Bifurcation in Parallel-Connected Buck Converters Under a Democratic Current Sharing Scheme

H. H. C. Iu and C. K. Tse

Department of Electronic and Information Engineering, Hong Kong Polytechnic University, Hong Kong

herbert@eie.polyu.edu.hk & cktse@eie.polyu.edu.hk

Abstract — This paper describes the bifurcation phenomena of a system of parallel-connected dc/dc converters. The results provide useful information for the design of stable current sharing in a democratic (central-limit control) configuration. Computer simulations are performed to capture the effects of variation of some chosen parameters on the qualitative behaviour of the system. It is found that variation of the voltage feedback gains leads to standard period-doubling bifurcation. A discrete-time map is derived and analysis is presented to establish the possibility of the bifurcation phenomena.

I Introduction

Paralleling power converters allows high current to be delivered to loads without the need to employ devices of high power rating. The main design issue in parallel converters is the control of the sharing of current among the constituent converters. In practice, mandatory control is needed to ensure proper current sharing, and many effective control schemes have been proposed in the past [1]–[3]. One common approach is to employ an active control scheme to force the current in each converter to follow an average current which is obtained by taking the average value of all individual output currents. In essence, the controller needs to calculate the average current value continuously, and each converter compares its output current with the average current value and incorporates the *error* into the voltage feedback loop. In practice, in each converter, an additional current loop is used to incorporate the error current, which is the difference between the output current and the average current, into the main voltage feedback loop to provide the required current sharing. Such a scheme is commonly known as the *democratic* current-sharing scheme [1]–[3]. Nonlinear dynamics and bifurcation behaviour are important topics of investigation in power electronics [4]–[9]. In this paper, we attempt to probe into some nonlinear phenomena of a system of parallel-connected buck converters controlled under a democratic current-sharing scheme.

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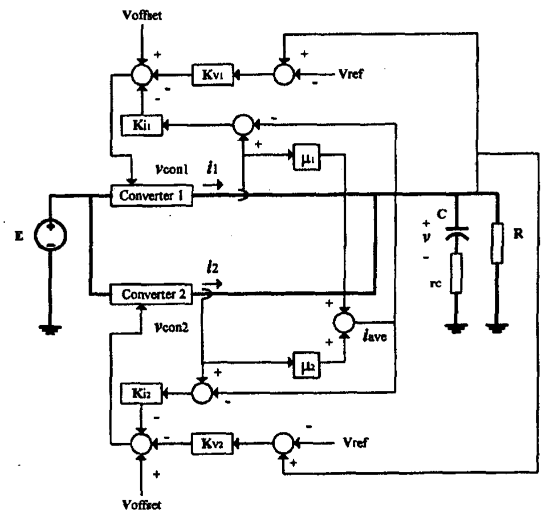

Fig. 1: Block diagram of parallel-connected dc/dc converters under a democratic current-sharing control

II Democratic Current-Sharing Controlled Parallel-Connected DC/DC Converters

The system under study consists of two dc/dc converters which are connected in parallel feeding a common load. The current drawn by the load is shared properly between the two buck converters by the action of a democratic current-sharing control scheme, as mentioned briefly in the preceding section. Figure 1 shows the block diagram of this democratic configuration.

Denoting the two converters as Converter 1 and Converter 2 as shown in Fig. 1, the operation of the system can be described as follows. Both converters are controlled via a simple pulse-width modulation (PWM) scheme, in which a control voltage v_{con} is compared with a sawtooth signal to generate a pulse-width modulated signal that drives the switch, as shown in Fig. 2. The sawtooth signal of the PWM generator is

$$v_{\text{ramp}} = V_L + (V_U - V_L)\frac{t \bmod T}{T}, \tag{1}$$

where V_L and V_U are the lower and upper voltage limits of the ramp, and T is the switching period. The PWM

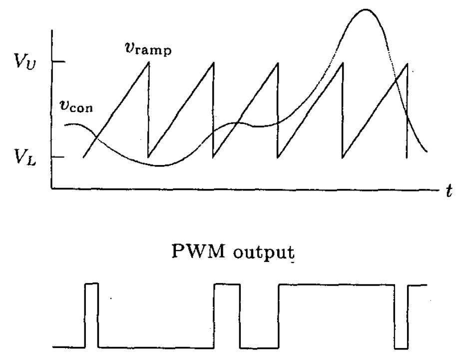

Fig. 2: Pulse-width modulation (PWM) showing relationship between the control voltage and the PWM output

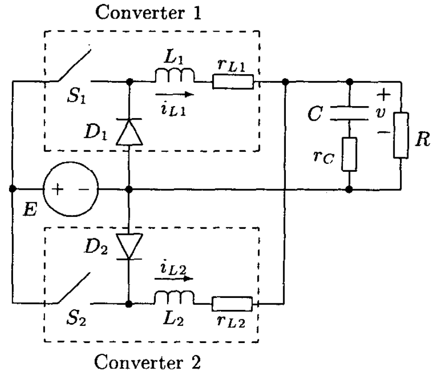

Fig. 3: Two parallel-connected buck converters

output is "high" when the control voltage is greater than v_{ramp}, and is "low" otherwise.

The control voltages of Converters 1 and 2 are given by the following equations:

$$v_{\text{con1}} = V_{\text{offset}} - K_{v1}(v - V_{\text{ref}}) - K_{i1}(i_1 - i_{\text{ave}}) \quad (2)$$

$$v_{\text{con2}} = V_{\text{offset}} - K_{v2}(v - V_{\text{ref}}) - K_{i2}(i_2 - i_{\text{ave}}) \quad (3)$$

where V_{offset} is a dc offset voltage that gives the steady-state duty cycle, V_{ref} is the reference voltage, K_{v1} and K_{v2} are the voltage feedback gains, K_{i1} and K_{i2} are the current feedback gains, and i_{ave} is an average current defined by

$$i_{\text{ave}} = \mu_1 i_1 + \mu_2 i_2, \quad (4)$$

where μ_1 and μ_2 are constants equal to $\frac{1}{n}$ (n = number of converters). Under this scheme, the output current of both Converter 1 and Converter 2 will follow the average current. As a result, we expect equal current sharing.

III State Equations for Two Parallel Buck Converters

Figure 3 shows two buck converters connected in parallel. The presence of four switches (S_1, S_2, D_1 and D_2) allows a total of sixteen possible switch states, and in each switch state the circuit is a linear third-order circuit.

When the converters are operating in continuous conduction mode (CCM), diode D_i is always in complementary state to switch S_i, for $i = 1, 2$. That is, when S_i is on, D_i is off, and vice versa. Hence, only four switch states are possible during a switching cycle, namely (i) S_1 and S_2 are on; (ii) S_1 is on and S_2 is off; (iii) S_1 is off and S_2 is on; (iv) S_1 and S_2 are off. The state equations corresponding to these switch states can be written as

$$\begin{array}{ll} \dot{x} = A_1 x + B_1 E & \text{for } S_1 \text{ and } S_2 \text{ on} \\ \dot{x} = A_2 x + B_2 E & \text{for } S_1 \text{ on and } S_2 \text{ off} \\ \dot{x} = A_3 x + B_3 E & \text{for } S_1 \text{ off and } S_2 \text{ on} \\ \dot{x} = A_4 x + B_4 E & \text{for } S_1 \text{ and } S_2 \text{ off,} \end{array} \quad (5)$$

where E is the input voltage, x is the state vector defined as $x = [v \ i_1 \ i_2]^T$ and the A's and B's are the system matrices. Note that the sequence of switch states, in general, takes the order as written in (5), i.e., starting with "S_1 and S_2 on" and ending with "S_1 and S_2 off" in a switching cycle. However, either "S_1 on S_2 off" or "S_1 off S_2 on" (not both) goes in the middle, depending upon the duty cycles of S_1 and S_2. In the case where S_1 has a larger duty cycle, we should omit the third equation in (5), and likewise for the case where S_2 has a larger duty cycle.

IV Selected Bifurcation Phenomena by Computer Simulations

We now begin our investigation with computer simulations. In particular, the gains K_{v1}, K_{v2}, K_{i1} and K_{i2} present themselves as design parameters that can be changed at will. We will henceforth focus on variation of these parameters. Our simulation is based on the exact state equations derived in Section III. Essentially, for each set of parameter values, time-domain cycle-by-cycle waveforms are generated by solving the appropriate linear equation in any sub-interval of time, according to the states of the switches which are determined from values of the control voltages v_{con1} and v_{con2}. Steady-state trajectories are obtained. Sampled data are then collected at $t = nT$ in the steady state. With sufficient number of sets of steady-state data, we can construct the bifurcation diagrams as required. The circuit parameters used in our simulations are shown in Table 1.

A large number of trajectories and bifurcation diagrams have been obtained. In the following, only representative trajectories and bifurcation diagrams are shown, which serve to exemplify the main findings concerning the bifurcation behaviour of a system of parallel buck converters under a democratic sharing scheme.

We first keep K_{v2} constant and vary K_{v1}. The bifurcation diagram, as shown in Fig. 4 (a), manifests period-doubling bifurcations. Next, we keep K_{v1} constant and

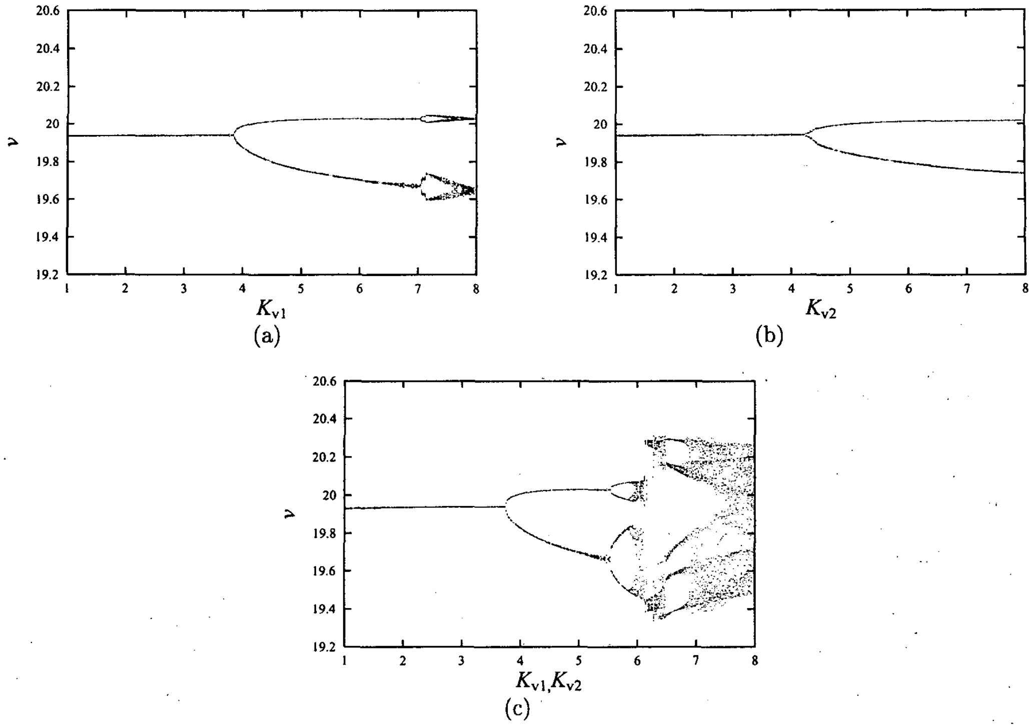

Fig. 4: (a) Bifurcation diagram with K_{v1} as bifurcation parameter ($K_{v2} = 3.5$, $K_{i1} = 1$ and $K_{i2} = 1$); (b) Bifurcation diagram with K_{v2} as bifurcation parameter ($K_{v1} = 3.5$, $K_{i1} = 1$ and $K_{i2} = 1$); (c) Bifurcation diagram with K_{v1} and K_{v2} as bifurcation parameters varying simultaneously ($K_{i1} = 1$, $K_{i2} = 1$)

Circuit Components	Values
Switching Period T	$400\mu s$
Input Voltage E	48V
Output Voltage v	24V
Offset Voltage V_{offset}	5V
Inductance L_1, ESR r_{L1}	0.02H, 0.05Ω
Inductance L_2, ESR r_{L2}	0.04H, 0.2Ω
Capacitance C, ESR r_C	$47\mu F$, 0.01Ω
Load Resistance R	10Ω

Table 1: Component values and steady-state voltages used in simulation

vary K_{v2}. The bifurcation diagram, as shown in Fig. 4 (b), again manifests a period-doubling bifurcation. Finally, we vary K_{v1} and K_{v2} simultaneously, and the corresponding bifurcation diagram is shown in Fig. 4 (c). Period-doubling bifurcations and chaos is observed.

We also capture the trajectories for some periodic and chaotic orbits when we vary K_{v1} and K_{v2} simultaneously. Figure 5(a) shows a period-1 orbit and Fig. 5(b) shows a period-2 orbit. Figure 5(c) shows a period-4 orbit and Fig. 5(d) shows a chaotic orbit.

In studying the bifurcation behaviour in respect of current gain variation, we keep K_{v1}, K_{v2} and K_{i2} constant, and vary K_{i1}. It is found that the system remains in stable period-1 operation irrespective of the choice of K_{i1}. Same observation is obtained when we vary K_{i2} and keep other parameters constant. Basically K_{i1} and K_{i2} only determine how close each converter current follows the average current.

V Analysis of Period-Doubling Bifurcation

From the foregoing simulation study, we have seen some bifurcation phenomena in a system of parallel buck converters when the voltage feedback gains are varied. In this section we analyze these bifurcations in terms of a suitable discrete-time model [8]–[9].

A Derivation of the Discrete-Time Map

We let x be the state variables as defined previously, and further let d_1 and d_2 be the duty cycle of Converter 1 and Converter 2 respectively. The discrete-time map in the neighbourhood of the T-periodic state that we aim to find takes the following form:

$$x_{n+1} = f(x_n, d_{1,n}, d_{2,n}) \tag{6}$$

where subscript n denotes the value at the beginning of the nth cycle, i.e., $x_n = x(nT)$. For the closed-loop system, we need also to find the feedback equations that relate $d_{1,n}$ and $d_{2,n}$ to x_n.

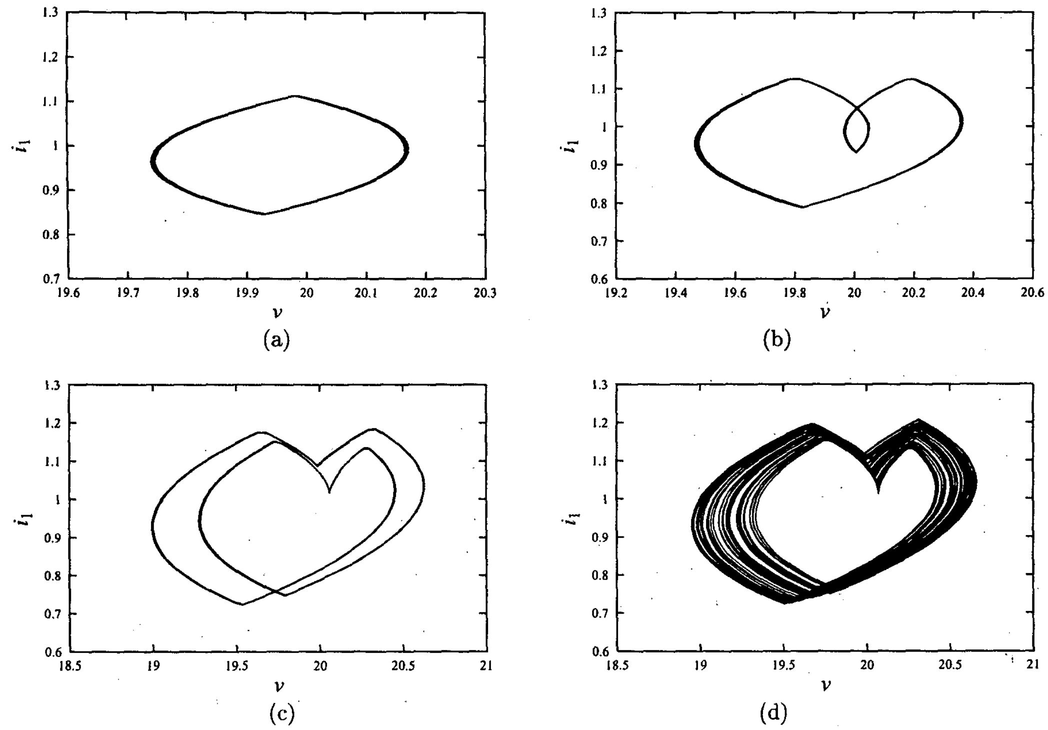

Fig. 5: (a) Trajectory of a period-1 orbit ($K_{v1} = 1$, $K_{v2} = 1$, $K_{i1} = 1$ and $K_{i2} = 1$); (b) Trajectory of a period-2 orbit ($K_{v1} = 4$, $K_{v2} = 4$, $K_{i1} = 1$ and $K_{i2} = 1$); (c) Trajectory of a period-4 orbit ($K_{v1} = 5.7$, $K_{v2} = 5.7$, $K_{i1} = 1$ and $K_{i2} = 1$); (d) Trajectory of a chaotic orbit ($K_{v1} = 6$, $K_{v2} = 6$, $K_{i1} = 1$ and $K_{i2} = 1$)

The state equations are given in (5) for different switch states. The order in which the system toggles between the switch states depends on d_1 and d_2. Thus, we need to assume that $d_2 > d_1$ in the neighbourhood of T-periodic state, or otherwise, in order to derive the discrete-time model. In particular the assumption $d_2 > d_1$ is consistent with our simulation study since r_{L1} has a lower value than r_{L2}. Note that such an assumption loses no generality.

Recall that if $d_2 > d_1$, the state "S_1 on and S_2 off" should be omitted. Hence, we have three switch states:

1. For $0 < t \leq d_1 T$, both S_1 and S_2 are on.
2. For $d_1 T < t \leq d_2 T$, S_1 is off and S_2 is on.
3. For $d_2 T < t \leq T$, both S_1 and S_2 are off.

In each switch state, the describing state equation is $\dot{x} = A_j x + B_j E$, where $j = 1, 3, 4$. (Note that $j = 2$ does not appear here.) For each state equation we can derive the solution, and by stacking up the solutions, x_{n+1} can be expressed in terms of x_n, $d_{1,n}$ and $d_{2,n}$, i.e.,

$$\begin{aligned} x_{n+1} \; = \; & \Phi_4((1-d_{2,n})T)\Phi_3((d_{2,n}-d_{1,n})T)\Phi_1(d_{1,n}T)x_n \\ & + \Phi_4((1-d_{2,n})T)\Phi_3((d_{2,n}-d_{1,n})T) \\ & \times (\Phi_1(d_{1,n}T) - \mathbf{1})A_1^{-1}B_1E + \Phi_4((1-d_{2,n})T) \\ & \times (\Phi_3((d_{2,n}-d_{1,n})T) - \mathbf{1})A_3^{-1}B_3E \\ & + (\Phi_4((1-d_{2,n})T) - \mathbf{1})A_4^{-1}B_4E, \end{aligned} \tag{7}$$

where $\mathbf{1}$ is the unit matrix, and $\Phi_j(\xi)$ is the transition matrix corresponding to A_j.

For parallel-connected buck converters, we let $A = A_1 = A_2 = A_3 = A_4$ and $\Phi(\xi) = \Phi_1(\xi) = \Phi_2(\xi) = \Phi_3(\xi) = \Phi_4(\xi)$. Hence, (7) can be rewritten as

$$\begin{aligned} x_{n+1} \; = \; & \Phi(T)x_n + \Phi(T)A^{-1}B_1E \\ & + \Phi((1-d_{1,n}T)A^{-1}(B_3 - B_1)E \\ & + \Phi((1-d_{2,n})T)A^{-1}(B_4 - B_3)E - A^{-1}B_4E. \end{aligned} \tag{8}$$

Our next step is to find the feedback relations that connect the duty cycles and the state variables. The control voltages v_{con1} and v_{con2}, as given before by (2) and (3), are

$$v_{\mathrm{con1}} = U_1 + \kappa_1^T x \quad \text{and} \quad v_{\mathrm{con2}} = U_2 + \kappa_2^T x \tag{9}$$

where U_1 and U_2 are constants, and the gain vectors κ_1 and κ_2 are $\kappa_1^T = [-K_{v1} \;\; \frac{-K_{i1}}{2} \;\; \frac{K_{i1}}{2}]$ and $\kappa_2^T = [-K_{v2} \;\; \frac{K_{i2}}{2} \;\; \frac{-K_{i2}}{2}]$. The ramp function can also be rewritten simply as $v_{\mathrm{ramp}} = \alpha + \beta(t \bmod T)$, where α and β are constants. To find the defining equations for the duty cycles, we first note that the switches are turned off when $v_{\mathrm{con1}} = v_{\mathrm{ramp}}$ and $v_{\mathrm{con2}} = v_{\mathrm{ramp}}$. Now, define $s_1(x_n, d_{1,n})$ and $s_2(x_n, d_{1,n}, d_{2,n})$ as

$$\begin{aligned} s_1(x_n, d_{1,n}) \; & \stackrel{\mathrm{def}}{=} \; v_{\mathrm{con1}} - v_{\mathrm{ramp}} \\ & = \; U_1 + \kappa_1^T x(d_{1,n}T) - (\alpha + \beta d_{1,n}T) \end{aligned}$$

$$s_2(x_n,d_{1,n},d_{2,n}) \stackrel{\text{def}}{=} v_{\text{con2}} - v_{\text{ramp}} = U_2 + \kappa_2^T x(d_{2,n}T) - (\alpha + \beta d_{2,n}T)$$

Thus, S_1 and S_2 are turned off, respectively, when

$$s_1(x_n,d_{1,n}) = 0 \text{ and } s_2(x_n,d_{1,n},d_{2,n}) = 0 \tag{10}$$

Solving (10), $d_{1,n}$ and $d_{2,n}$ can be obtained. Combining with (9), we have the discrete-time iterative map for the closed-loop system.

B *Derivation of the Jacobian*

The Jacobian plays an important role in the study of dynamical systems [10]. The essence of using a Jacobian in the analysis of dynamical systems lies in the capture of the dynamics in the small neighbourhood of an equilibrium point or orbit (stable or unstable). We will make use of this conventional method to examine the bifurcation phenomena in Section C.

Suppose the equilibrium point is given by $x(nT) = X_Q$. The Jacobian of the discrete time map evaluated at the equilibrium point can be written as follows:

$$\begin{aligned} J(X_Q) &= \frac{\partial f}{\partial x_n} - \frac{\partial f}{\partial d_{1,n}}\left(\frac{\partial s_1}{\partial d_{1,n}}\right)^{-1}\left(\frac{\partial s_1}{\partial x_n}\right) \\ &\quad - \frac{\partial f}{\partial d_{2,n}}\left(\frac{\partial s_2}{\partial d_{2,n}}\right)^{-1}\left[\left(\frac{\partial s_2}{\partial x_n}\right)\right. \\ &\quad \left.\left. + \frac{\partial s_2}{\partial d_{1,n}}\left(\frac{\partial s_1}{\partial d_{1,n}}\right)^{-1}\left(\frac{\partial s_1}{\partial x_n}\right)\right]\right|_{x_n = X_Q}, \end{aligned} \tag{11}$$

where

$$\frac{\partial f}{\partial x_n} = \begin{bmatrix} \frac{\partial f_1}{\partial v_n} & \frac{\partial f_1}{\partial i_{1,n}} & \frac{\partial f_1}{\partial i_{2,n}} \\ \frac{\partial f_2}{\partial v_n} & \frac{\partial f_2}{\partial i_{1,n}} & \frac{\partial f_2}{\partial i_{2,n}} \\ \frac{\partial f_3}{\partial v_n} & \frac{\partial f_3}{\partial i_{1,n}} & \frac{\partial f_3}{\partial i_{2,n}} \end{bmatrix} \tag{12}$$

$$\frac{\partial f}{\partial d_{1,n}} = \begin{bmatrix} \frac{\partial f_1}{\partial d_{1,n}} & \frac{\partial f_2}{\partial d_{1,n}} & \frac{\partial f_3}{\partial d_{1,n}} \end{bmatrix}^T \tag{13}$$

$$\frac{\partial s_1}{\partial x_n} = \begin{bmatrix} \frac{\partial s_1}{\partial v_n} & \frac{\partial s_1}{\partial i_{1,n}} & \frac{\partial s_1}{\partial i_{2,n}} \end{bmatrix} \tag{14}$$

$$\frac{\partial f}{\partial d_{2,n}} = \begin{bmatrix} \frac{\partial f_1}{\partial d_{2,n}} & \frac{\partial f_2}{\partial d_{2,n}} & \frac{\partial f_3}{\partial d_{2,n}} \end{bmatrix}^T \tag{15}$$

$$\frac{\partial s_2}{\partial x_n} = \begin{bmatrix} \frac{\partial s_2}{\partial v_n} & \frac{\partial s_2}{\partial i_{1,n}} & \frac{\partial s_2}{\partial i_{2,n}} \end{bmatrix} \tag{16}$$

Using (10) and (9), we can find all the derivatives in (11), i.e.,

$$\frac{\partial f}{\partial x_n} = \Phi(T) \tag{17}$$

$$\frac{\partial f}{\partial d_{1,n}} = -T\Phi((1-d_{1,n})T)(B_3 - B_1)E \tag{18}$$

$$\frac{\partial f}{\partial d_{2,n}} = -T\Phi((1-d_{2,n})T)(B_4 - B_3)E \tag{19}$$

$$\frac{\partial s_1}{\partial x_n} = \kappa_1^T \Phi(d_{1,n}T) \tag{20}$$

$$\frac{\partial s_1}{\partial d_{1,n}} = T\kappa_1^T \Phi(d_{1,n}T)(Ax_n + B_1E) - \beta T \tag{21}$$

$$\frac{\partial s_2}{\partial x_n} = \kappa_2^T \Phi(d_{2,n}T) \tag{22}$$

$$\begin{aligned} \frac{\partial s_2}{\partial d_{2,n}} &= T\kappa_2^T \Phi(d_{2,n}T)(Ax_n + B_1E) \\ &\quad + T\kappa_2^T \Phi((d_{2,n} - d_{1,n})T)(B_3 - B_1)E - \beta T \end{aligned} \tag{23}$$

$$\frac{\partial s_2}{\partial d_{1,n}} = -T\kappa_2^T \Phi((d_{2,n} - d_{1,n})T)(B_3 - B_1)E \tag{24}$$

Now, putting all the derivatives into (11) gives

$$\begin{aligned} J(X_Q) &= \Phi(T) - \frac{-\Phi((1-d_{1,n})T)(B_3-B_1)E\kappa_1^T\Phi(d_{1,n}T)}{\kappa_1^T\Phi(d_{1,n}T)(Ax_n+B_1E)-\beta} \\ &\quad - \frac{-\Phi((1-d_{2,n})T)(B_4-B_3)E\left[\kappa_2^T\Phi(d_{2,n}T)+\xi(\cdot)\right]}{\kappa_2^T\Phi(d_{2,n}T)(Ax_n+B_1E)+\kappa_2^T\Phi((d_{2,n}-d_{1,n})T)(B_3-B_1)E-\beta}, \end{aligned} \tag{25}$$

where

$$\xi(.) = \frac{-\kappa_2^T\Phi((d_{2,n}-d_{1,n})T)(B_3-B_1)E\kappa_1^T\Phi(d_{1,n}T)}{\kappa_1^T\Phi(d_{1,n}T)(Ax_n+B_1E)-\beta}. \tag{26}$$

Numerical algorithms can now be developed for computing $J(X_Q)$ and hence the characteristic multipliers, as will be shown in the next subsection.

C *Characteristic Multipliers and Period-Doubling*

The Jacobian derived in the foregoing subsection provides a means to evaluate the dynamics of the system. We will, in particular, study the loci of the characteristic multipliers, the aim being to find out possible bifurcation scenarios as the voltage feedback gains are varied. To find the characteristic multipliers, we solve the following polynomial equation in λ, whose roots actually give the characteristic multipliers.

$$\det[\lambda \mathbf{1} - J(X_Q)] = 0 \tag{27}$$

We will take note of any crossing from the interior of the unit circle to the exterior. In particular, if a real characteristic multiplier goes through -1 as it moves out of the unit circle, a period-doubling occurs [11].

Using (25), we can generate the loci of characteristic multipliers numerically. To maintain conciseness, we exemplify here the typical loci in Tables 2 and 3, which are graphically illustrated in Figs. 6 and 7. Both loci indicate a period-doubling bifurcation as K_{v1} and K_{v2} vary. This agrees with our simulation results in Section IV.

VI CONCLUSION

In this paper we focus on a parallel system of two buck converters which share current under a democratic current-sharing control scheme. It has been found that period-doubling bifurcations are possible when voltage feedback gains are varied. These results are useful for practical design of parallel converter systems to ensure stable period-1 operation in the expected stable region. Similar studies of the parallel converters under the master-slave current sharing scheme have been reported earlier [12]–[13].

K_{v1}	Char. Mult.	Remarks
2.70	-0.575$\pm j$0.334, 0.998	Stable 1T
2.90	-0.600$\pm j$0.273, 0.998	Stable 1T
3.10	-0.624$\pm j$0.195, 0.998	Stable 1T
3.30	-0.647, -0.647, 0.998	Stable 1T
3.40	-0.779, -0.537, 0.998	Stable 1T
3.60	-0.899, -0.459, 0.998	Stable 1T
3.80	-0.983, -0.414, 0.998	Stable 1T
3.85	-1.000, -0.405, 0.998	Period-doubling

Table 2: Characteristic multipliers for different values of K_{v1}

K_{v2}	Char. Mult.	Remarks
2.20	-0.538$\pm j$0.282, 0.998	Stable 1T
2.50	-0.573$\pm j$0.231, 0.998	Stable 1T
2.80	-0.605$\pm j$0.165, 0.998	Stable 1T
3.10	-0.634, -0.634, 0.998	Stable 1T
3.30	-0.774, -0.529, 0.998	Stable 1T
3.60	-0.875, -0.478, 0.998	Stable 1T
3.90	-0.950, -0.449, 0.998	Stable 1T
4.15	-1.000, -0.431, 0.998	Period-doubling

Table 3: Characteristic multipliers for different values of K_{v2}

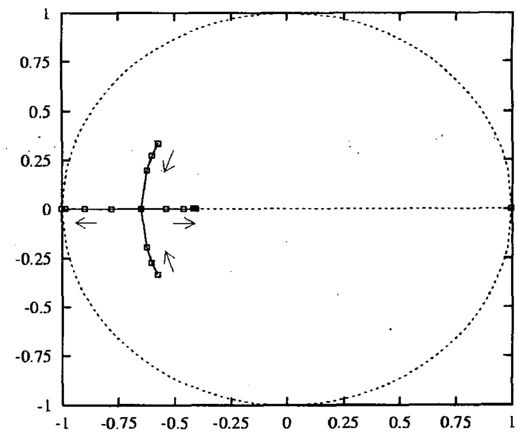

Fig. 6: Loci of characteristic multipliers as K_{v1} varies. Arrows indicate increasing K_{v1}

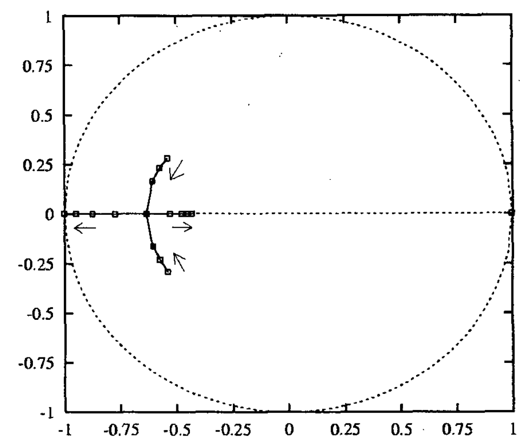

Fig. 7: Loci of characteristic multipliers as K_{v2} varies. Arrows indicate increasing K_{v2}

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