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A NEW TIME-DEPENDENT TRADING STRATEGY FOR SECURITIZED REAL ESTATE AND EQUITY INDICES

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Abstract. The "buy-and-hold" strategy based on the EMH has been adopted by many investors for long. However, the global financial crisis in 2008 caused more doubt to be cast on EMH. Therefore, many scholars have attempted to create a trading strategy which can outperform the "buy-and-hold" strategy. In this study, we use the Shiryaev-Zhou index to derive a new generalized time-dependent strategy of which the moving-window size can be changed to see how the moving-window size affects the resulting profit of our strategy. We test our strategy on the securitized real estate and general equity indices of six economies, and find the optimal moving-window size for our strategy on each stock index. The results show that when the optimal moving-window size is used, our strategy outperforms the "buy-and-hold" strategy for most cases. Furthermore, during stock market downturns, it's advisable to adopt our strategy, preferably with larger moving-window sizes, to prevent losses when the stock prices fall rapidly. However, during long periods of booms, it's better to adhere to the "buy-and-hold" strategy. This implies that we should switch strategies when market fundamentals changes significantly. Property practitioners can also apply this strategy for a better portfolio management to increase their profit.

Keywords: Shiryaev-Zhou index, "Buy-and-hold", moving-window size, transaction cost, securitized real estate index.

1. Introduction

Traditionally, the well-known "buy-and-hold" strategy is supported by many investors. This strategy is supported by the efficient market hypothesis (EMH) which says that at all times, stock prices fully reflect all available information and hence is fairly priced, so there is no point to trade, i.e. one should adhere to the "buy-and-hold" strategy. The EMH is supported by a number of studies like Malkiel and Fama (1970), Malkiel (2003, 2005), Barber and Odean (2000). However, recently, the financial markets of different nations have become more and more interrelated due to globalization. Hence the global financial market has become more volatile. On September 15, 2008, the bankruptcy of Lehman Brothers led to the break out of the global financial crisis, which is the most severe financial crisis since the Great Depression. The impact of the global financial crisis was worldwide as many studies showed that there was significant contagion across different nations (e.g. Hui, Chan 2012, 2013, 2014b; Hui, Chen 2012). Many scholars raised doubts on whether that the EMH and the "buy-and-hold" strategy still work. The EMH was blamed for leading investors to underestimate the danger of asset bubble bursting and to believe too much in rational expectations and market efficiencies. With the truthfulness of the EMH in doubt, some investors look for an alternative trading strategy. In particular, it would be perfect if we can buy a stock at the minimum price and sell it at the maximum price, earning the maximum profit. If such strategies exist, then the weak form of EMH is false. This is the motivation of our study.

To solve this problem, we make use of the Shiryaev-Zhou index to develop a new time-dependent trading strategy with a variable moving-window size, which was then applied on a number of securitized real estate indices and general equity indices. We make this comparison because the real estate market has become more and more important recently, especially in Hong Kong and U.S. The property sector (i.e. Hang Seng Property Index)

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has maintained a weight of over 20% of the Hang Seng Index (HSI) for years, reflecting that the real estate market has played a significant role in Hong Kong's economy for a long time. The Centa-City Leading Index (CCL), which is the most representative property price index in Hong Kong, surged from a record low of 31.77 in August 2003 to a historical high of 146.92 in September 2015, which was almost a fivefold increase. This revealed that the Hong Kong real estate market has been on a boom over the past decade. U.S. also experienced a real estate market boom before. The average sales prices of new homes sold in the U.S. doubled during the period 1993–2007. Subprime mortgages became common and real estate bubbles built up. On September 15, 2008, the Lehman Brothers went bankruptcy. The real estate bubbles burst, causing the breakout of the global financial crisis. Hence real estate played an important role in the global financial crisis, and the real estate prices became more volatile during the crisis. However, real estate has lower liquidity and higher transaction costs (Cheng *et al.* 2010). Furthermore, according to Hui and Zheng (2012), real estate can serve as a type of consumption goods as well as an investment tool. Hence securitized real estate indices, which can reflect the performances of real estate markets, may behave differently from general equity indices, so the optimal investment strategy may be different, causing differences in the results. Therefore, we compare the performance of our strategy on securitized real estate indices with that on general equity indices (benchmarks).

Hui and Yam (2014) derived a trading strategy from the Shiryaev-Zhou index, and found that their strategy outperformed the "buy-and-hold" strategy in general. This strategy was applied by Hui *et al.* (2014) and Hui and Chan (2014a) (with minor modifications) later. All of these three studies used the same moving-window size 130 for their strategies. However, in reality, stock prices are volatile and fluctuate a lot. The moving-window size 130 may not be optimal. Different stocks/stock indices may have different moving-window sizes, too. To bridge this gap, in this study, we construct a new, generalized time-dependent trading strategy with variable moving-window size, and apply the generalized strategy on securitized real estate indices and general equity indices of six economies: Hong Kong, Japan, U.S., U.K., France and Germany, during the period December 29, 1995–December 31, 2014. We compare the resulting profits with the profits arising from the "buy-and-hold" strategy, investigate how the change in moving-window size affects the profit of our strategy, and find the optimal moving-window size of the strategy for each stock index. The corresponding strategy will be called the optimal strategy. (Strictly speaking, the strategy/moving-window size is optimal for the period December 29, 1995–December 31, 2014 only) Our methodology has the following features:

1) Due to different economic conditions between different economies, the trends of stock indices of different economies are different, resulting in variation in performances of our strategy and hence different moving window sizes between stock indices of different economies. This study compares the optimal moving-window sizes of our strategy across different economies.
2) As explained before, the real estate market has become increasingly important, and securitized real estate indices may behave differently from general equity indices, resulting in different optimal investment strategies. Therefore, for each economy, we compare the optimal moving-window sizes of our strategy on the securitized real estate index of that economy with that on the general equity index of that economy. This can compare the performance of our strategy on the securitized real estate and general stock markets of that economy.
3) In reality, transaction costs exist. This will reduce the resulting profit of our strategy. When a different moving-window size is used, the increase in transaction cost may reduce the profit of our strategy by a different extent, so different amounts of transaction costs may yield different optimal moving-window sizes. Hence in this study, scenarios of scales of transaction costs will be considered to see how transaction costs affect the optimal window size.
4) The stock prices are volatile and are always fluctuating. Therefore, at some time our strategy may beat "buy-and-hold", but during other times "buy-and-hold" may outperform our strategy. The optimal moving-window size may also be different from time to time. To investigate this issue, the performance of our strategy (using different moving-window sizes) and the "buy-and-hold" strategy will be tracked along the whole timeline to find out the optimal moving-window sizes at different times along the timeline. If the results show that the optimal moving-window size (and hence the optimal strategy) changes along the timeline, this implies that, in fact, investors should adjust their strategies from time to time.

This study has an implication to investors that they can follow our trading strategy to earn more profit. The change of optimal strategy from time to time also implies that investors should review their portfolio regularly and adjust their strategies according to the market condition. The same applies to strategic property management. Property practitioners often hold real estate stocks or funds in their portfolio. They should also review their portfolio constantly and adjust their strategies when the market condition changes.

This paper proceeds as follows: Section 2 reviews previous literature on related topics. Section 3 describes the formula of the Shiryaev-Zhou index and its statistical estimation. In Section 4, we construct a new trading strategy of which the moving-window size can be varied. The data source is described in Section 5. Section 6 displays the results. Finally, we draw a conclusion in Section 7.

2. Literature review

There are many studies about optimal trading strategies in the past. Markowitz (1952) was the first to work on portfolio optimization, and introduced the mean-variance modern portfolio theory (MPT). Since then many dynamic investment models like the Merton portfolio (Samuelson 1969; Merton 1971) and the continuous-time Markowitz model (Richardson 1989) have been developed. These models do not lead to the pure "buy-and-hold" strategy. There were also related studies in recent years. Krystalogianni and Tsolacos (2004) investigated the yield structure between broad asset classes and the implications for portfolio allocation decisions and real estate investment, and derived a Markov switching strategy which was superior to the "buy-and-hold" strategy. Cheng *et al.* (2010) presented a theoretical model to determine the optimal holding period for real estate investment. They found that higher illiquidity and transaction costs led to longer holding periods, while higher return volatility implied shorter holding periods, ceteris paribus. Gallo *et al.* (2013) applied cointegration methods to create globally diversified real estate portfolios which outperformed the mean-variance optimized portfolio.

However, most of the trading strategies derived above did not consider that past stock price trends may affect future trends. Lempérière *et al.* (2014) took this into account and derived a back-tested strategy: to buy those assets of prices above their five-month averages, and to go short on those below their averages. If prices fell below (or rose above) their five-month averages, then one should switch positions. Their strategy not only provided a positive return over all long periods and over each decade in the sample, but also outperformed the "buy-and-hold" strategy. Our idea in this study is similar to Lempérière *et al.* (2014)'s idea. However, instead of simply taking the five-month averages of stock prices, we make use of the Shiryaev-Zhou index, which is derived from the problem of finding the optimal selling time to minimize the expected relative error between the selling price and the maximum price of a stock. Shiryaev *et al.* (2008) derived a "goodness index" γ of a stock to find the optimal time to sell the stock by the probabilistic approach, and showed that the optimal selling time t is determined by $t = T$ (T is the end of the period) when $\gamma \geq \frac{1}{2}$, and $t = 0$ when $\gamma \leq 0$ (this type of strategies are called "bang-bang" strategies). For the case $0 < \gamma < \frac{1}{2}$, Shiryaev *et al.* (2008) claimed that $t = 0$, and referred to the PDE approach of Dai *et al.* (2008). Du Toit and Peskir (2008) proved the same result by another probabilistic approach. Yam *et al.* (2009, 2012a, 2012b) applied the techniques in solving the secretary problem to resolve the same problem and derive the Shiryaev-Zhou index, which is smaller than the "goodness index" by 1/2.

However, Shiryaev *et al.* (2008), Du Toit and Peskir (2008) and Yam *et al.* (2009, 2012a, 2012b)'s methods all have the same disadvantage: they all assumed the drift (or return) and volatility to be constants. However, the market fundamentals are always varying in reality, especially during and after the global financial crisis in 2008. Therefore, the parameters vary in time is a more reasonable assumption. Wong *et al.* (2012) developed a dynamic bang-bang strategy in which the parameters varied over time. Provided that only returns from the relatively recent past was used to estimate λ (in this way, Wong *et al.* (2012)'s method is similar to Lempérière *et al.* (2014)'s), their strategy outperformed the "buy-and-hold" strategy on the CRSP, FTSE 100 and Hang Seng indices. λ has the same sign as the Shiryaev-Zhou index (Wong *et al.* 2012) and determines the optimal buying/selling time of a stock. Combining the dynamic bang-bang strategy of Wong *et al.* (2012) with the Shiryaev-Zhou index provides the theoretical and conceptual framework of our study.

Hui *et al.* (2012) first put the Shiryaev-Zhou index into practice, applying the Shiryaev-Zhou index on a number of Hong Kong listed real estate stocks. They only listed the latest selling dates of each stock, but did not calculate the resulting profit. Hui and Yam (2014) applied the Shiryaev-Zhou index to derive a trading strategy, and tested the strategy on four securitized real estate indices in Europe and North America. Their strategy beat the "buy-and-hold" strategy in general. Hui *et al.* (2014) tested the same strategy on six Asian securitized real estate indices and found that the strategy generally outperformed the "buy-and-hold" strategy, too. However, Hui and Chan (2014a) produced mixed results: the strategy outperformed "buy-and-hold" for the Hang Seng Index and the Hang Seng Property Index, but the results varied when the strategy was tested on individual stocks listed in Hong Kong: the strategy was still superior to "buy-and-hold" in general for property stocks, but underperformed "buy-and-hold" for most of the non-property stocks, especially when there were transaction costs.

However, the method of Hui and Yam (2014), Hui *et al.* (2014) and Hui and Chan (2014a) have the common drawback that they used a fixed moving-window size ($n = 130$) to calculate the estimator of the Shiryaev-Zhou index and hence derive their trading strategy. Hence, for a particular stock or stock index, assuming that transaction costs remain constant, the resulting profit is fixed. In reality, the stock price is volatile and fluctuates frequently. Thus changing the moving-window size may alter the estimator of the Shiryaev-Zhou index and hence the resulting profit derived by the strategy. In that case, what is the optimal moving-window size, i.e., what size of moving-window yields the maximum profit for the strategy? Up till now, no one has solved this question. To fill in this gap, we construct a new time-dependent trading strategy with variable moving-window size in this study, and find out the optimal moving-window size for each stock index.

3. The Shiryaev-Zhou index and its statistical estimation

The Shiryaev-Zhou index is derived from the problem of minimizing the time between the selling and maximum prices of the stock, and its formula is given by (Yam *et al.* 2009, 2012a, 2012b; Hui *et al.* 2012, 2014; Hui, Yam 2014; Hui, Chan 2014a):

$$\mu = (\alpha - 0.5\sigma^2)/\sigma^2 = \alpha/\sigma^2 - 0.5, \quad (3.1)$$

where: α, σ are the annual growth rate and the annual volatility of the stock respectively (α, σ are constants).

The parameters α and σ in (3.1) are constants. However, in reality, these parameters are always varying. Normally, we do not know their exact values. Hence we adopt the following moving-window approach (Hui *et al.* 2014):

Let S_i be a stock's closing price on day i, the continuously compounded daily return of the stock on day i, r_i ($i \geq 2$) is given by:

$$r_i = \log\left(\frac{S_i}{S_{i-1}}\right). \quad (3.2)$$

The sample mean is used to estimate the mean of the stock's daily return on day i ($i > n$):

$$\overline{r_i}(n) = \frac{1}{n}\sum_{j=1}^{n} r_{i-n+j}. \quad (3.3)$$

Assume that there are 250 trading days in one year, the estimator of α on day i is:

$$\hat{\alpha}_i(n) = 250\overline{r_i}(n) \quad (3.4)$$

The sample variance is used to estimate the daily variance on day i ($i > n$):

$$s_i^2(n) = \frac{1}{n-1}\sum_{j=1}^{n}\left(r_{i-n+j} - \overline{r_j}(n)\right)^2. \quad (3.5)$$

Hence the estimator of the variance σ^2 on day i ($i > n$) is:

$$\hat{\sigma}_i^2(n) = 250 s_i^2(n). \quad (3.6)$$

The estimator of the Shiryaev-Zhou index μ on day i ($i > n$) is:

$$\hat{\mu}_i(n) = \frac{\hat{\alpha}_i(n) - 0.5\hat{\sigma}_i^2(n)}{\hat{\sigma}_i^2(n)} = \frac{\hat{\alpha}_i(n)}{\hat{\sigma}_i^2(n)} - 0.5. \quad (3.7)$$

This study has a new feature that in formula (3.7), the estimator of the Shiryaev-Zhou index is, in fact, a function of n, which is the moving-window size. Hui and Yam (2014), Hui *et al.* (2014) and Hui and Chan (2014a) fixed n to be 130, but this methodology has the drawback that the resulting profit of their strategy would be fixed, assuming that the scale of transaction costs remains unchanged. Since the stock price is always fluctuating, changing the moving-window size may alter the sign of $\hat{\mu}_i(n)$, so the resulting profit of the strategy (which depends on the sign of $\hat{\mu}_i(n)$) may be different. In this study, we allow the moving-window size to vary, thus forming a new, generalized time-dependent trading strategy. Its profit depends on the moving-window size n, and hence we can find the moving-window size which maximizes the profit of our strategy, i.e. the optimal moving-window size. Our corresponding strategy is then the optimal strategy.

4. Our trading strategy

Applying the estimator of the Shiryaev-Zhou index derived by formula (3.7), we construct a trading strategy. We make the following two assumptions:

1) The transaction price (buying and selling price) of a stock index is its closing price on that day.
2) The amount of cash held at time $t = 0$ is adequate to cover all transactions during the period.

Our trading strategy is as follows (Hui, Chan 2014a):

1. On Day 1, if $\hat{\mu}_1(n) \geq 0$, buy one unit of the stock index. Otherwise, take no action.
2. From Day 2 to the second last day of the period, trade the stock index according to Table 1:

Table 1. Our trading strategy from Day 2 to the second last day

$\hat{\mu}_{i-1}(n)$	$\hat{\mu}_i(n)$	Action
≥ 0	≥ 0	No action (keep holding one unit of the stock/stock index)
≥ 0	< 0	Sell the entire one unit of the stock/stock index we hold
< 0	≥ 0	Buy one unit of the stock/stock index
< 0	< 0	No action (keep holding entire cash)

3. On the last day of the period, sell the entire one unit of the stock index if one is still holding the one unit of the stock index. Otherwise, do not take any action.

Since the profit of our strategy depends on the sign of the estimator of the Shiryaev-Zhou index $\hat{\mu}_i(n)$, which is a function of n, the profit of our strategy also depends on n, which is the moving-window size. Hence our trading strategy is a time-dependent strategy.

From Table 1, we can see that our strategy can be simplified as follows: on day i($i \geq 2$), if $\hat{\mu}_{i-1}(n) \geq 0$, hold one unit of the stock index (the periods of which $\hat{\mu}_{i-1}(n) \geq 0$ are called "holding periods"). Otherwise, hold entire cash (the periods of which $\hat{\mu}_{i-1}(n) < 0$ are called "non-holding periods"). Hence without transaction costs, the profit on day *i* is the same for both "buy-and-hold" and our strategy if $\hat{\mu}_{i-1}(n) \ge 0$. It is the stock price movements on the days of which $\hat{\mu}_{i-1}(n) < 0$ that makes the difference between the profits of our strategy and the "buy-and-hold" strategy: if $\hat{\mu}_{i-1}(n) < 0$, but the stock index rises on day i, then our strategy underperforms the "buy-and-hold" strategy on day *i*. On the other hand, if $\hat{\mu}_{i-1}(n) < 0$ and the stock index falls on day *i*, then our strategy outperforms "buy-and-hold" on day *i*. Summing up the differences between the profits of our strategy and "buy-and-hold" on the days of which $\hat{\mu}_{i-1}(n) < 0$, we can see whether our strategy outperforms "buy-and-hold" or not.

We consider the following three scenarios:

1) No transaction costs.
2) 0.1% transaction costs.
3) 0.2% transaction costs.

We test our trading strategy on the 12 stock indices described in Section 5. Our test is divided into three parts. Firstly, we select the following 6 moving-window sizes *n*: 40, 80, 120, 160, 200, 240, and test our trading strategy for these 6 moving window sizes on each stock index for zero, 0.1% and 0.2% transaction costs. In the second part, for each stock index and each scenario of amount of transaction costs (0%, 0.1% and 0.2%), we test our strategy for all cases of moving-window sizes *n* under the constraint $n \le 240$ (without this constraint, we have to test infinite number of moving-window sizes, which is impossible). We find out the moving-window size which gives the maximum profit for our strategy, i.e. the optimal moving-window size. The corresponding strategy will be called the optimal strategy. Finally, assuming no transaction costs and using the 6 selected moving-window sizes in the first part, we track our strategy and "buy-and-hold" along the whole period of observation to compare the difference between the profits of our strategy and "buy-and-hold" at different times in the period.

5. Data

We select the timeline and data for our tests. We set our period of observation as December 29, 1995 – December 31, 2014, a total of 4959 observations. This 19-year period is long enough for a time series analysis, and covers the recent major financial crises like the Asian financial crisis in 1997–98 and the global financial crisis in 2008. As described in Section 4, since the largest moving-window size we choose is 240, the calculation of the estimated value of Shiryaev-Zhou index $\hat{\mu}_i(n)$ on day i using the moving-window size $n = 240$ requires the stock price on day $i - 240$ to be known. Hence we trace back the timeline by 240 days, i.e. back to January 27, 1995.

Next, we select stock indices for our tests. We select 6 major, developed economies: Hong Kong, Japan, U.S., U.K., France and Germany. All of them have a securitized real estate index which belongs to the FTSE EPRA/NAREIT Global Real Estate Index Series and can be dated back to January 27, 1995 or before. For each economy, we select one securitized real estate index and one general equity index from Bloomberg, making up a total of 12 stock indices. Table 2 shows the 12 stock indices chosen. Note that the code in the bracket next to the index indicates the Bloomberg code of that index. The six general equity indices consist of the most frequently traded equities in the corresponding economies. They are commonly accepted as benchmarks of performance of stock markets of the corresponding economies. All of the six securitized real estate indices belong to the FTSE EPRA/NAREIT Global Real Estate Index Series so that they are compatible. They consist of the most frequently traded real estate stocks in the corresponding economies and thus can truly reflect the performance of the overall real estate market of the corresponding economies.

Table 2. The stock indices we choose

Economy	General equity index	Securitized real estate index
Hong Kong	Hang Seng Index (HSI)	FTSE EPRA/NAREIT Hong Kong Index (ELHK)
Japan	Tokyo Stock Exchange Tokyo Price Index Topix (TPX)	FTSE EPRA/NAREIT Japan Index (ELJP)
U.S.	S&P 500 Index (SPX)	FTSE EPRA/NAREIT US Index (UNUS)
U.K.	FTSE 100 Index (UKX)	FTSE EPRA/NAREIT UK Index (ELUK)
France	CAC 40 Index (CAC)	FTSE EPRA/NAREIT France Index (EPFR)
Germany	DAX Index (DAX)	FTSE EPRA/NAREIT Germany Index (EPGR)

6. The results

6.1. The optimal moving-window size

Here we apply the "buy-and-hold" strategy and the trading strategy described in Section 4 on the 12 stock indices chosen in Section 5 during the during the whole period of observation. We consider all the three scenarios of different amounts of transaction costs mentioned in Section 4 (0%, 0.1% and 0.2%). We select the following 6 different moving-window sizes *n* for our strategy described in Section 4: 40, 80, 120, 160, 200, 240. We determine which moving-window size *n* yields the maximum amount of profit for our strategy for a particular amount of transaction costs for each stock index. The results are shown in the Table 3 (note that for our trading strategy, the base for calculating the percentage profit is the initial cost. Furthermore, the cases of which our strategy outperforms "buy-and-hold" are highlighted in red):

From Table 3, the performance of our strategy varies among different stock indices. The best performance index are ELUK and EPGR, where our strategy outperforms "buy-and-hold" for all 6 selected moving-window sizes, and for all three scenarios of different amounts of transaction costs. The worst performing is UKX, where our strategy underperforms "buy-and-hold" for all cases except when the moving-window size is 240 and there are no transaction costs. Furthermore, our strategy outperforms "buy-and-hold" on the six securitized real estate indices for 65 out of 108 cases, compared with 49 out of 108 cases on the six general equity indices, showing that our strategy is more effective on the securitized real estate markets than on the general equity markets. Our strategy beats "buy-and-hold" for over half (114) of the total of 216 cases, reflecting that our strategy is slightly superior.

Table 3 show that the profit of our strategy changes as the moving-window size varies. A similar trend can be found: as the moving-window size increases, the profit of our strategy increases first, but then eventually decreases. The reason is explained by Hui and Chan (2014a): Increasing the moving-window size would result in a "smoothing effect", reducing the fluctuation of the Shiryaev-Zhou index, so our strategy would be more profitable, especially when transaction costs exist. However, if the moving-window size is too large, the estimated value of Shiryaev-Zhou index $\hat{\mu}_i(n)$ would lag behind the stock price even more, so the chance that the stock price is rising when $\hat{\mu}_i(n)$ is negative increases, reducing the profits of our strategy. Therefore, an optimal moving-window size for our strategy (which maximizes the profit of our strategy, which is then the optimal strategy) exists, which will be found later in this section.

Table 3 also shows that the amount of transaction costs would affect the resulting profit of our strategy. From the table, an increase in transaction costs reduces the profit of our strategy by a much larger extent than the profit when applying the "buy-and-hold" strategy. This is explained by Hui and Yam (2012), Hui and Chan (2014a) and Hui *et al.* (2014). If we look closer into the profit of our strategy under different moving-window sizes, we can see that in general, when the moving-window size increases, an increase in transaction costs would reduce the profit of our strategy by a smaller extent. In addition, the number of times of buying (or selling) the stock index for our strategy generally decreases as the moving-window size increases, except for a few exceptions (see Table 3). The reason can be explained as follows: from (3.4), (3.6) and (3.7), $\hat{\mu}_i(n)=\frac{\hat{\alpha}_i(n)}{\hat{\sigma}_i^2(n)}-0.5=\frac{\overline{r_i}(n)}{s_i^2(n)}-0.5$, while $\overline{r_i}(n)$ is the mean of the daily return of the stock (or stock index) from day $i-n+1$ to day *i* (see (3.4)). Therefore, increasing the moving-window size would result in a "smoothing effect", reducing the fluctuation of $\hat{\mu}_i(n)$. From Section 4, according to our trading strategy, we buy one unit of the stock index when $\hat{\mu}_i(n)$ turns from negative to positive, and sell one unit of it vice versa (see Table 1). Hence the number of times of buying (or selling) the stock index for our strategy is equal to the number of times $\hat{\mu}_i(n)$ turns from negative to positive (or from positive to negative). For a larger moving-window size, since $\hat{\mu}_i(n)$ fluctuates less frequently, the number of times of buying (or selling) the stock index for our strategy decreases. Thus when the amount of transaction costs increases, the resulting profit of our strategy decreases by a smaller extent. By this rationale, the optimal moving-window size for our strategy should either remain unchanged or increase when there are more transaction costs.

If $\hat{\mu}_i(n)$ fluctuates more frequently, i.e. the number of times of buying (or selling) the stock index for our strategy increases, then, according to Hui and Chan (2014a), the length of "holding periods" and "non-holding periods" are longer, so there is a higher chance that $\hat{\mu}_{i-1}(n)<0$, but the stock price is still rising on day *i*. Therefore, our strategy would be more likely to be outperformed by the "buy-and-hold" strategy. This can be seen from Table 3 that UKX has the largest average no. of times of buying (or selling) the stock index for our strategy, and the performance of our strategy is the worst on this index (our strategy beats "buy-and-hold" for one case only). On the other hand, ELUK has the smallest average no. of times of buying (or selling) the stock index for our strategy, while our strategy outperforms "buy-and-hold" for all cases on this index.

Then we take a step further to find the optimal moving-window size for our strategy on each stock index, under 0%, 0.1% and 0.2% transaction costs, with the constraint $n\le 240$. The Table 4 show the optimal moving-window sizes for our strategy on the 12 stock indices under the three different amounts of transaction costs (the cases of which our strategy outperforms "buy-and-hold" are highlighted in red).

Table 3. Comparison between the profits of our strategy and "buy-and-hold"

Moving-window size	40	80	120	160	200	240	buy-and-hold
ELHK							
No transaction costs	2558.97 395.54%	2314.01 357.67%	2359.59 364.72%	2166.15 334.82%	1426.32 220.46%	1653.75 255.62%	1311.13 202.66%
0.1% transaction costs	2232.43 344.72%	2059.63 318.04%	2191.84 338.45%	2041.22 315.19%	1305.54 201.59%	1523.42 235.24%	1308.52 202.06%
0.2% transaction costs	1905.89 294.00%	1805.26 278.48%	2024.09 312.24%	1916.28 295.61%	1184.76 182.76%	1393.09 214.90%	1305.92 201.45%
No. of times of buying (or selling) the stock index for our strategy	129	104	69	53	52	49	
Average no. of times of buying (or selling) the stock index for our strategy: 76.00							
ELJP							
No transaction costs	1853.30 124.93%	1259.93 84.93%	960.02 64.72%	2478.72 167.09%	1078.16 72.68%	1234.94 83.25%	1724.53 116.25%
0.1% transaction costs	1313.05 88.43%	856.49 57.68%	611.31 41.17%	2184.58 147.12%	790.67 53.25%	967.06 65.13%	1719.84 115.82%
0.2% transaction costs	772.80 51.99%	453.05 30.48%	262.61 17.67%	1890.44 127.18%	503.17 33.85%	699.18 47.04%	1715.15 115.39%
No. of times of buying (or selling) the stock index for our strategy	149	112	98	80	85	79	
Average no. of times of buying (or selling) the stock index for our strategy: 100.50							
UNUS							
No transaction costs	1868.59 197.34%	1478.99 156.19%	1785.52 188.56%	2076.63 219.31%	2083.05 219.99%	1490.08 157.36%	1902.39 200.91%
0.1% transaction costs	1374.17 144.98%	1127.01 118.90%	1496.70 157.91%	1852.31 195.42%	1909.21 201.43%	1348.26 142.24%	1898.59 200.31%
0.2% transaction costs	879.76 92.72%	775.03 81.69%	1207.89 127.31%	1627.99 171.59%	1735.38 182.90%	1206.43 127.15%	1894.80 199.71%
No. of times of buying (or selling) the stock index for our strategy	138	96	78	63	48	39	
Average no. of times of buying (or selling) the stock index for our strategy: 77.00							
ELUK							
No transaction costs	2071.87 194.87%	1806.24 169.88%	2415.24 221.24%	2214.02 208.24%	1877.77 151.50%	1485.02 139.67%	715.00 67.25%
0.1% transaction costs	1684.99 158.32%	1522.78 143.08%	2233.04 204.35%	2077.31 195.18%	1776.70 166.94%	1386.20 130.25%	712.16 66.91%
0.2% transaction costs	1298.12 121.85%	1239.31 116.33%	2050.84 187.49%	1940.60 182.16%	1675.62 157.28%	1287.38 120.84%	709.32 66.58%
No. of times of buying (or selling) the stock index for our strategy	137	98	69	55	43	41	
Average no. of times of buying (or selling) the stock index for our strategy: 73.83							
EPFR							
No transaction costs	3938.5 610.53%	2945.7 456.63%	3378.81 523.77%	3218.53 488.83%	2580.17 399.97%	2447.07 379.34%	2940.92 455.89%
0.1% transaction costs	3390.66 525.09%	2577.67 399.18%	3049.83 472.30%	2906.54 441.01%	2398.35 371.41%	2227.59 344.97%	2936.69 454.78%

Continued of Table 3

Moving-window size	40	80	120	160	200	240	buy-and-hold
0.2% transaction costs	2842.82 439.81%	2209.64 341.85%	2720.84 420.94%	2594.55 393.28%	2216.52 342.91%	2008.12 310.67%	2932.46 453.67%
No. of times of buying (or selling) the stock index for our strategy	140	87	75	70	43	55	
Average no. of times of buying (or selling) the stock index for our strategy: 78.33							
EPGR							
No transaction costs	576.43 122.04%	1035.08 124.03%	861.27 193.79%	859.48 100.64%	904.69 191.14%	1126.03 226.13%	–75.71 –9.14%
0.1% transaction costs	363.52 76.88%	879.09 105.23%	747.07 167.93%	752.89 88.07%	814.35 171.88%	1060.54 212.76%	–77.29 –9.32%
0.2% transaction costs	150.61 31.82%	723.10 86.47%	632.87 142.12%	646.30 75.53%	724.01 152.66%	995.04 199.43%	–78.87 –9.50%
No. of times of buying (or selling) the stock index for our strategy	158	110	84	72	65	53	
Average no. of times of buying (or selling) the stock index for our strategy: 90.33							
HSI							
No transaction costs	24826.65 246.46%	11077.55 109.97%	13622.53 135.23%	12201.14 121.12%	13942.80 138.41%	16892.37 167.69%	13531.65 134.33%
0.1% transaction costs	20474.63 203.05%	7277.52 72.17%	10243.78 101.59%	9827.16 97.46%	11681.05 115.84%	15483.45 153.55%	13497.97 133.86%
0.2% transaction costs	16122.62 159.73%	3477.49 34.45%	6865.03 68.01%	7453.19 73.84%	9419.31 93.32%	14074.54 139.44%	13464.29 133.40%
No. of times of buying (or selling) the stock index for our strategy	132	115	93	66	63	37	
Average no. of times of buying (or selling) the stock index for our strategy: 84.33							
TPX							
No transaction costs	–197.99 –12.55%	764.72 48.47%	945.43 59.92%	141.01 8.94%	348.39 22.08%	906.29 57.44%	–170.19 –10.79%
0.1% transaction costs	–599.45 –37.96%	498.78 31.58%	776.57 49.17%	–53.35 –3.38%	182.82 11.58%	766.20 48.52%	–173.18 –10.97%
0.2% transaction costs	–1000.90 –63.31%	232.83 14.73%	607.71 38.44%	–247.72 –15.67%	17.26 1.09%	626.11 39.61%	–176.16 –11.14%
No. of times of buying (or selling) the stock index for our strategy	163	114	72	84	72	62	
Average no. of times of buying (or selling) the stock index for our strategy: 94.50							
SPX							
No transaction costs	504.40 81.89%	659.63 107.09%	1465.49 237.93%	1507.33 244.72%	1441.38 234.02%	1443.81 234.41%	1442.97 234.27%
0.1% transaction costs	55.56 9.01%	381.04 61.80%	1300.02 210.86%	1377.07 223.35%	1323.52 214.67%	1341.26 217.54%	1440.30 233.61%
0.2% transaction costs	–393.28 –63.72%	102.45 16.60%	1134.55 183.83%	1246.81 202.02%	1205.67 195.36%	1238.72 200.71%	1437.62 232.94%
No. of times of buying (or selling) the stock index for our strategy	181	112	68	53	49	42	
Average no. of times of buying (or selling) the stock index for our strategy: 84.17							

End of Table 3

Moving-window size	40	80	120	160	200	240	buy-and-hold
UKX							
No transaction costs	–936.59	–413.28	2342.94	548.81	2474.24	2927.64	2876.79
	–25.39%	–11.20%	63.51%	14.88%	67.07%	79.35%	77.98%
0.1% transaction costs	–2957.58	–1928.93	1255.41	–538.99	1492.85	2261.69	2866.53
	–80.09%	–52.23%	33.99%	–14.60%	40.42%	61.24%	77.62%
0.2% transaction costs	–4978.56	–3444.58	167.88	–1626.80	511.46	1595.75	2856.28
	–134.68%	–93.18%	4.54%	–44.01%	13.84%	43.17%	77.27%
No. of times of buying (or selling) the stock index for our strategy	183	138	95	96	86	56	
Average no. of times of buying (or selling) the stock index for our strategy: 109.00							
CAC							
No transaction costs	2169.71	3144.29	3788.46	4311.59	4273.04	4461.57	2400.78
	115.91%	167.97%	196.17%	219.50%	228.26%	238.34%	128.25%
0.1% transaction costs	707.37	2309.08	3123.30	3807.25	3795.10	4110.67	2394.64
	37.75%	123.23%	161.57%	193.63%	202.53%	219.37%	127.79%
0.2% transaction costs	–754.96	1473.87	2458.15	3302.91	3317.15	3759.77	2388.49
	–40.25%	78.58%	127.03%	167.81%	176.85%	200.44%	127.34%
No. of times of buying (or selling) the stock index for our strategy	179	102	87	67	62	45	
Average no. of times of buying (or selling) the stock index for our strategy: 90.33							
DAX							
No transaction costs	5908.41	5155.23	8598.13	9540.06	9012.15	9731.41	7544.86
	261.35%	225.63%	380.33%	422.00%	398.65%	430.46%	333.74%
0.1% transaction costs	3934.89	3692.69	7686.61	9092.21	8604.62	9513.73	7532.79
	173.88%	161.45%	339.67%	401.79%	380.24%	420.41%	332.87%
0.2% transaction costs	1961.37	2230.16	6775.09	8644.31	8197.08	9296.05	7520.73
	86.59%	97.41%	299.09%	381.61%	361.87%	410.38%	332.01%
No. of times of buying (or selling) the stock index for our strategy	163	123	77	40	37	20	
Average no. of times of buying (or selling) the stock index for our strategy: 76.67							

Table 4 shows that the optimal moving-window size for our strategy varies among the 12 stock indices. The indices ELHK and HSI have the smallest optimal moving-window sizes. The optimal moving-window size for our strategy on ELHK is 25 for all three cases, while the optimal moving-window size for our strategy on HSI is 19 without transaction costs, and 45 with 0.1% or 0.2% transaction costs. Other indices have much larger optimal moving-window sizes, five of which have optimal moving-window sizes of greater than 200. To explain this result, we can refer back to Table 3. From the table, when the moving-window size is 40, ELHK has the least number of times of buying (or selling) the stock index for our strategy among the six securitized real estate indices, while HSI has the least number of times of buying (or selling) the stock index for our strategy among the six general equity estate indices. This means that $\hat{\mu}_i(n)$ changes sign less frequently, and the length of "holding periods" and "non-holding periods" are longer, so there is a lower chance that $\hat{\mu}_{i-1}(n)<0$, but the stock price is still rising on day *i*. Therefore, our strategy is more likely to outperform the "buy-and-hold" strategy for smaller moving-window sizes for the indices HSI and ELHK, resulting in their smaller optimal moving-window sizes. Japan's securitized real estate index ELJP has the third smallest optimal moving-window size of 90. Overall, the indices of Asian economies have smaller optimal moving-window sizes than the indices of western economies do.

Table 4. The optimal moving-window sizes for our strategy on the 12 indices

Transaction cost	buy-and-hold	Optimal strategy	Optimal moving-window size	No. of times of buying (or selling) the stock index for the optimal strategy
ELHK				
0%	1311.13 202.66%	3762.8 581.61%	25	158
0.1%	1308.52 202.06%	3381.28 522.12%	25	158
0.2%	1305.92 201.45%	2999.75 462.74%	25	158
ELJP				
0%	1724.53 116.25%	3480.43 234.62%	90	100
0.1%	1719.84 115.82%	3116.15 209.85%	90	100
0.2%	1715.15 115.39%	2751.87 185.14%	90	100
UNUS				
0%	1902.39 200.91%	2899.79 306.24%	131	49
0.1%	1898.59 200.31%	2729.80 288.00%	131	49
0.2%	1894.80 199.71%	2559.80 269.80%	131	49
ELUK				
0%	715.00 67.25%	2886.04 264.37%	108	69
0.1%	712.16 66.91%	2695.57 246.68%	108	69
0.2%	709.32 66.58%	2505.09 229.02%	108	69
EPFR				
0%	2940.92 455.89%	3623.38 561.69%	121	71
0.1%	2936.69 454.78%	3306.47 512.05%	121	71
0.2%	2932.46 453.67%	2990.54 462.66%	122	68
EPGR				
0%	–75.71 –9.14%	1215.58 244.48%	234	52
0.1%	–77.29 –9.32%	1151.95 231.45%	234	52
0.2%	–78.87 –9.50%	1091.74 219.13%	233	49

Transaction cost	buy-and-hold	Optimal strategy	Optimal moving-window size	No. of times of buying (or selling) the stock index for the optimal strategy
HSI				
0%	13531.65 134.33%	28271.91 280.66%	19	236
0.1%	13497.97 133.86%	23016.58 228.26%	45	135
0.2%	13464.29 133.40%	18535.70 183.64%	45	135
TPX				
0%	–170.19 –10.79%	1276.8 80.93%	222	61
0.1%	–173.18 –10.97%	1141.07 72.25%	222	61
0.2%	–176.16 –11.14%	1005.35 63.60%	222	61
SPX				
0%	1442.97 234.27%	1621.92 263.33%	210	45
0.1%	1440.30 233.61%	1514.21 245.60%	210	45
0.2%	1437.62 232.94%	1406.51 227.90%	210	45
UKX				
0%	2876.79 77.98%	3095.24 83.90%	220	60
0.1%	2866.53 77.62%	2425.34 65.67%	222	55
0.2%	2856.28 77.27%	1795.85 48.58%	222	55
CAC				
0%	2400.78 128.25%	5789.28 303.36%	231	48
0.1%	2394.64 127.79%	5395.64 282.45%	231	48
0.2%	2388.49 127.34%	5002.01 261.58%	231	48
DAX				
0%	7544.86 333.74%	10929.67 483.47%	140	54
0.1%	7532.79 332.87%	10280.72 454.31%	140	54
0.2%	7520.73 332.01%	9631.76 425.20%	140	54

Table 5. No. of fluctuations of the 12 stock indices during the period of observation

Index	ELHK	ELJP	UNUS	ELUK	EPFR	EPGR	HSI	TPX	SPX	UKX	CAC	DAX
No. of fluctuations	2309	2330	2341	2304	2450	2416	2434	2330	2539	2444	2521	2443

In particular, the indices EPGR and TPX yield a negative return throughout the whole period of observation, meaning that the "buy-and-hold" strategy incurs a loss. The optimal moving-window sizes for our strategy on these two indices are very large (222 to 234). We will use a graphical method to explain this result in the next sub-section. From Table 4, we can see that the optimal strategy outperforms the "buy-and-hold" strategy for almost all cases (33 out of 36). The exceptional cases are the indices SPX and UKX. The optimal strategy underperforms "buy-and-hold" on SPX with 0.2% transaction costs, and on UKX with 0.1% or 0.2% transaction costs. This shows that if we choose a suitable moving-window size, our strategy can really beat the "buy-and-hold" strategy for most cases. All the three cases which the optimal strategy underperforms "buy-and-hold" are on general equity indices. The optimal strategy outperforms "buy-and-hold" for all cases on the six securitized real estate indices. When the optimal moving-window size is used, our strategy still works better on securitized real estate indices than on general equity indices. This may be a consequence of liquidity.

For each economy, the optimal moving-window size for our strategy is much larger on the general equity index of the economy than on the securitized real estate index of that economy, except for Hong Kong with no transaction costs, and for Germany for all three cases. To explain this, we find the number of fluctuations, i.e. the number of times the daily return changes sign, of the 12 stock indices during the whole period of observation as shown in Table 5. From the table, we can see that for each economy, the number of fluctuations of the general equity index of the economy is more than that of the securitized real estate index of that economy (except Japan, of which the number of fluctuations is the same for both indices). With more fluctuations, $\hat{\mu}_i(n)$ fluctuates more frequently as well. Since $\hat{\mu}_i(n)$ lags behind the stock index itself, there is a greater chance that $\hat{\mu}_{i-1}(n) < 0$ even though the stock index is rising on day *i*, especially when using smaller moving-window sizes. However, when a larger moving-window size is used, the "smoothing effect" would lower the chance of this mismatch. Therefore, the optimal moving-window size would tend to be larger.

Table 4 also shows the relationship between the amount of transaction costs and the optimal moving-window size for our strategy. For eight of the 12 stock indices, the optimal moving-window size for our strategy remains unchanged as the amount of transaction costs varies. For the remaining four indices, three of them (EPFR, HSI, UKX), the optimal moving-window size increases as the amount of transaction costs increases. For example, the optimal moving-window size for our strategy on HSI is 19 without transaction costs, but the optimal moving-window size increases to 45 with 0.1% or 0.2% transaction costs. Taking a closer look into Table 4, we can see that the number of times we buy (or sell) HSI for our strategy is 236 when the moving-window size is 19, but this number falls significantly to 135 when the moving-window size increases to 45. As explained before, a smaller number of times of buying (or selling) the stock index for our strategy would cause the resulting profit of our strategy to drop by a smaller extent when the amount of transaction costs increases. This causes the optimal moving-window size to increase. The index EPGR is the only exception, with an optimal window size of 234 when there are zero or 0.1% transaction costs, and 233 with 0.2% transaction costs. The reason is that the number of times of buying (or selling) EPGR for our strategy is greater with a moving-window size of 234 (52 times) than with a moving-window size of 233 (49 times). This is one of the few exceptions that the number of times of buying (or selling) the stock index for our strategy increases when the moving-window size increases.

6.2. The resulting profits of our strategy at different times throughout the period

The optimal strategy described in the previous sub-section considers the whole time period. However, during different sections of time period, the optimal strategy (and hence the optimal moving-window size) may be different. To investigate how the optimal strategy varies with time, in this sub-section, we compare the resulting profits of the "buy-and-hold" strategy and our strategy using the 6 selected moving-window sizes (40, 80, 120, 160, 200, 240) at different times along the whole period of observation. For the sake of convenience, we only consider the case without transaction costs. The results are shown in Figures 1–12.

From Figures 1–12 there are some periods that our strategy and "buy-and-hold" move in the same direction. These periods correspond to "holding periods" where $\hat{\mu}_{i-1}(n) \geq 0$. During other periods, our strategy moves horizontally, while the "buy-and-hold" strategy moves as normal. These periods correspond to "non-holding periods" where $\hat{\mu}_{i-1}(n) < 0$ (refer to Section 4 for the definition of "holding period" and "non-holding period"). Hence it is the movement of stock price during the "non-holding period" which makes the difference in profits between the three strategies. Note that for our strategy, for different moving-window sizes, the "holding periods" and "non-holding periods" are different. Since a larger moving-window size would result in a "smoothing effect", the "non-holding periods" for our strategy are generally

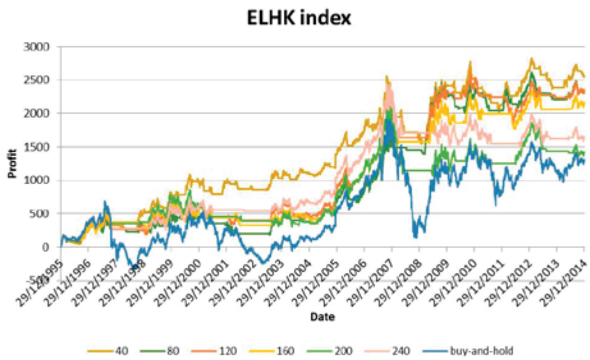

Figure 1. Profits of our strategy and "buy-and-hold" on ELHK index (without transaction costs)

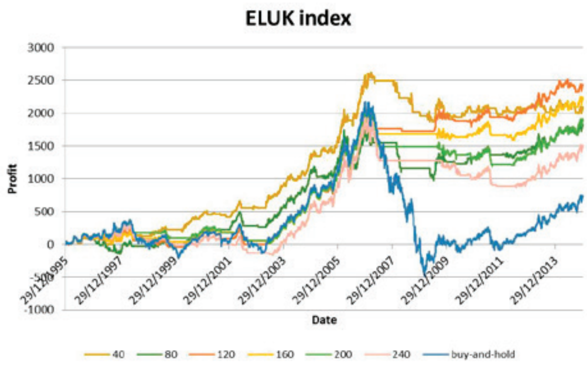

Figure 4. Profits of our strategy and "buy-and-hold" on ELUK index (without transaction costs)

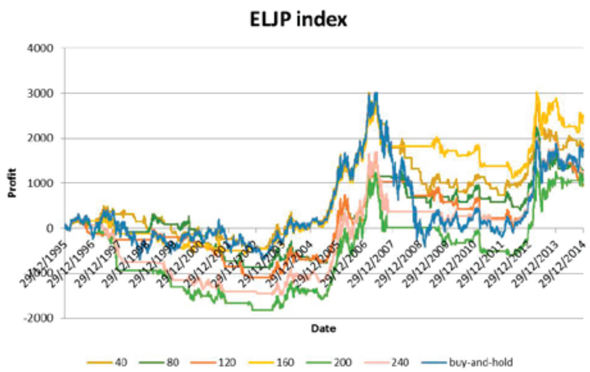

Figure 2. Profits of our strategy and "buy-and-hold" on ELJP index (without transaction costs)

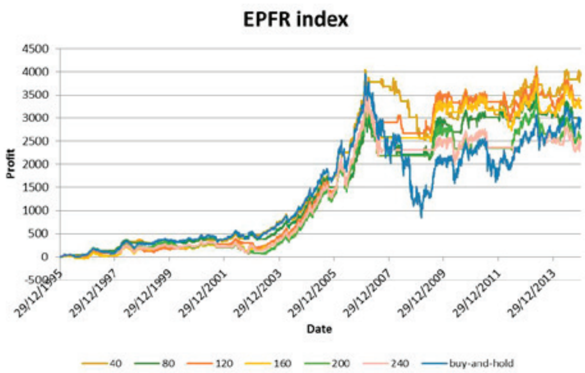

Figure 5. Profits of our strategy and "buy-and-hold" on EPFR index (without transaction costs)

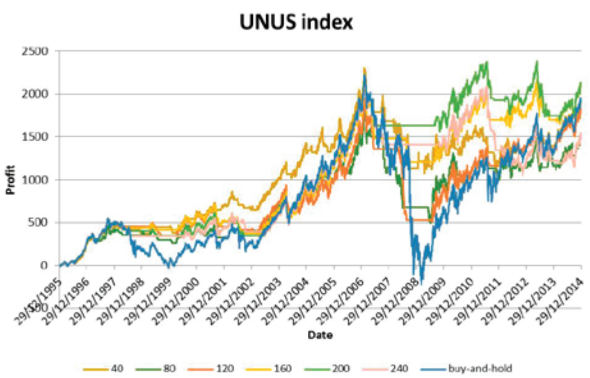

Figure 3. Profits of our strategy and "buy-and-hold" on UNUS index (without transaction costs)

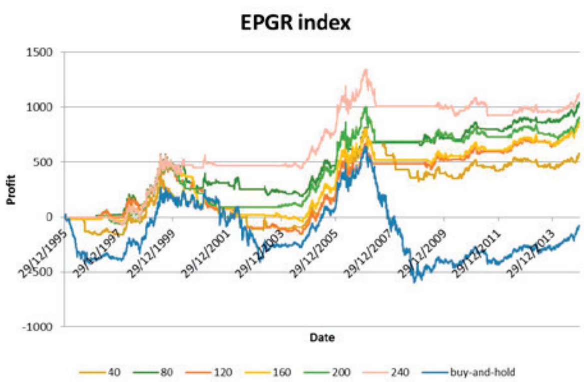

Figure 6. Profits of our strategy and "buy-and-hold" on EPGR index (without transaction costs)

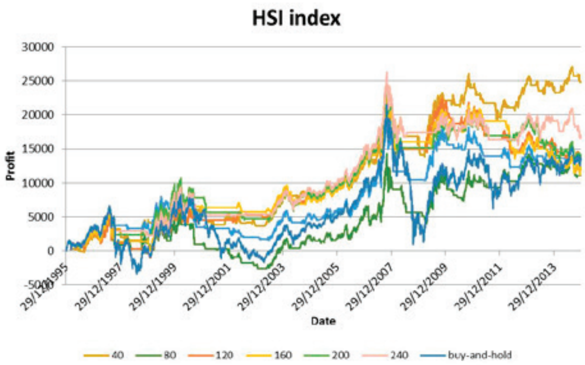

Figure 7. Profits of our strategy and "buy-and-hold" on HSI index (without transaction costs)

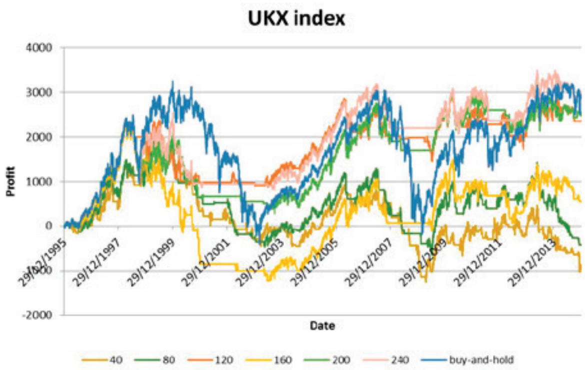

Figure 10. Profits of our strategy and "buy-and-hold" on UKX index (without transaction costs)

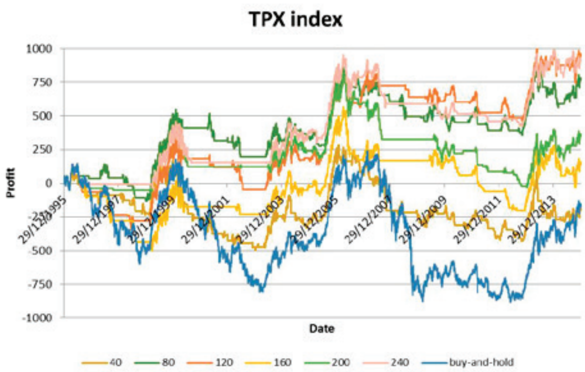

Figure 8. Profits of our strategy and "buy-and-hold" on TPX index (without transaction costs)

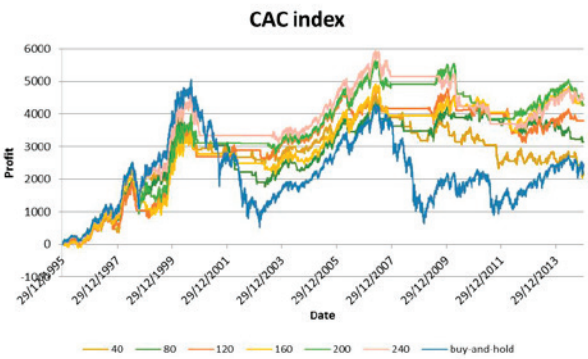

Figure 11. Profits of our strategy and "buy-and-hold" on CAC index (without transaction costs)

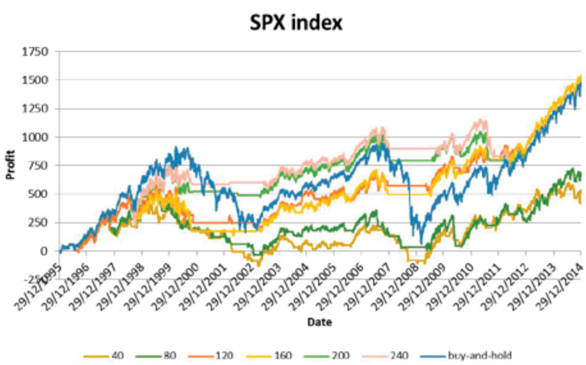

Figure 9. Profits of our strategy and "buy-and-hold" on SPX index (without transaction costs)

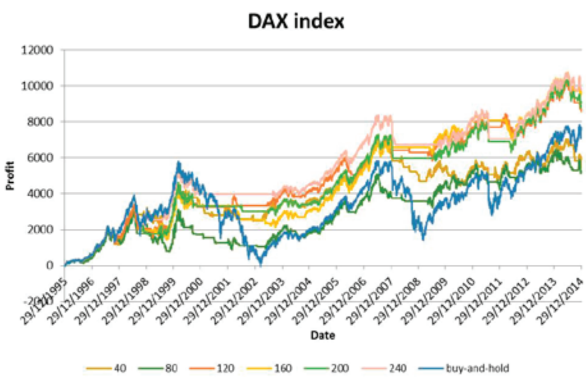

Figure 12. Profits of our strategy and "buy-and-hold" on DAX index (without transaction costs)

longer when using larger moving-window sizes. We can also see that our strategy outperform "buy-and-hold" during some periods, but underperform during other periods. The "optimal" moving-window size for our strategy also varies from time to time.

In particular, for most of the 12 stock indices, our strategy outperforms "buy-and-hold" the most during the period late 2008 – early 2009, especially when the moving-window size is 240. This period corresponds to the time when the global financial crisis was most severe when many stock markets fell sharply to a trough. Since the stock indices keeps falling, $\hat{\mu}_i(n)$ stays negative for a long time (so this period is a "non-holding period"). Thus we need not hold the stock indices according to our strategy, avoiding losses by holding the indices when applying the "buy-and-hold" strategy. Therefore, our strategy outperforms "buy-and-hold" during this period. In particular, if we use a larger moving-window size, our strategy would beat "buy-and-hold" by a larger extent during this period. The reason is that the stock index movements are not perfectly smooth. During a long period of bust, the stock index seldom keeps on falling continuously. There are always short periods when the stock price is rising. If the moving-window size is small, then there is a larger chance that $\hat{\mu}_i(n) \geq 0$, i.e., "holding periods" exist. Since $\hat{\mu}_i(n)$ lags behind the stock price, the stock price is often still falling during those "holding periods", so our strategy would suffer a loss. However, if a larger moving-window size is used, the "smoothing effect" would lower the chance that $\hat{\mu}_i(n) \geq 0$. Thus the "non-holding period" is longer and hence our strategy can outperform the "buy-and-hold" strategy by a larger extent. This is seen from Figures 1–12 that from 2008 to early 2009, for larger moving-window sizes (200 and 240), our strategy is represented by a long horizontal line, indicating a long "non-holding period". However, for shorter moving-window sizes (40 and 80), our strategy is represented by short "non-holding periods" accompanied with some "holding periods" during which the stock index is usually falling. Therefore, using larger moving-window sizes for our strategy would earn a larger profit than using smaller moving-window sizes from 2008 to early 2009. Similar result is found for the period 2001–2003, when the U.S. IT bubble burst, followed by the SARS outbreak in Hong Kong in early 2003. Our strategy is most effective around the trough of an economic cycle. On the other hand, in late 2007, for the majority of the 12 stock indices, our strategy underperforms "buy-and-hold" for most moving-window sizes. Looking into Figures 1–12, most of the stock indices attain a peak at this time after a long period of boom during 2003–2007. During this period, the 12 stock indices are on a rising trend in general, but there are several short periods when the stock indices fall. Hence $\hat{\mu}_i(n)$ remains positive most of the time, but there are short periods when $\hat{\mu}_i(n)$ turns negative. However, since $\hat{\mu}_i(n)$ lags behind the stock price, the stock index is often still rising during those "non-holding periods". In this case our strategy would be outperformed by "buy-and-hold". This is reflected in Figures 1–12 that for most of the 12 stock indices, our strategy outperforms "buy-and-hold" for most moving-window sizes in 2003, but as time goes on, the gap narrows gradually and our strategy is outperformed by "buy-and-hold" for more and more moving-window sizes. In late 2007, when the stock indices attain a peak, our strategy even underperforms "buy-and-hold" for most moving-window sizes. This means that for the period 2003–2007, the true optimal strategy is, in fact, the "buy-and-hold" strategy. Our strategy is less effective around the peak of an economic cycle. Hence the optimal strategy varies from time to time. During busts, our strategy with large moving window sizes would earn the maximum profit. However, during booms, the "buy-and-hold" strategy is optimal. This can also explain why the indices EPGR and TPX, which yield a negative return throughout the period, have very large optimal moving-window sizes for our strategy. Figures 6 and 8 show that both indices have major busts periods during the period. In particular, the EPGR index fell by over 1000 points during early 2007–early 2009, causing EPGR to be the best performing index, where our strategy outperforms "buy-and-hold" significantly for all 12 selected moving-window sizes. This reflects that our strategy is particularly effective on adverse-performing stocks or stock indices, similar to Hui and Chan (2014a)'s result that their strategies outperformed "buy-and-hold" the most for the stock New World, which fell significantly during the period. What we add to Hui and Chan (2014) is that for those adverse-performing stocks or stock indices, the optimal moving-window sizes for our strategy tend to be larger.

7. Conclusion

In this study, we test our generalized time-dependent strategy with different moving-window sizes and the "buy-and-hold" strategy on the securitized real estate and general equity indices of six economies: Hong Kong, Japan, U.S., U.K., France, Germany, during the period December 29, 1995–December 31, 2014, and find the optimal moving-window sizes for our strategy on the 12 stock indices. The following lists out our main results:

1) In general, when transaction costs increase, the optimal moving-window size for our strategy remains unchanged or increases.
2) For each economy, the optimal moving-window size for our strategy is generally larger on the general equity index than on the securitized real estate index.
3) The indices of Asian economies have smaller optimal moving-window sizes than the indices of western economies do.
4) Our strategy outperforms "buy-and-hold" by a larg-

er extent during busts than during booms. Furthermore, for adverse performing stock indices, larger moving-window size leads to greater profit for our strategy.

Compared with previous studies, Hui and Yam (2014), Hui *et al.* (2014) and Hui and Chan (2014a) all used the same moving-window size (130) for their trading strategies. Hence, for a particular stock or stock index, fixing the amount transaction costs also fixes the resulting profit. We are unable to know whether 130 is really the optimal moving-window size. To solve this problem and hence fill in the gap of knowledge, this study constructs a generalized time-dependent strategy of which the moving-window size can be varied. When the optimal moving-window size is chosen, our strategy outperforms the "buy-and-hold" strategy for most cases, and is also superior to Hui and Yam (2014), Hui *et al.* (2014) and Hui and Chan (2014a)'s strategy (as none of the 12 indices has an optimal moving-window size of 130). Furthermore, by tracking the profit of our strategy at different times along the whole period, we find that our strategy is more effective during busts than during booms. The results in Sub-section 6.2 give the following suggested investment strategy: during stock market downturns, it's advisable to adopt our strategy, preferably with larger moving-window sizes, to prevent losses when the stock prices fall rapidly. However, during long periods of booms, it's better to adhere to the "buy-and-hold" strategy. This strategy is adoptable to property practitioners in trading real estate stocks/funds, too. This implies that in reality, we have to switch strategies regularly in order to earn more profit. The time to switch strategies depends on the market fundamentals like economic conditions and profitability of the companies (for individual stocks). When there are significant changes in such factors, then investors should adjust their strategies. To find out the time of switching strategies which maximizes the profit is an interesting topic for future research. Besides, one can also test our strategy and attempt to find the optimal moving-window size for our strategy on stock indices in other economies and individual stocks as well.

Although our strategy is found to outperform the "buy-and-hold" strategy for more than half of the cases, this does not imply that the EMH is false. This is because according to EMH, no other strategies can beat "buy-and-hold" in the long run. However, we use a finite time period in this study, so the performance of our strategy outside the period of observation is unknown. More evidence is needed to prove whether the EMH is true or not.

The major limitation of this study is that instead of individual stocks, non-tradable stock indices are used. Although there are now ETFs tracking the stock indices, it was only until the 2000s that the ETF market became active. We still use stock indices because they can reflect the performances of the overall markets (in this study, one of our objectives is to test our strategy on the overall markets) and hence are representative. One can replicate our trading strategy on individual stocks as well. This is another possible scope of future research.

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