r}(t) = u(t) - f_c\dot{r}(t) - F_2 \\ m\ddot{r}_x(t) = F_2 \\ m\ddot{r}_x y(t) = F_1 - mg \\ 0 = r(t) - r_x(t) + l\sin\theta(t) \\ 0 = r_y(t) - l\cos\theta(t) \end{cases}$$

where $\theta(t) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ denotes the angle of the pen dulum from the vertical, $r(t)$ denotes the displacement of the cart, $r_x(t)$ and $r_y(t)$ denote the horizontal and vertical positions of the pendulum centre, respectively, $g = 9.8\ \mathrm{m/s^2}$ is the gravity constant, $M = 1.3282$ kg is the mass of the cart, $m = 0.22$ kg is the mass of the pendulum, $fc = 22.915$ N is the friction factor of the cart, $fp = 0.007056\ \mathrm{kg \cdot m^2}$ is the frictional resisting moment factor of the pendulum, $l = 0.304$ m is the length from the pendulum centre of mass to the shaft axis, $J = 0.004963\ \mathrm{kg \cdot m^2}$ is the moment of inertia of the pendulum around the centre of mass, and u is the force applied to the cart. In addition, F_1 and F_2 represent the opposite and equal interactional forces between the cart and the pendulum. The physical experimental model of cart and inverted pendulum and free body diagrams are shown in Fig. 4.

Let

$$\begin{aligned} x =\ & [x_1, x_2, x_3, x_4, x_5]^{\mathrm{T}} \\ =\ & [\theta(t), \dot{\theta}(t), r(t), \dot{r}(t), r_x(t) - r(t)]^{\mathrm{T}} \end{aligned}$$

we can get the following nonlinear descriptor systems:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \\ \dfrac{\begin{bmatrix} f_c m l x_4 \cos x_1 - f_p(M+m)x_2 - ml\cos x_1 u \\ +(M+m)mgx_5 - m_2 l x_2^2 x_5 \cos x_1 \end{bmatrix}}{(M+m)(J+ml^2) - m^2 l^2 \cos^2 x_1} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{\begin{bmatrix} f_p m l x_2 \cos x_1 - f_c(J+ml^2)x_4 + (J+ml^2)u \\ -m^2 l g x_5 \cos x_1 + (J+ml^2)m x_2^2 x_5 \end{bmatrix}}{(M+m)(J+ml^2) - m^2 l^2 \cos^2 x_1} \\ 0 = l\sin x_1 - x_5 \end{cases}.$$

Let $\xi(t) = x_1(t)$, and construct the auxiliary system by two rule fuzzy model as follows:

Rule 1: if $x_1(t)$ is about 0, then

$$E\dot{x} = A_1 x + B_1 u$$

Rule 2: if $x_1(t)$ is about $\pm\frac{\pi}{3}$, then

$$E\dot{x} = A_2 x + B_2 u$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & a_{122} & 0 & a_{124} & a_{125} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & a_{142} & 0 & a_{144} & a_{145} \\ a_{151} & 0 & 0 & 0 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ b_{12} \\ 0 \\ b_{14} \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & a_{222} & 0 & a_{224} & a_{225} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & a_{242} & 0 & a_{244} & a_{245} \\ a_{251} & 0 & 0 & 0 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ b_{22} \\ 0 \\ b_{24} \\ 0 \end{bmatrix}$$

$a_{122} = -f_p(M+m)/a_1$, $a_{124} = f_c ml/a_1$, $a_{125} = (M+m)mg/a_1$, $a_{142} = f_p ml/a_1$, $a_{144} = -f_c(J+ml^2)/a_1$, $a_{145} = -m^2 lg/a_1$, $a_{151} = l$, $b_{12} = -m*l/a_1$, $b_{14} = (J+ml^2)/a_1$, $a_1 = (M+m)(J+ml^2) - m^2 l^2$, $a_{222} = -f_p(M+m)/a_2$, $a_{224} = f_c ml/2a_2$, $a_{225} = (M+m)mg/a_2$, $a_{242} = f_p ml/2a_2$, $a_{244} = -f_c(J+ml^2)/a_2$, $a_{245} = -m^2 lg/2a_2$, $a_{251} = 3\sqrt{3}l/2\pi$, $b_{22} = -m*l/2a_2$, $b_{24} = (J+ml^2)/a_2$, $a_2 = (M+m)(J+ml^2) - m^2 l^2/4$.

Let choose membership functions for Rules 1 and 2:

$$h_1(x_1) = \frac{1 - \frac{1}{1+e^{-7(x_1+\pi/6)}}}{1+e^{-7(x_1+\pi/6)}}, h_2(x_1) = 1 - h_1(x_1).$$

Choosing $\alpha_1 = \alpha_2 = 1$, $\beta_{11} = \beta_{22} = 1$, $\beta_{21} = \beta_{12} = 0.25$, $\phi_1 = \phi_2 = -14$, the part of solutions to the LMIs of Theorems III.3–III.6 are shown in Table II. To demonstrate the effectiveness of the design method, assuming the initial condition is $x_0 = [0.2617, 0, 0, 0, 0.0787]^{\mathrm{T}}$ and based on

TABLE II
SOLUTIONS OF THEOREMS III.3–III.6

Methods	Solutions
Theorem III.3	$P_1 = \begin{bmatrix} 0.0162 & -0.0569 & -0.0141 & -0.0090 & 0 \\ -0.0569 & 0.3409 & 0.0425 & -0.1182 & 0 \\ -0.0141 & 0.0425 & 0.2552 & -0.1027 & 0 \\ -0.0090 & -0.1182 & -0.1027 & 0.3332 & 0 \\ 0.0009 & 0.0008 & -0.0030 & -0.0068 & 0.0002 \end{bmatrix}$ $P_2 = \begin{bmatrix} 0.0162 & -0.0568 & -0.0141 & -0.0090 & 0 \\ -0.0568 & 0.3411 & 0.0429 & -0.1192 & 0 \\ -0.0141 & 0.0429 & 0.2555 & -0.1032 & 0 \\ -0.0090 & -0.1192 & -0.1032 & 0.3344 & 0 \\ 0.0008 & 0.0016 & -0.0028 & -0.0085 & 0.0002 \end{bmatrix}$ $K_1 = \begin{bmatrix} 7.2033 & 4.8396 & 0.8246 & 24.3522 & 76.2082 \end{bmatrix}$ $K_2 = \begin{bmatrix} 16.2671 & 10.8151 & 1.9912 & 26.4132 & 162.8625 \end{bmatrix}$
Theorem III.4	$P_1 = \begin{bmatrix} 1.9518 & -7.9962 & 0.0569 & -1.3512 & 0 \\ -7.9962 & 38.9966 & -0.0598 & 0.0644 & 0 \\ 0.0569 & -0.0598 & 26.9413 & -2.1413 & 0 \\ -1.3512 & 0.0644 & -2.1413 & 6.3603 & 0 \\ 0.0195 & 90.3474 & 0.0239 & -35.3221 & 0.0300 \end{bmatrix}$ $P_2 = \begin{bmatrix} 1.8969 & -8.3950 & -0.0235 & -0.8546 & 0 \\ -8.3950 & 40.4444 & -0.1663 & 0.4441 & 0 \\ -0.0235 & -0.1663 & 26.8911 & -1.7791 & 0 \\ -0.8546 & 0.4441 & -1.7791 & 4.2655 & 0 \\ -0.1598 & 40.7266 & 0.0444 & -31.6669 & 0.0238 \end{bmatrix}$ $N_1 = 10^3 \times \begin{bmatrix} -0.0188 & 4.5384 & -0.0479 & -0.1013 & -0.0467 \end{bmatrix}$ $N_2 = 10^3 \times \begin{bmatrix} -0.0089 & 4.1371 & -0.0354 & -0.0399 & -0.0466 \end{bmatrix}$
Theorem III.5	$\widehat{P}_1 = \begin{bmatrix} 7.7192 & -25.0850 & -8.2757 & -5.5342 & 0 \\ -25.0850 & 150.8375 & 21.8681 & -60.1569 & 0 \\ -8.2757 & 21.8681 & 146.6303 & -54.2179 & 0 \\ -5.5342 & -60.1569 & -54.2179 & 185.3114 & 0 \\ 0.4962 & 0.4915 & -1.7875 & -3.9092 & 0.1177 \end{bmatrix}$ $\widehat{P}_2 = \begin{bmatrix} 7.7192 & -25.0850 & -8.2757 & -5.5342 & 0 \\ -25.0850 & 150.8375 & 21.8681 & -60.1569 & 0 \\ -8.2757 & 21.8681 & 146.6303 & -54.2179 & 0 \\ -5.5342 & -60.1569 & -54.2179 & 185.3114 & 0 \\ 0.4587 & 0.9538 & -1.6685 & -5.0382 & 0.1009 \end{bmatrix}$ $K_1 = \begin{bmatrix} 6.2339 & 4.5973 & 0.6564 & 24.1920 & 72.1343 \end{bmatrix}$ $K_2 = \begin{bmatrix} 14.5999 & 10.4765 & 1.6417 & 26.0963 & 156.4483 \end{bmatrix}$
Theorem III.6	$\widehat{P}_1 = \begin{bmatrix} 46.7907 & -187.8119 & 1.8058 & -46.9021 & 0 \\ -187.8119 & 861.8591 & 2.0977 & 76.1100 & 0 \\ 1.8058 & 2.0977 & 758.8265 & -77.4824 & 0 \\ -46.9021 & 76.1100 & -77.4824 & 173.1133 & 0 \\ -9.1552 & -646.3700 & -1.7195 & 224.0576 & 1.1861 \end{bmatrix}$ $\widehat{P}_2 = \begin{bmatrix} 46.7907 & -187.8119 & 1.8058 & -46.9021 & 0 \\ -187.8119 & 861.8591 & 2.0977 & 76.1100 & 0 \\ 1.8058 & 2.0977 & 758.8265 & -77.4824 & 0 \\ -46.9021 & 76.1100 & -77.4824 & 173.1133 & 0 \\ -9.0553 & -304.1327 & -0.3199 & 216.2612 & 0.8159 \end{bmatrix}$ $N_1 = 10^4 \times \begin{bmatrix} -0.1067 & -3.0416 & -0.1841 & 0.4310 & 0.0364 \end{bmatrix}$ $N_2 = 10^4 \times \begin{bmatrix} -0.0911 & -2.8251 & -0.1598 & 0.3022 & 0.0362 \end{bmatrix}$

Theorems III.3–III.6, the trajectories of this system under the PDC and non-PDC controllers are shown in Fig. 5.

V. CONCLUSION

In this paper, a new relaxed admissibility condition for T–S fuzzy descriptor systems has been given by Nonquadratic fuzzy Lyapunov function. This condition fully uses the information of fuzzy membership functions, e.g., $h_k(\xi) \le \alpha_k \le 1$, $h_k(\xi)h_l(\xi) \le \beta_{kl} \le 1$, $\dot{h}_k(\xi) \ge \phi_k$ and $\sum_{i=1}^{r} h_k(\xi) = 1$. Meanwhile, this condition breaks the limitation of every fuzzy subsystem to be stable. Importantly, the technique is particularly well suited to other control problems of T–S fuzzy descriptor systems. In order to relieve the difficulty caused by time derivatives of fuzzy membership functions, a sufficient condition without time derivatives of fuzzy basis functions has been proposed, which ensures that fuzzy descriptor systems are admissible. Based on these analysis results both PDC and non-PDC controllers have been designed. New conditions have been represented in the form of strict LMIs, so they can be easily checked by solving by Matlab. Some numerical examples show how the feasibility region larger than the previous work.

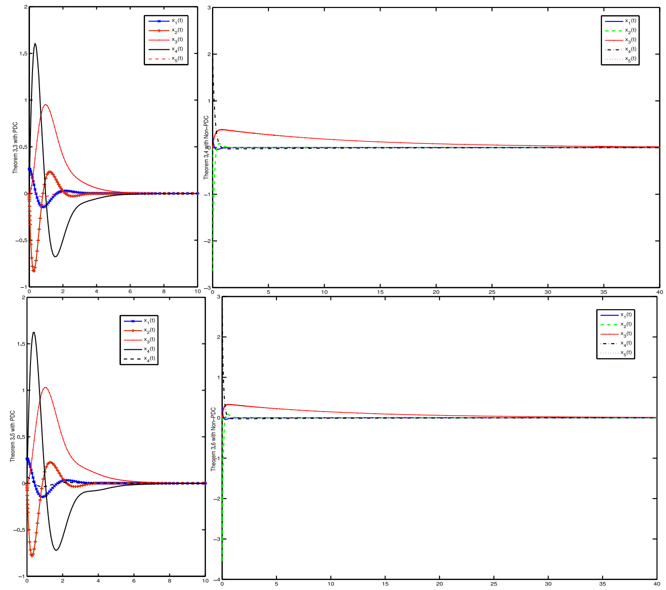

Fig. 5. Trajectory of the closed-loop fuzzy descriptor system.

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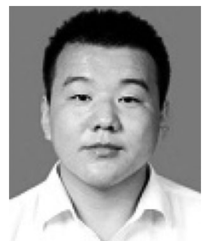

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