Traffic Congestion Analysis in Complex Networks

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***Abstract*— The problem of traffic congestion in complex networks is studied. Two kinds of complex network structures, namely random graphs and scale-free networks, are considered. In terms of the structure of connection, random graphs are homogeneous networks whereas the scale-free networks are heterogeneous networks. For both types of networks, we introduce an additional scale-free feature in the load generation process such that a small number of nodes are more heavily loaded than others. A traffic model similar to the routing algorithm in computer networks is used in our simulation study. We show how the network structures and parameters influence the traffic congestion status.**

I. Introduction

Studies of traffic flow in networks are in the focus of research in different areas. Examples include packet flows in the Internet [1], [2], telephone calls in telephone networks [3], [4], electrical power transmitted in power grids [5], and people or goods transported in transportation networks [6]. These networks are so large in scale that we need to build models to describe the topology of these systems before we can study them.

The random graph model is a natural candidate for the study of the topology of large scale networks with equivalent units. In the random graph model developed by Erdos and Renyi [7], each pair of nodes are connected with equal probability. The proposed network is a homogeneous network in which the effect of each node is statistically identical. This model is widely used to describe large scale networks in different disciplines.

Thanks to the advent of high-speed computers, many empirical data from real-world networks can be readily collected and available for analysis. These data indicate that topologies of many real networks are quite different from the random graph model. In the random network model, the effect of each node is similar. But many real networks show heterogeneous properties. Furthermore, recent research on the complex networks has shown that these networks have a small-world scale-free feature [8], [9]. Since the network topology has significant effects on the traffic in the network, the comparative traffic analysis for different network structures is of great practice relevance.

In our previous work, we have shown how the complex user behavior influences the traffic in telephone networks [4]. In the telephone network, nodes are end users (each with a specific number) and a connection between two nodes means that these two users may call each other. Telephone network is a circuit-switching system [10], in which a connection is established between the caller node and receiver node. The traffic load is only between these two nodes during the phone call, and no other node is involved. Unlike the telephone network, many real networks are packet-switched systems. One typical example is the Internet. In this huge computer network, nodes are computers/routers and a connection is a physical link that joins two nodes together. Information is broken down into packets and transmitted through a path in the network. Not only the source and destination nodes, but also many other nodes along the path are involved in the transmission process. In reality, the packet generation (loading) process is also scale-free in the sense that a small number of nodes are more heavily loaded. The traffic in packet-switching networks is thus more complex, and the traffic analysis is more challenging.

Our Aim: In this paper, we study the traffic congestion in random and scale-free networks with a scale-free loading process. Each node has a buffer with finite size, which is the maximum number of packets that can be accommodated in a node at any one time. We will show how the complex network structure and scale-free loading contribute to traffic congestion.

II. Network Structures

In this paper we consider two kinds of complex networks, namely the *random graph* and the *scale-free network*. The performance comparison between them can help us understand the important role of network structure on traffic status.

A. Random Graphs

The construction of random graphs is quite simple. Assume a network with N nodes. Each pair of nodes are connected with probability p. Consequently, the total number of connections in the network is a random variable with a mean value equal to $E(n) = p\frac{N(N-1)}{2}$. For each node, its number of connections (called *degree*) k is also a random variable. If N is large enough, the degree distribution follows a Poisson distribution [8], i.e.,

$$P(k) = C_n^k p^k (1-p)^{N-k} = \frac{\langle k \rangle^k e^{-\langle k \rangle}}{k!}, \qquad (1)$$

where $\langle k \rangle = p(N-1) \approx pN$ is the mean value of k.

Figure 1 shows an example of the degree distribution of a random graph with $\langle k \rangle = 2.9$. In the figure, the distribution curve decays very fast as the number of connections increases. The maximum number of connections is only 11 and only one node has such a number of connections. On the other hand, most of nodes have few connections. Since each pair of nodes

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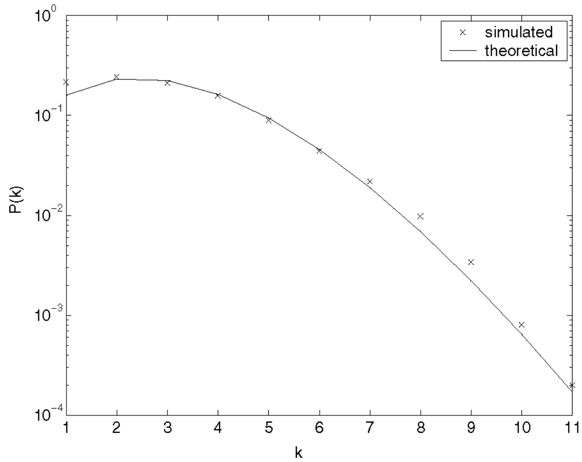

Fig. 1. Degree distribution of a random graph.

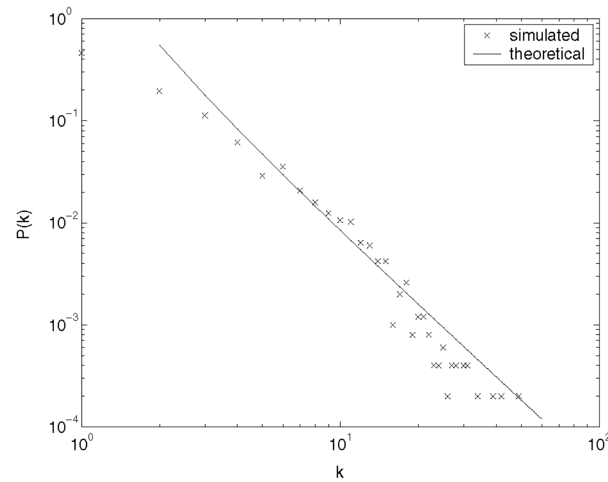

Fig. 2. Degree distribution of a scale-free network.

are connected with equal probability, the random graph is a homogeneous network.

B. Scale-free Networks

We use the evolution algorithm to construct the scale-free network [11]. The network starts with m_0 isolated nodes. At each time step, one of the following three processes is done according to some probabilities.

1) With probability p, m ($m < m_0$) new connections are added into the network. To do that, for each new connection we randomly choose a node as one end. The other end of the connection is selected with a probability

$$\prod(k_i) = \frac{k_i + 1}{\sum_j (k_j + 1)}. \tag{2}$$

2) With probability q, m existing connections are rewired. To do that we randomly choose a node (say node i) and a connection from this node (say the connection from node i to node j). We remove this connection and rewire it between node i to j', where j' is randomly chosen with probability $\prod(k_{j'})$ given by Eq. (2).
3) With probability $1 - p - q$, a new node with m new connections is added into the network. The m new connections are connected to existing node j with probability $\prod(k_j)$ given by (2).

This algorithm simulates the evolution of real networks such as the Internet or airline networks. Consider the airline network as an example. At first, there are only few airports in the network. Some new airlines are set up between existing airports. With the development of air-travel market, new airports are built and connected to existing airports with new airlines; more airlines are also added between existing airports; some existing airlines may be cancelled and reconnected to other airports. All these processes contribute to today's airline network.

Based on this construction method, the degree distribution of the evolved network has a generalized power-law form [11]:

$$P(k) \propto [k + \kappa(p, q, m)]^{-\gamma(p,q,m)}, \tag{3}$$

where $\kappa(p, q, m) = 1 + (p - q)\left(\frac{2m(1-q)}{1-p-q} + 1\right)$ and $\gamma(p, q, m) = 1 + \frac{2m(1-q)+1-p-q}{m}$. Figure 2 gives an example of the degree distribution of a scale-free network with $\gamma = 2.4$. The average number of connections is $\langle k \rangle = 2.9$, which is identical with that in Fig. 1. The degree distribution of a scale-free network has a power-law tail. In this network, a small number of nodes have a very large number of connections. It indicates that the effects of different nodes in the scale-free networks are not identical. In other words, the scale-free network is a heterogeneous network.

III. Traffic Model

The traffic model we used in this paper is similar to the routing algorithm in computer networks. The routing algorithm is as follows:

1) Each node in the network has a buffer with length B.
2) At each time step, node i transmits the first μ_i packets in its buffer one step toward its destination. If a packet reaches its destination, it is removed from the network. Otherwise, the buffer status of its next node is checked. If the buffer is not full, the packet is added into the buffer. If the buffer is full, the packet is dropped.
3) At each time step, new packets are generated. The probability for node i to generate a packet is λ_i. When a packet is generated, its destination is chosen randomly. Then, the shortest path from the source to the destination is determined. If there are more than one shortest path, we randomly choose one. The newly generated packet is placed at the end of the buffer of this node. If the buffer is already full, the newly generated packet is dropped.

In this traffic model, a packet drop indicates that congestion happens. The characteristic that measures the system performance is the packet drop probability. A high drop probability indicates that a large percentage of packets cannot reach their destinations. Then, the quality of service is poorer.

In the traffic model, the packet generation rates for different nodes are different. The more popular the node is, the more packets are generated from this node. In our model, the number of connections can be considered as the index of how popular the node is. Thus, we assume that the packet generation probability for node i is

$$\lambda_i = \alpha k_i, \tag{4}$$

where α is a constant coefficient and k_i is the degree of node i. Similarly, we assume that the packet delivery rate is also proportional to the number of connections

$$\mu_i = \lfloor 1 + \beta k_i \rfloor, \tag{5}$$

where β is a constant coefficient.

IV. Simulation Results

In the first simulation, the numbers of generated packets and blocked packets per time step are shown in Fig. 3. The parameters in the simulation are set as follows

- Network construction: $N = 5000$, $\langle k \rangle = 2.9$.
- Traffic model: $B = 10$, $\alpha = 0.01$, $\beta = 0.1$.

When we construct the scale-free network we assure that $k_i < 100$ (see Fig. 2) so that the generation probability $\lambda_i = \alpha k_i = 0.01 k_i < 1$.

Because of the random nature of the simulation, the actual numbers of newly generated packets vary with time. The average number of newly generated packets at each time step can be derived as

$$\sum_{i=1}^{N} \lambda_i = \alpha \sum_{i=1}^{N} k_i = \alpha N \langle k \rangle = 145. \tag{6}$$

Since the random graph and the scale-free network have identical average number of connections $\langle k \rangle$, their average numbers of newly generated packets are identical.

On the other hand, the numbers of dropped packets in the two networks have quite different features. The scale-free network has a much higher drop rate, as it is a heterogeneous network in which a small number of popular nodes, which have many more connections than most of the others, play an influential role in the traffic. These nodes have much heavier traffic loads so that their buffers are easily overflowed. However, a large number of packets are still delivered to these popular nodes. As a result, many packets have to be dropped. At the same time, a large number of nodes, each of which has only few connections, are idle for most of the time. The unbalanced traffic load is the cause for the high drop probability in scale-free network. On the contrary, the random graph is a homogeneous network, in which the effect of each node is similar. The traffic loads are equally distributed to a large number of nodes. So the number of dropped packets is much lower.

In the traffic model, several parameters have significant effects on the system performance. Figures 4, 5, and 6 show the effects of B, α, and β, respectively. In the figures, the drop probability P_d is defined as

$$P_d = \frac{\text{average number of dropped packets}}{\text{average number of generated packets}}. \tag{7}$$

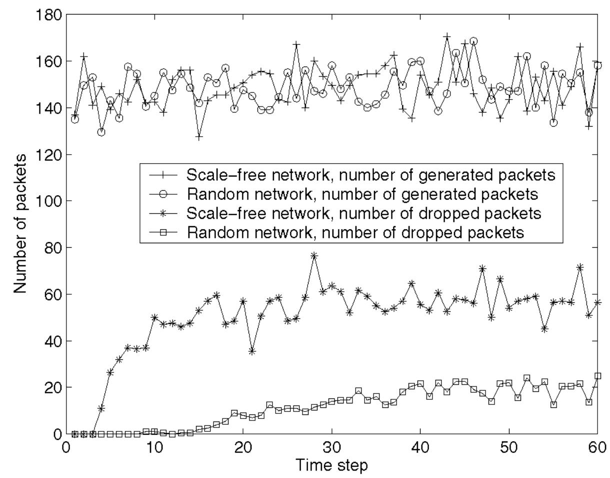

Fig. 3. Number of generated packets and dropped packets per time step.

Here the average number of dropped packets is calculated by averaging the numbers after the network has come to a steady state. In order to show the effect of each parameter, in each simulation, only the value of the corresponding parameter is changed. Other parameters are set as they were in Fig. 3.

The results are consistent with our intuition. In Fig. 4 increasing buffer length B implies more packets can be added into the buffer instead of being dropped. So P_d drops as the value of B increases. Increasing α implies more packets being generated with the packet delivery rate unchanged. Thus, the traffic congestion becomes more severe, and the drop probability increases, as shown in In Fig. 5. In Fig. 6, as β increase, nodes have a higher ability to deliver packets to their destinations. Hence, the number of packets waiting in buffers decreases. The drop probability also decreases. Furthermore, in all three cases, the drop probability of the scale-free network is much higher than that of the random graph because of the accumulation of traffic load in the popular nodes.

Finally, we show the effect of $\langle k \rangle$. When $\langle k \rangle$ increases, both λ_i and μ_i increase. As shown in Figs. 5 and 6, λ_i and μ_i have opposite effects on P_d. Moreover, increasing $\langle k \rangle$ also increases the number of connections in the network, providing more shortcuts for packets transmission and causing a smaller value of γ. Combining all these factors, the total effect of $\langle k \rangle$ is shown in Fig. 7.

V. Conclusions and Future Works

We live in a world which is composed of complex networks. Examples include the Internet, networks of telephone users, airline networks, power grids, etc. The network structure has a profound effect on the behavior of the network and its intended performance if it is designed for an engineering purpose. In this paper we study in particular the traffic congestion problem in complex networks. Two complex network models, i.e., the random graph and the scale-free network, are considered. In terms of the connection structure, the random graph is a homogeneous network, whereas the scale-free network is a heterogeneous network. For both types of networks, we introduce an additional scale-free feature in the load generation

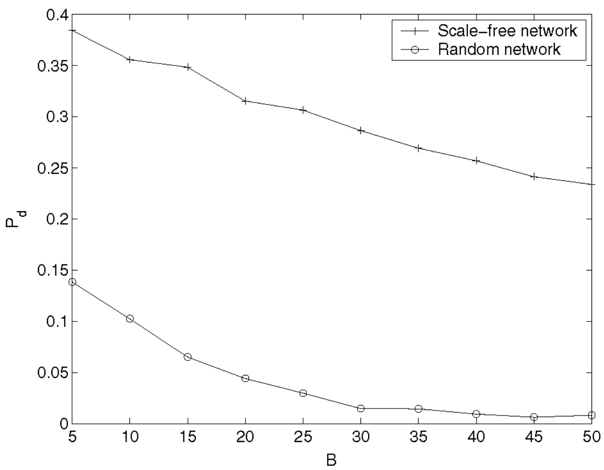

Fig. 4. Effect of B on the drop probability.

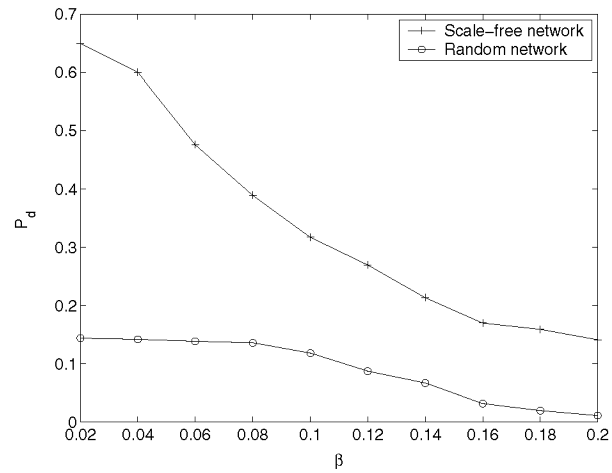

Fig. 6. Effect of β on the drop probability.

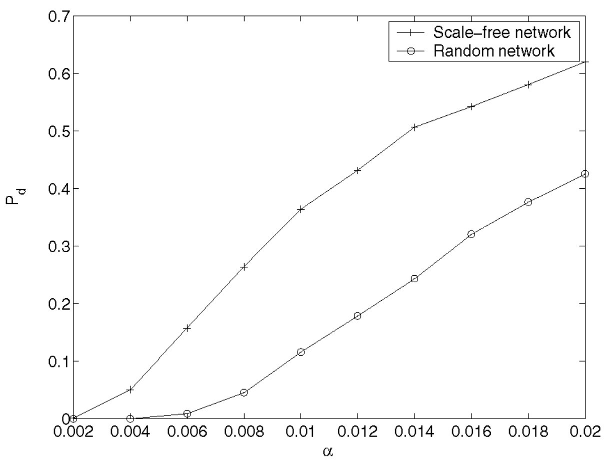

Fig. 5. Effect of α on the drop probability.

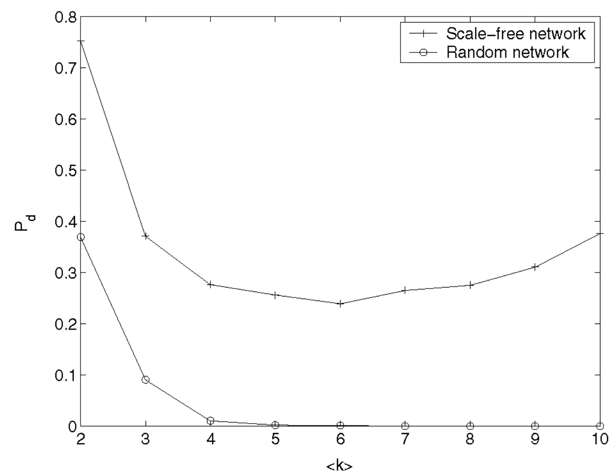

Fig. 7. Effect of $\langle k \rangle$ on the drop probability.

process such that a small number of nodes are more heavily loaded than the rest of the others, satisfying a power-law equation. Due to the unbalanced nature of scale-free networks, the traffic load assembles in the most popular nodes and causes severe traffic congestions. On the contrary, the traffic load in random graphs is distributed among all nodes, making the traffic congestion rate low. The simulated results also show that several system parameters have significant effects on the network traffic. Changing the values of some parameters may drastically change the traffic status.

This paper shows a preliminary study on the traffic congestion analysis in complex networks. There is still a lot of works need to be done. For example, we use a simple traffic model, in which the packet generation probability and packet delivery rate of one node are both proportional to its number of connections. In the future these assumptions can be revised based on some realistic network circumstances.

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