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Probabilistic Fuzzy Regression Approach for Preference Modeling

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Abstract

Two types of uncertainty, namely, randomness and fuzziness, exist in preference modeling. Fuzziness is mainly caused by human subjective judgment and incomplete knowledge, and randomness often originates from the variability of influences on the inputs and outputs of a preference model. Various techniques have been utilized to develop preference models. However, only few previous studies have addressed both fuzziness and randomness in preference modeling. Among these limited studies, none have considered the randomness caused by particular independent variables. To fill this research gap, this study proposes probabilistic fuzzy regression (PFR), a new approach for preference modeling. PFR considers both the fuzziness of data sets and the randomness caused by independent variables. In the proposed approach, probability density functions (PDFs) are adopted to model randomness. The parameter settings of the PDFs are determined using a chaos optimization algorithm. The probabilistic terms of the PFR models are generated according to the expected value functions of the random variables. Fuzzy regression analysis is employed to determine the fuzzy coefficients for all the terms of the PFR models. An industrial case study of a tea maker design is used to illustrate the applicability of PFR and evaluate its effectiveness. Modeling results obtained from PFR are compared with those obtained from statistical regression, fuzzy regression, and fuzzy least-squares regression. Results of the training and validation tests show that PFR outperforms the other approaches in terms of training and validation errors.

Keywords: Probabilistic fuzzy regression, Preference modeling, Chaos optimization algorithm

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1. Introduction

Survey/experimental data is often used to develop empirical models that relate the inputs and outputs of a system or process. Various approaches for developing empirical models have been attempted. These approaches include quantification theory I (Chang, 2008), ordinal logistic regression (Barone et al., 2007), artificial neural networks (Lai et al., 2005), fuzzy logic approach (Lau et al., 2006), multiple statistical regression (Han et al., 2000), fuzzy linear regression (Sekkeli et al., 2010), particle swarm optimization-based fuzzy regression (Chan et al., 2011a), neural fuzzy systems (Kwong et al., 2009), kernel-based nonlinear fuzzy regression (Su et al., 2013), fuzzy polynomial regression based on fuzzy neural networks (Otadi, 2014), fuzzy regression models using fuzzy distances (de Hierro et al., 2016), and fuzzy regression models based on least absolute deviation (Li et al., 2016). Development of empirical models using survey/experimental data often involves both fuzziness and randomness. Fuzziness is mainly caused by human subjective judgment and incomplete knowledge, and randomness often originates from the variability of influences on the inputs and outputs of a system or process. Only few previous studies have examined both fuzziness and randomness in empirical modeling. Watada et al. (2009) proposed a confidence-interval-based fuzzy random regression approach to address the uncertainties caused by fuzziness and randomness in modeling. In their study, variables were regarded as known fuzzy numbers and probabilities. Kwong et al. (2008) proposed a fuzzy least-squares regression approach to capture fuzziness and randomness simultaneously in modeling manufacturing processes. However, the approach does not specifically address the randomness caused by independent variables.

Preference modeling is aimed at developing models to relate customer preferences and design parameters where customer surveys are commonly adopted to understand customers' preferences and the survey results are used to generate preference models. A number of studies have been conducted to develop preference models via survey and experimental data. Various statistical techniques, such as partial least squares analysis (Nagamachi, 2008) and statistical linear regression (Han et al. 2000; You et al., 2006), have been adopted to model customer preference. However, in customer surveys, customers' responses are always imprecise such as "quite good" and "not very well". Thus, survey results unavoidably contain a high degree of fuzziness. Numerous fuzzy approaches for preference modeling have been employed to address the fuzziness in preference modeling. These approaches include fuzzy inference techniques (Liu et al., 2007; Fung et al., 1999), fuzzy rule-based approach (Lau et al., 2006; Park and Han, 2004; Fung et al., 1998), fuzzy logic approach (Lin et al., 2007), fuzzy linear regression

(Sekkeli et al., 2010; Shimizu and Jindo, 1995; Chen et al., 2004), nonlinear programming-based fuzzy regression (Chen and Chen, 2006), genetics-based fuzzy regression (Chan et al., 2011b), chaos-based fuzzy regression (Jiang et al., 2013), a stepwise-based fuzzy regression (Chan et al., 2015), and a forward selection-based fuzzy regression (Chan and Ling, 2016). However, all these techniques can only be utilized to deal with either randomness or fuzziness in preference modeling. Kwong et al. (2010) proposed a generalized fuzzy least-squares regression approach to address both fuzziness and randomness in preference modeling. In their proposed approach, Kwong et al. assumed that the estimation error is random and the objective function minimizes the sum of the squares of the residual error (Chang, 2001). However, the approach does not consider the randomness caused by independent variables.

In the current study, a new approach to preference modeling, namely, probabilistic fuzzy regression (PFR), is proposed. PFR can address the fuzziness caused by human subjective judgment and the randomness caused by random variables. Probability density functions (PDFs) are adopted in the proposed approach to model the randomness of independent (random) variables. A chaos optimization algorithm (COA) is employed to determine the parameter settings of the PDFs, and PDFs are then generated. The expected value functions of the random variables based on the PDFs are then generated and incorporated into the PFR models. Fuzzy regression analysis is then conducted to determine the fuzzy coefficients for all the terms of the PFR model.

The remainder of the paper is organized as follows. Section 2 presents the proposed PFR. Section 3 describes a case study on modeling consumer preference based on the proposed approach. Section 4 presents the validation of the proposed approach, and Section 5 provides the conclusions.

2. Probabilistic Fuzzy Regression (PFR)

The general form of a fuzzy linear regression model can be expressed as follows:

$$\tilde{Y}_i = \tilde{A}_0 + \tilde{A}_1 x_{i1} + \cdots + \tilde{A}_k x_{ik} = \tilde{A} x_i \qquad (1)$$

where $\tilde{Y}_i$, $i = 1,2,\cdots,n$, is the predicted output, which is a fuzzy number; n is the number of data sets; x_{ij} , $j = 0,1,2,\cdots,k$ is the jth independent variable of the ith data set; k is the number of independent variables; and $\tilde{A}_j$ is the fuzzy coefficient of the jth independent variable. $\tilde{A}_j = (s_j^L, a_j^c, s_j^R)$, where a_j^c , s_j^L , and s_j^R are the central value, left-, and right-side spreads of the fuzzy coefficients, respectively. If the fuzzy coefficients are symmetric fuzzy numbers, $s_j^L = s_j^R$; $x_i = [x_{i0}, x_{i1}, \cdots, x_{ik}]$ is a vector of the independent variables and $x_{i0} = 1$, and $\tilde{A} = [\tilde{A}_0, \tilde{A}_1, \cdots, \tilde{A}_k]^T$ is a vector of the fuzzy coefficients. The fuzzy regression model, Eq. (1), can be rewritten as follows:

$$\tilde{Y}_i = (\tilde{Y}_i^{sL}, \tilde{Y}_i^c, \tilde{Y}_i^{sR}) = (s_0^L, a_0^c, s_0^R) + (s_1^L, a_1^c, s_1^R)x_{i1} + \cdots + (s_k^L, a_k^c, s_k^R)x_{ik} \quad (2)$$

The predicted output of Eq. (1) can be presented as $\tilde{Y}_i = (\tilde{Y}_i^{sL}, \tilde{Y}_i^c, \tilde{Y}_i^{sR})$, where $\tilde{Y}_i^c$, $\tilde{Y}_i^{sL}$, and $\tilde{Y}_i^{sR}$ are the center, left-, and right-side spread values of the output, respectively. The major processes of PFR are described in the following subsections.

2.1. Determination of parameter settings of PDFs

The uncertainty of a random variable can be described by a PDF, $f(x)$, which is a function defined in the interval $[xmin, xmax]$ and has the following properties.

(a) $f(x) \geq 0$ for all x.

(b) $\int_{xmin}^{xmax} f(x)dx = 1$.

$xmin$, $xmax$, or both can be infinite.

The form of $f(x)$ depends on the probability distribution of a continuous random variable. Several PDFs, such as uniform, triangular, Gaussian, and exponential functions, are commonly used. The parameter settings of PDFs are determined using COA. COA is a stochastic search algorithm in which chaos is introduced into the optimization strategy to accelerate the optimum seeking operation and determine the global optimal solution (Ren and Zhong, 2011). COA employs chaotic dynamics to solve optimization problems and it has been applied successfully in various areas such as robot optimization control, function optimization and supply chain optimization (Mishra et al., 2008). Compared with conventional optimization methods, COA has faster convergence and can search for better solutions (Nanba et al., 2002). This algorithm also has an improved capacity to seek for the global optimal solution of an optimization problem and can escape from a local minimum. Chaos has dynamic properties, including ergodicity, intrinsic stochastic properties, and sensitive dependence on initial conditions. The characteristic of randomness ensures the capability for a large-scale search. Ergodicity allows COA to traverse all possible states without repetition and overcome the limitations caused by ergodic searching in general random methods. COA uses the carrier wave method to linearly map the selected chaos variables onto the space of optimization variables and then searches for the optimal solutions based on the ergodicity of the chaos variables. The processes of applying COA in this study are described as follows.

First, the number of iterations of COA is defined. Each chaos variable represents the parameter settings of PDFs, and the number of elements in a chaos variable is equal to the number of parameters to be determined. The chaos variable is initialized in which the values are selected randomly in the range [0, 1]. The ranges of parameters $[a,b]$ are initialized, in which a and b are the lower and upper limits of the optimization variable, respectively.

Second, the iteration number is set as $m=1$. Based on the initialized chaos variable, the logistic model used in COA is shown in Eq. (3), and logistic mapping can generate chaos variables through iteration.

$$c_m = f(c_{m-1}) = uc_{m-1}(1-c_{m-1}), \tag{3}$$

where u is a control parameter; $c_m \in [0,1]$ is the mth iteration value of the chaos variable c; and c_0 is the initialized chaos variable.

The linear mapping for converting chaos variables into optimization variables is formulated as follows:

$$q_m = a + (b-a)\cdot c_m, \tag{4}$$

where q_m is the optimization variable and the value of q_m is the parameter settings of PDFs. Based on the iteration, the chaos variables traverse between $[0,1]$, and the corresponding optimization variables traverse in the corresponding range $[a,b]$. In this case, the optimal solution can be identified in the area of feasible solutions.

Based on the values of q_m, PDFs, $f(x)$, are generated. The model can be developed based on $f(x)$ and fuzzy coefficients by which the predicted output $\tilde{Y}_i = (\tilde{Y}_i^{sL}, \tilde{Y}_i^{c}, \tilde{Y}_i^{sR})$ can be obtained. The predicted crisp output of $\tilde{Y}_i$ is denoted as $\hat{y}_i$, which is equal to the center value $\tilde{Y}_i^{c}$ if symmetric triangular member functions are used in PFR. The mean absolute percentage error ($MAPE$) is defined as the average of percentage errors, which is scale-independent and is a popular measure for evaluating prediction accuracy (Gilliland et al., 2015; Kim and Kim, 2016). Thus, MAPE was adopted in this study as the fitness function in COA., which is defined as follows:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|\hat{y}_i - y_i\right|}{y_i}\cdot 100 \tag{5}$$

where n is the number of data sets; $\hat{y}_i$ is the ith predicted crisp output of $\tilde{Y}_i$ and y_i is the ith actual crisp output based on survey data. The values of $MAPE$ and q_m in the first iteration are recorded as the best fitness value $fv^* = MAPE_1$ and the best solution $q^* = q_1$, respectively.

Third, the iteration continues by $m+1 \rightarrow m$. The chaos variable and optimization variable are updated by (3) and (4), respectively. The $MAPE$ in the $m+1$ iteration, $MAPE_{m+1}$, is obtained using (5). If $MAPE_{m+1} \leq fv^*$, then $fv^* = MAPE_{m+1}$ and $q^* = q_{m+1}$. Otherwise, fv^* and q^* remain the same. Finally, after the number of iterations reaches the predefined number, the iteration of COA stops. fv^* is the best fitness value and the values of q^* are the determined parameter settings of PDFs.

2.2. Generation of PFR models

With the PDFs obtained from Section 2.1, the expected value function of a random variable X, $E[X]$, can be generated as shown in Eq. (6) to replace the corresponding random variables in the model shown in Eq. (2) and become a probabilistic term.

$$E[X] = \int_{xmin}^{xmax} x f(x) dx \tag{6}$$

Considering the random variables, the model in Eq. (2) can be rewritten as follows:

$$\tilde{Y}_i = (\tilde{Y}_i^{sL}, \tilde{Y}_i^{c}, \tilde{Y}_i^{sR}) = (s_0^L, a_0^c, s_0^R) + (s_1^L, a_1^c, s_1^R) x_{i1}^{'} + \cdots + (s_k^L, a_k^c, s_k^R) x_{ik}^{'}, \tag{7}$$

where $x_{ij}^{'} = E(x_{ij})$ if x_{ij} is a random variable and is defined as a probabilistic term; otherwise, $x_{ij}^{'} = x_{ij}$, $i = 1, 2, \cdots, n$, and $j = 0, 1, 2, \cdots, k$. For example, if five variables, x_1 to x_5, are involved in preference modeling and x_1 and x_4 are random variables, the PFR model to be generated can be expressed as follows:

$$\begin{aligned} \tilde{Y} = & (s_0^L, a_0^c, s_0^R) + (s_1^L, a_1^c, s_1^R) E(x_1) + (s_2^L, a_2^c, s_2^R) x_2 + (s_3^L, a_3^c, s_3^R) x_3 \\ & + (s_4^L, a_4^c, s_4^R) E(x_4) + (s_5^L, a_5^c, s_5^R) x_5 \end{aligned} . \tag{8}$$

Fuzzy regression analysis is employed to determine the fuzzy coefficients for each term of the PFR model. The predicted output of Eq. (7), $\tilde{Y}_i = (\tilde{Y}_i^{sL}, \tilde{Y}_i^{c}, \tilde{Y}_i^{sR})$, are calculated as follows:

$$\tilde{Y}_i^c = \sum_{j=0}^{k} a_j^c x_{ij}^{'}, \tag{9}$$

$$\tilde{Y}_i^{sL} = s_0^L + \sum_{\substack{j=1 \\ x_{ij}^{'} \geq 0}}^{k} s_j^L x_{ij}^{'} + \sum_{\substack{j=1 \\ x_{ij}^{'} < 0}}^{k} s_j^R (-x_{ij}^{'}) \ , \tag{10}$$

$$\tilde{Y}_i^{sR} = s_0^R + \sum_{\substack{j=1 \\ x_{ij}^{'} \geq 0}}^{k} s_j^R x_{ij}^{'} + \sum_{\substack{j=1 \\ x_{ij}^{'} < 0}}^{k} s_j^L (-x_{ij}^{'}) \ . \tag{11}$$

The asymmetric fuzzy coefficients with central point a_j^c and spread values s_j^L and s_j^R can be determined by solving the following linear programming (LP) problem (Ishibuchi and Nii, 2001; Fung et al., 2005).

$$Min\ J = \sum_{j=0}^{k} \left((s_j^L + s_j^R) \sum_{i=1}^{n} \left| x_{ij}^{'} \right| \right) \tag{12}$$

subject to

$$-\sum_{j=0}^{k} a_j^c x_{ij}^{'} + (1-h)\tilde{Y}_i^{sL} \geq -\left[y_i \right]_{hL} \quad i = 1, 2, \cdots, n \tag{13}$$

$$\sum_{j=0}^{k} a_j^c x_{ij}^{'} + (1-h)\tilde{Y}_i^{sR} \geq \left[y_i \right]_{hR} \quad i = 1, 2, \cdots, n \tag{14}$$

$$s_j^L, s_j^R \geq 0, a_j^c \in R, j = 0, 1, 2, \cdots k \tag{15}$$

$$x_{i0}^{'} = 1 \text{ for all } i \text{ and } 0 \leq h \leq 1 , \tag{16}$$

where J is the objective function that represents the total width of the fuzzy outputs of the model shown in Eq. (7); $1 + k$ is the number of terms of the fuzzy regression model; $x_{ij}^{'}$ is the jth term of the ith data set in the model; $|.|$ refers to the absolute value of $x_{ij}^{'}$; $\left[y_i \right]_{hL}$ and $\left[y_i \right]_{hR}$ are the values of the h-level of the ith output of the data sets; and h refers to the degree to which the fuzzy model fits the given data and is located between 0 and 1.

The constraints in Eqs. (13) and (14) set the upper and lower boundaries of the estimated output, respectively, and the constraint in Eq. (15) ensures that s_j^L and s_j^R are non-negative. The basic idea of this LP problem is similar to that in Tanaka's fuzzy regression analysis (Tanaka, 1987).

2.3. Algorithm of PFR

The algorithm of the proposed PFR is summarized below.

Step 1: The parameters are initialized, including the number of iterations, number of PDFs for each random variable, number of PDF parameters, initialized chaos variables, and ranges of parameters.

Step 2: The structure of the PFR model is generated using Eq. (7). The number of terms in the PFR model is $1+k$, where k is the number of independent variables.

Step 3: The iteration begins from $m=1$. The chaos variables c_m are generated based on the logistic model in Eq. (3) and transformed into optimization variables q_m using Eq. (4).

Step 4: The interval that a random variable belongs to is defined based on the experimental data, and the corresponding PDF is selected. The expected value functions of the random variables are then generated based on Eq. (6) and the values of q_m. The random variables are substituted by their corresponding expected value functions, and the probabilistic terms of the PFR models are generated.

Step 5: The fuzzy coefficient of each term of the PFR model is determined by solving the LP problem shown in Eqs. (9) to (16). The fuzzy coefficients $\tilde{A}_j = (s_j^L, a_j^c, s_j^R)$ are assigned to all the terms of the PFR model according to the generated structure.

Step 6: Predicted output $\hat{y}_i$ is calculated with the developed PFR model. $MAPE$ between $\hat{y}_i$ and actual value y_i for all data sets can then be obtained using Eq. (5) as the fitness value of the iteration m.

Step 7: The iteration is continued by $m+1 \rightarrow m$ and stops after the number of iterations reaches the predefined value. The values of $MAPE$ are obtained for each iteration and compared. The solution with the smallest fitness value is selected based on step 3 in Section 2.1. A PFR model with the smallest error is then generated.

3. Preference modeling through PFR

An industrial case study on a tea maker design was conducted to evaluate the effectiveness of the proposed approach in preference modeling. A new tea maker prototype was built by the case company. In the case study, experiments were conducted using the prototype to investigate the relationships between the product variables and consumer preference. PFR was then introduced to generate preference models for the tea maker based on the experimental and survey data. Two dimensions of consumer preference, namely, aroma and texture, denoted as y_A and y_T, respectively, were studied. The processes of brewing tea using the prototype are as follows.

First, 3.5 L of fresh water is poured into container II of the tea maker and heated to 93 °C or above. Second, a certain amount of tea leaves is poured into the tea infuser, which is then placed in container I of the tea maker. The original temperature of the water decreases because of the heat loss brought about by the immersion of the cold tea infuser. Hence, the water needs to be reheated to maintain the temperature at a certain level, which is called the reheating temperature (x_1). Third, the tea is brewed in the first brewing cycle after the water is reheated. The tea infuser is dropped into the water for a certain number of times to release the chemical contents from the tea. The tea infuser is immersed in water for a number of seconds each time, and another few seconds elapse before the next drop. The number of drops and the immersion time are denoted as x_2 and x_3, respectively. Finally, the second brewing cycle is initialized to release additional chemical contents from the tea into the water. Similar to the first brewing cycle, this cycle involves the immersion of the tea infuser into the water for a certain number of times. For each drop, the tea infuser is immersed in water for a certain amount of time, and a few seconds elapse before the next drop. The number of times the tea infuser is immersed into the water and the immersion time in the second brewing cycle are denoted as x_4 and x_5, respectively.

To design an experimental plan, the case company defined the level settings of the five product variables (i.e., x_1 to x_5). These settings are shown in Table 1.

Table 1. Level settings of the product variables

Product variables	Reheating temperature (°C)	Number of drops in the first brewing cycle	Immersion time in the first brewing cycle (min)	Number of drops in the second brewing cycle	Immersion time in the second brewing cycle (s)
	x_1	x_2	x_3	x_4	x_5
Level 1	93	1	8.5	2	10
Level 2	95	2	9	3	20
Level 3	97	3	9.5	4	30
Level 4	99	4	10	5	40

Given that five product variables had to be examined and each of them has four levels, 1024 (4^5) experiments need to be conducted under a full factorial design; this large number of experiments involves a large amount of resources in terms of time and manpower. An orthogonal array, $L16$ (4^5), was selected for the experimental design to reduce the experimental resources while enabling a balanced study of the significance of each product variable. Table 2 shows the settings of the 16 experiments.

Table 2. $L16$ (4^5) orthogonal array for the tea maker design

Experiments	Reheating temperature (°C)	Number of drops in the first brewing cycle	Immersion time in the first brewing cycle (min)	Number of drops in the second brewing cycle	Immersion time in the second brewing cycle (s)
	x_1	x_2	x_3	x_4	x_5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	3	3	3	3
4	1	4	4	4	4
5	2	1	2	3	4
6	2	2	1	4	3
7	2	3	4	1	2
8	2	4	3	2	1
9	3	1	3	4	2
10	3	2	4	3	1
11	3	3	1	2	4
12	3	4	2	1	3
13	4	1	4	2	3
14	4	2	3	1	4
15	4	3	2	4	1
16	4	4	1	3	2

The same amount of tea was brewed in each experiment according to the processes described in the preceding paragraph. The same volume of milk was added to produce milk tea. The milk tea samples were given to a tasting panel composed of five milk tea experts. These experts were asked to assess the samples in terms of aroma (A) and texture (T) by using five linguistic descriptions, which are "very bad", "bad", "moderate", "good", and "very good", respectively. Table 3 shows the assessment results obtained from the tasting panel. In the table, "/" means no result was provided by the corresponding milk tea expert.

Table 3. Assessment results from the tasting panel

Experiments	First expert		Second expert		Third expert		Forth expert		Fifth expert	
	A	T	A	T	A	T	A	T	A	T
1	very bad	bad	bad	moderate	bad	good	very bad	very good	/	/

2	bad	bad	very bad	mod-erate	bad	bad	very bad	very good	mod-erate	mod-erate
3	bad	very bad	good	good	/	/	mod-erate	mode rate	mod-erate	good
4	bad	very bad	good	mod-erate	mod-erate	bad	mod-erate	good	good	good
5	very bad	very bad	good	good	bad	very bad	bad	good	/	/
6	very bad	mode rate	good	very bad	/	/	very bad	good	mod-erate	very bad
7	very bad	bad	mod-erate	mode rate	/	/	very bad	good	good	good
8	very bad	bad	mod-erate	good	/	/	bad	good	good	good
9	very bad	mode rate	good	good	mod-erate	mod-erate	bad	very good	/	/
10	bad	mode rate	/	/	bad	mod-erate	mode rate	good	/	/
11	bad	bad	mod-erate	good	bad	mod-erate	bad	good	/	/
12	very bad	bad	mod-erate	good	mode rate	mod-erate	mode rate	mode rate	/	/
13	very bad	mode rate	mod-erate	good	bad	mod-erate	very bad	good	good	good
14	very bad	very bad	/	/	bad	mod-erate	bad	good	bad	mod-erate
15	bad	bad	good	good	/	/	mode rate	mod-erate	mod-erate	good
16	very bad	bad	good	good	mod-erate	bad	/	/	mode rate	good

Note: A and T means aroma and taste respectively.

The experts assessed the milk tea based on their subjective judgment, thus leading to a high degree of fuzziness in the survey data. As a result, the linguistic variables were denoted as fuzzy numbers. Different types of membership functions such as triangular, trapezoidal, Gaussian, bell-shaped, sigmoidal and polynomial-based membership functions with symmetrical shapes and equal spreads were compared in previous research and the results indicated that triangular membership functions exhibit the best performance (Zhao and Bose, 2002). In addition, triangular membership functions possess a consistency property, and it has been shown that a specific class of fuzzy systems with triangular membership functions presents the universal approximation property (Sciascio and Carelli, 1995). Therefore, in this research, triangular membership functions are adopted. Fig. 1 shows the membership functions of the linguistic variables used in this case study. The linguistic variables are represented by the following fuzzy numbers:

"very bad" = (0, 1, 2); "bad" = (1, 2, 3); "moderate" = (2, 3, 4);

"good" = (3, 4, 5); and "very good" = (4, 5, 6).

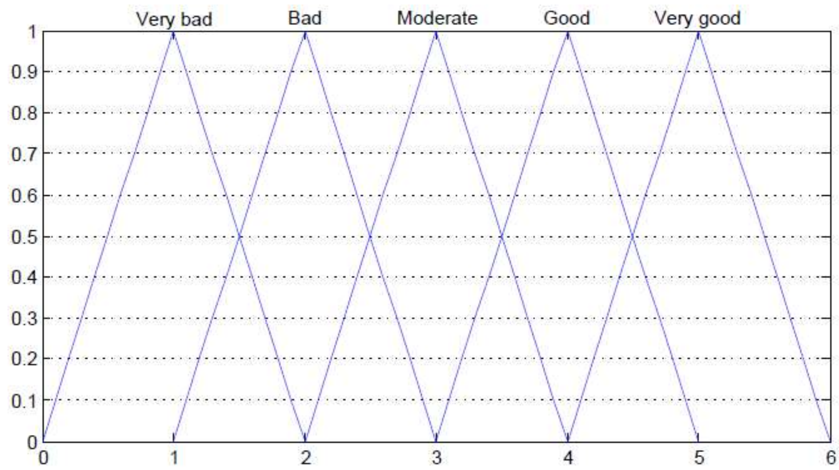

Fig. 1. Membership function for the linguistic variables.

Based on Table 3 and the above fuzzy numbers for the linguistic variables, the means of the assessment results were calculated for aroma and texture, respectively. The experimental plan and the means of the assessment results are shown in Table 4.

Table 4. Experimental settings and the means of assessment results

Experiments	x_1	x_2	x_3	x_4	x_5	Means of assessment results	
						A	T
1	93	1	8.5	2	10	(0.5, 1.5, 2.5)	(2.5, 3.5, 4.5)
2	93	2	9	3	20	(0.8, 1.8, 2.8)	(2, 3, 4)
3	93	3	9.5	4	30	(2, 3, 4)	(2, 3, 4)
4	93	4	10	5	40	(2.2, 3.2, 4.2)	(1.8, 2.8, 3.8)
5	95	2	8.5	4	40	(1.25, 2.25, 3.25)	(1.5, 2.5, 3.5)
6	95	1	9	5	30	(1.25, 2.25, 3.25)	(1.25, 2.25, 3.25)
7	95	4	9.5	2	20	(1.25, 2.25, 3.25)	(2.25, 3.25, 4.25)
8	95	3	10	3	10	(1.5, 2.5, 3.5)	(2.5, 3.5, 4.5)
9	97	3	8.5	5	20	(1.5, 2.5, 3.5)	(2.75, 3.75, 4.75)
10	97	4	9	4	10	(1.3333, 2.3333, 3.3333)	(2.3333, 3.3333, 4.3333)
11	97	1	9.5	3	40	(1.25, 2.25, 3.25)	(2.25, 3.25, 4.25)
12	97	2	10	2	30	(1.5, 2.5, 3.5)	(2, 3, 4)
13	99	4	8.5	3	30	(1.2, 2.2, 3.2)	(2.6, 3.6, 4.6)
14	99	3	9	2	40	(0.75, 1.75, 2.75)	(1.75, 2.75, 3.75)
15	99	2	9.5	5	10	(2, 3, 4)	(2.25, 3.25, 4.25)
16	99	1	10	4	20	(1.75, 2.75, 3.75)	(2, 3, 4)

With the assistance of the case company, x_1, x_3 and x_5 were identified as random variables. Their PDFs were then generated. With reheating temperature x_1 as an example, the randomness of x_1 is mainly caused by the variations in performance of the thermostats and heaters related to the reheating temperature. In a real-world environment, the measured reheating temperatures of different tea makers (with the same brand and model) exhibit several differences even under the same temperature setting. A number of thermostat and heater samples, which were of the same brand and model but sampled from different batches, were collected and randomly combined to conduct experiments and investigate the randomness of the reheating temperature. The experimental results were then utilized to generate the PDFs for the reheating temperature. Given that the experimental results generally followed a normal distribution to a certain extent, Gaussian functions were used to generate the PDFs for the random product variables, as shown below,

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2}), \tag{17}$$

where μ and σ are the mean and standard deviations of the distribution, respectively.

Each level of a random product variable has its own PDF that involves two parameters, namely, μ and σ. Considering that this case study involved three random product variables, with each variable having four levels, a total of 24 (3×4×2) parameters needed to be determined using COA. The number of iterations for a chaotic search in COA was set to 1000. The ranges of x_1 were set to [92.1, 93.9], [94.4, 95.9], [96.1, 97.9], and [98.2, 100] for 93 °C, 95 °C, 97 °C, and 99 °C, respectively. The ranges of x_3 were set to [8.3, 8.7], [8.8, 9.2], [9.3, 9.7], and [9.8, 10.2] for 8.5, 9, 9.5, and 10 min, respectively. The ranges of x_5 were set to [6, 14], [15, 25], [26, 34], and [35, 45] for 10, 20, 30, and 40 s, respectively. The ranges of σ for x_1, x_3, and x_5 were [0.1, 0.5], [0.02, 0.12], and [1, 2] for all four levels, respectively. The value setting of h in PFR was determined by using different values within the range $[0,1]$. After a number of trials, the h value was set to 0.1 because it yielded the smallest training error of the PFR model. Development of the preference models for the tea maker based on the proposed approach was implemented using Matlab using the data sets shown in Table 4. With the "aroma" dimension, y_A, as an example, the PDFs of x_1 generated based on COA are shown in Fig. 2. The optimal parameter settings of the PDFs for x_1, x_3, and x_5 were obtained and are shown in Table 5. In the table, μ_{x_1}, μ_{x_3}, and μ_{x_5} as well as $\sigma_{x_1}, \sigma_{x_3}$, and σ_{x_5} denote the settings of μ and σ for x_1, x_3, and x_5, respectively.

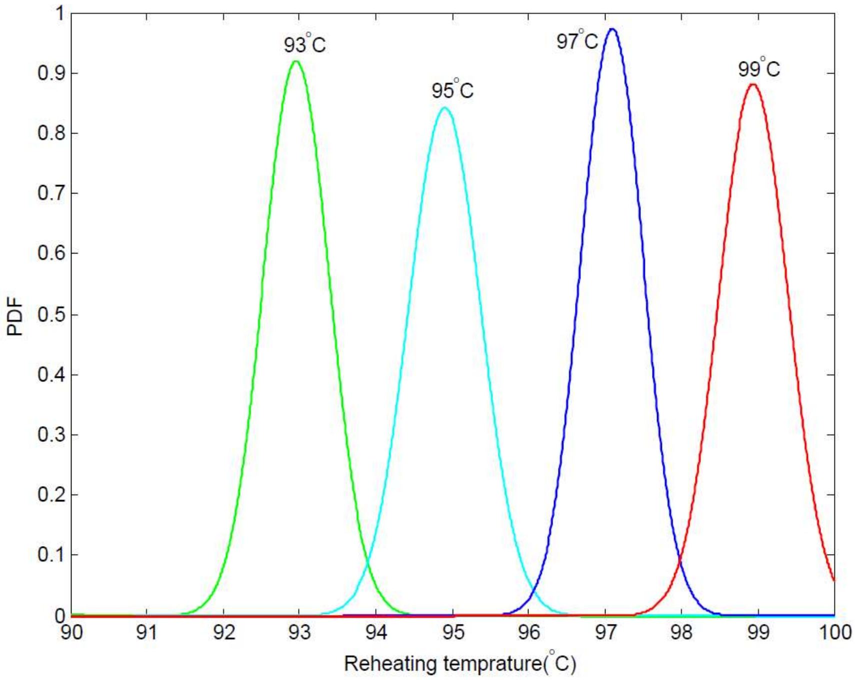

Fig. 2. PDFs of reheating temperature x_1.

Table 5. Parameter settings of PDFs obtained using COA

Product random variables	Levels	μ		σ	
Reheating temperature x_1	1	μ_{x_1}	92.9565	σ_{x_1}	0.4335
	2		94.9002		0.4732
	3		97.0906		0.4096
	4		98.9407		0.4522
Immersion time in the first brewing cycle x_3	1	μ_{x_3}	8.5199	σ_{x_3}	0.0212
	2		9.0092		0.1100
	3		9.4940		0.0219
	4		9.9856		0.0908
Immersion time in the second brewing cycle x_5	1	μ_{x_5}	9.9567	σ_{x_5}	1.9921
	2		20.3760		1.0568
	3		29.7483		1.5189
	4		40.2206		1.8188

Since x_1, x_3, and x_5 are continuous random variables that can take on any value over the corresponding interval and the PDFs of individual level settings of the random variables were determined based on Eq. (17) and COA, their expected value functions were generated based on the integral using Eqs. (6) and (17) and are considered as the probabilistic terms of the PFR model. For illustrative purpose, Tanaka's fuzzy regression analysis (Tanaka, 1987) was employed to determine the fuzzy coefficients of a PFR model for y_A. The fuzzy coefficients were determined based on Eqs. (9) to (16), and the PFR model for aroma was generated as follows:

$$
\begin{aligned}
y_A &= \left(-4.2890\ \ 2.2392\times10^{-9}\right) + \left(0.0041, 0.0021\right)\ E\left(x_1\right) + \left(0.1110, 0.0114\right)\ x_2 \\
&\quad + \left(0.5579, 0.0901\right)\ E\left(x_3\right) + \left(0.2640, 0.0033\right)\ x_4 + \left(0.0028, 0.0026\right)\ E\left(x_5\right) \\
&= \left(-4.2890\ \ 2.2392\times10^{-9}\right) + \left(0.0041, 0.0021\right) \int_{x_1 min}^{x_1 max} \frac{x_1}{\sqrt{2\pi}\sigma_{x_1}} \exp\!\left(-\frac{(x_1-\mu_{x_1})^2}{2\sigma_{x_1}{}^2}\right) dx_1 \ , \qquad (18) \\
&\quad + \left(0.1110, 0.0114\right)\ x_2 + \left(0.5579, 0.0901\right) \int_{x_3 min}^{x_3 max} \frac{x_3}{\sqrt{2\pi}\sigma_{x_3}} \exp\!\left(-\frac{(x_3-\mu_{x_3})^2}{2\sigma_{x_3}{}^2}\right) dx_3 \\
&\quad + \left(0.2640, 0.0033\right)\ x_4 + \left(0.0028, 0.0026\right) \int_{x_5 min}^{x_5 max} \frac{x_5}{\sqrt{2\pi}\sigma_{x_5}} \exp\!\left(-\frac{(x_5-\mu_{x_5})^2}{2\sigma_{x_5}{}^2}\right) dx_5
\end{aligned}
$$

where $x_j min$ and $x_j max$, $j \in [1,3,5]$, are ranges of x_j.

For each data set, with the actual values of x_j, $j \in [1,3,5]$, the corresponding $x_j min$ and $x_j max$ were defined. Based on (18) and the parameter settings shown in Table 5, the expected values of x_j, $j \in [1,3,5]$, were obtained by computing the corresponding integral over x_j, $j \in [1,3,5]$ in the probabilistic terms. The values of x_j, $j \in [1,3,5]$, were then substituted by their corresponding expected values. For example, if the intended settings of x_1, x_3, and x_5 are close to the level 1 setting of x_1, level 2 setting of x_3, and level 4 setting of x_5, respectively, the PFR model can be generated as follows:

$$
\begin{aligned}
y_A &= \left(-4.2890\ \ 2.2392\times10^{-9}\right) + \left(0.0041, 0.0021\right) \int_{92.1}^{93.9} \frac{x_1}{0.4335\times\sqrt{2\pi}} \exp\!\left(-\frac{(x_1-92.9565)^2}{2\times0.4335^2}\right) dx_1 \\
&\quad + \left(0.1110, 0.0114\right)\ x_2 + \left(0.5579, 0.0901\right) \int_{8.8}^{9.2} \frac{x_3}{0.11\times\sqrt{2\pi}} \exp\!\left(-\frac{(x_3-9.0092)^2}{2\times0.11^2}\right) dx_3 \\
&\quad + \left(0.2640, 0.0033\right)\ x_4 + \left(0.0028, 0.0026\right) \int_{35}^{45} \frac{x_5}{1.8188\times\sqrt{2\pi}} \exp\!\left(-\frac{(x_5-40.2206)^2}{2\times1.8188^2}\right) dx_5
\end{aligned}
$$

(19)

Similarly, the PFR model for the dimension "texture," y_T, was generated, as shown in the last row of Table 6.

The modeling results of the proposed PFR were compared with those of statistical regressions (SR), fuzzy regression (FR), and fuzzy least-squares regression (FLSR) to evaluate the proposed method's effectiveness. $MAPE$ and the variance of errors (VoE) defined in Eqs. (5) and (20), respectively, were adopted to compare the modeling results of the four approaches.

$$VoE = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{\left|\hat{y}_i - y_i\right|}{y_i} - MAPE\right)^2, \tag{20}$$

The same survey data was utilized to develop preference models based on SR, FR, and FLSR. For SR, the center values of the means of assessment results in shown Table 4 were used as crisp outputs. The confidence interval for SR was set to 95%, which is a common setting for statistical regression analysis (Zar, 1984). To determine the h value for FR, different h values within the range of $[0,1]$ were used to generate FR models. The modeling errors of the models were then derived and compared, and the h value corresponding to the smallest error was selected. The h value of FR was set to 0.7. For FLSR, the prediction capability of models generally increases when a large value of h ($0 \le h < 1$) is selected (Kwong et al., 2010). In this study, the h value of FLSR was set to 0.9. The four approaches for modeling consumer preference were implemented using Matlab. Table 6 shows the developed models, training errors, and the variance of training errors for the "aroma" and "texture" dimensions based on the four approaches. From this table, it can be found that the coefficients of the models generated based on the SR are crisp and the predicted outputs of the SR models are all crisp values. The fuzzy relationships between the product variables and consumer preference cannot be addressed by SR. The models generated based on the FR, FLSR and PFR are all fuzzy models with fuzzy coefficients and only the PFR models can model the randomness of independent random variables. The table also shows that the values of $MAPE$ and VoE based on PFR are the smallest among all the values of the other approaches.

Table 6. Developed models based on the four approaches and their training results

Consumer preference	Approa-ches	Developed models	$MAPE$ (%)	VoE
Aroma	SR	$y_A = -4.5254 + 0.0117x_1 + 0.0975x_2 + 0.4933x_3 + 0.2608x_4 + 0.0025x_5$	6.1366	17.9919
	FR	$y_A = (-4.0139, 1.5427) + (-0.0019, 0)x_1 + (0.1360, 0.0274)x_2 + (0.5622, 0.0003)x_3 + (0.2727, 0.0385)x_4 + (0.0035, 0.0017)x_5$	7.1588	13.8830
	FLSR	$y_A = (0.8183, 1.5560) + (0.0097, 0.0172)x_1 + (0.1677, 0.0386)x_2 + (0.0049, 0.1577)x_3 + (0.0536, 0.2073)x_4 + (-0.0027, 0.0128)x_5$	15.3219	100.8771

	PFR	$y_A = (-4.2890, 2.2392\times 10^{-9}) + (0.0041, 0.0021) \int_{x_1 min}^{x_1 max} \frac{x_1}{\sqrt{2\pi}\sigma_{x_1}} \exp(-\frac{(x_1-\mu_{x_1})^2}{2\sigma_{x_1}^2}) dx_1 + (0.1110, 0.0114)\, x_2 + (0.5579, 0.0901) \int_{x_3 min}^{x_3 max} \frac{x_3}{\sqrt{2\pi}\sigma_{x_3}} \exp(-\frac{(x_3-\mu_{x_3})^2}{2\sigma_{x_3}^2}) dx_3 + (0.2640, 0.0033)\, x_4 + (0.0028, 0.0026) \int_{x_5 min}^{x_5 max} \frac{x_5}{\sqrt{2\pi}\sigma_{x_5}} \exp(-\frac{(x_5-\mu_{x_5})^2}{2\sigma_{x_5}^2}) dx_5$	2.8978	5.1376
Texture	SR	$y_T = 1.1184 + 0.0342x_1 + 0.1050x_2 - 0.0867x_3 - 0.0717x_4 - 0.0200x_5$	7.3394	34.9107
	FR	$y_T = (-2.0435, 3.1832\times 10^{-12}) + (0.0752, 0.0015)x_1 + (-0.0046, 0)x_2 + (-0.1201, 0)x_3 + (-0.0790, 0.2915)x_4 + (-0.0267, 0.0274)x_5$	9.1569	46.4488
	FLSR	$y_T = (1.5404, 1.6270) + (0.0141, 0.0160)x_1 + (0.2093, 3.0298\times 10^{-16})x_2 + (0.0044, 0.1338)x_3 + (-0.0179, 0.2035)x_4 + (-0.0055, 0.0287)x_5$	10.7285	76.9902
	PFR	$y_T = (3.2019, 7.0372\times 10^{-11}) + (0.0224, 0.0096) \int_{x_1 min}^{x_1 max} \frac{x_1}{\sqrt{2\pi}\sigma_{x_1}} \exp(-\frac{(x_1-\mu_{x_1})^2}{2\sigma_{x_1}^2}) dx_1 + (0.0791, 7.0486\times 10^{-12})\, x_2 + (-0.1485, 0.0305) \int_{x_3 min}^{x_3 max} \frac{x_3}{\sqrt{2\pi}\sigma_{x_3}} \exp(-\frac{(x_3-\mu_{x_3})^2}{2\sigma_{x_3}^2}) dx_3 + (-0.1254, 0.0324)\, x_4 + (-0.0221, 0.0019) \int_{x_5 min}^{x_5 max} \frac{x_5}{\sqrt{2\pi}\sigma_{x_5}} \exp(-\frac{(x_5-\mu_{x_5})^2}{2\sigma_{x_5}^2}) dx_5$	6.1845	12.5463

4. Validation

A total of 30 validation tests were conducted for the two dimensions of consumer preference, “aroma” and “texture”, to further validate the effectiveness of the proposed PFR. For each validation test, 11 data sets were randomly selected as training data sets from the 16 data sets to develop the preference models. The remaining five data sets were used as testing data sets. No data set was repeated in the validation tests. The parameter settings of the four approaches were same as those described in Section 3. $MAPE$ and VoE, which were obtained with the four approaches, were adopted to compare the validation results. Figures 3 and 4 show the $MAPE$ values of the 30 validation tests based on the four approaches for the dimensions "aroma" and "texture," respectively. The lines with "+", "*", "O", and the solid line "-" denote the validation results of SR, FR, FLSR, and PFR, respectively. Table 7 shows the mean $MAPE$ and VoE of the validation tests for the two dimensions based on the four approaches.

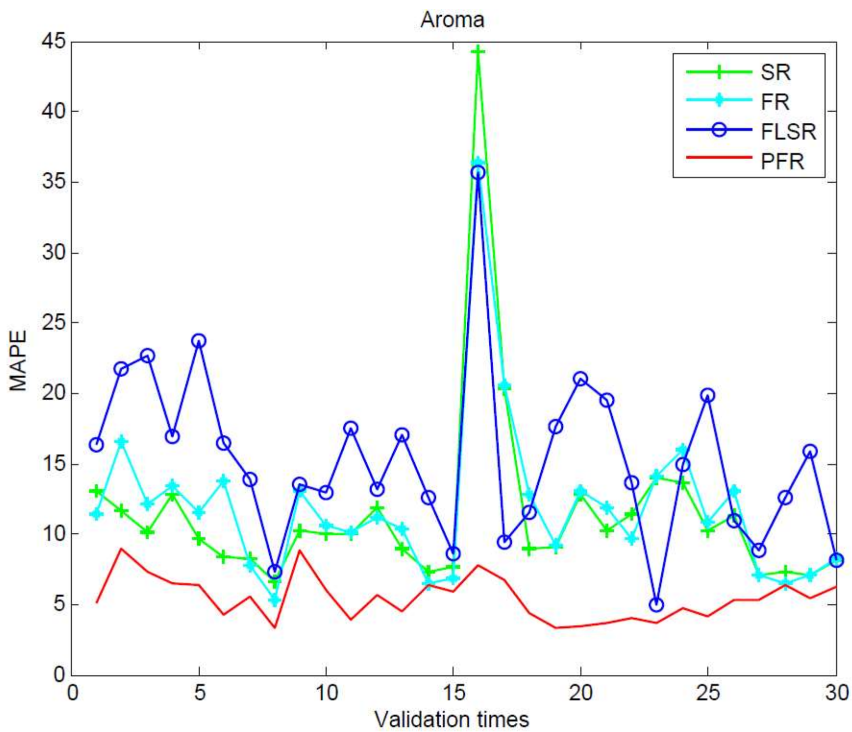

Fig. 3. *MAPE* of 30 validation tests based on the four approaches for the dimension "aroma."

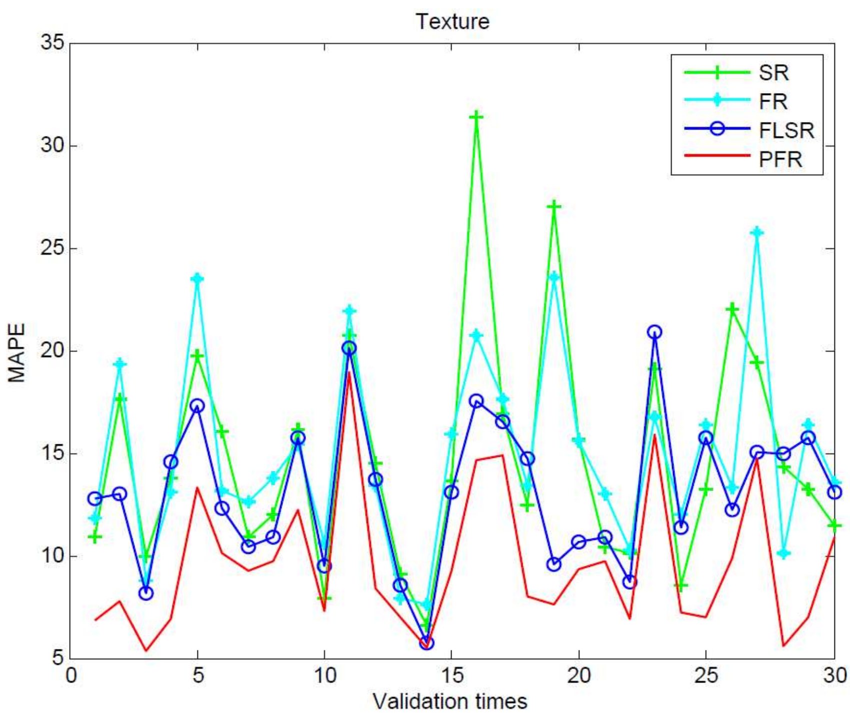

Fig. 4. *MAPE* of 30 validation tests based on the four approaches for the dimension "texture."

Table 7. Means and variances of the validation errors for the two dimensions based on the four approaches

Consumer preference	Validation error	SR	FR	FLSR	PFR
Aroma	$MAPE$ (%)	11.4296	11.9090	15.3128	5.4482
	VoE	100.2044	98.9313	157.1381	13.4715
Texture	$MAPE$ (%)	14.8327	14.9196	13.1174	9.5723
	VoE	118.9678	111.4238	112.7727	42.9447

Figures 3 and 4 show that the validation errors for all the 30 validation tests based on PFR are the smallest. Table 7 shows that PFR outperforms the other approaches in modeling preference in terms of the mean $MAPE$ and VoE for the two dimensions.

5. Conclusions

Empirical modeling is a popular approach to develop preference models for relating customer preference and design parameters based on survey/experimental data. The modeling which quite often involves both fuzziness and randomness. Only few previous studies have addressed the issues of both fuzziness and randomness in preference modeling. Among these studies, none have specifically considered the randomness caused by independent variables in the modeling. To fill this research gap, a novel PFR for preference modeling is proposed in this study. In the proposed approach, PDFs are adopted to model the randomness of independent random variables, and COA is employed to determine the parameter settings of the PDFs. The expected value functions are generated based on the PDFs and transformed into probabilistic terms of a PFR model. Fuzzy regression analysis is then conducted to determine the fuzzy coefficients for all the terms of the PFR model. The generated PFR model can address the fuzziness caused by human subjective judgment and the randomness caused by independent variables.

An industrial case study on a tea maker design was conducted to illustrate and validate the proposed approach. A total of 30 validation tests were performed. The test results indicate that PFR performs better than statistical regression, fuzzy regression, and fuzzy least-squares regression in modeling consumer preference in terms of training and validation errors. In this paper, only preference modeling is described, but the PFR approach can be applied in all other modeling problems with the following two characteristics; the relationships between dependent and independent variables are highly fuzzy as well as independent variables contain random variables. Future research would involve a study on generating PFR models that contain interaction and second-order or even higher-order terms. On the other hand, preference modeling quite often involves various dimensions of preference. For example, the tea maker design case described in Section 3 could involve several dimensions of consumer preference such as texture, aroma, smoothness and tea concentration. Preference weights of them need to be determined in order to derive proper settings of product variables for users. Future work could incorporate intuitionistic fuzzy relations (Xu and Liao, 2015; Liu and Liao, 2016) into preference modeling for determining the preference weights of individual dimensions.

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References

Barone, S., Lombardo, A., Tarantino, P., 2007. A weighted logistic regression for conjoint analysis and Kansei engineering. Quality and Reliability Engineering International 23(6), 689–706.

Chan, K.Y., Dillon, T., Kwong, C.K., 2011a. Modeling a liquid epoxy molding process using a particle swarm optimization based fuzzy regression. IEEE Transaction on Industrial Informatics 7(1), 148–158.

Chan, K. Y., Kwong, C. K., Dillon, T. S., Fung, K. Y., 2011b. An intelligent fuzzy regression approach for affective product design that captures nonlinearity and fuzziness. Journal of Engineering Design 22(8), 523–542.

Chan, K. Y., Lam, H. K., Dillon, T. S., Ling, S. H., 2015. A stepwise-based fuzzy regression procedure for developing customer preference models in new product development. IEEE Transactions on Fuzzy Systems 23 (5), 1728-1745.

Chan, K. Y., Ling, S. H., 2016. A forward selection based fuzzy regression for new product development that correlates engineering characteristics with consumer preferences. Journal of Intelligent and Fuzzy Systems 30 (3), 1869-1880.

Chang, C. C., 2008. Factors influencing visual comfort appreciation of the product form of digital cameras. International Journal of Industrial Ergonomics 38(11–12), 1007–1016.

Chang, Y. H. O., 2001. Hybrid fuzzy least-squares regression analysis and its reliability measures. Fuzzy Sets and Systems 119(2), 225–246.

Chen, Y., Chen, L., 2006. A nonlinear possibilistic regression approach to model functional relationships in product planning. International Journal of Advanced Manufacturing Technology 28(11–12), 1175–1181.

Chen, Y., Tang, J., Fung, R.Y. K., Ren, Z., 2004. Fuzzy regression-based mathematical programming model for quality function deployment. International Journal of Production Research 42(5), 1009–1027.

de Hierro, A. F. R. L, Martinez-Moreno, J., Aguilar- Pena, C., de Hierro, C. R. L., 2016. Estimation of a fuzzy regression model using fuzzy distances. IEEE Transactions on Fuzzy Systems 24(2), 344-359.

Fung, R.Y.K., Chen, Y, Chen, L., Tang, J., 2005. A fuzzy expected value-based goal programing model for product planning using quality function deployment. Engineering Optimization 37(6), 633–645.

Fung, R. Y. K., Law, D. S. T., Ip, W. H., 1999. Design targets determination for inter-dependent Product attributes in QFD using fuzzy inference. Integrated Manufacturing System 10(6), 376–383.

Fung, R. Y. K., Popplewell, K., Xie, J., 1998. An intelligent hybrid system for customer requirements analysis and product attribute targets determination. International Journal of Production Research 36(1), 13–34.

Gilliland, M., Tashman, L., Sglavo, U., 2015. Business Forecasting: Practical Problems and Solutions. John Wiley & Sons, New Jersey.

Han, S. H., Yun, M. H., Kim, K. J., Kwahk, J., 2000. Evaluation of product usability: development and validation of usability dimensions and design elements based on empirical models. International Journal of Industrial Ergonomics 26(4), 477–488.

Ishibuchi, H., Nii, M., 2001. Fuzzy regression using asymmetric fuzzy coefficients and fuzzified neural networks. Fuzzy Sets and Systems 119, 273–290.

Jiang, H., Kwong, C.K., Ip, W.H., Chen, Z., 2013. Chaos-based fuzzy regression approach to modeling customer satisfaction for product design. IEEE Transactions on Fuzzy Systems, 21(5), 926–936.

Kim, S., Kim, H., 2016. A new metric of absolute percentage error for intermittent demand forecasts. International Journal of Forecasting 32(3), 669-679.

Kwong, C. K., Chen, Y., Chan, K. Y., Luo, X., 2010. A generalised fuzzy least-squares regression approach to modeling relationships in QFD. Journal of Engineering Design 21(5), 601–613.

Kwong, C.K., Chen, Y., Chan, K.Y., Wong, H., 2008. The hybrid fuzzy least-squares regression approach to modeling manufacturing processes. IEEE Transactions on Fuzzy Systems 16(3), 644–651.

Kwong, C. K., Wong, T. C., Chan, K. Y., 2009. A methodology of generating customer satisfaction models for new product development using a neuro-fuzzy approach. Expert Systems with Applications 36(8), 11262–11270.

Lai, H. H., Lin, Y. C., Yeh, C. H., 2005. Form design of product image using grey relational analysis and neural network models. Computers & Operations Research 32(10), 2689–2711.

Lau, T. W., Hui, P. C. L., Ng, F. S. F., Chan, K. C. C., 2006. A new fuzzy approach to improve fashion product development. Computers in Industry 57(1), 82–92.

Li, J., Zeng, W., Xie, J., Yin, Q., 2016. A new fuzzy regression model based on least absolute deviation. Engineering Applications of Artificial Intelligence 52(2016), 54-64.

Lin, Y. C., Lai, H. H., Yeh, C. H., 2007. Consumer-oriented product form design based on fuzzy logic: A case study of mobile phones. International Journal of Industrial Ergonomics 37(6), 531–543.
Liu, W. and Liao, H., 2016. A bibliometric analysis of fuzzy decision research during 1970-2015, International Journal of Fuzzy Systems, 19(1), 1-14.
Liu, X., Zeng, X., Xu, Y., Koehl, L., 2007. A fuzzy model for customer satisfaction index in e-commerce. Mathematics and Computers in Simulation 77(5–6), 512–521.
Mishra, N., Choudhary, A. K., Tiwari, M. K., 2008. Modeling the planning and Scheduling across the outsourcing supply chain: a Chaos-based fast Tabu-SA approach. International Journal of Production Research 46(13), 3683-3715.
Nagamachi, M., 2008. Perspectives and the new trend of Kansei/affective engineering. The TQM Journal 20(4), 290–298.
Nanba, R., Hasegawa, M., Nishita, T., Aihara, K., 2002. Optimization using chaotic neural network and its application to lighting design. Control and Cybernetics 31(2), 249–269.
Otadi, M, 2014. Fully fuzzy polynomial regression with fuzzy neural networks. Neurocomputing 142(2014), 486-493.
Park, J., Han, S. H., 2004. A fuzzy rule-based approach to modeling affective user satisfaction towards office chair design. International Journal of Industrial Ergonomics 34 (1), 31–47.
Ren B., Zhong, W., 2011. Multi-objective optimization using chaos based PSO. Information Technology Journal 10(10), 1908–1916.
Sciascio, F. D., Carelli, R., 1995. Fuzzy basis functions for triangle-shaped membership functions: Universal approximation - MISO case. In: Proceedings of Sixth International Fuzzy Systems Association World Congress (IFSA'95), 11, pp. 439-442.
Sekkeli, G., Koksal, G., Batmaz, I., Bayrak, O. T., 2010. Classification models based on Tanaka's fuzzy linear regression approach: The case of customer satisfaction modeling. Journal of Intelligent and Fuzzy systems 21(5), 341–351.
Shimizu, Y., Jindo, Y., 1995. A fuzzy logic analysis method for evaluation human sensitivities. International Journal of Industrial Ergonomics 15(1), 39–47.
Su, Z., Wang, P., Song, Z., 2013. Kernel based nonlinear fuzzy regression model. Engineering Applications of Artificial Intelligence 26(2013), 724-738.
Tanaka, H., 1987. Fuzzy data analysis by possibilitic linear model. Fuzzy Sets and Systems 24(3), 363–375.
Watada, J., Wang, S., Pedrycz, W., 2009. Building Confidence-Interval-Based Fuzzy Random Regression Models. IEEE Transactions on Fuzzy Systems 17(6), 1273–1283.
Xu, Z., Liao, H., 2015. A survey of approaches to decision making with intuitionistic fuzzy preference relations. Knowledge-Based Systems 80(2015), 131-142.
You, H., Ryu, T., Oh, K., Yun, M. H., Kim, K. J., 2006. Development of customer satisfaction models for automotive interior materials. International Journal of Industrial Ergonomics 36(4), 323–330.
Zar, J. H., 1984. Biostatistical Analysis. USA: Prentice Hall International, New Jersey.
Zhao, J., Bose, B. K., 2002. Evaluation of membership functions for fuzzy logic controlled induction motor drive. In: Proceeding of IEEE 2002 28th Annual Conference of the

Industrial Electronics Society, 1, pp. 229–234.