Modelling Joint Activity-Travel Pattern Scheduling Problem in Multi-modal Transit Networks

Xiao Fu *

School of Transportation, Southeast University

Nanjing 210096, China

Telephone: (+86)18652070293

E-mail: reneefu93@gmail.com; fuxiao@seu.edu.cn

William H.K. Lam

Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University

Hung Hom, Kowloon, Hong Kong

E-mail: william.lam@polyu.edu.hk

* Corresponding author

Abstract

Over the past decades, many activity-based travel behaviour models have been proposed based on individuals' independent decision making. The modelling of individuals' joint activity/travel choices, however, has received less attention. In reality, both independent and joint activities/travels form individual's normal daily activity-travel patterns. Travel surveys have indicated that joint activity/travel constitutes an important part in individuals' daily activity-travel patterns. On this basis, explicit modelling of joint activity/travel choices is an essential component for long-term transport planning. In this study, an activity-based network equilibrium model is proposed for scheduling two-individual joint activity-travel patterns (JATPs) in congested multi-modal transit networks. The proposed model can be used to comprehensively investigate individuals' activity choices (e.g. activity start time and duration, activity sequence) and travel choices (e.g. departure time, route and mode) in multi-modal transit networks, including both independent ones and joint ones. The time-dependent JATP choice problem is converted into an equivalent static user equilibrium model by constructing a joint-activity-time-space (JATS) super-network platform. Joint travel benefit is modelled by incorporating a commonality factor in the JATP utility. A solution algorithm without prior JATP enumeration is proposed to solve the JATP scheduling problem on the JATS super-network. Numerical results show that individuals' independent and joint activity/travel choices can be simultaneously investigated by the proposed model. The impacts of joint travel benefit on individuals' independent and joint activity-travel choices are explicitly investigated.

Keywords
activity-based approach; user equilibrium; joint activity-travel pattern; joint travel benefit

1. Introduction

Travel demands are derived from the desire of people to participate in various financially and socially stimulated activities such as work, eating and shopping. Over the past decades, to perceive the underlying motivation of trip making, increased attention has been given to the activity-based approach in travel behaviour modelling (Hirsh et al. 1986; Kitamura 1988; Axhausen and Gärling 1992; Recker 1995; Yamamoto and Kitamura 1999; Bhat et al. 2004; Timmermans 2005; Goulias et al. 2012; Chow and Recker 2012). It is widely recognized that the activity-based approach can reflect temporal and spatial constraints, household influence, interdependencies of trips, scheduling of activities, and also the linkage between activities and trips.

Many activity-based travel behaviour models are based on individual decision making but joint decisions on activity/travel choices are not explicitly considered in network equilibrium models. In reality, however, both independent and joint activities/travels form essential parts of individuals' daily activity-travel patterns. For example, household members meet at subway stations after work, then travel jointly to have dinner in a shopping mall. With the rapid development of information and telecommunication technology, such joint activity constitutes an ever-increasing share of an individual's daily activity-travel pattern (Ronald et al. 2012). Travel surveys indicate that joint travel has now become a significant portion of travel within regions (Vovsha et al. 2003). From such findings, the importance of explicit analysis and modelling of joint activity/travel choices for long-term transport planning and policy analysis is clear (Bhat et al. 2012).

Currently, a number of transportation studies have investigated the joint activity-travel choice problem with consideration of inter-personal dependencies. The complex nature of inter-personal dependencies results in many studies using the simulation technique. For example, Miller and Roorda (2003) proposed a micro-simulation model to generate daily activity-travel patterns for all individuals in a household on the basis of a conventional trip diary survey. Arentze and Timmermans (2009) developed a need-based model of activity generation for a multi-day planning period taking account of household members' interactions. Dubernet and Axhausen (2013) included joint travels in a multi-agent micro-simulation.

Apart from simulation models, a number of econometric models have also been proposed with the aim of exploring the intra-household behavioural interactions in relation to activity-travel choice behaviour, using structural equation modelling or the random utility approach. For example, the study of out-of-home activities and travel durations by Globe and McNally (1997), a time allocation model for two-individual households that accounts for joint activity participation by Gliebe and Koppelman (2002), the work of Zhang et al. (2009) in which different household utility functions are introduced to represent household members' joint decision making interactions, and the study of household member's work location choice by Gupta et al. (2015).

Compared to the development of activity-based simulation models and econometric models, fewer studies have been devoted to developing activity-based mathematical analytical models such as network equilibrium models to consider intra-household interactions. Activity-based network equilibrium models can provide a comprehensive understanding of individuals' activity-travel choice behaviour, and present more accurate traffic conditions in a congested transportation network. Most existing studies on activity-based network equilibrium, however, are on the basis of one individual level (Lam and Yin 2001; Lam and Huang 2002; Huang and Lam 2005; Li et al. 2010; Ramadurai and Ukkusuri 2010; Ouyang et al. 2011; Fu and Lam 2014; Liu et al. 2015; Chow et al. 2015). These studies do not consider individuals' joint activity-travel choices and ignore intra-household behavioural interactions. As joint participation in activities and travels represent a substantial portion of individuals' daily activity-travel patterns, it is of important interest to develop network equilibrium models which can comprehensively consider individuals' independent and joint activity/travel choices in congested transportation networks.

In many Asian cities such as Hong Kong, most daily travel is conducted using various public transit modes (over 90% in Hong Kong) rather than privately owned cars. Joint travels using public transit may benefit individuals by satisfying a need for communal activity or by offering pleasurable travel experience. The consideration of joint activity-travel choices in long-term transit planning is an important research area, as yet largely unexplored. Hence, a network equilibrium model for scheduling joint activity-travel patterns (JATPs) in multi-modal transit networks is proposed in this study. Joint travel benefit and the impacts of the benefit on various activity/travel choices are explicitly explored by the proposed model.

The problem of coupling constraints, which is a major challenge in JATP modelling, is solved in this study by proposing a novel joint-activity-time-space (JATS) multi-modal super-network. Using the JATS super-network platform, both the independent activity/travel choice and joint choice can be modelled simultaneously. The relationship between activity choices and travel choices can be effectively captured by solving the user equilibrium (UE) problem on the JATS super-network platform. This paper extends the existing theories by developing a comprehensive framework to capture independent and joint activity/travel choices in multi-modal transit networks. A network equilibrium model with consideration of joint travel benefit is explicitly proposed. The ultimate aim of the proposed model is to be used for future long-term strategic planning.

2. Problem Statement and Network Representation

2.1. Joint activity-travel pattern (JATP)

In this study, a JATP concept is proposed to model the activity-travel choices of a two-individual household within the study time period. A JATP consists of

- Individuals' independent activity choices;
- Individuals' joint activity choices;
- Individuals' independent travel choices;
- Individuals' joint travel choices.

Fig. 1 shows an example of a two-individual (i.e. individual A and individual B) JATP from 6:00 to 24:00. The two individuals' all independent and joint activity/travel choices (e.g. time and space coordination, activity sequence and location, activity start time and duration, route and mode choices) throughout the whole time period are depicted as a JATP. It can be seen that the activity sequence of this JATP is home-work-shopping-home. The activity start/end time, activity duration, activity location can be traced. Several trips are conducted between different activities. The travel time of each trip, route choice and mode choice can also be found. Note that both independent and joint activities/travels are included in the JATP. In example Fig. 1, the two individuals shop together after independent work activities, then jointly travel home. The activity-travel choice problem of a two-individual household is termed the JATP scheduling problem. This problem is solved in this study.

Fig. 1 An illustration of a two-individual JATP

2.2. Model assumptions

In order to facilitate essential ideas without loss of generality, the following assumptions are made in this study.

A1: It is assumed that the total population in the study network consists of homogeneous households. Each household is composed of two full-time workers. The two individuals in a household jointly make activity-travel decisions, and the joint decision-making process seeks to maximize the utility of the entire household (Lam and Yin 2001; Ouyang et al. 2011; Fu et al. 2014).

A2: In urban areas such as those in Hong Kong, most individuals remain in the work place during the noon period. In this study, the JATP scheduling problem is divided into two time periods (i.e. morning period before 12:00 noon and afternoon period after 12:00 noon) to reduce the size of super-network. Individuals are assumed to be at work place at 12:00 noon.

A3: The proposed model falls within the static model category for long-term planning at the strategic level. Therefore, it is assumed that individuals have perfect knowledge of traffic conditions throughout the whole network (Fu and Lam 2014).

A4: The JATP is considered in a fixed study horizon, divided into K equally spaced time intervals (Lam and Yin 2001; Zhang et al. 2005; Ouyang et al. 2011; Fu et al. 2014).

A5: No vehicle capacity constraint exists. Crowding discomforts in vehicles and at activity locations are modelled.

Three types of activities are investigated and described in this study: work, shopping, and home activities. Work is considered as an independent activity, while shopping and home activities can be conducted independently or jointly. Home and work are considered as compulsory activities, while shopping is non-compulsory activities (Fu and Lam 2014). The activity choices, including activity sequence, activity location, activity start time and duration are not fixed.

2.3. A joint-activity-time-space (JATS) super-network platform

The synchronization problem (i.e. the temporal and spatial co-ordination among individuals) poses a challenge in joint activity-travel modelling. To produce consistent activity/travel choices, some studies concerned with synchronization of joint activity/travel participation (Dubernet and Axhausen 2013; Fang et al. 2011; Liao et al. 2013b). Liao et al. (2013b) developed joint multi-state super-networks to address the independent and joint mode/route choices of two interacting household members. The above study is the first attempt to extend individual multi-state super-networks (Liao et al. 2013a) to joint multi-state super-networks. The synchronization of mode/route choice, where and when to meet or depart can be represented by the proposed super-network of Liao et al. (2013b). Liao et al.'s multi-state super-network, however, has difficulty in tackling the non-linear fare structures of public transit systems, such as the system in Hong Kong. In Liao et al.'s model, activity duration has to be pre-determined, and link cost is independent of the influence of crowding effect. Furthermore, the joint travel benefit has not been considered in their model. Therefore, as presented in this paper, a novel super-network platform is proposed to incorporate independent and joint activity/travel choices, non-linear fare structures, flexible activity start time and duration, and the crowding effects in the multi-modal transit network.

Consider a multi-modal transit network $M = (U, V)$, where $U = \{i\}$ and $V = \{v\}$ are, respectively, the set of physical nodes and the set of physical links. The multi-modal transit network M can be divided into w sub-networks $M_b = (U_b, V_b)$, $b \in B$, $U_b \subseteq U$, $V_b \subseteq V$, $w = |B|$, where $b \in B$ is a specified transit mode, and U_b and V_b, respectively, are the set of nodes and the set of links associated with the sub-network M_b. The sub-networks are combined and represented by a strongly connected graph $G = (N, A)$ through a state-augmentation approach (Bertsekas 1995), where N is a set of nodes and A is a set of links. The resultant network G is termed the state-augmented multi-modal (SAM) network (Lo et al. 2003). In the SAM network, direct in-vehicle links which may consist of more than one consecutive physical link are used to represent transit fares on a node to node basis, so the SAM network can be used to model the non-linear fare structures in multi-modal transit networks.

Based on the formation of the SAM network, presented in this paper, a JATS super-network expansion approach is proposed to represent individuals' independent and joint activity choices and travel choices over a multi-modal transit network. In this approach, the SAM network is further developed by incorporating time-space coordinates and activity links. This augmentation produces the JATS super-network. The study horizon is divided into K equally spaced time intervals. Let $k = 1, 2, ..., K, K+1$ be the start time of a node or link. The framework of the JATS super-network is given below.

Nodes: Each node in the SAM network is augmented into $K+1$ nodes in JATS super-network. Each node in the JATS super-network is described as JATS node $(ind, (i, l), k)$, where ind is the individual(s) indicator. The value of ind is equal to 1 (2) indicating individual A (B) is at the node, and the value of ind is equal to 12 indicating both A and B are at the node. The JATS nodes with $ind = 12$ are called joint nodes, while the ones with $ind = 1$ or 2 are independent nodes. i is the physical location of the node. l is the alight or aboard indicator. The value of l is equal to 1 (0) indicating that the individual is at the beginning (end) of an in-vehicle link. k indicates the time of the node.

Links: Links in the JATS super-network are classified into five categories, i.e. $A = A_a \cup A_t \cup A_d \cup A_w \cup A_m$.

- A_a is the set of activity links. A_a is constructed between the augmented nodes with the same individual(s) and at the same location to indicate that a particular activity is conducted for one time interval. Each $a_a \in A_a$ is characterised by individual(s), activity location, activity type, and activity start time. The activity time window is not required as the activity utility by time of day is adopted in this study (Lam and Yin 2001). The process of route searching in the JATS super-network can lead to realistic and more generalized results regarding the times to perform activities during the study period. $A_{\mathrm{a}} = A_{\mathrm{a}}^{\mathrm{indep}} \cup A_{\mathrm{a}}^{\mathrm{joint}}$, where $A_{\mathrm{a}}^{\mathrm{indep}}$ denotes the set of independent activity links and $A_{\mathrm{a}}^{\mathrm{joint}}$ denotes the set of joint activity links.

- A_{t} is the set of transfer links. Each transfer link $a_{\mathrm{t}} \in A_{\mathrm{t}}$ is constructed to indicate a transfer between different transit lines/modes. The duration of a transfer link is assumed to be zero in this study but transfer dis-utility is considered.
- A_{d} is the set of direct in-vehicle links made up of physical links. Each in-vehicle link $a_{\mathrm{d}} \in A_{\mathrm{d}}$ represents a direct in-vehicle movement. At the end of each in-vehicle link, an activity can be conducted. It should be noted that a direct in-vehicle link may consist of more than one consecutive physical link. In this way, non-linear fares can be directly represented on a node to node basis.
- A_{w} is the set of waiting links. Each $a_{\mathrm{w}} \in A_{\mathrm{w}}$ is constructed between the independent nodes at the same location to indicate an individual waiting for the other individual for one time interval.
- A_{m} is the set of meeting links. Each $a_{\mathrm{m}} \in A_{\mathrm{m}}$ is constructed between an independent node and a joint node at the same location to represent individuals meet with each other at the node. The duration of a meeting link is assumed to be zero (Liao et al. 2013b).

Access and egress links such as walking links are not considered in this paper, which can be incorporated in further study. It is also necessary to extend the two-individual JATS super-network to a general multi-individual JATS super-network.

Based on previous studies, the recurrent joint activity/travel may be mainly occurred in the AM peak and/or PM peak during a typical weekday. Thus, in this paper, only one episode of joint activity/travel is considered in the morning/afternoon period. Although there may be non-recurrent joint activity/travel particularly during weekend and/or public holidays, it is out of the scope for the current study but can be considered for further investigation.

Fig. 2 shows an example of the JATS super-network consisting of two transit modes (i.e. subway and bus) and two activities (i.e. work and shopping). The two individuals (A and B) work at different places and shop together after work. In this example, the study horizon is divided into three equally spaced time intervals. The travel time of each physical link is one interval.

2.4. JATS super-network expansion algorithm

A rule-based algorithm is proposed to generate the JATS super-network for two-individual household JATP scheduling. With this rule-based algorithm, the conventional multi-modal transit network can be automatically transformed into the

JATS super-network.

Fig. 2 is an example of the network expansion result for a time period after work. Each joint route from the two origins (i.e. one origin for one individual) to the destination (i.e. the same destination for the two individuals) in the JATS super-network represents a feasible JATP in the afternoon period. The detailed steps of the proposed JATS super-network expansion algorithm for the afternoon period are presented in Appendix A.

Following the model assumption A2, individuals are assumed to start the morning period with joint home activity and end that period with independent work activities. In the afternoon period, individuals are assumed to start with independent work activities and end that afternoon period with joint home activity. By splitting the whole time period into two periods, please note that individuals may conduct duplicate activities (e.g. shop in the two periods). The duplication problem should be solved in further research. In the JATS super-network for the morning period, each joint route from the origin (at home) to two destinations (at work places) represents a feasible JATP. The network expansion steps for the morning period are similar and not listed in this paper. With the use of the JATS super-network, individuals' activity choices (i.e. activity locations, sequence and durations) and travel choices (i.e. route and mode choices, transfers), including both independent ones and joint ones, are explicitly represented by different links in the proposed super-network platform. The relationships between activity choices and travel choices are reflected by the JATS super-network topology. The non-linear fare structures in multi-modal transit networks can be explicitly modelled. Flexible activity start time and duration are incorporated. Each joint route in the JATS super-network platform represents a feasible JATP.

Fig. 2 An illustration of the JATS super-network

2.5. Link utility/dis-utility in JATS super-network

In this study, marginal activity utility is specified for different individuals. Considering the crowding discomfort at activity locations, the utility of individual ind performing independent activity link a_{a} is expressed by

$$u_{a_{\text{a}}}^{ind} = \left(1 + \beta'_{a_{\text{a}}} \left(\frac{f_{a_{\text{a}}}}{\kappa_{a_{\text{a}}}}\right)^{\theta'_{a_{\text{a}}}}\right) \cdot \int_{k}^{k+1} \bar{u}_{a_{\text{a}}}^{ind}(\omega) d\omega, \qquad a_{\text{a}} \in A_{\text{a}}^{\text{indep}},\ ind = 1,2 \tag{1}$$

where $\bar{u}_{a_{\text{a}}}^{ind}$ denotes the marginal activity utility of individual ind; $f_{a_{\text{a}}}$ is the passenger flow on the activity link; $\kappa_{a_{\text{a}}}$ is the capacity of the activity location; $\beta'_{a_{\text{a}}}$ and $\theta'_{a_{\text{a}}}$ are model parameters relevant to activity type. $\beta'_{a_{\text{a}}}$ is equal to 0 for compulsory activities (e.g. home and work), as the utility of compulsory activities is assumed not affected by the crowding at the activity locations.

As regards joint activity utility, a group utility function proposed by Zhang et al. (2002) is adopted to represent the preference for performing joint activities with consideration of intra-household interactions. The utility of joint activity link a_{a} is expressed by

$$u_{a_a}^{12} = w_1 \cdot u_{a_a}^1 + w_2 \cdot u_{a_a}^2 + \chi \cdot u_{a_a}^1 \cdot u_{a_a}^2, \quad a_a \in A_a^{\text{joint}} \tag{2}$$

The joint activity utility is the summation of weighted individuals' utilities and a weighted interaction effect. $u_{a_a}^1$ and $u_{a_a}^2$ are independent utilities of two individuals which can be obtained from Equation (1). w_{ind} is individual ind's weight parameter. $w_1 \geq 0$, $w_2 \geq 0$, and $w_1 + w_2 = 1$. w_{ind} $(ind = 1, 2)$ can be interpreted as a measure of the individual ind's power in household's decision making. The parameter χ moderates the effect of joint activity benefit and reflects household members' concern for joint activity. A detailed interpretation of this function and other types of household utility function can be found in Zhang et al. (2002, 2009). In further study, Equation (2) should be extended to express intra-household interaction for multi-individual household (Zhang et al. 2009). Interactions of various activities between different household members should be investigated.

The dis-utility of physical link v with start time interval k (denoted as $disu_v(k)$) is expressed to represent in-vehicle crowding discomfort (Spiess 1983; Nielsen 2000; Lo et al. 2003):

$$disu_v(k) = -\text{vot} \cdot t_v^0 \left(1 + \beta_b \left(\frac{f_v(k)}{h_b \cdot g_b} \right)^{\theta_b} \right), \quad v \in V_b \tag{3}$$

where t_v^0 is the free-flow travel time of physical link v; h_b is the vehicle capacity of mode b; g_b denotes the frequency of mode b; vot is the value of time; β_b and θ_b are model parameters relevant to mode b. $f_v(k)$ is the passenger flow on the physical link v at time interval k, which can be expressed as the summation of passenger flows on all in-vehicle links consisting of this physical link:

$$f_v(k) = \sum_{a_d \in A_d} f_{a_d} \cdot \delta(a_d, v), \tag{4}$$

where $\delta(a_d, v)$ is equal to 1 if physical link v is in direct in-vehicle link a_d; 0 otherwise.

The in-vehicle link dis-utility can be obtained by the summation of related physical link dis-utilities and transit fare:

$$disu_{a_d} = \sum_{v \in V} disu_v \cdot \delta(a_d, v) \text{-} \rho_{a_d}, \tag{5}$$

where ρ_{a_d} is the transit fare with respect to the direct in-vehicle link a_d. In this way, non-linear fares can be directly represented by node-to-node basis.

As regards transfer links by mode, the link dis-utility can be expressed as

$$disu_{a_t} = -\text{vot} \cdot \frac{1}{2g_b}, \quad a_t \in A_t \tag{6}$$

where g_b is the frequency of the transit line to which individuals transfer on the transfer link concerned.

In the JATS super-network, each waiting link indicates waiting for one time interval, thus, the dis-utility of waiting link (denoted as $disu_{a_w}$) can be expressed as

$disu_{a_w} = -\text{vot}$. The dis-utility of meeting link $a_m \in A_m$ is assumed to be zero (Liao et al. 2013b).

3. The JATP Scheduling Model

3.1. Impact of joint travel length

Individuals are known to gain benefits from joint travel and joint activity. To comprehensively model individuals' independent and joint activity/travel choice behaviour, individuals' preference for joint activity/travel should be investigated. However, the modelling of joint travel benefit is still largely unexplored in the literature.

Activity-travel surveys indicate individuals are willing to travel further and pursue activities for longer durations when the activity-travel is being pursued jointly with family or friends. According to the findings of Srinivasan and Bhat (2008), joint episodes are often of long durations. In this study, the benefits gained from joint activity is modelled by incorporating interaction parameter in joint activity utility function (refer to Equation (2)). The benefit from joint travel is modelled by considering the joint travel length. In general, individuals make JATP choices based on the consideration of different joint travel lengths (refer to Fig. 3). For example, individuals can meet at the nearest subway station, and jointly take a lengthy journey to the shopping mall (as is shown in Fig. 3(a)). They can also meet at a subway station near the shopping mall, and take a short joint journey (as Fig. 3(b) shows).

Although a short joint bus/train journey to the shopping mall (i.e. Fig. 3(b)) is a possibility, in reality, it is not likely. Individuals are likely to conduct such travel independently. Faced with waiting either at the subway station or a shopping mall, prior to meeting, a traveller is more likely to avoid the short joint train journey preferring to meet at the shopping mall destination. Thus, the length of joint travel should be explicitly considered in modelling individuals' JATP choice behaviour.

Fig. 3 Comparison of two JATPs with different joint travel lengths

A JATP scheduling model with explicit consideration of joint travel benefit is described in this study. A measure of JATP utility is proposed. The impact of joint travel length is modelled in the JATP utility by using a commonality factor (Cascetta et al. 1996; Zhou et al. 2012).

The commonality factor was first proposed by Cascetta et al. (1996) in the C-logit model. In the C-logit model, the commonality factor is added into the route utility function to account for the similarities between overlapping routes. Utilities of overlapping routes are modified by incorporating this factor in the C-logit model. As joint travel brings individuals benefit and increases individuals' utility, in this study, the concept of commonality factor is adopted to enable the consideration the similarity of individuals' routes (i.e. the proportion of joint travel in total travel). A measure of JATP utility with commonality factor is proposed in this study.

The JATP scheduling model can be interpreted as an implicit JATP availability choice model, where the JATP utility with commonality factor can be considered as a normalised measure of the availability of a JATP as an alternative for a generic household. The commonality factor is calculated based on the joint travel proportion in total travel. The JATP utility increases with the proportion of joint travel. In this paper, only one class of household membership (i.e. two-individual household) is modelled, so the specification of commonality factor proposed in this paper is applicable for two-individual households. As regards other classes of household membership (e.g. three-individual household), the specifications of commonality factor should be proposed and calibrated with empirical data in further research.

Let $P = \{p\}$ be the joint route set in the JATS super-network (i.e. JATP set). The proposed network equilibrium model for JATP scheduling keeps the mathematical structure of conventional UE model, but with a modified route utility. In this study, a concept of JATP utility is proposed to represent the household utility gain from all independent and joint choices. The JATP utility (denoted as φ_p), is defined as the sum of the overall activity utility of JATP p (denoted as $u^p_{activity}$), and overall travel dis-utility of JATP p (denoted as $disu^p_{travel}$) times a commonality factor (denoted as cf_p):

$$\varphi_p = u^p_{activity} + disu^p_{travel} \cdot cf_p. \tag{7}$$

By considering the act of waiting for another individual as an activity, the overall activity utility of the JATP p (i.e. $u^p_{activity}$) can be expressed by summing the weighted utilities of activity links and dis-utilities of waiting links:

$$u^p_{activity} = \sum_{a_\mathrm{a} \in A_\mathrm{a}} w_{ind} \cdot u^{ind}_{a_\mathrm{a}} \cdot \delta(p, a_\mathrm{a}) + \sum_{a_\mathrm{w} \in A_\mathrm{w}} w_{ind} \cdot disu_{a_\mathrm{w}} \cdot \delta(p, a_\mathrm{w}). \tag{8}$$

The overall travel dis-utility of the JATP p (i.e. $disu^p_{travel}$) can be obtained by summing the weighted dis-utilities of in-vehicle links and transfer links:

$$disu^p_{travel} = \sum_{a_\mathrm{d} \in A_\mathrm{d}} w_{ind} \cdot disu_{a_\mathrm{d}} \cdot \delta(p, a_\mathrm{d}) + \sum_{a_\mathrm{t} \in A_\mathrm{t}} w_{ind} \cdot disu_{a_\mathrm{t}} \cdot \delta(p, a_\mathrm{t}), \tag{9}$$

where $\delta(p,a)$ is the incidence relationship between JATP and link; $\delta(p,a)$ is equal to 1 indicates that this link is used in the JATP, 0 otherwise. w_{ind} is the individual weight parameter concerning the link is conducted by which individual(s).

The commonality factor cf_p of JATP p is directly proportional to the joint travel proportion in individuals' overall travel. The role played by cf_p is as follows: a JATP with a large proportion of joint travel has a smaller cf_p, thus a larger JATP utility with respect to a JATP with a small proportion of joint travel.

The commonality factor can be specified in different functional forms, resulting in different JATP utility. One possible way to specify the commonality factor is:

$$cf_p = 1 / e^{\beta_{cf} \cdot \frac{L^p_{joint}}{L^p_{total}}}, \quad p \in P \tag{10}$$

where L^p_{joint} is the "length" of joint travel; L^p_{total} gives the overall "lengths" of individuals' all travels in JATP p; $\frac{L^p_{joint}}{L^p_{total}}$ indicates the proportion of joint travel in total travel. β_{cf} is the commonality factor parameter. It is greater than or equal to 0. If β_{cf} is equal to 0, the commonality factor is equal to 1. This indicates that the joint travel benefit is not considered and the proposed JATP scheduling model collapses to become the conventional activity-travel pattern scheduling model.

It can be shown that the above specification of the commonality factor cf_p has the following properties: (i) If JATP p does not include any joint travel (i.e. all travels in the JATP are independent), L^p_{joint} is equal to 0 and cf_p is equal to 1. Thus, the JATP utility is equal to the simple summation of activity utility and travel dis-utility. The indication is that there is no benefit from joint travel. (ii) If individuals take only joint travel in the JATP (i.e. no independent travel), $L^p_{joint} = L^p_{total}$ and cf_p is equal to $1/e^{\beta_{cf}}$. As $\beta_{cf} \geq 0$, $0 < 1/e^{\beta_{cf}} \leq 1$, the JATP utility is increased, which means individuals obtain benefit from joint travel. (iii) It is not difficult to see that $L^p_{total} \geq L^p_{joint}$, thus, $0 \leq \frac{L^p_{joint}}{L^p_{total}} \leq 1$ and $1/e^{\beta_{cf}} \leq cf_p \leq 1$.

The effect of the commonality factor in the JATP utility is exemplified in a simple JATP shown by Fig. 4(a). Assuming β_{cf} is equal to 1 and the total travel dis-utility ($disu^p_{travel}$) is equal to HK\$ -10, it can be found from Fig. 4(b) that, φ_p is equal to HK\$ -10 regardless of joint travel length if the commonality factor is not incorporated. However, if joint travel benefit is considered by using the proposed commonality factor cf_p, φ_p increases with the proportion of joint travel in total travel. For example, φ_p increases to HK\$ -6.07 if the proportion of joint travel (i.e. $\frac{L^p_{joint}}{L^p_{total}}$) is equal to 0.5. It can be seen from Fig. 4 that when $\frac{L^p_{joint}}{L^p_{total}}$ increases from 0 to 1, the JATP utility varies from HK\$ -10 to HK\$ -3.7. The specification of commonality factor proposed in this paper neglects the fact that individuals may not prefer to travel jointly. However, calibration of commonality factor with empirical data is required for practical applications in further study. Different specifications of commonality factor for multi-class household membership should also be investigated.

Fig. 4 Effect of commonality factor on JATP utility

In Equation (10), the "length" can either be a flow-independent attribute (e.g. free-flow travel time) or flow-dependent attribute (e.g. travel dis-utility). The former case is clearly a special case of the latter when the congestion effect is negligible. The selection of appropriate attributes in the commonality factor depends on the specifics of the intended scenarios. For example, for individuals who have better knowledge of network conditions such as commuters equipped with traveller information, it would be more appropriate to choose a flow-dependent commonality factor. On the other hand, a flow-independent commonality factor would be more suitable for modelling route choice behaviour of individuals who have little information about the network conditions. As regards the flow-independent case, the JATP utility can be expressed as

$$\varphi_p = u_{activity}^p + disu_{travel}^p / e^{\beta_{cf} \cdot \frac{t_{joint}^p}{t_{travel}^p}}, \tag{11}$$

where t_{joint}^p denotes the individuals' joint travel time during the whole JATP, while t_{travel}^p denotes the individuals' total travel time (including joint travel time and independent travel time) during the whole JATP. As regards the flow-dependent case, the JATP utility can be expressed as

$$\varphi_p = u_{activity}^p + disu_{travel}^p / e^{\beta_{cf} \cdot \frac{disu_{joint}^p}{disu_{travel}^p}}, \tag{12}$$

where $disu_{joint}^p$ denotes the individuals' joint travel dis-utility. In this study, the proposed model, in nature, falls into the category of a static UE model for long-term transit planning at the strategic level, and individuals are assumed to have perfect knowledge of traffic conditions throughout the whole network (refer to model assumption A3). Thus, the flow-dependent case as Equation (12) shows is adopted in this study.

3.2. Model formulation

With the use of the proposed JATS super-network, both individuals' independent and joint activity/travel choices are explicitly represented by different links in the proposed JATS super-network platform. The time-dependent relationships between activity choices and travel choices can be modelled by the JATS super-network topology. Each joint route from origin to destination in the JATS super-network represents a feasible JATP. Therefore, the proposed time-dependent JATP scheduling problem is equivalent to a static multi-modal transport assignment model on the JATS super-network.

The proposed model falls into the category of static transport network equilibrium model in nature for long-term planning at the strategic level. It is thus postulated that all households would have a UE activity-travel choice pattern: for each day, the utilities of all used joint JATPs are the largest and equal, and all unused JATPs have smaller utilities. Denote π as the optimal route (i.e. the optimal JATP) with the largest utility in the JATS super-network. The UE condition can be formally expressed as

$$f_p(\varphi_\pi - \varphi_p) = 0, \tag{13}$$

$$q = \sum_{p \in P} f_p, \tag{14}$$

$$\varphi_\pi - \varphi_p \geq 0, \tag{15}$$

$$f_p \geq 0, \tag{16}$$

where f_p denotes the household flow on JATP p, and q denotes the total household number in the study network. Each household is assigned in the JATS super-network as a whole, so the proposed UE model can distinguish activity-travel patterns between individuals from different households.

The previously mentioned UE problem can be further expressed as the following gap function formulation:

$$\min\ GAP = \sum_{p \in P} f_p(\varphi_\pi - \varphi_p). \quad (17)$$

The gap function refers to the overall gap capturing the complementary slackness conditions of the proposed UE model. The gap function is non-negative, $GAP \geq 0$.

The above UE condition can also be formulated as a variational inequality (VI):
Find $f_p^* \in \Omega$ such that

$$\sum_{p \in P} \varphi_p^*(f_p^* - f_p) \geq 0, \quad \forall f_p \in \Omega \quad (18)$$

where Ω is the feasible set of JATP flows defined by (14) and (16).

Theorem 1. The solution of the VI problem (18) is equivalent to the UE condition (13) - (16).
Proof
For the proof, readers are referred to Smith (1979) on the route VI formulation for the static traffic assignment problem.

Theorem 2. At least one solution of the VI problem (18) exists.
Proof
According to Facchinei and Pang (2003), the proof can be completed by the following two properties: (a) The JATP utility is continuous; (b) the feasible set Ω is compact and convex.

In general, the uniqueness of the solution depends on the monotone property of VI formulation. According to the definition of the JATP utility and commonality factor, the uniqueness of the solution cannot be guaranteed due to the non-additive form of JATP utility and the non-separable flow-dependent commonality factor.

4. Solution Algorithm

In this section, a route searching algorithm to determine the optimal JATP is first presented in Section 4.1. Based on this algorithm, a solution algorithm for solving the UE model is proposed and given in Section 4.2.

4.1. Solution algorithm for searching the optimal JATP

Household members schedule their independent and joint activities/travels to maximize their JATP utility. Such actions are the equivalent of finding the route with maximum JATP utility from origin to destination in the JATS super-network. Therefore, the JATP searching problem can be converted into a shortest route problem by using the JATS super-network. It can be seen from Equation (12) that the JATP utility cannot be calculated by simple summation of the link (dis-)utilities. This non-additive property indicates that a sub-route between any pair of nodes on the shortest route may not be the shortest route itself. Therefore, conventional single-criterion shortest route algorithms such as the Dijkstra's algorithm and the Bellman-Ford algorithm cannot be adapted for finding the optimal JATP. The JATP searching problem can be formulated as a multi-criterion problem with respect to three decision variables, i.e. total activity utility in JATP p (i.e. $u_{activity}^p$), total travel dis-utility (i.e. $disu_{travel}^p$), and joint travel dis-utility (i.e. $disu_{joint}^p$). It is unlikely that a single optimal pattern can be found because of the conflicting criteria in the multi-criterion shortest route problem, but a set of non-dominated routes can be obtained in the JATS super-network. The definition of non-dominated routes is that, it is not possible to find another route with a better value in one criterion without worsening another criterion. The JATP dominant condition can be defined as follows (Chen et al. 2011):

Definition 1 (JATP dominant condition)**.** Given two JATPs $p_i \neq p_j \in P$, p_i dominates p_j, if p_i and p_j satisfy

(i) $u_{activity}^{p_i} > u_{activity}^{p_j}$ and $disu_{travel}^{p_i} \geq disu_{travel}^{p_j}$ and $disu_{joint}^{p_i} \leq disu_{joint}^{p_j}$, or

(ii) $u_{activity}^{p_i} \geq u_{activity}^{p_j}$ and $disu_{travel}^{p_i} > disu_{travel}^{p_j}$ and $disu_{joint}^{p_i} \leq disu_{joint}^{p_j}$, or

(iii) $u_{activity}^{p_i} \geq u_{activity}^{p_j}$ and $disu_{travel}^{p_i} \geq disu_{travel}^{p_j}$ and $disu_{joint}^{p_i} < disu_{joint}^{p_j}$.

The proposed JATP dominant condition is extended from the dominant condition used in Chen et al. (2011) for shortest path finding. A label-selection label-correcting method (Guerriero and Musmanno 2001; Chen et al. 2011) is adopted in the development of an efficient solution algorithm for finding the optimal JATP in multi-modal transit networks. Following the model assumption A2, in this study, the JATP scheduling problem is divided into two time periods (i.e. morning period before 12:00 noon and afternoon period after 12:00 noon). In the morning period, two individuals start journeys from the same node in the JATS super-network, and end at different destinations. The morning JATP search is from the origin to the two destinations. Regarding the afternoon period, the destination of the two individuals in the JATS super-network is arrival at the same node, thus the proposed afternoon JATP searching algorithm looks for the optimal JATP backwards, that is from the destination point to the origin points (i.e. consider the JATP destination node d as the route searching origin, and the two JATP origin nodes y_A and y_B as the route search destinations). The JATP searching algorithm for the afternoon period is given in Appendix B. The algorithm for the morning period is similar and not shown in this paper.

This paper focuses on recurrent joint activity/travel during a typical weekday. Thus, at most one episode of joint activity/travel is considered in morning/afternoon period. In further research, the proposed JATS super-network and JATP searching algorithm should be extended to be more general for modelling both recurrent and non-recurrent joint activity/travel during weekday, weekend and public holidays. Multiple episodes of joint activity/travel, which pose challenges in joint route searching algorithm, is worthwhile for further investigation.

4.2. Solution algorithm for solving the UE problem

In this section, a path-based solution algorithm is proposed for solving the UE problem by using the JATP searching algorithm proposed in Section 4.1. The path set (i.e. JATP set) is generated by the column generation technique using the JATP searching algorithm. This avoids the burden of enumerating a pre-defined set of JATPs. Most conventional solution algorithms cannot be used to solve the proposed UE model as it is difficult to determine the descent direction for solving the JATP scheduling problem in multi-modal transit network. The widely used method of successive average (MSA) is a heuristic method with a forced convergence property.

Thus, a solution algorithm based on MSA is proposed for solving the JATP scheduling problem.

The UE solution accuracy level is measured by the relative gap $RGAP$ as

$$RGAP = GAP / \sum_{p \in P} f_p \varphi_p . \tag{19}$$

The GAP in Equation (19) refers to Equation (17). The smaller $RGAP$ value indicates better approximation of the UE solution.

The solution algorithm for solving the JATP scheduling problem is outlined as follows.

Step 1. Transform the traditional multi-modal transit network into the JATS super network by using the JATS super-network expansion algorithm.

Step 2. Initialization. Let $n = 0$. Call the JATP searching algorithm proposed in Section 4.1 to find the optimal path $\pi \in P$ in the JATS super-network (i.e. JATP) with the largest JATP utility. Assign all households on π. Update link flows and link (dis-)utilities.

Step 3. Column generation. Call the JATP searching algorithm proposed in Section 4.1 to find the optimal JATP $\pi \in P$. If φ_π is larger than any φ_p in P, add π into P.

Step 4. Flow update. Perform an all-or-nothing assignment based on JATP utilities, and yield auxiliary JATP flows. Obtain updated JATP flows using an MSA process. Update link flows and link (dis-)utilities.

Step 5. Convergence test. For an acceptable convergence level τ, if $RGAP \le \tau$, stop. Otherwise let $n = n + 1$ and go back to Step 3.

5. Numerical Examples

The point of the numerical examples is to illustrate: (a) application of the proposed model and solution algorithm; (b) the effects of joint travel benefit on individuals' activity-travel choices.

It is believed that various activity participations have different preferred times. Activity participation usually starts with a warming up phase in which the marginal activity utility increases. After reaching a maximum point, the marginal utility decreases. In this study, the following marginal utility function proposed by Ettema and Timmermans (2003) is adopted.

$$\bar{u}_{a_a}(k) = \frac{\gamma_{a_a} \beta_{a_a} u_{a_a}^{\max}}{\exp[\beta_{a_a}(k - \alpha_{a_a})]\left\{1 + \exp[-\beta_{a_a}(k - \alpha_{a_a})]\right\}^{\gamma_{a_a}+1}}, \tag{20}$$

where k is the time of day; $u_{a_a}^{\max}$ is the maximum accumulated utility of activity a_a, and α_{a_a}, β_{a_a}, γ_{a_a} are the activity-specific parameters to be estimated. These parameters can be estimated on the basis of survey data (Ettema and Timmermans 2003; Ashiru et al. 2004). Table 1 shows the given parameters in the marginal utility function for the numerical examples in this study. The value of time was HK$ 60.00/hour. Other parameters were set as $w_1 = 0.5$, $w_2 = 0.5$, $\chi = 0.01$, $\beta'_{a_a} = 0.001$, $\theta'_{a_a} = 2$, $\beta_b = 0.1$, $\theta_b = 2$.

Table 1 Given parameters in the marginal utility function

	Work (A) (morning)	Work (B) (morning)	Work (A) (afternoon)	Work (B) (morning)	Home (morning)	Home (afternoon)	Shopping
$u_{a_a}^{\max}$ (HK$)	1800	1700	1800	1700	1000	2500	800
α_{a_a}	600	600	900	900	360	1320	1140
β_{a_a}	0.021	0.021	0.021	0.021	0.0048	0.0048	0.018
γ_{a_a}	0.8	0.8	0.8	0.8	1.8	1.8	1

5.1. A small network

The study time period for the small example was from 5:00 p.m. to 7:00 p.m. and was equally divided into 12 intervals (i.e. 10 minutes per interval). Fig. 5 depicts a simple multi-modal transit network. One subway line and two bus lines served in the network. Included are three nodes and three physical links. The three nodes represent three study zones: work place of individual A, work place of individual B, and shop area. Three activities (i.e. work (A), work (B), shopping) can be performed at the respective three nodes. Link L2 is an overlapping link on which individuals can conduct joint travel.

Fig. 5 A small multi-modal transit network

The travel time of bus link L1 was 20 minutes, and the travel time of bus link L2 was 40 min. The travel time of using subway from node 1 to node 3 was the same as the time using bus. The bus fare was HK$ 2.00 per physical link. The subway fare was HK$ 12.00. The total household number in this small network was 2000 (i.e. 2000 individuals at work place of A and 2000 individuals at work place of B).

Fig. 6 illustrates the representative JATP which most households choose under the UE condition. It is seen that using the proposed novel super-network, individuals' independent and joint activity choices can be traced. Such choices include activity start/end time, independent and joint activity duration. Individuals' independent and joint travel choices can also be found, such as departure time, route/mode choice, and meeting time/location. Fig. 6(a) depicts the resultant JATP without considering joint travel benefit ($\beta_{cf} = 0$) and Fig. 6(b) illustrates the resultant JATP with consideration of joint travel benefit ($\beta_{cf} = 1$). A comparison of these two JATPs indicates that without considering joint travel benefit explicitly, individuals tend to depart earlier after work (i.e. 17:00), and meet at shopping location after independent travels. It can be seen from Fig. 6(b) that if joint travel benefit is considered, individual B's work time is extended by 20 min (i.e. the departure time changes from 17:00 to 17:20). Individual B waits individual A at the work place, and then they travel jointly to shop, hence obtaining maximum JATP utility. Fig. 6 illustrates that individuals' preference towards joint travel can be effectively captured by the proposed JATP scheduling model. Individuals' travel and activity choices, including departure time, route choice, activity start time, and activity duration, are affected by JATP utility.

Fig. 6 Comparison of JATP choice with and without considering joint travel benefit

The effects of in-vehicle travel time on travel choice are investigated by the proposed model as shown in Fig. 7. In the small network, two individuals can conduct joint travel at overlapping link L2. The model results are compared by changing the travel time of the overlapping link. It can be seen from the figure that with the JATP scheduling model, the increase of link travel time has resulted in an increase in the number of people choosing joint travel. For example, if L2 link travel time is 20 min, about 52.17% people choose joint travel. If the travel time is 70 min, the percentage of joint travel increases to 63.70%. This is due to people can obtain much benefit from joint travel if the travel time is long.

Fig. 7 Effects of in-vehicle travel time on travel choice behaviour

Table 2 shows the proportion of people choosing joint travel and average joint travel time and equilibrium JATP utility from tests with different commonality factor parameters (β_{cf}). From the table, the higher the commonality factor parameter, the larger the joint travel proportion and thus larger average joint travel time per person. $\beta_{cf}=0$ indicates that joint travel benefit is not considered explicitly. With the increase of β_{cf}, the joint travel proportion increases from 2.27% to 99.00%, and the average joint travel time per person is from 0.9 min to 39.9 min. The equilibrium JATP utility is as large as HK$ 229.33 under $\beta_{cf}=3$ compared to HK$168.46 under $\beta_{cf}=0$. This illustrates the benefits gained from joint travel decisions.

Table 2 Joint travel choices under different commonality factors

	Proportion of joint travel	Average joint travel time per person	Equilibrium JATP utility
$\beta_{cf}=0$	**2.27%**	**0.9 min**	**HK$ 168.46**
$\beta_{cf}=1$	59.09%	23.6 min	HK$ 194.05
$\beta_{cf}=2$	83.33%	33.3 min	HK$ 214.18
$\beta_{cf}=3$	**99.00%**	**39.9 min**	**HK$ 229.33**

5.2. The Sioux-Falls network

The proposed model and algorithm was also tested using the Sioux-Falls network, shown in Fig. 8. The study period was from 6:00 a.m. to 9:00 p.m. Two assignments were conducted, one for the morning period and the other for the afternoon period. To reduce the size of super-network, in this example, it was assumed that individuals stayed at the work places from 10:00 a.m. until 5:00 p.m. Transit lines in the network were created based on the work of Szeto and Jiang (2014). Bus line number 10 in Szeto and Jiang (2014) was considered as a subway line in this example. The in-vehicle travel time of each physical link in the network was one time interval. The non-linear fares were set as: using less than or equal to 4 physical links costs HK$ 5.00; using more than 4 physical links costs HK$ 8.00. The total household number in the network was 8000.

Fig. 8 Sioux-Falls network

The convergence characteristics of the proposed UE solution algorithm are illustrated in Fig. 9. It can be seen that the UE condition at the relative gap (as shown in Equation (19)) of 0.01 has been achieved within 100 iterations using less than 0.5 h. This result indicates that the proposed MSA solution algorithm can solve the UE problem for this typical network with an acceptable accuracy level.

By using the proposed JATS super-network platform, activities in different time periods, route and mode choice can be automatically captured. However, the construction of the novel complicated super-network platform presents difficulties in using this approach. When constructing the JATS super-network platform, activity links for each time period should be included, with travel links built with different start times for each mode. The JATS super-network construction results in a huge complicated network to represent a multi-modal transport network which, initially, was one of a relatively small size. Thus, the reduction of the size of the super-network is worthy of further study for applying the proposed model in larger networks. Activity time windows can be predetermined and fixed, and uneven time periods can be specified for different activities so as to reduce the size of the super-network. Efficient solution algorithms should also be examined for solving the network equilibrium models in real-size transport networks.

Fig. 9 Convergence result for the Sioux-Falls network

Fig. 10 presents the temporal population distribution for different activities (home, work, and shopping) and travels (independent travel and joint travel) under four scenarios with different link travel times. Scenarios 1 and 3 are model results without considering joint travel benefit explicitly ($\beta_{cf} = 0$). Scenarios 2 and 4 are results with joint travel benefit ($\beta_{cf} = 1$). By comparing the four scenarios, it was found that under Scenario 2 and 4 (i.e. considering joint travel benefit), individuals tend to conduct joint travels (JT) to work in the morning and after work in the afternoon. The average daily joint travel time is 67.9 min per person under Scenario 1, compared to 122.2 min per person in Scenario 2.

Comparing Scenario 2 to Scenario 1, more individuals conduct the shopping activity (S) jointly after work. However, it is noted that if traffic congestion on the network reaches the point of doubling all link travel times (Scenarios 3 and 4), individuals will leave home earlier in the morning, and in addition not conduct non-compulsory shopping to ensure they can return home as early as possible. Thus, previous activity-travel scheduling models which do not explicitly consider joint travel benefit, may underestimate individuals' joint travel choices. Individuals' departure times, activity start times and durations may also be biased.

Fig. 10 Temporal population distributions under different link travel times with and without joint travel benefit

6. Conclusions

This paper presents an activity-based network equilibrium model for scheduling two-individual JATPs in congested multi-modal transit networks. A JATS super-network is proposed for simultaneously addressing individuals' independent and joint activities/travels. It is shown that the JATP scheduling problem can be transformed into a static traffic assignment problem on the proposed JATS super-network. A solution algorithm without prior enumeration of JATPs is proposed for solving the JATP scheduling problem on the JATS super-network. Numerical results show that both individuals' independent and joint activity/travel choices can be simultaneously investigated by the proposed model. Included are such as activity start time, activity duration, departure time of each person, independent and joint travel routes/modes.

The benefit from joint travel is explicitly modelled in this study by incorporating a commonality factor in the JATP utility. It was found that the joint travel benefit significantly influences individuals' activity/travel choices. If joint travel benefit is not considered, the estimation of individuals' activity-travel choices (e.g. departure time, activity duration, joint/independent travel duration) would be biased. It was also found that the in-vehicle travel time also affects individuals' JATP choice. With the given commonality factor, people tend to conduct joint travel when the in-vehicle travel time is relatively longer.

In this paper, only one behaviourally homogeneous group is considered for facilitation of presentation of the essential ideas. Different types of households can be considered as an extension of the model proposed in this paper, such as two full-time workers with children, non-worker or part-time worker, and two retired persons (Vovsha et al. 2004). To take account of the heterogeneity, a more generalized super-network platform and a new model formulation are required. Additionally, the size of JATS super-network increases exponentially with the number of household members. The computational burden is the major difficulty for modelling households with three or more individuals. However, as more individuals involve in the joint activity/travel, stronger constraints may be adopted to limit the size of the potential JATPs. The number of individuals meeting/departing points can also be reduced by location choice models so as to make the proposed approach still feasible despite the complexity (Liao et al. 2013b).

Further study is required for calibration and validation of the utility functions and commonality factor with empirical data (Chow and Recker 2012). Statistical methods such as maximum likelihood method can be employed to calibrate the parameters of the activity-based model (Fu et al. 2015). Time-series data are required for calibration of the proposed model. The dataset should include household members' activity choices and geographic locations over time. The extension of the proposed model to multi-modal transport networks including road networks is also worthwhile for further investigation. Logistical problem of sharing limited resources such as vehicle sharing problem can be considered to make the JATP scheduling model more applicable for multi-modal transport networks (Xu et al. 2015). In addition, the proposed model can be extended to incorporate multi-class household membership (i.e. one-individual household and multi-individual household). Non-recurrent joint activity/travel particularly during weekend and/or public holidays can also be considered for further investigation.

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Appendix A

The detailed steps of the proposed JATS super-network expansion algorithm for the afternoon period are given in this appendix. The algorithm for the morning period is similar and not given here.

Input: a multi-modal transit network M, two origin locations for individual A and B (i_A and i_B), one destination location (i_{AB}), activity locations ($i_{\mathrm{a}} \in I_{\mathrm{a}}$), and number of time intervals K.
Output: the JATS super-network.
Step 1. Node augmentation.
For each node $i \in U$, expand the node into JATS node $\left(ind,(i,l),k\right)$, $ind = 1,2,12$, $l = 0,1$, $k = 1,2,...,K,K+1$. Denote the JATS node set as N.
Step 2. Construction of JATS activity links.
Scan all nodes in set N. Construct JATS activity links $a_{\mathrm{a}} \in A_{\mathrm{a}}$ between $\left(ind,(i_{\mathrm{a}},0),k\right)$ and $\left(ind,(i_{\mathrm{a}},0),k+1\right)$.
Step 3. Construction of JATS transfer links.
Scan all nodes in set N. Construct JATS transfer links $a_{\mathrm{t}} \in A_{\mathrm{t}}$ between $\left(ind,(i_{\mathrm{a}},0),k\right)$ and $\left(ind,(i_{\mathrm{a}},1),k\right)$.
Step 4. Construction of JATS direct in-vehicle links.
Find all in-vehicle links in network M on the basis of physical travel links.
For each $i \in U$, find all $i' \in U$ which are connected to i by in-vehicle links. Obtain the travel time $t_{a_{\mathrm{d}}}^0$ of each in-vehicle link.

For each i', construct JATS direct in-vehicle links between $\left(ind,(i,1),k\right)$ and $\left(ind,(i',0),k+t_{a_{\mathrm{d}}}^0\right)$.

Step 5. Construction of JATS waiting links.
Scan all nodes in set N. Construct JATS waiting links $a_{\mathrm{w}} \in A_{\mathrm{w}}$ between $\left(ind,(i,0),k\right)$ and $\left(ind,(i,0),k+1\right)$.
Step 6. Construction of JATS meeting links.
Scan all nodes in set N. Construct JATS meeting links $a_{\mathrm{m}} \in A_{\mathrm{m}}$ between $\left(1,(i,0),k\right)$ and $\left(12,(i,0),k\right)$, and between $\left(2,(i,0),k\right)$ and $\left(12,(i,0),k\right)$.
Step 7. Simplification of the super-network.
Delete the augmented nodes which are not two-way connected except for the origin nodes (i.e. $\left(1,(i_A,0),1\right)$ and $\left(2,(i_B,0),1\right)$) and the destination node (i.e. $\left(12,(i_{AB},0),k+1\right)$). Delete the redundant links.

Appendix B

In this appendix, the detailed steps of JATP searching algorithm for the afternoon period are presented. The algorithm for the morning period is similar and not shown here.

Let $P^{dy_A y_B}$ be a set of non-dominated routes maintained at nodes y_A and y_B, and the non-dominated routes from destination d to all node pairs are maintained in a scan eligible set, denoted as SE. At each iteration, one non-dominated route $p_i^{dy_A y_B}$ is selected from SE in a first-in-first-out (FIFO) order for route extension. A temporary route is constructed by extending the selected route $p_i^{dy_A y_B}$ to its successor link whose end node is y_A or y_B (y_A for example here, and the temporary route is denoted as $p_j^{dy_A' y_B}$). The dominant relationship between the newly generated route $p_j^{dy_A' y_B}$ and the set of non-dominated routes $P^{dy_A' y_B}$ at nodes y_A' and y_B is determined based on JATP dominant condition (Definition 1). If $p_j^{dy_A' y_B}$ is a non-dominated route at nodes y_A' and y_B, it is then inserted into $P^{dy_A' y_B}$ and SE. As the newly generated route $p_j^{dy_A' y_B}$ may also dominate some routes in $P^{dy_A' y_B}$, these dominated routes should be eliminated from $P^{dy_A' y_B}$ and SE. The proposed algorithm continues the route searching process until SE becomes empty. At the last step of this algorithm, the optimal JATP can be determined by choosing the route with the largest JATP utility.

The detailed steps of the proposed algorithm for finding the optimal joint route in JATS super-network for the afternoon period are listed as follows.
Inputs: destination node d
Returns: the optimal joint route in the JATS super-network (i.e. the optimal JATP)
Step 1. Initialization:
Create a route p_i^{ddd} from node d to itself, and set $u_{activity}^{p_i^{ddd}} = 0, disu_{travel}^{p_i^{ddd}} = 0, disu_{joint}^{p_i^{ddd}} = 0$.
Add p_i^{ddd} into label-vector P^{ddd} and the list of candidate labels SE.
Step 2. Label selection:
Take label $p_i^{dy_A y_B} \in P^{dy_A y_B}$ from SE in FIFO order. If $SE = \phi$, go to Step 4; otherwise go to Step 3.
Step 3. Route extension:
If $y_A = y_B$ (denoted as y for uniformity), go to Step 3.1.; otherwise go to Step 3.2.

Step 3.1. For every link a (with start node x) whose end node is y: If link a is a meeting link, go to Step 3.1.1; If link a is an activity/waiting link, go to Step 3.1.2; If link a is an in-vehicle/transfer link, go to Step 3.1.3.

Step 3.1.1. Find the corresponding meeting link a' (with start node x') of the other individual. Generate a new label $p_j^{dxx'} \in P^{dxx'}$. Set $u_{activity}^{p_j^{dxx'}} = u_{activity}^{p_i^{dyy}}$, $disu_{travel}^{p_j^{dxx'}} = disu_{travel}^{p_i^{dyy}}$, and $disu_{joint}^{p_j^{dxx'}} = disu_{joint}^{p_i^{dyy}}$.

Step 3.1.2. Generate a new label $p_j^{dxx} \in P^{dxx}$. Set $u_{activity}^{p_j^{dxx}} = u_{activity}^{p_i^{dyy}} + u_a$, $disu_{travel}^{p_j^{dxx}} = disu_{travel}^{p_i^{dyy}}$, and $disu_{joint}^{p_j^{dxx}} = disu_{joint}^{p_i^{dyy}}$.

Step 3.1.3. Generate a new label $p_j^{dxx} \in P^{dxx}$. Set $u_{activity}^{p_j^{dxx}} = u_{activity}^{p_i^{dyy}}$,

$disu_{travel}^{p_j^{dxx}} = disu_{travel}^{p_i^{dyy}} + u_a$, and $disu_{joint}^{p_j^{dxx}} = disu_{joint}^{p_i^{dyy}} + u_a$.

Step 3.2. For every link a (with start node x) the end node of which is y_A or y_B (denoted as y for simplicity)

If link a is an activity/waiting link, generate a new label $p_j^{dxy} \in P^{dxy}$, set $u_{activity}^{p_j^{dxy}} = u_{activity}^{p_i^{dyy}} + u_a$, $disu_{travel}^{p_j^{dxy}} = disu_{travel}^{p_i^{dyy}}$, and $disu_{joint}^{p_j^{dxy}} = disu_{joint}^{p_i^{dyy}}$;

If link a is an in-vehicle/transfer link, generate a new label $p_j^{dxy} \in P^{dxy}$, set $u_{activity}^{p_j^{dxy}} = u_{activity}^{p_i^{dyy}}$, $disu_{travel}^{p_j^{dxy}} = disu_{travel}^{p_i^{dyy}} + u_a$, and $disu_{joint}^{p_j^{dxy}} = disu_{joint}^{p_i^{dyy}}$.

If the new label $p_j^{dxx'}$ (or p_j^{dxx} or p_j^{dxy}) is a non-dominated route under the JATP dominant condition, then insert the new label into $P^{dxx'}$ (or P^{dxx} or P^{dxy}) and SE , and remove all routes dominated by the new label from $P^{dxx'}$ (or P^{dxx} or P^{dxy}) and SE .

Go back to Step 2.

Step 4. Determine the optimal JATP with the largest JATP utility. Stop.

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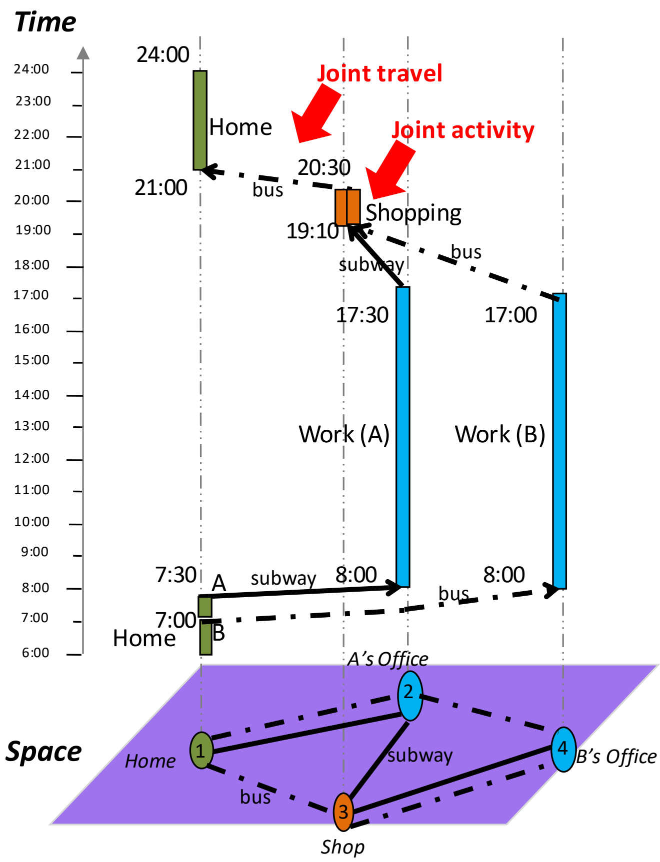


Fig. 1 An illustration of a two-individual JATP

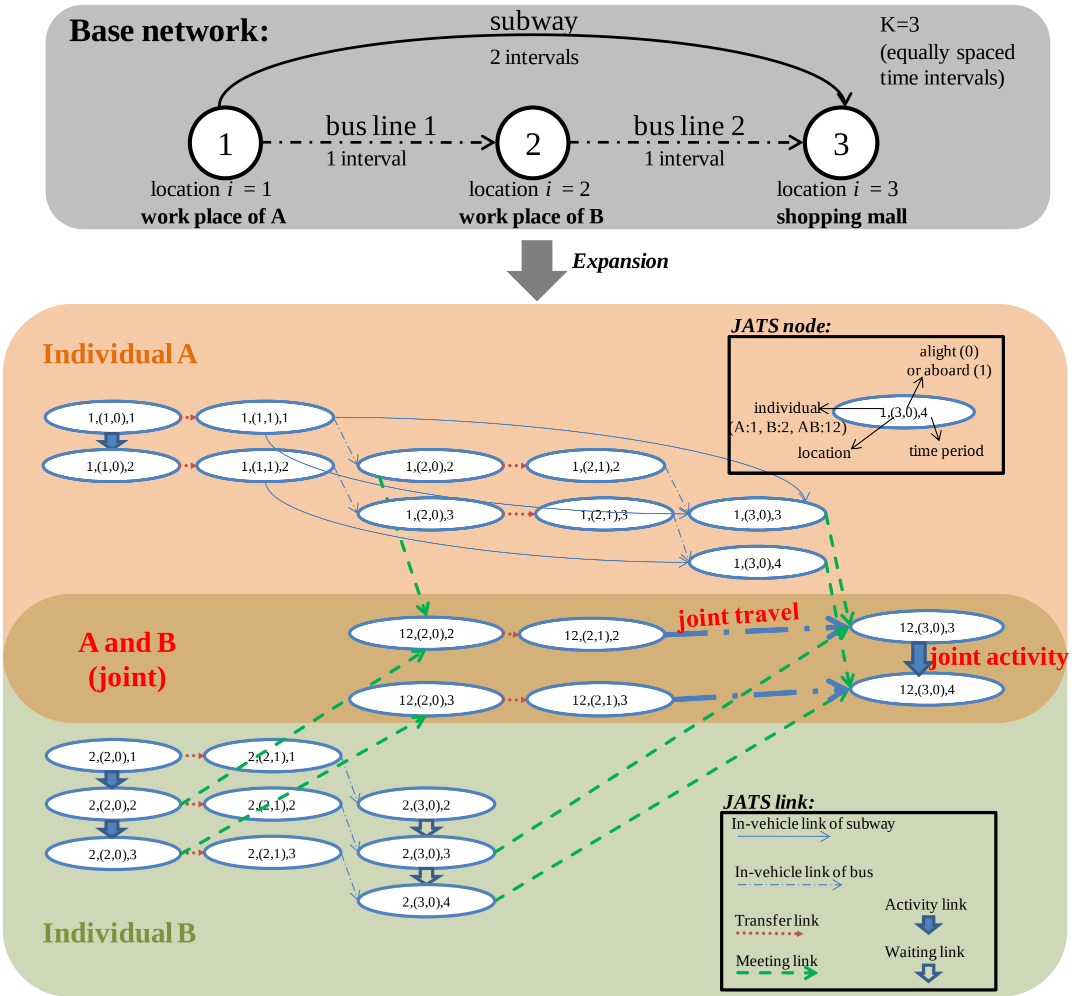
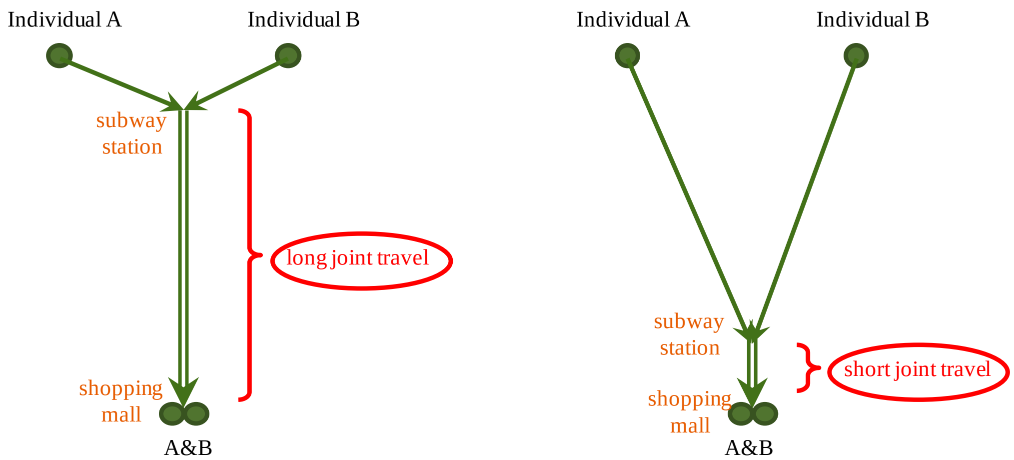
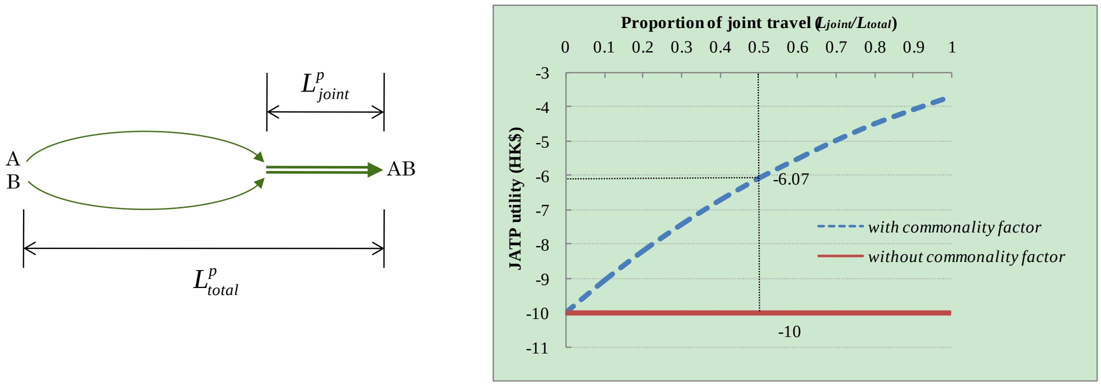


Fig. 2 An illustration of the JATS super-network



(a) long joint travel (b) short joint travel

Fig. 3 Comparison of two JATPs with different joint travel lengths



(a) a small JATP (b) the JATP utility

Fig. 4 Effect of commonality factor on JATP utility

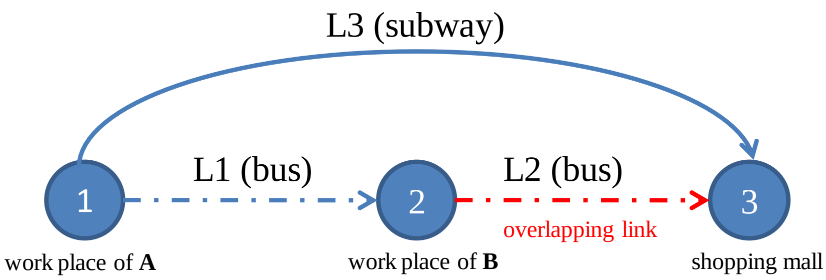


Fig. 5 A small multi-modal transit network

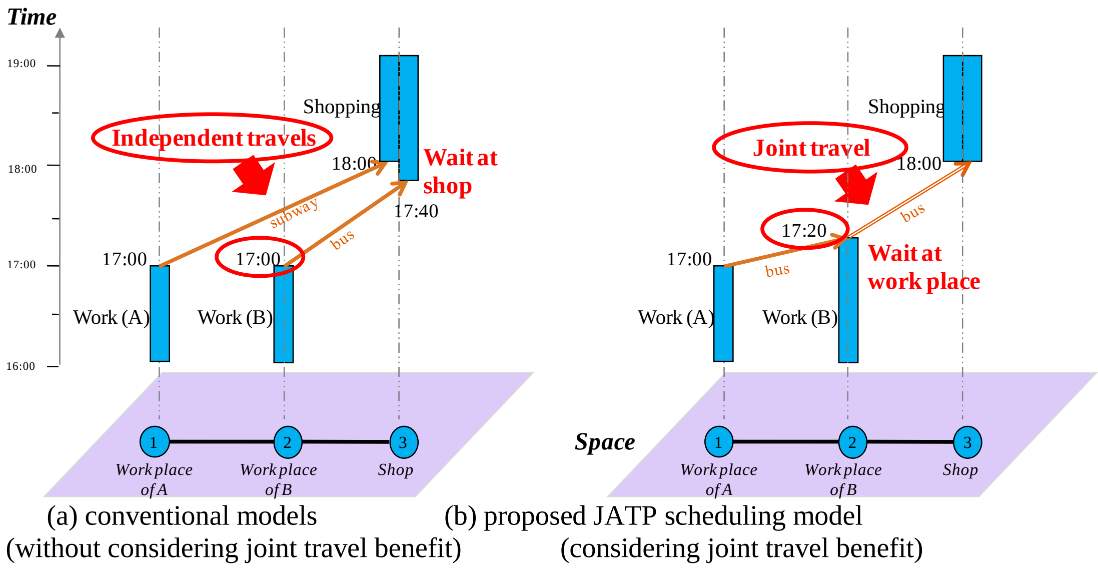


Fig. 6 Comparison of JATP choice with and without considering joint travel benefit

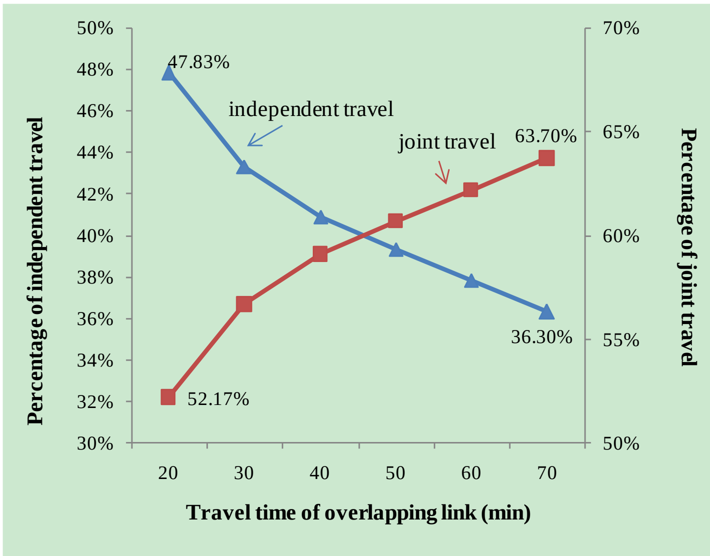


Fig. 7 Effects of in-vehicle travel time on travel choice behaviour

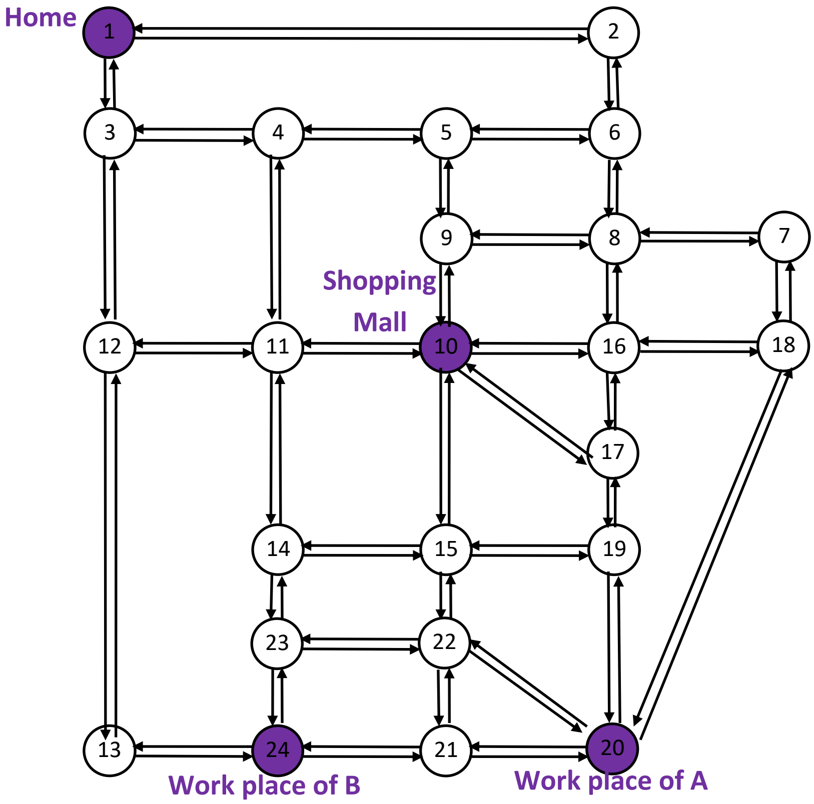


Fig. 8 Sioux-Falls network

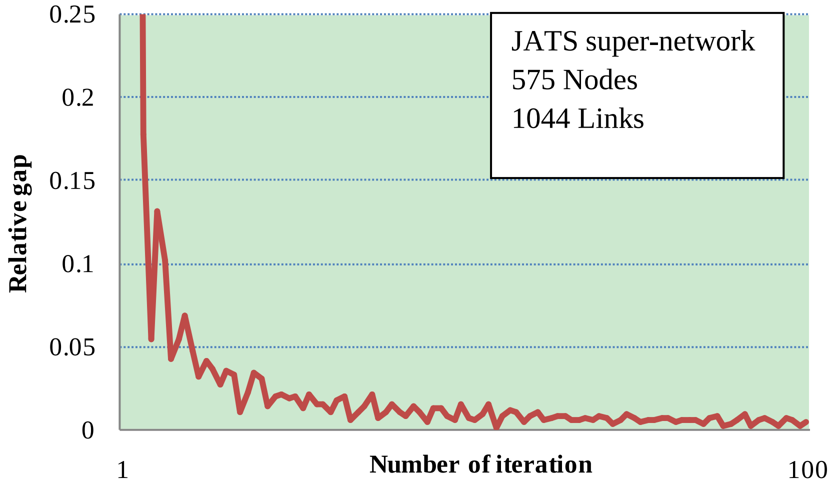


Fig. 9 Convergence result for the Sioux-Falls network

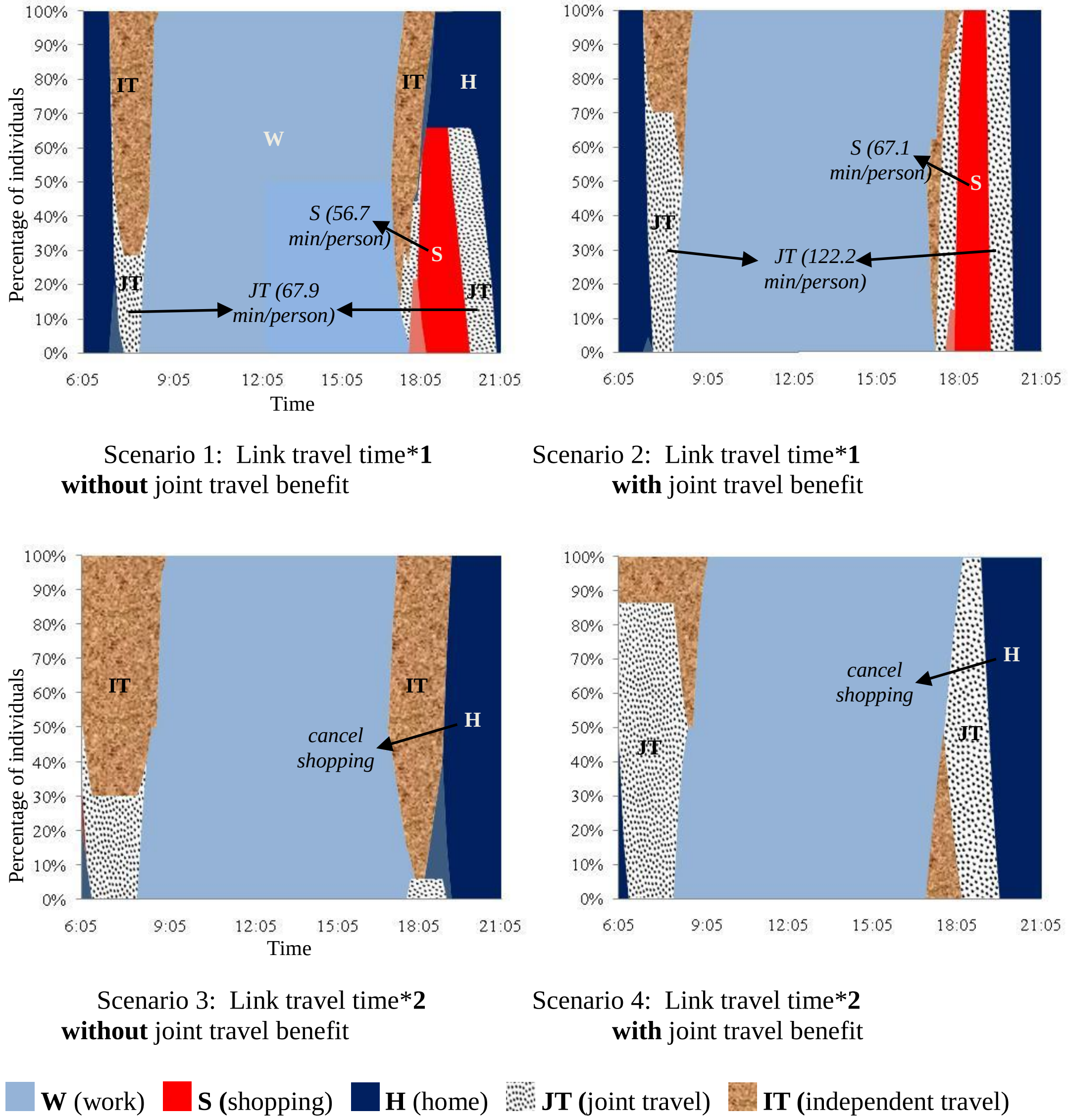


Fig. 10 Temporal population distributions under different link travel times with and without joint travel benefit