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Procurement Strategies with Unreliable Suppliers under Correlated Random Yields

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Problem Definition: This paper studies the sourcing of a monopoly firm that procures from multiple unreliable suppliers to meet its deterministic/price-dependent demand. The suppliers' production processes are unreliable and are modeled by correlated proportional random yields. **Academic/Practical Relevance**: As a proactive risk mitigation tool, supply diversification has been widely studied in the literature with the primary focus on independent supply risks. However, supply risks in practice may be correlated in nature for various reasons. By accounting for yield correlation among suppliers' production processes, our work aims to help firms better manage their supply base and fully exploit the benefit of risk pooling through diversification. **Methodology**: Stochastic optimization. **Results**: We formulate the firm's problem in a general n-supplier setting and prove its structural properties. For a two-supplier case, we fully characterize the firm's optimal sourcing decision and provide a unified measurement to quantify how yield correlation and characteristics jointly affect the supply base selection. Specifically, we show that when the two suppliers are highly positively correlated, the firm may sole source from the supplier with higher effective procurement cost (the procurement cost per expected delivered unit) and also higher reliability. In addition, as the production yields become more positively correlated, supply diversification becomes less likely and the firm's profit decreases. Moreover, assuming multivariate normally distributed yields, we generalize those results and relevant insights to the multiple-supplier case. We uncover the critical role played by yield correlation and illustrate the insufficiency of using effective procurement cost alone to qualify a supplier. Finally, we incorporate demand uncertainty to confirm the robustness of our findings. **Managerial Implications**: Our results urge caution in selecting the optimal supply base when the yield risks are correlated. Particularly, yield correlation, effective procurement costs, and supplier reliability should be jointly taken into account; otherwise, ignoring any one of these factors may lead to suboptimal outcomes.

Key words: supply diversification, correlated random yields, newsvendor model, pricing, multivariate dependence order

1. Introduction

Supply risk is prevalent in today's global economy. As documented in Gyorey et al. (2010), nearly two thirds of the executives who responded to a McKinsey global survey reported that the risks of their supply chains had increased over the past three years, and supply uncertainty was listed among the top three most significant risks by the survey participants. Among supply risks, production yield risk is a significant one for many industries. For example, in the semiconductor industry, the per batch yield is often less than the initial lot size[1] due to complicated production processes and strict production specifications (see Tang and Kouvelis 2014). In agribusiness, per farmed acre crop yield depends on many unpredictable factors, such as weather conditions, rainfall levels, pesticides used, farming techniques, etc. (see Jones et al. 2003). In vaccine manufacturing, the quantity of vaccine that can be obtained per chicken egg exhibits variability due to the uncertainty in the growing conditions of the viral strains. The uncertain yield in vaccine production is a primary planning concern for both the manufacturers and the policy makers (see Chick et al. 2008).

Many operational strategies can be utilized to effectively mitigate yield uncertainty, among which supply diversification is widely adopted. When facing production yield risk, firms usually spread their procurement orders among multiple suppliers to achieve risk pooling at reasonable costs. For example, Yousuf (2012) documents that besides procuring oranges solely from Florida, the largest citrus-growing area in United States, PepsiCo, Inc., also diversified their orange sourcing and juice production between California and Brazil, to hedge against the potential yield loss. Similarly, Monsanto, the world's largest seed producer, stated in its 2013 annual report (Monsanto 2013) that the company attempted to manage the weather-induced yield risk by producing seeds at multiple growing locations, such as North and South America. In addition, as discussed in Strom (2015), many major food companies and restaurant chains, such as Post Holdings and McDonald's, diversify their sourcing of raw and processed eggs from multiple suppliers throughout the country.

Supply diversification has been widely studied in the existing literature. Under the assumption of *independent* supply yield risks, Dada et al. (2007) have summarized a general principle of supplier selection: "a given supplier will be selected only if all less-expensive suppliers are selected, regardless of the given supplier's reliability level". Due to its simplicity, this rule has greatly helped the management with the diversification decision. Instead of considering both cost and reliability of a supply source when selecting suppliers in the first place, the firm can simply treat cost as an "order qualifier" and reliability as an "order winner". In other words, cost takes precedence over reliability as far as supplier selection is concerned. It is noteworthy that the above well-known principle relies on the premise of independent supply risks, which captures the situation where

[1] The Berkeley benchmarking study (Leachman 2002) reports that DRAM yields varied from 35% to 95% across multiple fabs and production lines in 1997-98. Most of the data clustered within the 85%-95% range.

the suppliers have little interaction with each other. The oranges grown in the United States and Brazil and the seeds produced in North and South America in the previous examples fit in such a situation well. However, consider the following example. In 2015, the breakout of bird flu seriously affected the yield of egg production of many major egg suppliers nationwide and resulted in a significant shortage for many food companies and restaurant chains, despite the diversified sourcing strategies they adopted (Strom 2015). Clearly, the diversification strategies here would face possible systematic and correlated yield shortages across suppliers, which may be attributed to them being geographically close and/or having common upper-tier suppliers. In this case, the aforementioned rule of supplier selection needs reexamination; and, more importantly, the impact of yield correlation on the firm's optimal sourcing strategy should be studied to help firms better manage their supply base and fully exploit the benefit of risk pooling through diversification.

The objective of the paper is to understand the impact of yield correlation on a firm's procurement strategies. How should a firm optimally source and diversify its supply base when procuring from a set of unreliable suppliers under correlated yield risks? What is the impact of yield correlation and characteristics on the firm's supplier selection decision? More specifically, we study the sourcing and diversification decisions of a monopoly firm that procures from multiple unreliable suppliers to meet the downstream demand. The suppliers can be either external suppliers or internal production facilities of the firm. The suppliers' production processes are unreliable and are modeled by correlated proportional random yields following a general joint distribution. We consider two demand models: (1) fixed demand model, in which the firm procures to satisfy a fixed amount of demand; and (2) responsive pricing model, in which the firm is endowed with the pricing power to determine demand and it sets the price after yields realization. The two demand models can be unified in a common framework and therefore generate the same set of results and insights concerning the impact of yield correlation on the sourcing and supplier selection decisions. For each model, we consider the general setting (n suppliers with generally distributed yields) and formulate the firm's problem as a convex optimization.

We then delve into the case of two correlated unreliable suppliers and fully characterize the firm's optimal procurement strategy. Our results uncover an interesting relationship among procurement cost, supplier reliability, and yield correlation. In particular, when the two suppliers are either negatively or weakly positively correlated, the *effective* procurement cost, i.e., the procurement cost per expected delivered unit, may continue to act as an "order qualifier", whereas the supplier reliability remains a secondary consideration. However, when the two suppliers are highly positively correlated, the firm may sole source from the more expensive supplier,[2] provided that it is more reliable. This is because including the less reliable supplier in the supply base decreases the combined

[2] Hereafter, we refer to "expensive" ["cheap"] supplier as the one with high [low] effective procurement cost.

reliability in this case. As a result, the supplier reliability may take precedence over the effective procurement cost to qualify a supplier. Therefore, when yields are correlated, the guiding rule for supplier selection may become more sophisticated than that derived under the independent yields assumption. In fact, both effective procurement cost and supplier reliability could be the primary considerations to determine the supply base. Aside from the firm's diversification decisions, we also conduct comparative statics analysis to understand the impact of yield correlation on the firm's optimal order quantities and profit. Among other results, we show that as yields become more positively correlated, diversification becomes less likely, which results in less risk-pooling benefit, and the firm earns a lower expected profit.

Assuming multivariate normally distributed random yields, we further show how the aforementioned results and insights may be generalized to the case with more than two unreliable suppliers. Based on the closed-form optimality conditions, we find that, in the presence of yield correlation, using effective procurement cost alone may not be sufficient to qualify a supplier. In fact, a more expensive supplier may be selected over a less expensive one, and the former may even be less reliable than the latter. Hence, effective procurement cost, supplier reliability, and yield correlation must be taken into account altogether when selecting the supply base. Furthermore, the intertwining relationship among the three factors can be characterized by a linear inequality as a necessary condition for the optimal supplier selection. In particular, the effective procurement costs of the selected suppliers (as a vector), after a linear transformation, is smaller than that of the unselected suppliers; moreover, the linear transformation is dictated by the reliability of the selected suppliers, as well as the correlation between the selected and unselected suppliers. These results further uncover and emphasize the critical role of yield correlation in the general n-supplier setting.

Finally, we incorporate demand uncertainty into our models to check and confirm the robustness of the findings. Under the assumption that the random demand is independent of the random yields, we manage to replicate the analysis for the two-supplier model, and obtain the same set of results as when demand is deterministic. Specifically, when the yields are highly positively correlated, the firm may sole source from the more expensive but more reliable supplier. Hence, whether demand uncertainty exists or not, the firm should always pay close attention to the supply base correlation when making its sourcing and diversification decisions.

To conclude, our paper contributes to the existing literature by incorporating yield correlation into the supplier selection problem, and provides a unified framework to quantify how yield correlation, effective procurement cost, and supplier reliability jointly affect the firm's procurement strategies. Complementary to the well-known principle of effective cost ranking for supplier selection under independent yields, we show that, with correlated yields, the rule of supplier selection becomes more sophisticated. In fact, yield correlation, effective procurement cost, and supplier reliability must altogether be the primary considerations; ignoring any of the three factors may lead to suboptimal outcomes.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 sets up the model. Section 4 investigates the impact of yield correlation on the firm's sourcing decisions when facing two unreliable suppliers. Section 5 generalizes the main results to the settings with more than two suppliers under multivariate normally distributed yields. Section 6 extends the analysis with demand uncertainty. Conclusions and directions for future research are presented in Section 7. All proofs and additional results are relegated to the appendices.

2. Literature Review

This paper is mainly related to the literature of supply risk management. Depending on the distinct nature, there are three main types of supply risks studied in the literature: random capacity, random disruption, and random yield. This paper primarily contributes to the broad literature on supply random yield (see Yano and Lee 1995, Grosfeld-Nir and Gerchak 2004, for comprehensive reviews of this literature). The primary focus in this line of research is on how to design operational strategies to effectively mitigate yield risk (see, e.g., Tang 2006, for general discussions). For example, when facing production yield uncertainty, firms could either inflate production and/or hold extra inventory to hedge against it (see Henig and Gerchak 1990), or exert effort to improve their suppliers' production reliability (see Tang et al. 2014). Moreover, emergent/backup production can also help mitigate the impact of yield losses (see Xu and Lu 2013, Kouvelis and Li 2013).

Supply diversification is an effective risk mitigation tool that has been extensively studied in the existing literature. For example, Anupindi and Akella (1993) first study a newsvendor's sourcing decision with two suppliers subject to random yield. In both single-period and multiple-period settings, they show that it is never optimal for the firm to sole source from the more expensive supplier alone. Dada et al. (2007) study a newsvendor's procurement problem when sourcing from multiple unreliable suppliers. They provide a unified framework of modeling supply risk and derive a general guidance on the optimal supplier selection: "a given supplier will be selected only if all less-expensive suppliers are selected, regardless of the given supplier's reliability level". This simple and useful rule is summarized as "cost is an order qualifier, reliability is an order winner". Federgruen and Yang (2008, 2009, 2011, 2014) confirm this rule for single-period, multi-period, and infinite horizon settings with a general pool of random yield suppliers, and provide efficient algorithms to compute the optimal policy. Dong et al. (2016) study the impact of pricing timing on the firm's diversification and supplier selection decision. On the other hand, Swaminathan and Shanthikumar (1999) and Chen et al. (2001) respectively show that this rule may be violated when demand follows discrete distributions or the procurement cost is nonlinear and concave.

Although supply diversification is commonly used to mitigate the supplier's production risk, Chod et al. (2019) recently provide an alternative theory, which attributes the buyer's default risk as the underlying factor that may drive supply diversification.

While all of the above papers assume independent supply uncertainties, there are a few works that consider correlated yield risks in the sourcing problems. Among them, Babich et al. (2007) investigate the buyer's optimal sourcing decision in a decentralized procurement setting, and Lu et al. (2015) study the effect of supply disruption on the robust facility location design. Both papers focus on the case of correlated disruption risks, which belong to discrete random yield model with all-or-nothing delivery outcome. In the context of correlated continuous random yields, Tang and Kouvelis (2011) analyze competing duopoly firms' equilibrium choices between sole and dual sourcing. They primarily focus on the case of symmetric suppliers so that the buying firm will always opt to dual source whenever the two suppliers are both available. We differ by considering a general set of suppliers with distinct costs and yield characteristics to fully explore the impact of yield correlation on a monopoly firm's supplier selection decision under different demand models. Mak and Shen (2014) investigate the impact of inventory pooling in a multi-location inventory system with correlated demand and yields (with both full and partial information), and show that the expected system costs are increasing in the degree of positive dependence between two types of risks. By contrast, our study is from a different angle and primarily focuses on the effect of yield correlation on the firm's diversification decision, rather than its impact on the total operating cost. Finally, our research topic has been briefly brought up by Federgruen and Yang (2011) in their extension section (page 1039), where they comment that, when yields are correlated, "it is no longer true that the optimal set of suppliers, in any given period, is consecutive in the effective cost rates, i.e., consists of those whose effective cost rate is below a given benchmark rate" in a finite horizon setting. Our work substantially furthers their comment by studying the sourcing decision under a newsvendor setting and provides a thorough analysis to quantitatively characterize when and how yield correlation alters the supplier selection decisions based on effective cost ranking.

This paper is also related to the vast literature of the joint pricing and inventory management, see Yano and Gilbert (2003), Chan et al. (2004), and Chen and Simchi-Levi (2012), for a comprehensive review. For the multi-period setting, Federgruen and Heching (1999) show that a base-stock/list-price policy is optimal. Chen and Simchi-Levi (2004a,b, 2006) study inventory control and pricing strategies with fixed setup costs and show the optimality of (s, S, p) policy for the finite horizon, the infinite horizon and the continuous review models. By incorporating supply uncertainty, Li and Zheng (2006) and Feng (2010) revisit the dynamic pricing and inventory management problem with stochastic proportional yield and stochastic capacity, respectively, and show that a reorder-point/list-price policy is optimal. Shen et al. (2018) derive sufficient conditions for the optimality of base-stock-list-price policy under lost-sales model. For the single period setting, Petruzzi and Dada (1999) review and extend the classic price-setting newsvendor problem under demand uncertainty. Kocabıyıkoğlu and Popescu (2011) offer a unifying perspective on the price-setting newsvendor problem and characterize the structural results under general assumptions on the elasticity of stochastic demand. In the context of supply uncertainty, Serel (2008) investigates the inventory and pricing decision between a retailer and a manufacturer when there is competition from a second supplier under disruption risk. Pan and So (2010) study an assembler's pricing and inventory decisions with random yield. Kouvelis et al. (2017) provide a general set of conditions for the unimodality of the price-setting newsvendor problem under random yield.

There is another related stream of literature that investigates the effect of responsive pricing on the firm's operational decisions. Van Mieghem and Dada (1999) are the first to study the impact of the pricing timing on the inventory decision when demand is uncertain. Granot and Yin (2008) investigate the impact of order and price postponement in a decentralized supply chain with a price-setting retailer. Tang and Yin (2007) demonstrate that responsive pricing improves the profit of a firm under random yield. Kouvelis et al. (2020) analyze price postponement under risk aversion and random yield. Our paper is particularly related to Chod and Rudi (2005) and Chod et al. (2010). To be more specific, Chod and Rudi (2005) study the impact of responsive pricing on the adoption of flexible resources to satisfy two correlated demand classes. They show that the value of flexibility is the most significant if demands are highly variable and negatively correlated. Chod et al. (2010) investigate the setting where a manufacturer procures components under demand uncertainty and operates in a responsive pricing, make-to-order environment. They study the impact of demand correlation and show that the value of production flexibility and the expected profit increase with demand correlation if and only if commonality between the products is not too high. Instead of studying demand correlation, we focus on yield correlation and investigate its impact on the firm's sourcing and supplier selection decisions. Our results show that the value of supply diversification is more significant when yields are more negatively correlated and high supply risk correlation may call for more sophisticated rules on a monopoly firm's supplier selection decision.

To sum up, we contribute to the aforementioned streams of literature by demonstrating the impact of yield correlation on the firm's sourcing and supplier selection decisions in both a fixed-demand model and a responsive pricing model. Our work provides grounded and unified insights into the firm's supplier selection decision when facing potentially correlated yield risks.

3. Model Setup

In this section, we discuss the model setup in details. Specifically, the supply side and demand side elements are characterized in Sections 3.1 and 3.2, respectively. We formulate our problem and discuss its structural property in Section 3.3, and provide an overview of our analytical roadmap in Section 3.4.

3.1. Supply Side

Consider a firm that procures from $n \geq 2$ suppliers, which can be either external suppliers or internal production facilities. The supply processes are unreliable in the sense that, for supplier i, only a random fraction $\xi_i \in [0,1]$ of the order quantity is delivered, $i = 1, 2, \cdots, n$. Moreover, these random variables are possibly correlated. Let μ_i and σ_i be the mean and standard deviation of yield factor ξ_i, respectively. Hence, the coefficient of variation (c.v.), $v_i := \frac{\sigma_i}{\mu_i}$, is used to measure supplier i's reliability in production yield.

The unit procurement cost from supplier i is c_i $(i = 1, 2)$. Define the corresponding effective unit procurement cost (shorthanded as *effective cost* hereafter) as the procurement cost per expected delivered unit, i.e., c_i/μ_i. In spite of the unreliable supply processes, we assume that the firm pays for the entire order quantity regardless of the yield realizations. This payment scheme is applicable to the scenario where the supplier is the firm's internal production facility. It is also commonly adopted when the firm procures from external suppliers whose supply yields are affected by uncontrollable factors. For example, in agribusiness, production yield is usually affected by many exogenous factors, such as weather conditions, temperatures, rainfall levels, etc. When procuring from agri-producers, one common way to settle payment is referred to as "acreage agreement", which stipulates that the buyer pays for the total planting land acreage with a per acreage agreed payment and the producers are required to deliver all the available crop to the buyer after harvest (see Scott 2003). The pay-for-order-quantity agreement assumes that the buyer bears the supply risk and is, thus, appropriate for the scenarios with strong buyers. Firms such as PepsiCo and Monsanto, who are willing to control and absorb supply risk through their procurement decisions, epitomize this type of buyers. In addition, this payment scheme is also widely adopted in the random yield literature (see Federgruen and Yang 2009, for detailed discussions). We further remark that, even if the firm only pays for the actual delivered amount, the same analysis can carry through once we rescale the unit procurement cost c_i to the corresponding effective cost c_i/μ_i.[3] Finally, without loss of generality, we assume there is no salvage value or lost-sales goodwill cost, as these costs can be easily incorporated without qualitatively changing the results.

3.2. Demand Side

The firm sells to the retail market as a monopoly and faces a deterministic demand function. Excluding demand uncertainty from the analysis allows us to better focus on the supply yield risk;

[3] See Remark 2.2 in Li and Zheng (2006) for the detailed transformation and discussion between these two payment schemes under supply random yield.

such an approach is often seen in the literature on pricing under supply uncertainty (see, e.g., Tang and Yin 2007, Tang and Kouvelis 2011). In Section 6, we incorporate demand uncertainty into our model and confirm the robustness of the results.

Suppose that the retail price is p and the demand in the retail market is d. Depending on the price sensitivity of the demand, we consider two demand models: (1) the *fixed demand model*; and (2) the *price-dependent demand model*. In the first model, the demand is insensitive to price. Hence, the firm procures to satisfy a fixed amount of demand, $d = d_0 > 0$, and the retail price, $p = p_0$, is also exogenously given. In the second model, the demand $d = d(p)$ is assumed to be a strictly decreasing function of the retail price; hence the inverse demand function $p(d)$ exists. In this case, the firm is endowed with the pricing power, and is able to decide the optimal price over an interval $[0, p_{max}]$, where $p_{max} < +\infty$ and $d(p_{max}) = 0$. We assume that the revenue function $r(d) = p(d)d$ is smooth and concave in d. In general, two pricing schemes are possible, depending on whether the pricing decision is made ex ante or ex post to the realization of yield uncertainties; in fact, both have been studied in literature (e.g., Chod and Rudi 2005, Tang and Yin 2007). In our main paper, we will focus on the ex post pricing, and refer to it as *responsive pricing* to be consistent with the previous works (Tang and Yin 2007). The ex ante pricing scheme is discussed in Appendix C as an extension to confirm the robustness of our results.

3.3. Problem Formulation

Next, we formulate the sourcing and supply diversification problem for the firm. Throughout the paper, notations in the bold font represent (column) vectors. Unless otherwise specified, all the expectations are taken with respect to the multivariate random yield variables $\boldsymbol{\xi} \in [0,1]^n$, whose joint distribution is characterized by a cumulative distribution function (c.d.f.) $G(\mathbf{x})$. As mentioned, we consider two scenarios, namely, *fixed demand* and *responsive pricing*, and therefore a subscript $s \in \{f, r\}$ is added to indicate the scenario whenever necessary.

The main decision the firm needs to make is the order quantities from the n unreliable suppliers, denoted by $\mathbf{q} \geq 0$. The objective is to maximize the expected profit from selling the delivered order, $\mathbf{q}^T\boldsymbol{\xi}$, to the retail market. In the fixed demand scenario, the firm's objective function is given by

$$\Pi_f(\mathbf{q}) = p_0 \mathbf{E} \min\{d_0, \mathbf{q}^T\boldsymbol{\xi}\} - \mathbf{q}^T\mathbf{c}. \tag{1}$$

In the responsive pricing scenario, after deciding the order quantities, the firm also optimally sets the retail price after the yields realize in order to maximize its revenue. This two-stage decision making process is captured by the following optimization problem:

$$\Pi_r(\mathbf{q}) = \mathbf{E}[\pi_r(\mathbf{q}^T\boldsymbol{\xi})] - \mathbf{q}^T\mathbf{c}, \tag{2}$$

$$\text{where} \qquad \pi_r\left(\mathbf{q}^T\boldsymbol{\xi}\right) = \max_{p\in[0,p_{max}]} p \min\left\{d(p), \mathbf{q}^T\boldsymbol{\xi}\right\}.$$

Our first result below shows that the above two optimization problems are well behaved. In fact, the following lemma ensures that the Karush–Kuhn–Tucker (KKT) condition is both sufficient and necessary to solve for the optimal order quantities.

LEMMA 1. *For scenario $s \in \{f, r\}$, the objective function $\Pi_s(\mathbf{q})$ is differentiable and jointly concave in $\mathbf{q}$; moreover, $\Pi_s(\mathbf{q})$ is twice continuously differentiable if $\boldsymbol{\xi}$ follows a continuous distribution.*

To avoid the uninteresting case of no production, we do not consider large procurement costs. Indeed, from problems (1) and (2), it is straightforward to find an upper bound for the firm's profit, i.e., $\Pi_s \le \mathbf{q}^T(\alpha_s \boldsymbol{\mu} - \mathbf{c})$, where $\alpha_f = p_0$ in scenario f, and $\alpha_r = p_{max}$ in scenario r. Therefore, we focus on the procurement cost space $\mathbf{c} \in \mathcal{C}_s := \{0 \le c_i \le \alpha_s \mu_i; i = 1, \cdots, n.\}$ in scenario $s \in \{f, r\}$. Additionally, to make the results cleaner, we will use linear demand function, i.e., $d(p) = a - bp$, $a > 0, b > 0$ in our subsequent analysis; so $p_{max} = a/b$.

3.4. Analytical Approach Overview

Our formulation of the firm's problems in (1) and (2) admits quite general settings, and Lemma 1 further guarantees the use of KKT conditions to search for optimality. However, with arbitrarily many suppliers and general correlated yield distribution, it is difficult to obtain general closed-form solutions for this challenging problem and derive meaningful insights. Hence, to further our understanding of the firm's procurement strategies, we adopt the following analysis roadmap.

First, in Section 4, we focus on the case wherein the firm's supply source consists of two unreliable suppliers, whose yield factors follow a general correlated bivariate distribution. Indeed, many firms in reality often procure from two suppliers (Tang and Kouvelis 2011). Analyzing the two-supplier case will help us obtain the key results and insights regarding how the yield correlation affects the firm's sourcing and supplier selection decisions. Then, in Section 5, we investigate the case with a general set of $n > 2$ unreliable suppliers whose yields follow a multivariate normal distribution. Under this facilitating assumption, we show how the results obtained from the two-supplier case may be generalized to higher dimensions. When working with multivariate normal distribution, we further assume that $\mathbf{Prob}(\boldsymbol{\xi} \in [0,1]^n) > 1 - \epsilon$ to deal with the issue of unrealistic yield factors – the event that the realized random yield is not a fraction number is negligible when ϵ is small.[4] Such a treatment is widely accepted in the literature (e.g. Chod and Rudi 2005).

[4] This is not a restrictive assumption as it can be satisfied by properly selecting combinations of (μ_i, σ_i), $i = 1, 2, ..., n$. For example, we may require $\mu_i - k\sigma_i > 0$ and $\mu_i + k\sigma_i < 1$ for some large k, so that the probability of non-fractional realizations is negligible. This is equivalent to the fact that σ_i is not too large, i.e., $\sigma_i < \min\{\frac{1-\mu_i}{k}, \frac{\mu_i}{k}\}$. Admittedly, this may exclude the distributions with high variability from consideration. Nevertheless, from our subsequent analysis, it is the relative yield reliability comparison and the correlation structure, rather than the absolute yield reliability, that matters in the supplier selection process. Even under this assumption, we are still able to select (μ_i, σ_i) that constitutes the entire feasible range of such relative comparison.

Finally, we introduce the following lemma, which serves as an essential building block of our analysis on the one hand, and connects the results in Sections 4 and 5 on the other. It is clear from its proof that Lemma 2(i)-(ii) applies to any non-negative correlated bivariate random variable. Due to its generality, we expect Lemma 2 to be useful in future studies involving correlated risks.

LEMMA 2. *Let $\boldsymbol{\xi} = (\xi_1, \xi_2)^T$ be a correlated bivariate random variable with $\xi_i \geq 0$, $i = 1, 2$, and correlation coefficient ρ. Without loss of generality, assume $\frac{\sigma_1}{\mu_1} < \frac{\sigma_2}{\mu_2}$. Let $\rho^* := \frac{\sigma_1}{\mu_1} / \frac{\sigma_2}{\mu_2} < 1$. The following statements hold:*

(i) $\rho \in [-1, \rho^*)$ if $\frac{E(\xi_2|\xi_1)}{\xi_1}$ strictly decreases in ξ_1.

(ii) $\rho \in (\rho^*, 1]$ if $\frac{E(\xi_2|\xi_1)}{\xi_1}$ strictly increases in ξ_1.

(iii) Suppose $\boldsymbol{\xi}$ follows a bivariate normal distribution. In this case,

(a) the sufficient condition in (i) and (ii), respectively, is also necessary;

(b) $\rho = \rho^*$ if and only if $\frac{E(\xi_2|\xi_1)}{\xi_1}$ is a constant independent from ξ_1;

(c) $\frac{E(\xi_1|\xi_2)}{\xi_2}$ strictly decreases in ξ_2.

The importance of Lemma 2 lies in the linkage it establishes between the monotonicity of the conditional expectation, $\frac{E(\xi_2|\xi_1)}{\xi_1}$, and the correlation coefficient ρ of $(\xi_1, \xi_2)^T$. Specifically, if the conditional expectation is strictly increasing in ξ_1, then ξ_1 and ξ_2 are highly positively correlated, i.e., $\rho > \rho^*$; otherwise, they are either negatively or weakly positively correlated, i.e., $\rho < \rho^*$. Moreover, the unique threshold, ρ^*, has a meaningful statistical interpretation that allows us to further include the reliability into the picture. To wit, since ρ^* is the ratio between the c.v.'s of the random variables, $\rho^* < 1$ indicates that the random variable ξ_1 is more reliable. Finally, Lemma 2(iii) strengthens the linkage with the assumption of bivariate normal distribution, under which the monotonicity of $\frac{E(\xi_2|\xi_1)}{\xi_1}$ becomes equivalent to the magnitude of ρ relative to ρ^*.

We further remark that bivariate normal distribution is not the only distribution that satisfies Lemma 2(iii). Based on its proof (see Appendix B for details), the sufficient condition is that the joint distribution has linear conditional expectations, i.e., $E(\xi_i|\xi_{3-i}) = s\xi_{3-i} + m$, $i = 1, 2$, for some s and m. Put differently, it is adequate to require the conditional expectation to have a linear form in order for Lemma 2 to hold. The bivariate normal distribution happens to be one such example. Therefore, all of our results that follow from Lemma 2(iii) are in fact valid for any non-negative bivariate random variables with linear conditional expectations.

4. The Two-Supplier Model

In this section, we study the firm's procurement strategies for the two-supplier model under both the fixed demand and the responsive pricing scenarios. When making sourcing and supply diversification decisions, the firm should consider two factors about the supply source, namely, the procurement cost and the supplier reliability. Therefore, our primary question is how the cost and reliability information about the suppliers guide the firm with its sourcing decisions. When the yields are independent, simple criterion such as "cost is an order qualifier and reliability is an order winner" has been shown to successfully answer the question (Dada et al. 2007, Federgruen and Yang 2011). When the yields are correlated, however, the question requires reexamination.

To make our subsequent discussion clear, we precisely define the two characteristics of the suppliers, i.e., cost and reliability, in our context. As previously mentioned, the firm pays for the entire order quantity by our assumption. Hence, consistent with the literature (Federgruen and Yang 2011), we focus on the per unit *effective cost* $\frac{c_i}{\mu_i}$ $(i=1,2)$ of each supplier. Thus, when comparing the costs of different suppliers, we mainly refer to the comparison of the effective costs. On the other hand, the reliability of the supplier is measured by the coefficient of variation of the random yield. Without loss of generality, we assume that supplier 1 is more reliable than supplier 2, i.e., $\frac{\sigma_1}{\mu_1} < \frac{\sigma_2}{\mu_2}$, throughout this section, which implies $\rho^* := \frac{\sigma_1}{\mu_1} / \frac{\sigma_2}{\mu_2} < 1$.

4.1. Cost, Reliability, and Correlation

In this subsection, we investigate the critical role played by the yield correlation when the firm selects its supply base, show that effective cost ranking alone may not be sufficient to provide full guidance to the management, and propose that more sophisticated criterion involving both cost and reliability needs to be considered.

By Lemma 1, the firm's objective function is well-behaved. As such, we are able to fully characterize the firm's optimal sourcing and diversification decision. We further define a cost threshold for each supplier, above which no order will be placed towards that supplier. Then, we will use the properties of the thresholds to derive managerial insights into the supply base selection. The results are summarized below.

Proposition 1. *Suppose there are two unreliable suppliers. Under scenario* $s \in \{f, r\}$*, there exist two increasing cost threshold functions* $C_s^2(c_1) \le C_s^1(c_1)$ *for* $0 \le c_1 \le \alpha_s \mu_1$*, where* $\alpha_f = p_0$ *and* $\alpha_r = p_{max}$*; moreover, for* $i = 1, 2$*, the boundary values* $C_s^i(0) = 0$ *and* $C_s^i(\alpha_s \mu_1) = \alpha_s \mu_2$*. For any* $c_1 \in [0, \alpha_s \mu_1]$*, the firm sole sources from supplier 1 if* $c_2 \in [C_s^1(c_1), \alpha_s \mu_2]$*, sole sources from supplier 2 if* $c_2 \in [0, C_s^2(c_1)]$*, and dual sources otherwise.*

In both the fixed demand model and the responsive pricing model, Proposition 1 completely depicts the firm's optimal sourcing decisions based on the procurement cost comparison against certain thresholds. Specifically, the upper cost threshold $C_s^1(c_1)$ can be interpreted as the marginal revenue brought by supplier 2 given that the firm optimally sole sources from supplier 1 (see the proof for details). If the marginal revenue exceeds the corresponding marginal procurement cost c_2, then the firm should include supplier 2 into the supply base and utilize dual sourcing. Otherwise, the firm should sole source from supplier 1. Similarly, there exists a cost threshold that measures the marginal revenue of adding supplier 1 when the firm optimally sole sources from supplier 2. This threshold, after rewritten as a function of c_1, is the lower cost threshold $C_s^2(c_1)$.

Based on the above characterized cost thresholds, we want to further understand how precisely they may provide implications for the supply base selection. In particular, is the supplier with higher effective cost never selected to be the sole source of supply for the firm? The following proposition shows that the answer depends on the yield correlation. Thus, compared to the simple criterion that depends on effective cost ranking alone, which works well in the absence of yield correlation, the rules for supplier selection in our setting could be more sophisticated.

PROPOSITION 2. *Suppose there are two unreliable suppliers. Under scenario $s \in \{f, r\}$, the following statements hold for any $c_1 \in (0, \alpha_s \mu_1)$, where $\alpha_f = p_0$ and $\alpha_r = p_{max}$.*

(i) If $\frac{\mathbf{E}(\xi_i|\xi_{3-i})}{\xi_{3-i}}$ strictly decreases in ξ_{3-i}, $i = 1, 2$, then $C_s^1(c_1) > \frac{\mu_2}{\mu_1} c_1 > C_s^2(c_1)$.

(ii) If $\frac{\mathbf{E}(\xi_2|\xi_1)}{\xi_1}$ strictly increases in ξ_1, then $\frac{\mu_2}{\mu_1} c_1 > C_s^1(c_1) > C_s^2(c_1)$.

(iii) Assume $\boldsymbol{\xi}$ follows a bivariate normal distribution. In this case,

(a) if $\rho \in [-1, \rho^*)$, then $C_s^1(c_1) > \frac{\mu_2}{\mu_1} c_1 > C_s^2(c_1)$;

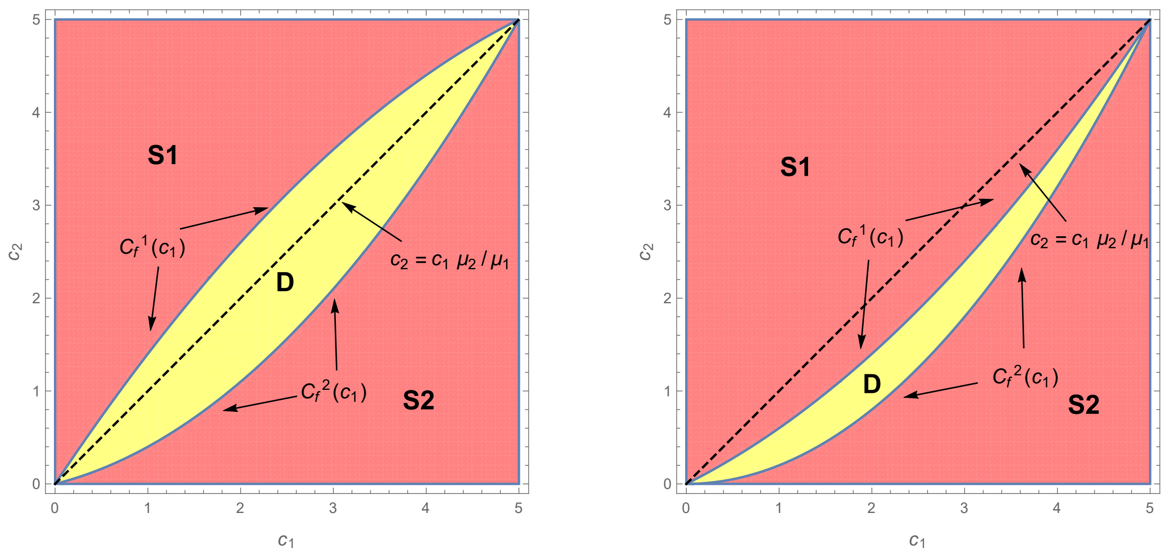
(b) if $\rho = \rho^*$, then $C_s^1(c_1) = \frac{\mu_2}{\mu_1} c_1 \geq C_s^2(c_1)$; and

(c) if $\rho \in (\rho^*, 1]$, then $\frac{\mu_2}{\mu_1} c_1 > C_s^1(c_1) > C_s^2(c_1)$.

The first observation from Proposition 2 is that the cost thresholds have different order relationship with respect to the straight line $c_2/\mu_2 = c_1/\mu_1$ on the $\mathcal{C}_s$ plane. To visually facilitate the understanding of the results, we provide Figure 1, which illustrates the two possible orderings of the cost thresholds. Note that, although the specific threshold functions in the respective scenario $s = \{f, r\}$ are different, the curves have similar shapes; hence, in line with our unifying approach, we use Figure 1 to represent both scenarios.

The diagonal line, $c_2 = \frac{\mu_2}{\mu_1} c_1$, divides $\mathcal{C}_s$ into two parts where the suppliers have opposite effective cost ranking. More specifically, supplier 1 is less expensive with a lower effective cost ($\frac{c_1}{\mu_1} < \frac{c_2}{\mu_2}$) in the upper part, and is more expensive with a higher effective cost ($\frac{c_1}{\mu_1} > \frac{c_2}{\mu_2}$) in the lower part. Hence, put together with C_s^1 and C_s^2, the diagonal line links the cost thresholds to the cost comparison of the two suppliers. For example, when $C_s^1(c_1) > \frac{\mu_2}{\mu_1} c_1 > C_s^2(c_1)$ (left panel of Figure 1), the firm either sole sources from the less expensive supplier or dual sources, but never sole sources from the more expensive supplier – echoing with the well-known principle of effective cost ranking. On the other hand, when $\frac{\mu_2}{\mu_1} c_1 > C_s^1(c_1) > C_s^2(c_1)$ (right panel of Figure 1), there exists a non-empty region between C_s^1 and the diagonal line. As a result, if the suppliers' costs fall into that region, then supplier 1 is more expensive than supplier 2 on the one hand, yet the firm sole sources from

Figure 1 Optimal Sourcing Decision with Cost Thresholds.



(a) Correlation coefficient $\rho \in [-1, \rho^*]$. (b) Correlation coefficient $\rho \in (\rho^*, 1)$.

Legends: ***S1*** *= sole source from supplier 1;* ***S2*** *= sole source from supplier 2;* ***D*** *= dual source.*

supplier 1 (by Proposition 1) on the other. In this case, therefore, the effective cost ranking alone is insufficient to guide supply base selection.

Proposition 2 offers further insights into the above interesting observation. In fact, whether the cost threshold C_s^1 is above or below the diagonal line depends on the monotonicity of the conditional expectation $\frac{\mathbf{E}(\xi_2|\xi_1)}{\xi_1}$, which contains information on the yield correlation. In other words, there exists a link between correlation and effective cost comparison in the context of supply diversification. Moreover, the linkage is further strengthened by Lemma 2, where the monotonicity of the conditional expectation is shown to imply the magnitude of the coefficient of correlation. To be specific, if $\frac{\mathbf{E}(\xi_2|\xi_1)}{\xi_1}$ strictly increases in ξ_1, then $\rho > \rho^*$ by Lemma 2; and, at the same time, $\frac{\mu_2}{\mu_1}c_1 > C_s^1(c_1)$ by Proposition 2. Note that $\rho^* = v_1/v_2 < 1$ by our assumption. Therefore, we deduce the following important insight that connects effective cost, supplier reliability and yield correlation: the firm may sole source from the more expensive, but more reliable, supplier if the yield correlation is highly positive. Finally, Proposition 2(iii) shows that, under a bivariate normal distribution, the comparison of ρ and ρ^* can be used to order the cost thresholds and the diagonal line. Hence, it is more direct to see that reliability may preempt effective cost to be the primary consideration in supplier selection when ρ is sufficiently large.

To better understand the above rationale, we now examine the combined reliability of the supply base. To start, we measure the supply risk using the coefficient of variation of the combined supply. For any sourcing quantities $\mathbf{q} = (q_1, q_2)^T$, let

$$CV(q_1, q_2) = \frac{\sqrt{q_1^2\sigma_1^2 + q_2^2\sigma_2^2 + 2\rho\sigma_1\sigma_2 q_1 q_2}}{q_1\mu_1 + q_2\mu_2}.$$

Clearly, $CV(q_1,0)=\frac{\sigma_1}{\mu_1}$ and $CV(0,q_2)=\frac{\sigma_2}{\mu_2}$. In the following lemma, we characterize the moderating role of yield correlation by studying the impact of including an additional supplier into the supply base on the combined reliability.

LEMMA 3. *Consider a supplier base consisting of one unreliable supplier. Adding a more reliable supplier always leads to a lower combined supply risk, i.e., $CV(q_1,q_2)<CV(0,q_2)$ for all $q_1>0$, $q_2>0$ and $\rho\in[-1,1]$. However, adding a less reliable supplier has consequences described below.*

(i) If $\rho\in[-1,\rho^*)$, then $\forall q_1>0$, there exists a unique threshold $q_2^*(q_1)=\frac{\mu_1(\rho^*-\rho)}{\mu_2((\rho^*)^{-1}-\rho)}q_1>0$ such that $CV(q_1,q_2)$ strictly decreases in q_2 if $q_2<q_2^*(q_1)$ and strictly increases in q_2 if $q_2>q_2^*(q_1)$.

(iii) If $\rho=\rho^*$, then $\forall q_1>0$, $CV(q_1,q_2)$ increases in q_2 with $\frac{\partial CV(q_1,q_2)}{\partial q_2}|_{q_2=0}=0$.

(iv) If $\rho\in(\rho^*,1]$, then $\forall q_1>0$, $CV(q_1,q_2)$ strictly increases in q_2 with $\frac{\partial CV(q_1,q_2)}{\partial q_2}|_{q_2=0}>0$.

The overall reliability of the combined supply is naturally influenced by the addition of either supplier into the base. However, the specific effect depends on the supplier's reliability as well as their correlation. While including the more reliable supplier (i.e., supplier 1) always makes the combined supply more reliable, it is not necessarily the same for the less reliable supplier (i.e., supplier 2). Lemma 3 shows that sourcing from an additional less reliable supplier may actually make the combined supply riskier. The critical moderator here is the yield correlation. When the yields are either negatively or weakly positively correlated, sourcing a small amount ($q_2<q_2^*$) from supplier 2, which acts as a hedging tool, can alleviate the overall supply risk; however, the unreliability of supplier 2 grows to overshadow the risk mitigation effect when sourcing too much $q_2>q_2^*$. On the other hand, when the supply yields are highly positively correlated (i.e., $\rho\in[\rho^*,1]$), adding supplier 2 to the supply base always puts the firm in a riskier position. In this case, the hedging strategy may easily fail due to large ρ, and including a less reliable supplier will negatively impact the combined supply reliability.

Now, Propositions 1 and 2 together with Lemmas 2 and 3 provide a clear view over the relationship among effective cost, supplier reliability, and yield correlation. They reveal the insufficiency of using the effective cost ranking alone to guide supplier selection when yields are correlated, and characterize the underlying driving forces of supplier reliability invoked by yield correlation. To present a more complete picture of these relationships, we recapitulate the rationale behind the firm's optimal procurement strategy using the results in these propositions and lemmas.

As previously discussed, the cost thresholds in Proposition 1 are measured by the marginal revenue brought by the addition of a supplier into the supply base. In the meantime, adding a supplier has impact on the reliability of the supply base, as characterized in Lemma 3. Hence, the firm must trade off these two factors. Since including the more reliable supplier (supplier 1) always increases the reliability of the combined supply regardless of the yield correlation (Lemma 3), its marginal value is enhanced so that the firm will never sole source from the more expensive but less reliable supplier (i.e., $C_s^2(c_1) \le \frac{\mu_2}{\mu_1} c_1$ as shown in Proposition 2).

When we consider the marginal value of adding the less reliable supplier (supplier 2) into the supply base, the yield correlation is interestingly involved. In particular, when $\frac{\mathbf{E}(\xi_2|\xi_1)}{\xi_1}$ strictly decreases in ξ_1, the yield correlation $\rho < \rho^*$ is low (Lemma 2(i)). Then, Lemma 3 indicates that including supplier 2 into the supply base may initially increase the overall reliability, and thus enhances the marginal value of supplier 2. Indeed, we have $C_s^1(c_1) > \frac{\mu_2}{\mu_1} c_1$ in this case by Proposition 1. On the other hand, when $\frac{\mathbf{E}(\xi_2|\xi_1)}{\xi_1}$ strictly increases in ξ_1, the supply yields are highly positively correlated, i.e., $\rho > \rho^*$ (Lemma 2(ii)). As a result, adding the less reliable supplier decreases the reliability of the combined supply (Lemma 3(iii)), a negative impact on the marginal revenue of sourcing from supplier 2. This echoes with Proposition 2(ii), where the cost threshold is lower than the diagonal line, i.e., $C_s^1(c_1) < \frac{\mu_2}{\mu_1} c_1$. In this case, it is possible for the firm to sole source from the more expensive and more reliable supplier.

Finally, Lemma 2(iii) ensures that, when the yields follow a bivariate normal distribution, the above relationship between the marginal value of adding a supplier and its corresponding impact on the overall supply risk becomes even stronger and more direct, as the correlation is represented by its coefficient ρ alone. We will discuss more of this result in Section 5 later, when we assume multivariate normally distributed yield for the case with $n > 2$ suppliers. In addition, we further remark that Proposition 2(iii) continues to hold for any non-negative bivariate yield distribution given it has linear conditional expectations. See the discussions of Lemma 2 for details.

4.2. Impact of Yield Correlation

The next question that naturally arises is how yield correlation affects the firm's optimal profit and the associated optimal order quantities. To answer the question and obtain relevant insights, we utilize the concept of *supermodular* (SM) order (see Shaked and Shanthikumar 2007, Chapter 9). Let $\boldsymbol{\xi}$ and $\hat{\boldsymbol{\xi}}$ be two multivariate random variables. We say $\boldsymbol{\xi}$ is smaller than $\hat{\boldsymbol{\xi}}$ in the SM order, denoted by $\boldsymbol{\xi} \le_{sm} \hat{\boldsymbol{\xi}}$, if $\mathbf{E}[\kappa(\boldsymbol{\xi})] \le \mathbf{E}[\kappa(\hat{\boldsymbol{\xi}})]$ for any supermodular function $\kappa(\cdot)$. The SM order is a popular multivariate stochastic order and is closely related to the notion of correlation. In particular, for bivariate random variables, if $\boldsymbol{\xi} \le_{sm} \hat{\boldsymbol{\xi}}$, then their respective coefficients of correlation are ordered as $\rho \le \hat{\rho}$ (see Shaked and Shanthikumar 2007, p.389-395). Moreover, the reverse is also true if the random variables follow normal distribution (Shaked and Shanthikumar 2007, Muller and Scarsini 2000). Just like the monotonicity of conditional expectation studied in Section 4.1, we use such a stochastic order here as an important indicator of the correlation of random variables. Facilitated by the concept of the SM order, we now study the impact of yield correlation on the firm's dual sourcing strategy and its optimal profit.

PROPOSITION 3. *Consider the sourcing problem with two unreliable suppliers. Assume* $\boldsymbol{\xi} \leq_{sm} \hat{\boldsymbol{\xi}}$*. Under scenario* $s = \{f, r\}$*, the following statements hold.*

(i) $C_s^1 \geq \hat{C}_s^1$ *and* $C_s^2 \leq \hat{C}_s^2$*.*

(ii) $\Pi_s(\mathbf{q}) \geq \hat{\Pi}_s(\mathbf{q})$ *for all feasible* $\mathbf{q}$*; therefore, the optimal profits satisfy* $\Pi_s^* \geq \hat{\Pi}_s^*$*.*

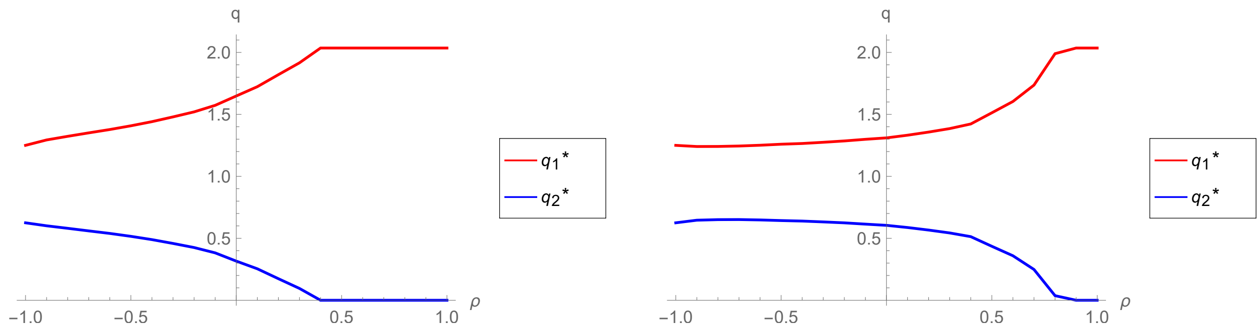
(iii) Suppose $\boldsymbol{\xi}$ *follows a bivariate normal distribution with coefficient of correlation* ρ*. Then,* C_s^1 *decreases in* ρ *and* C_s^2 *increases in* ρ*; and the optimal profit* Π_s^**, as well as the profit function* $\Pi_s(\mathbf{q})$ *with any fixed* $\mathbf{q}$*, decrease in* ρ*.*

Hence, if the random yield becomes larger in the SM order, Proposition 3(i) and (ii) show that the dual sourcing region on the feasible cost plane, measured by the gap between the cost thresholds, shrinks and the firm's optimal profit decreases. Indeed, as the firm dual sources less often, the benefit of diversification is weakened and the profit is negatively impacted. Since increasing in the SM order also implies that the coefficient of correlation becomes larger, the increased yield correlation may be somehow associated with the above negative impact on profit. Particularly, when the yield follows a bivariate normal distribution, the impact of correlation on both the dual sourcing region and the profit becomes more direct. This is because the SM order is equivalent to the correlation comparison in this case.

In addition to the firm's optimal profit, we also examine the impact of yield correlation on the firm's optimal order quantity decisions. Instead of analytical approach, we conduct numerical experiments to study the patterns of firm's order with respect to the coefficient of yield correlation. More specifically, let $\boldsymbol{\xi}$ follow a bivariate normal distribution with $\boldsymbol{\mu} = (0.5, 0.6)^T$, $\sigma_1 = 0.06$ and $\sigma_2 = 0.12$ (supplier 1 is more reliable with lower coefficient of variation). Furthermore, we consider two pairs of procurement costs $\mathbf{c}_L = (0.2, 0.25)^T$ and $\mathbf{c}_R = (0.2, 0.23)^T$. It can be easily verified that under the first [second] cost pair, supplier 1 has lower [higher] effective cost. In Figure 2, we plot the optimal order quantities as functions of ρ with the two cost pairs, respectively. It is worth mentioning that the figure is obtained under the fixed demand model where we set $d_0 = 1$; however, under responsive pricing model, plots with similar trends can be obtained and the relevant insights remain unchanged.

The left panel of Figure 2 uses the cost parameter $\mathbf{c}_L$, and thus focuses on the situation where supplier 1 is more reliable and also has lower effective cost. It clearly shows that as the yield becomes more positively correlated, the firm decreases the order quantity from supplier 2 and increases that from supplier 1. As the correlation further increases, specifically when $\rho > 0.39$, the firm sole sources from the less expensive and more reliable supplier 1. The right panel of Figure 2 considers another cost parameter $\mathbf{c}_R$, which represents the situation where the more reliable supplier 1 is also more expensive. In this case, the firm has to consider cost and reliability jointly

Figure 2 Impact of Coefficient of Yield Correlation on the Optimal Order Quantities.



(a) Procurement cost $\mathbf{c}_L = (0.2, 0.25)^T$ (b) Procurement cost $\mathbf{c}_R = (0.2, 0.23)^T$

Note: In this example, we set $\mu_1 = 0.5, \sigma_1 = 0.06$, $\mu_2 = 0.6$, and $\sigma_2 = 0.12$. The demand is fixed to be 1.

in making its ordering decision. As a result, there are some non-monotone trends in the ordering quantities when ρ is highly negative (e.g., $\rho < -0.5$). However, when the yields become positively correlated, the firm always increases the order quantity from supplier 1 and decreases that from supplier 2. In particular, when $\rho > 0.81$, the firm sole sources from the more expensive but more reliable supplier, which confirms our finding in Proposition 2 that supplier reliability may take precedence over effective cost in selecting the supply base when facing highly positively correlated yields.

5. The n-Supplier Model with Multivariate Normal Yield Distribution

Can the main results on supplier selection derived from the previous two-supplier model be generalized to the situation with more-than-two unreliable suppliers? In this section, we attempt to answer this question by studying the n-supplier model. Due to the complexity of the problem, we will use a different approach from that in Section 4 to show how correlation matters. In the sequel, we assume that the yields follow a multivariate normal distribution, i.e., $\boldsymbol{\xi} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Possible negative realization notwithstanding, the multivariate normal distribution has been a popular assumption in the literature (e.g., Chod and Rudi 2005) due to its tractability. Similar to the previous works, we further assume that $\mathbf{Prob}(\boldsymbol{\xi} \in [0,1]^n) > 1 - \epsilon$ for a small ϵ so that the unrealistic realizations are negligible.

The multivariate normal premise conveniently leads to a helpful observation in our setting: the total delivered quantity follows a normal distribution, i.e. $Q = \mathbf{q}^T \boldsymbol{\xi} \sim \mathcal{N}(\mathbf{q}^T \boldsymbol{\mu}, \mathbf{q}^T \boldsymbol{\Sigma} \mathbf{q})$. We write its c.d.f. as $F(y;\mathbf{q}) = \mathbf{Prob}(Q \le y)$ and its p.d.f. as $f(y;\mathbf{q}) = F'(y;\mathbf{q})$ (the distribution of Q depends on the order quantities). Then, we can write out the firm's objective functions in (1) and (2). For the fixed demand scenario,

$$\Pi_f(\mathbf{q}) = p_0 \left(\int_{-\infty}^{d_0} y dF(y;\mathbf{q}) + \int_{d_0}^{+\infty} d_0 dF(y;\mathbf{q}) \right) - \mathbf{q}^T \mathbf{c}.$$

For the responsive pricing scenario with linear demand $d(p) = a - bp$,

$$\Pi_r(\mathbf{q}) = \int_{-\infty}^{a/2} \frac{(a-y)y}{b} dF(y;\mathbf{q}) + \int_{a/2}^{+\infty} \frac{a^2}{4b} dF(y;\mathbf{q}) - \mathbf{q}^T\mathbf{c}.$$

To proceed, we utilize the properties of the normal distribution to derive the KKT conditions in a more specific form, as shown in the next proposition.

PROPOSITION 4. *Assume that the yields follow a multivariate normal distribution. The optimal order quantities* $\mathbf{q}^*$ *satisfy the following conditions. For fixed demand model,*

$$p_0\left(F^*(d_0)\boldsymbol{\mu} - f^*(d_0)\boldsymbol{\Sigma}\mathbf{q}^*\right) - \mathbf{c} \le \mathbf{0};$$

and for responsive pricing,

$$\left(\frac{a-2m^*}{b}F^*(a/2) + \frac{2{\sigma^*}^2 f^*(a/2)}{b}\right)\boldsymbol{\mu} - \frac{2F^*(a/2)}{b}\boldsymbol{\Sigma}\mathbf{q}^* - \mathbf{c} \le \mathbf{0}.$$

In the above, $F^*(y) = F(y;\mathbf{q}^*)$*,* $f^*(y) = f(y;\mathbf{q}^*)$*,* $m^* = {\mathbf{q}^*}^T\boldsymbol{\mu}$ *and* ${\sigma^*}^2 = {\mathbf{q}^*}^T\boldsymbol{\Sigma}\mathbf{q}^*$*. Moreover, for every* $i = 1, 2, \cdots, n$*, the corresponding inequality becomes equality if and only if* $q_i^* > 0$*.*

Proposition 4 allows us to have a unified form of the KKT conditions in both the fixed demand and the responsive pricing scenarios. Based on the above expressions, the optimal order quantities in $s \in \{f, r\}$, respectively, must satisfy

$$A_s(\mathbf{q}^*)\boldsymbol{\mu} - B_s(\mathbf{q}^*)\boldsymbol{\Sigma}\mathbf{q}^* - \mathbf{c} \le \mathbf{0}; \quad B_s > 0. \tag{3}$$

It is worth noting that, A_s and B_s in (3) are both *scalar* functions of the optimal order quantities $\mathbf{q}^*$; and B_s is strictly positive in both scenarios. Therefore, A_s and B_s actually have the same impact on the optimality conditions across all suppliers. On the contrary, a supplier's mean yield factor, its variance and covariance with other suppliers, and its per unit procurement cost are the differentiating parameters in term of the optimality condition for that particular supplier. More importantly, after factoring out the mean yield factor, condition (3) could link a supplier's effective cost to the yield correlation and the supplier reliability.

Our following study will use the optimality condition in an ex post manner. That is, instead of solving for the optimal order quantities, we assume that the optimal decision, $\mathbf{q}^*$, has been made and the suppliers are chosen, given the exogenous cost and reliability portfolio. Without loss of generality, let $J = \{1, 2, \cdots, j\}$ be the active (selected) suppliers and $J^c = \{j+1, \cdots, n\}$ be the inactive (not selected) suppliers. The analysis is certainly identical with respect to any permutation of the supplier indexes. Hereafter, we use the subscripts 1 (YES) and 0 (NO) to denote the sourcing status (they are used only with vector/matrix variables), i.e., $\mathbf{q}_1 > 0 = \mathbf{q}_0$. Accordingly, we partition the mean vector and variance-covariance matrix as

$$\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_0 \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{10} \\ \boldsymbol{\Sigma}_{01} & \boldsymbol{\Sigma}_{00} \end{pmatrix}.$$

Before delving into the study on yield correlation, we first show that, in the special case of uncorrelated random yield, the unified optimality condition (3) may be simplified to derive the well-known supplier selection criterion, i.e., the effective cost ranking rule.

COROLLARY 1. *Suppose that the yields follow a multivariate normal distribution and are mutually uncorrelated. Then,*

$$\frac{c_i}{\mu_i} < \frac{c_k}{\mu_k}, \quad \forall i \in J, \forall k \in J^c.$$

Corollary 1 echoes with the established results in the presence of independent random yield, where effective cost takes precedence over reliability and serves as the primary consideration to qualify a supplier. However, when the yields are correlated, we may not be able to recover the same result. In fact, the criterion used to qualify suppliers and choose supply base becomes more sophisticated than the simple rule based on the effective cost alone. Among other characteristics, yield correlation plays a critical role. To elaborate, we investigate the relationship between the supplier selection and the effective cost ranking, and how the relationship is moderated by yield correlation.

To prepare and facilitate our study, we first define a new variable $\mathbf{z}$ such that $z_i = A_s - \frac{c_i}{\mu_i}$ for scenario $s = \{f, r\}$ and $i = 1, \cdots, n$. This variable can be used as an equivalence to rank the effective costs, because for two suppliers i and k, $\frac{c_i}{\mu_i} < \frac{c_k}{\mu_k}$ if and only if $z_i > z_k$. Recall that $0 \leq c_i \leq \alpha_s \mu_i$, and thus feasible $\mathbf{z} \in \mathcal{Z} := [A_s - \alpha_s, A_s]^n$, which is an n dimensional cube. Given the above definition, we now apply Proposition 4 and condition (3) to relate the supplier selection to the effective cost ranking, from which the impact of yield correlation is investigated.

PROPOSITION 5. *Assume that the yields follow a multivariate normal distribution, and the supply base is optimally chosen to be $J = \{1, 2, \cdots, j\}$. Then, there exists an $(n-j) \times j$ matrix $\mathbf{W}$, such that*

$$\mathbf{W}\mathbf{z}_1 > \mathbf{z}_0. \tag{4}$$

For $k = 1, \cdots, n-j, i = 1, \cdots, j$, the entries of matrix $\mathbf{W}$ are specifically given by $W_{ki} = \frac{\tau_{ki}\mu_i}{\mu_{j+k}}$, where τ_{ki} are the entries of the $(n-j) \times j$ matrix $\boldsymbol{\Sigma}_{01}\boldsymbol{\Sigma}_{11}^{-1}$.

Proposition 5 reveals the critical role played by the yield correlation in establishing the relationship between the supplier selection and the effective cost. To be specific, note that the matrix $\boldsymbol{\Sigma}_{11}^{-1}$ represents the precision of the random yields of all the active suppliers and $\boldsymbol{\Sigma}_{01}$ contains the information on the yield correlation between the active and inactive suppliers. Therefore, the matrix $\mathbf{W}$ contains information on the yield correlation as well as the coefficients of variation of the suppliers. It is noteworthy that the variance-covariance matrix $\mathbf{\Sigma}_{00}$ for the inactive suppliers is absent in condition (4). In other words, for those inactive suppliers, it is more because of their correlation with the active suppliers rather than the correlation among themselves that causes their not being selected.

Next, let us take a closer look at the effective cost comparison between the active suppliers and each inactive supplier; specifically, write condition (4) as

$$\sum_{i \in J} W_{t-j,i} z_i > z_t, \, \forall t \in J^c.$$

Hence, the effective cost comparison between the active suppliers and every inactive supplier is less explicit than one may intuit, especially when the yields are correlated. For every inactive supplier t, z_t must be smaller than a linear combination of z_i's from all active suppliers. By the definition of $\mathbf{z}$, we can deduce that every inactive supplier's effective cost must be larger than some linear transformation of all the active suppliers' effective costs. Moreover, the linear transformation depends on the reliability of the suppliers (precision of the selected suppliers and coefficients of variation of all suppliers) as well as the correlation between the active and inactive suppliers. Being different from a simple one-to-one comparison, the above inequality conveys the possibility that a more expensive supplier is selected over a less expensive supplier in the presence of yield correlation. The following proposition formalizes the condition under which this will happen.

PROPOSITION 6. *Suppose that, for some $l \in J$ and $t \in J^c$, $c_l > 0$ and*

$$\sum_{i \in J} W_{t-j,i} z_i > z_l.$$

Then, there exists a z_t such that supplier l is more expensive than supplier t, yet the former is active and the latter is inactive.

Proposition 6 has an intuitive geometric interpretation. Consider the n-dimension cube $\mathcal{Z} = [A_s - \alpha_s, A_s]^n$; then, $\mathbf{W}\mathbf{z}_1$ is a hyperplane that cuts through the cube. The condition in Proposition 6 simply implies that at least some part of this hyperplane must be "higher" than the diagonal of the cube. When this occurs, there exists some point in between the hyperplane and the diagonal such that the corresponding costs can be used to construct examples in which the less expensive supplier is inactive. This is a generalization of the two-supplier model with respect to the graphical illustration in Figure 1. Instead of considering the space of procurement costs, the geometric meaning here is based on the $\mathbf{z}$ space defined previously.

Moreover, our findings have uncovered the intertwining relationship among yield correlation, effective cost and supplier reliability. As an application of Propositions 5 and 6, we discuss two simple examples, which showcase how yield correlation plays a role in shaping the firm's procurement strategy. In the first example, the reliability may preempt the effective cost as the primary criterion for a firm to select the supply base. In the second, neither the supplier reliability nor the effective cost is sufficient enough to be the deciding factor for the firm to qualify a supplier. Consider n suppliers indexed in a descending order of their effective costs; i.e., supplier 1 is the most expensive and supplier n is the least expensive.

Example 1: The firm sole sources from the most expensive supplier; i.e., $q_1 > 0$ and $q_t = 0$ for $t > 1$. In this case, we have

$$\boldsymbol{\Sigma}_{01}\boldsymbol{\Sigma}_{11}^{-1} = (\rho_{12}\sigma_1\sigma_2, \cdots, \rho_{1n}\sigma_1\sigma_n)^T \sigma_1^{-2},$$

and $z_1 = \mu_1^{-1} B_s \sigma_1^2 q_1 > 0$. Therefore, applying Proposition 6, we conclude that the firm sole sources from the most expensive supplier when $W_{t-1,1} z_1 > z_1 \Leftrightarrow W_{t-1,1} > 1 \Leftrightarrow \rho_{1t} > \rho_t^*$ for all $t > 1$, where $\rho_t^* := v_1/v_t$ and $v_t = \frac{\sigma_t}{\mu_t}$. Furthermore, for $\rho_{1t} > \rho_t^*$ to hold, we must have $\rho_t^* < 1$, which means that supplier 1 has the smallest coefficient of variation, i.e., supplier 1 is the most reliable.

Taken together, this example shows that sole sourcing from the most expensive supplier is possible when that supplier is also the most reliable, and when the yield correlation between that supplier and any other less expensive supplier is large enough. Recall that in Propositions 1 and 2, we have constructed the cost threshold $C_s^1(c_1)$ and compared it with the diagonal line $c_2/\mu_2 = c_1/\mu_1$ to obtain similar counter-intuitive results in the two-supplier model. Those results can be easily recovered from the above example, which is clearly a higher dimensional generalization.

Example 2: The firm sources from all but the least expensive supplier; i.e., $q_n = 0$ and $q_t > 0$ for $t < n$. According to Proposition 5, we have

$$\left(\frac{\rho_{n1}\sigma_1^2}{\rho_1^*}, \cdots, \frac{\rho_{n,n-1}\sigma_{n-1}^2}{\rho_{n-1}^*}\right) \boldsymbol{\Sigma}_{11}^{-1} (z_1, \cdots, z_{n-1})^T > z_n,$$

where $\rho_i^* = v_i/v_n$ for all $i < n$ is the c.v. ratio between each active supplier and the least expensive but inactive supplier. Note that σ_i^2 $(i < n)$ are the diagonal entries of $\boldsymbol{\Sigma}_{11}$; hence the above condition is plausibly a higher dimensional analogue of the condition in the two-supplier model ($\frac{\rho}{\rho^*} z_1 > z_2$). However, unlike the $n = 2$ case, here the effective cost, the c.v. ratios among suppliers, and the correlation between each active supplier and the inactive supplier are inextricably intertwined, resulting in an even more counter-intuitive optimal composition of the supplier base. Let us further demonstrate this point with a numerical example of $n = 3$, where the only inactive supplier is both the least expensive and the most reliable. Consider the fixed demand scenario with $p_0 = d_0 = 1$. The yields distribution is characterized by $\boldsymbol{\mu} = (0.5, 0.6, 0.4)^T$, $\sigma_1 = 0.10$, $\sigma_2 = 0.12$, $\sigma_3 = 0.07$, $\rho_{12} = -0.3$, $\rho_{31} = 0.5$, and $\rho_{32} = 0.6$. Moreover, the unit procurement costs are $\mathbf{c} = (0.26, 0.31, 0.20)^T$. In this case, the optimal order quantities are computed to be $(0.9625, 0.8330, 0)^T$; i.e., supplier 3 is inactive. However, it is direct to verify that the effective costs are $(0.52, 0.52, 0.50)^T$ and the coefficients of variation are $(0.20, 0.20, 0.18)^T$. That is, the firm completely forgoes the seemingly ideal supplier and only sources from the more expensive and less reliable suppliers.

The reasons for the firm's unexpected sourcing behavior in this example are twofold. First, since suppliers 1 and 2 are negatively correlated, the firm may hedge against the yield risk and enhance the overall reliability by sourcing from them, even if they are more expensive. Second, the correlations between supplier 3 and the other two suppliers are relatively high. Similar to the main results uncovered by Lemma 3, with highly positive yield correlation, adding supplier 3 may decrease the overall reliability of the firm's combined supply.

Given the importance of yield correlation in the procurement decisions, its impact on the firm's profit performance also needs investigation. Similar to Section 4.2, where the concept of SM order is introduced to examine how yield correlation affects the optimal diversification choice, we may continue using this multivariate stochastic order as a tool to study how yield correlation affects the firm's profit. Even more convenient than dealing with general bivariate random variables in Section 4.2, we can now establish an equivalence relationship between the SM order and the entry-wise comparison of the variance-covariance matrix under the assumption of multivariate normal yield distribution. As a result, we obtain the following proposition, which shows the monotonicity of the firm's profit with respect to the correlation between any two suppliers' yield factors.

Proposition 7. *Consider the sourcing problem with n unreliable suppliers under scenario $s = \{f, r\}$. Suppose $\boldsymbol{\xi}$ follows a multivariate normal distribution with coefficients of correlation $-1 < \rho_{ij} < 1$ $(i \neq j)$. Then, the profit function $\Pi_s(\mathbf{q})$ for any given $\mathbf{q}$, as well as the optimal profit Π_s^*, decrease in ρ_{ij} for any pair of (i, j) with $i \neq j$.*

The above proposition is an analogue to Proposition 3. We make a couple of remarks on their relationship. First, Proposition 7 is in parallel of Proposition 3(ii) and (iii), both showing that if any of the coefficients of correlation for the random yield becomes larger, then the firm's profit, given any procurement quantities, will never increase. Note that an increase of ρ_{ij} $(i \neq j)$ means that supplier i and supplier j become either less negatively correlated or more positively correlated. The former case means that the firm will perform no better with less opportunity to hedge the yield risk; and the latter case shows the negative impact of the positive yield correlation due to the potential decline in the overall reliability. In both cases, the yield correlation has considerable influences on the firm's optimal profit. Second, results similar to Proposition 3(i) are no longer easy to derive when there are more than two unreliable suppliers, because the yield correlation affects the firm's sourcing behavior in a more complicated and involved manner now. Specifically, for some $k \in J$ and $t \in J^c$, if ρ_{tk} increases, then condition (4) may or may not hold, depending on the precision matrix and the costs of the active suppliers. Consequently, we need a case-specific analysis to determine the impact of yield correlation on the firm's diversification strategy.

In summary, our results for the n-supplier model with multivariate normal yield distribution reinforce the idea that yield correlation plays a crucial part in the decision process for supply diversification. Two main takeaways are highlighted in the following. First, to construct an optimal supply base, the sourcing firm should carefully strike a balance between the supplier reliability and the effective cost in accordance with the underlying yield correlation structure. Such a comprehensive consideration would help the firm to better exploit the benefit of risk pooling through supply diversification. Second, yield correlation is shown to have a major impact on the firm's profit, regardless of the specific sourcing quantities. As a result, correlation between different suppliers' random yield factors can serve as an important indicator that facilitates the firm's decision making to choose a successful procurement strategy.

6. Random Demand

Previously, we assume deterministic demand function to single out the impact of supply risks. In this section, we incorporate demand uncertainty to the problem formulation and show that our main results continue to hold. In particular, the yield correlation influences the firm's diversification decisions in the same way regardless of the demand variability, as long as the demand risk and supply risk are statistically independent.

Suppose that the random part of the demand is $D \geq 0$. In scenario $s = f$, the entire demand equals D (we will keep using the subscript f in this scenario, although the demand is no longer fixed); and in scenario $s = r$, let D be the intercept of the price-dependent linear demand, i.e., the demand equals $D - bp$. The firm's optimization problem will therefore include the expectation with respect to D and $\boldsymbol{\xi}$. Moreover, we assume that D is bounded above and that the demand risk and the supply risk are mutually independent.

Our objective is to show that yield correlation still has a substantial impact on the firm's diversification strategy even in the presence of demand uncertainty. To that end, let us focus on the two-supplier model (i.e., $n = 2$) with a general bivariate yield distribution. Under the assumption that the demand risk and the supply risk are mutually independent, the objective function in each scenario is respectively given below (recall that $Q = \mathbf{q}^T\boldsymbol{\xi}$):

$$\bar{\Pi}_f(\mathbf{q}) = \mathbf{E}_D \mathbf{E}_{\boldsymbol{\xi}} p_0 \left[Q \mathbf{1}_{\{Q \leq D\}} + D \mathbf{1}_{\{Q > D\}} \right] - \mathbf{q}^T \mathbf{c};$$

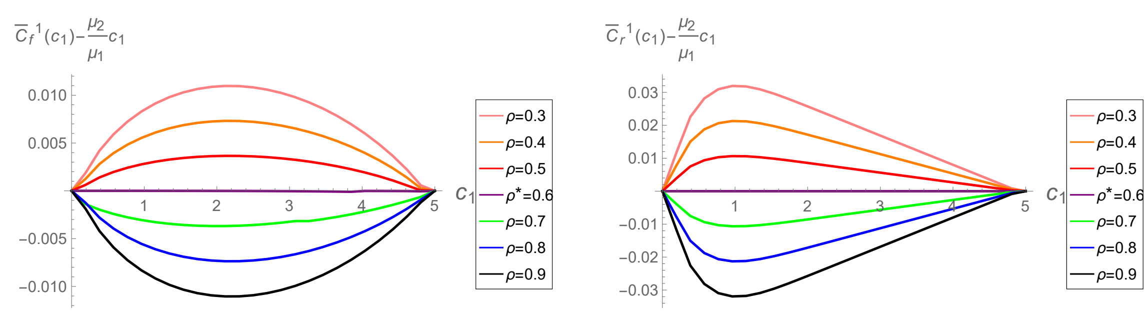
$$\bar{\Pi}_r(\mathbf{q}) = \mathbf{E}_D \mathbf{E}_{\boldsymbol{\xi}} \left[\frac{(D - Q)Q}{b} \mathbf{1}_{\{Q < D/2\}} + \frac{D^2}{4b} \mathbf{1}_{\{Q \geq D/2\}} \right] - \mathbf{q}^T \mathbf{c}.$$

These functions are concave in Q, and therefore jointly concave in $\mathbf{q}$ due to linearity. Hence, we can first conclude that the firm's objective function is still well-behaved to admit the KKT optimality condition as both sufficient and necessary. Moreover, the following proposition confirms that our main results in Section 4 concerning the cost thresholds and the impact of yield correlation on the firm's supplier selection decision continue to hold under demand uncertainty.

PROPOSITION 8. *In scenario $s = \{f, r\}$, respectively, consider demand uncertainty that is independent of the supply risk. Then, there exist two cost thresholds $\bar{C}_s^i(c_1)$ $(i = 1, 2)$, which satisfy all the properties described in Propositions 1 and 2.*

To depict the above findings, we provide a set of numerical examples as illustrations. Let $\boldsymbol{\xi}$ follow a bivariate normal distribution with $\boldsymbol{\mu} = (0.5, 0.6)^T$, $\sigma_1 = 0.06$ and $\sigma_2 = 0.12$; and let D follow a normal distribution with $\mu_D = 10$ and $\sigma_D = 1$. For scenario f, the retail price is set as $p_0 = 1$; for scenario r, the price sensitivity is set as $b = 1$. For both models, the threshold correlation coefficient $\rho^* = \frac{\sigma_1}{\mu_1} / \frac{\sigma_2}{\mu_2} = 0.6$. We further choose the correlation coefficient ρ within the set $\{0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ and plot the difference between the upper cost threshold $\bar{C}_s^1(c_1)$ $(s \in \{f, r\})$ and the diagonal curve $\frac{\mu_2}{\mu_1} c_1$ for the entire feasible region of c_1. For both scenarios, Figure 3 indicates that $\bar{C}_s^1(c_1) - \frac{\mu_2}{\mu_1} c_1 > 0$ when $\rho < \rho^*$, and $\bar{C}_s^1(c_1) - \frac{\mu_2}{\mu_1} c_1 < 0$ when $\rho > \rho^*$, $s \in \{f, r\}$.

Figure 3 The Difference Between the Upper Cost Threshold and the Diagonal Curve under Random Demands



(a) Scenario f: $\bar{C}_f^1(c_1) - \frac{\mu_2}{\mu_1} c_1$. (b) scenario r: $\bar{C}_r^1(c_1) - \frac{\mu_2}{\mu_1} c_1$.

Therefore, as long as the demand uncertainty and the supply uncertainty are independent, the demand variability does not affect the impact of yield correlation. As a result, the managerial insights regarding the supplier selection will not alter due to demand uncertainty. In other words, whether the demand risk exists or not, the firm should always pay close attention to yield correlation when making its sourcing and diversification decisions under supply risk. Hence, our results complement the findings in the existing literature such as Anupindi and Akella (1993), Dada et al. (2007), Federgruen and Yang (2008, 2009, 2011, 2014).

7. Conclusion

Supply diversification has been widely adopted in practice as an effective supply risk mitigation tool. Current understanding of supply diversification mainly focuses on independent supply risks and emphasizes on the determining role of procurement cost. With independent yield risks, "a given supplier will be selected only if all less-expensive suppliers are selected, regardless of the given supplier's reliability level" (Dada et al. 2007). However, it remains unclear whether and when this rule continues to hold under correlated supply yield risks.

In this paper, we study the sourcing and diversification decisions of a monopoly firm that procures from multiple unreliable suppliers to meet its deterministic/price-dependent demand. The suppliers' production processes are unreliable and are modeled by correlated proportional random yields, with a general joint distribution. Although the firm's sourcing decisions can be formulated as a convex optimization problem, solving it in the most general setting is difficult. Assuming a setting with two unreliable suppliers, we uncover an interesting interrelationship among effective cost, supplier reliability and yield correlation, which provides guiding insights into the firm's sourcing and supplier selection decisions. To be specific, when the two suppliers are either negatively or weakly positively correlated, the firm never sole sources from the more expensive supplier alone. In this case, the general rule of thumb derived from the independent yield setting continues to hold. However, when the yields are highly positively correlated, including a less reliable supplier into the supply base may reduce the combined reliability and becomes a suboptimal action even if that supplier is less expensive. In this case, the firm may sole source from the more expensive, but more reliable, supplier. Our results suggest that, when a firm sources from suppliers with correlated yields, cost and supplier reliability should be jointly considered in the diversification decisions. In addition, we find that the firm is less likely to dual source and gains a lower profit if the yield correlation increases.

Moreover, we also look at the n-supplier model with multivariate normally distributed random yields. Our analysis shows that the above main results can be generalized to higher dimensions. Particularly, the effective cost comparison between the active and inactive suppliers is affected by the supplier reliability and the yield correlation. In fact, we may use a linear inequality to characterize the intertwining relationship among the three factors (namely, correlation, reliability and cost). Hence, the firm should jointly consider all three factors when making the supplier selection decisions.

Finally, we incorporate demand uncertainty into our models. Our analysis suggests that, as long as the demand risk and the supply risk are statistically independent, the impact of yield correlation is still substantial. In particular, when the firm faces two unreliable suppliers, it will sole source from the more expensive but more reliable supplier when the yields are highly positively correlated, regardless of the demand variability arising in the firm's retail market. Hence, we confirm the robustness of our results with respect to the demand uncertainty.

There are multiple directions along which this paper can be extended. First, the price-setting firm may need to decide the price before the yield realization, resulting in the ex ante pricing model. The analysis in this model is technically challenging, so we provide some brief discussions and numerical results to confirm our main results in Appendix C, leaving the more rigorous studies to future research. Moreover, with an understanding of how yield correlation can complicate the supplier selection decision, one interesting direction for future research could focus on developing effective heuristic algorithms for this challenging problem. Finally, the suppliers' wholesale price can be set endogenously. Hence, its impact on the firm's optimal diversification decision under correlated yields would be an interesting direction to explore.

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