CO-EXISTENCE OF CHAOS-BASED AND CONVENTIONAL DIGITAL COMMUNICATION SYSTEMS

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ABSTRACT

This paper studies the performance of selected chaos-based communication systems whose spectrum overlaps with that of conventional narrowband systems. Such a scenario may occur in normal systems under study are the coherent chaos-shift-keying (CSK) system and the non-coherent differential chaos-shift-keying, binary phase-shift-keying. We consider a chaos-based communication system and a conventional system whose bandwidths overlap significantly. We refer to the whole system as combined chaos-based-conventional system, which can be represented by the block diagram shown in Fig. 1. Our analysis will proceed in a discrete-time fashion. At time $k$, denote the output of the chaos transmitter by $s_k$ and that of the conventional transmitter by $u_k$. These two signals are then added, as well as corrupted by noise $n_k$ in the channel, before they arrive at the receiver end. At the receiver, based on the incoming signal $r_k$, the receivers of the chaos-based system and the conventional system will attempt to recover their respective data streams. Coherent or non-coherent detection schemes may be applied in the receivers, depending upon the modulation methods used in the transmitter. Specifically, we will consider a “combined CSK-BPSK” system and a “combined DCSK-BPSK” system, and will attempt to develop analytical expressions for the bit error rates of the recovered data streams.

2. SYSTEM OVERVIEW

We consider a chaos-based communication system and a conventional system whose bandwidths overlap significantly. We refer to the whole system as combined chaos-based-conventional system, which can be represented by the block diagram shown in Fig. 1. Our analysis will proceed in a discrete-time fashion. At time $k$, denote the output of the chaos transmitter by $s_k$ and that of the conventional transmitter by $u_k$. These two signals are then added, as well as corrupted by noise $n_k$ in the channel, before they arrive at the receiver end. At the receiver, based on the incoming signal $r_k$, the receivers of the chaos-based system and the conventional system will attempt to recover their respective data streams. Coherent or non-coherent detection schemes may be applied in the receivers, depending upon the modulation methods used in the transmitter. Specifically, we will consider a “combined CSK-BPSK” system and a “combined DCSK-BPSK” system, and will attempt to develop analytical expressions for the bit error rates of the recovered data streams.

3. PERFORMANCE ANALYSIS

3.1. Combined CSK-BPSK Communication System

We first consider a discrete-time baseband equivalent model of a combined CSK-BPSK communication system. We assume that the CSK system and the BPSK system have identical bit rate and that their bit streams are synchronized. Also, the carrier frequencies of the two systems are identical and synchronized. Further, “−1” and “+1” occur with equal probabilities in the bit streams of both systems.

For simplicity, we consider here a CSK system in which one chaos generator is used to produce chaotic signal samples $\{x_k\}$ for $k = 1, 2, \ldots$. Suppose $\alpha_k \in \{-1, +1\}$ is the symbol to be sent during the $k$th bit period. Define the spreading factor $2\beta$, as the number of chaotic samples used to transmit one binary symbol. During the $k$th bit duration, i.e., for $k = 2\beta(l-1) + 1, 2\beta(l-1) + 2, \ldots, 2\beta l$, the output of the CSK transmitter is $s_k = \alpha_k x_k$.

In the BPSK system, we denote the $k$th transmitted symbol by $b_k \in \{-1, +1\}$. Moreover, the signal power is $P_a$. Thus, during the $k$th bit duration, i.e., for $k = 2\beta(l-1) + 1, 2\beta(l-1) + 2, \ldots, 2\beta l$, the transmitted signal is constant and is represented by $u_k = \sqrt{P_a} b_k$.

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Fig. 1: Block diagram of a combined chaos-based-conventional digital communication system.

Fig. 2: Block diagram of a coherent CSK receiver.

Fig. 3: Block diagram of a BPSK receiver.

The CSK and BPSK signals are combined and corrupted by an additive white Gaussian noise in the channel, before arriving at the receiving end. Thus, the received signal, denoted by $r_k$, is given by $r_k = s_k + u_k + q_k$ where $q_k$ is a Gaussian noise sample of zero mean and variance (power spectral density) $N_0/2$.

### 3.1.1. Performance of the CSK System

Assume that a correlator-type receiver is employed. Referring to Fig. 2, the correlator output for the $l$th bit, $y_l$, is given by

$$
y_l = \sum_{k=2^{l}(l-1)+1}^{2^{l+1}} \sum_{k=2^{l}(l-1)+1}^{2^{l+1}} x_k^2 + \sqrt{P_b} b_l \sum_{k=2^{l}(l-1)+1}^{2^{l+1}} x_k + \sum_{k=2^{l}(l-1)+1}^{2^{l+1}} \eta_k \delta_k \quad (1)
$$

The overall error probability of the $l$th transmitted symbol can be calculated from

$$
\text{BER}_{\text{CSK}}^{(l)} = \frac{1}{2} \left[ \text{BER}_{\text{CSK}}^{(l)} + \text{BER}_{\text{CSK}}^{(l)} \right] \quad (2)
$$

where

$$
\text{BER}_{\text{CSK}}^{(l)} = \text{Prob}(b_l = +1) \times \text{Prob}(y_l < 0 | (a_l = +1, b_l = +1)) + \text{Prob}(b_l = -1) \times \text{Prob}(y_l < 0 | (a_l = +1, b_l = -1))
$$

$$
= \frac{1}{2} \text{erfc} \left( \frac{E[y_l | (a_l = +1, b_l = +1)]}{\sqrt{2\text{var}[y_l | (a_l = +1, b_l = +1)]}} \right) + \frac{1}{2} \text{erfc} \left( \frac{E[y_l | (a_l = +1, b_l = -1)]}{\sqrt{2\text{var}[y_l | (a_l = +1, b_l = -1)]}} \right)
$$

$$
= \frac{1}{4} \text{erfc} \left( \frac{-E[y_l | (a_l = +1, b_l = +1)]}{\sqrt{2\text{var}[y_l | (a_l = +1, b_l = +1)]}} \right) + \frac{1}{4} \text{erfc} \left( \frac{-E[y_l | (a_l = +1, b_l = -1)]}{\sqrt{2\text{var}[y_l | (a_l = +1, b_l = -1)]}} \right)
$$

### 3.1.2. Performance of the BPSK System

In the BPSK receiver shown in Fig. 3, the incoming signal samples within a symbol period are summed to give $z_l$, i.e.,

$$
z_l = \sum_{k=2^{l}(l-1)+1}^{2^{l+1}} \sum_{k=2^{l}(l-1)+1}^{2^{l+1}} x_k + \sqrt{P_b} b_l \sum_{k=2^{l}(l-1)+1}^{2^{l+1}} x_k + \sum_{k=2^{l}(l-1)+1}^{2^{l+1}} \eta_k \delta_k \quad (5)
$$

Assuming that $z_l$ is normal, the error probability for the $l$th transmitted BPSK symbol can be found as

$$
\text{BER}_{\text{BPSK}}^{(l)} = \text{Prob}(b_l = +1) \times \text{Prob}(z_l < 0 | (b_l = +1)) + \text{Prob}(b_l = -1) \times \text{Prob}(z_l > 0 | (b_l = -1))
$$

$$
= \frac{1}{2} \text{erfc} \left( \frac{2\beta \sqrt{P_b}}{\sqrt{2\beta P_b + 2\beta N_0}} \right)
$$

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where $P_s = E[x_k^2]$ denotes the average chaotic signal power. Since $B_{E_b/N_0}^{CSK}$ is independent of $l$, it also represents the BER of the system. For a fixed chaotic signal power $P_s$, the BER can be improved by increasing the spreading factor $2b$ and/or increasing the BPSK signal power $P_B$.

### 3.2. Combined DCSK-BPSK Communication System

In this section, we move on to a combined DCSK-BPSK system. Figure 4 shows the block diagram of a DCSK transmitter and receiver pair [1]. Making the same assumptions as in Section 3.1, we obtain the transmitted DCSK signal in the $l$th bit duration as

$$s_k = \begin{cases} x_k & \text{for } k = 2b(l-1) + 1, \ldots, 2b(l-1) + 2b-1 \\ \alpha_k x_k & \text{for } k = 2b(l-1) + 2b + 1, \ldots, 2b(l-1) + 2b \end{cases}$$

(7)

The noisy received signal, $r_k$, is given by $r_k = s_k + u_k + \eta_k$. At the receiver, the detector essentially computes the correlation of the received reference and data slots of the same symbol, as shown in Fig. 4. The output of the correlator for the $l$th received bit equals $y_l = \sum_{k=2b(l-1)+1}^{2b(l-1)+2b} r_k x_k$. Using a likewise procedure as in Section 3.1, the BER of the DCSK system under a combined communication environment can be derived. Similarly, the output of the BPSK correlator can be computed and hence the BER is found.

#### 3.3. Example

Consider the case where a logistic map is used for chaos generation. The form of the map is $x_{k+1} = g(x_k) = 1 - 2x_k^2$ and the invariant probability density function of $\{x_k\}$ equals

$$p(x) = \begin{cases} \frac{1}{\sqrt{\pi}} e^{-x^2} & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$

(8)

For this case, the numerical values of the means and variances of $y_l$ $(\alpha = \pm 1, \beta = \pm 1)$ are obtained and substituted into (2) to obtain the BER of the CSK system, i.e.,

$$\text{BER}_{\text{CSK}} \approx \frac{1}{4} \text{erfc} \left( \frac{2\beta}{\sqrt{1 + 4P_B + 2N_0 + 4\sqrt{P_B}}} \right).$$

(9)

Moreover, for the BPSK system under a combined CSK-BPSK environment, we obtain

$$\text{BER}_{\text{BPSK-CSK}} = \frac{1}{4} \text{erfc} \left( \frac{2\beta P_B}{\sqrt{1 + N_0}} \right).$$

(10)

For the combined DCSK-BPSK system, it can also be shown that the BERs for the DCSK system and BPSK system are, respectively,

$$\text{BER}_{\text{DCSK}} = \frac{1}{8} \text{erfc} \left( \frac{0.5\beta + \beta P_B}{\sqrt{\zeta - 2\beta P_B}} \right)$$

$$+ \frac{1}{8} \text{erfc} \left( \frac{0.5\beta - \beta P_B}{\sqrt{\zeta + 2\beta P_B}} \right)$$

and

$$\text{BER}_{\text{BPSK-DCSK}} = \frac{1}{4} \text{erfc} \left( \frac{2\beta P_B}{2 + N_0} \right) + \frac{1}{4} \text{erfc} \left( \frac{2\beta P_B}{N_0} \right)$$

where

$$\zeta = 0.25\beta + 4\beta P_B + 2\beta P_B N_0 + \beta N_0 + \frac{\beta N_0^2}{2}.$$  

(13)

### 4. COMPUTER SIMULATIONS AND DISCUSSIONS

In this section we study the performances of the chaos-based and conventional digital communication systems under a combined environment by computer simulations. The logistic map described in Section 3.3 has been used to generate the chaotic sequences. For comparison, we also plot in each case the analytical BERs obtained in Section 3.3. Results are shown in Fig. 5 for the combined CSK-BPSK system, and in Fig. 6 for the combined DCSK-BPSK system. In general, computer simulations and analytical results are in good agreement. Also, as would be expected, the coherent CSK system generally performs better than the non-coherent DCSK system. Further observations are summarized as follows.

1. The BER of the chaos-based system in the combined environment generally deteriorates (increases) as $P_B/P_s$ increases for any given $E_b/N_0$. This is apparently due to the increasing power of the BPSK signal which causes more interference to the chaos-based system, thus giving a higher BER.

2. At a fixed $E_b/N_0$, the BER of the BPSK system in the combined environment improves as $P_B/P_s$ increases.

3. Comparing the two types of chaos-based communication systems, the performance of the DCSK system is degraded to a larger extent under the influence of a BPSK signal.
Fig. 5: BERs versus $E_b/N_0$ of the individual systems in a combined CSK-BPSK environment. Simulated BERs are plotted as points and analytical BERs plotted as lines. Spreading factor is 100. (a) Coherent CSK system; (b) BPSK system.

4. From Fig. 5, it can be observed that both CSK and BPSK systems can perform reasonably well for a $P_b/P_s$ value between $-5$ dB and $5$ dB. In fact, we find that the system capacity is maximum at a $P_b/P_s$ value of $0$ dB (not shown here because of space limitation).

5. In the case of the combined DCSK-BPSK system, satisfactory BER performance can be maintained for power ratio around $P_b/P_s = -5$ dB. The system capacity is found to be maximized under such a condition.

5. CONCLUSION

In this paper, the problem of co-existence of chaos-based communication systems and conventional narrowband communication systems is studied in terms of two specific sample systems, namely, a combined CSK-BPSK system and a combined DCSK-BPSK system. A parallel study of the co-existence problem of chaos-based conventional spread-spectrum systems can be found in [5].

6. REFERENCES


