Geo-spatial Information Science
Vol. 3, No. 1, p. 1~10 March 2000

IMPROVING 9-INTERSECTION MODEL BY REPLACING THE COMPLEMENT WITH VORONOI REGION

CHEN Jun
LI Chengming
LI Zhilin
Gold C M

KEY WORDS 9-intersection model; Voronoi region; topological relationship

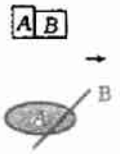
ABSTRACT 9-intersection model is the most popular framework used for formalizing the spatial relations between two spatial objects A and B. It transforms the topological relationships between two simple spatial objects A and B into point-set topology problem in terms of the intersections of A's boundary (∂A), interior (A^0) and exterior (A^-) with B's boundary (∂B), interior (B^0) and exterior (B^-). It is shown in this paper that there exist some limitations of the original 9-intersection model due to its definition of an object's exterior as its complement, and it is difficult to distinguish different disjoint relations and relations between complex objects with holes, difficult or even impossible to compute the intersections with the two object's complements ($\partial A \cap B^-$, $A^0 \cap \partial B^-$, $A^- \cap \partial B$, $A^- \cap B^0$ and $A^- \cap B^-$) since the complements are infinitive. The authors suggest to re-define the exterior of spatial object by replacing the complement with its Voronoi region. A new Voronoi-based 9-intersection (VNI) is proposed and used for formalizing topological relations between spatial objects. By improving the 9-interscetion model, it is now possible to distinguish disjoint relations and to deal with objects with holes. Also it is possible to compute the exterior-based intersections and manipulate spatial relations with the VNI.

1 Overview of the original 9-intersection model

The spatial relations between spatial entities are known as important as the entities themselves. It is therefore very essential to know what possible spatial relationships are and how they can be determined. The 9-intersection model is the most popular mathematical framework for formalizing spatial relations and have been widely used in spatial query languages(Engenhofer, 1991; Clementini*etal*., 1994; Mark *et al*., 1995). Using this model the topological relationships between two simple spatial objects A and B are transformed into point-set topology problem. That is, the topological relations between two objects A and B are defined in terms of the intersections of A's boundary (∂A), interior (A^0) and exterior (A^-) with B's boundary (∂B), interior (B^0) and exterior (B^-), as shown in Fig. 1. Firstly, it is necessary to corroborate whether the contents of the 9-intersection is empty or non-empty, a number of binary topological relations could be identified (Egenhofer and Franzosa, 1991). For instance, eight relations can be realized between two spatial regions in R^2, such as disjoint, meet, equal, inside, contains, covers, covered-by and overlap (Egenhofer and Sharma, 1993). It provides a complete

Project supported by the National Natural Science Foundation of China (No. 49471059)
CHEN Jun, Professor, National Geomatics Center of China, Baishengcun 1, Zizhuyuan, Beijing 100044, China
E-mail: chenjun@nsdi.gov.cn

coverage and the relations are mutually exclusive, that is, only one of them holds for any particular configurations at a time. Efforts had been made to examine the actual number of realizable topological relations between area-line, line-line, area-point, line-point as well as point-point (Egenhofer and Herring, 1991; Egenhofer, 1993; Sun *et al.*, 1993).



	∂B	B^0	B^-
∂A	$\partial A \cap \partial B$	$\partial A \cap B^0$	$\partial A \cap B^-$
A^0	$A^0 \cap \partial B$	$A^0 \cap B^0$	$A^0 \cap B^-$
A^-	$A^- \cap \partial B$	$A^- \cap B^0$	$A^- \cap B^-$

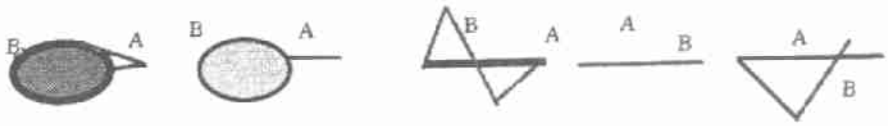
Fig. 1 Point-set topology-based 9-intersection framework

It was further found that the number of relations existing among objects depends on the dimension of the space with respect to the dimension of the objects and on topological properties of the objects embedded in that space (Egenhofer and Sharma, 1993). A dimension extended method was proposed by Clementini *et al.* to take the dimension of the intersection result into account instead of only distinguishing empty or non-empty intersections (Clementini *et al.*, 1993). The dimension extended method was also used in formalizing topological relations in 3D space (Guo and Chen, 1997). On the other hand, the above model deals with simple objects as the homogeneously 2-dimensional, connected areas and lines with exactly two end points (Egenhofer, 1993). In order to identify the topological relations between two regions with holes, each of these regions should be separated into its generalized region and its holes, and the combinatorial intersections of the generalized regions and holes would be examined (Egenhofer *et al.*, 1994). Moreover, the 9-intersection model has also been used or extended for examining the possible topological relations between regions in discrete space (Egenhofer and Sharma, 1993; Winter, 1995), modeling conceptual neighborhoods of topological line-region relations (Egenhofer and Mark, 1995), grouping the very large number of different topological relationships for point, line and area features into a small set of meaningful relations (Clementini *et al.*, 1993), describing the directional relationships between arbitrary shapes and flow direction relationships (Abdelmoty and Williams, 1994; Papaidas and Theodoridis, 1997), deriving the composition of two binary topological relations (Egenhofer, 1991), describing changes in topological relationships by introducing a Closest-Topological-Relationship-Graph and the concept of a topological distance (Egenhofer and Al-Taha, 1992), analyzing the distribution of topological relations in geographic datasets (Florence Ⅲ and Egenhofer, 1996), as well as formalizing the spatio-temporal relations between the father-son parcels during the process of land subdivision (Cheng and Chen, 1997). The findings of these investigations have significantly contributed to the development of state-of-art spatial data models and spatial query functionalities (Egenhofer and Mark, 1995; Mark *et al.*, 1995; Papadis and Theodoridis, 1997).

It is known that the 9-intersection model is an extension of the initially proposed 4-intersection model which has sets of intersections $\partial A \cap \partial B$, $\partial A \cap B^0$, $\partial B \cap A^0$ and $A^0 \cap B^0$. By adding the intersections with the two object's complements, the 9-intersection could distinguish among topological relations that would be considered as the same using the 4-intersection when two objects have co-dimension 0 (Egenhofer *et al.*). 33 different spatial relations are possible for two simple lines and 19 for a line and a region. However, it would be shown in section 2 that there exist some limitations due to the definition of an object's exterior as its complement. One limitation is that it is difficult to distinguish different disjoint relations and relations between complex object's complements ($\partial A \cap B^-$, $A^0 \cap B^-$, $A^- \cap \partial B$, $A^- \cap B^0$, and $A^0 \cap B^-$) since the complements are infinitive. The consequence is that although compound spatial relations could be defined in the SDTS with aggregates of two or more 9-intersection primitives (Mark *et al.*, 1995), it is difficult to calculate these primitives from spatial data. By re-defining the exterior of spatial object with its Voronoi region instead of its complement, a new Voronoi-based 9-intersection (VNI) was proposed in section 3. With the new VNI, it is possible to distinguish disjoint relations, to deal with objects with holes and to compute the five exterior-based intersections. Further investigations are also discussed in section 5.

2 Revisiting roles of complement in distinction of spatial relations

The complement of an object is the set of all points of R^2 not contained in that object (Egenhofer, 1991). There are five complement-related intersections in the 9-intersection model, $\partial A \cap \partial B^-$, $A^0 \cap B^-$, $A^- \cap \partial B$, $A^- \cap B^0$, and $A^- \cap B^-$. It was pointed out that the complement is useful for judging whether or not an object is completely included in another one if the co-dimension is greater than 0(Egenhofer *et al.*). In Fig. 2, we have two groups of line-line relations. The relations in each group have the same 4-intersection, but distinct 9-intersection. In order to revisit the roles of the complements A^- and B^-, we will examine how the complements behave in distinction of spatial relations for the area-area, line-line and line-area objects.



$$R_4(A,B) \quad \begin{bmatrix} -\varnothing & \varnothing \\ \varnothing & \varnothing \end{bmatrix} \qquad \begin{bmatrix} -\varnothing & \varnothing \\ \varnothing & -\varnothing \end{bmatrix}$$

$$R_9(A,B) \quad \begin{bmatrix} -\varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & -\varnothing \\ -\varnothing & -\varnothing & -\varnothing \end{bmatrix} \begin{bmatrix} -\varnothing & \varnothing & -\varnothing \\ \varnothing & \varnothing & -\varnothing \\ -\varnothing & -\varnothing & -\varnothing \end{bmatrix} \begin{bmatrix} -\varnothing & \varnothing & \varnothing \\ \varnothing & -\varnothing & -\varnothing \\ \varnothing & -\varnothing & -\varnothing \end{bmatrix} \begin{bmatrix} -\varnothing & \varnothing & \varnothing \\ \varnothing & -\varnothing & \varnothing \\ \varnothing & \varnothing & -\varnothing \end{bmatrix} \begin{bmatrix} -\varnothing & \varnothing & -\varnothing \\ \varnothing & -\varnothing & -\varnothing \\ -\varnothing & -\varnothing & -\varnothing \end{bmatrix}$$

Fig. 2 Form 4-intersection $R_4(A,B)$ to 9-intersection $R_9(A,B)$

It was shown in Fig. 3(a) that all the five complement-related sets remain non-empty when the relations between two area objects change gradually from disjoint, meet to partially overlap. When one area object falls into the another one (such as equal, covered-by and cover, contains and inside), some of the five complement-related sets would be empty. In fact, A's complement will not intersect with the B's boundary and interior when A and B are equal, and vice versa. When A is covered by object B, its boundary and interior fall into B's interior, and will not intersect with B's complement. It should be pointed out that the eight relations in Fig. 3(a) have distinct 4-intersection. It would say that the complements do not have critical roles in distinguishing region-region relations, and that is why the 9-intersection reveals the same set of region-region relations.

The 9-interscetion forms 33 distinct line-line relations in IR^2 and provides a much finer resolution than the 4-intersection (Egenhofer and Herring, 1991). When the two simple line objects have disjoint, meet, cross and overlap relations as shown in Fig. 3(b), all the five complement-related sets remain non-empty. When a line falls into another one, some of the complement-related sets would be empty. For instance, when A and B have a cover-relation, A's boundary (only two nodes) and interior are contained by B's interior and they do not intersect with the complement of B. $A^- \cap \partial B$ and $A^- \cap B^0$ are therefore empty. It is interested to see that the overlap and cover relations have the same 4-intesection, but have distinct 9-line intersection. Moreover, it can also be seen from Fig. 3(c) that five complement-related sets play roles when a line interior is falling into the other's.

It should be noted that $A^- \cap B^-$ always takes the non-empty value $-\varnothing$. The reason is that the complements of any two objects are largely overlapped. One of the consequences is that it is difficult to distinguish disjoint relations. For instance, there are four objects A, B, C and D in Fig. 4. These objects have different disjoint relations, but the 9-intersections R_{AB}, R_{AC} and R_{AD} are the same. Due to the infinitive complement, A's complement intersects with the boundaries, interiors and complements of object B, C and D, and it's boundary and interior also intersect with the complements of objects B, C and D. The five complement-related sets always take the non-empty value $-\varnothing$. In other words, the complements could not play roles in distinguishing these disjoint relations. Another example is shown in Fig. 5 where three different disjoint relations between object A and object B have the same 9-interscetion. It seems that the original point-set topology based 9-intersection is good for distinguishing the intersected spatial relations where objects intersect with each other. All the disjoint cases with be mapped into the same 9-intersection because the exterior of an object is defined as its complement. Since about 80% spatial relations are disjoint relations (Florence and Egenhofer, 1996), it is necessary to improve the original 9-intersection model to distinguish and describe these disjoint relations.

The second consequence is that it is difficult or

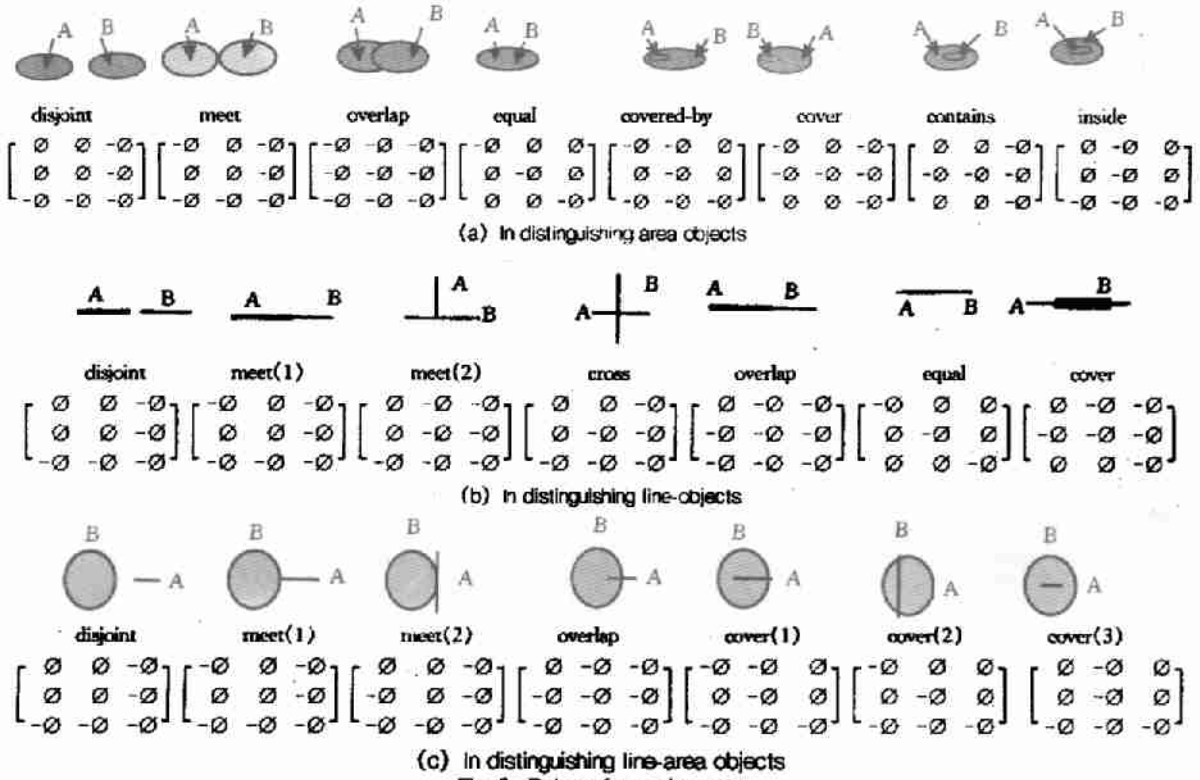


Fig. 3 Roles of complements

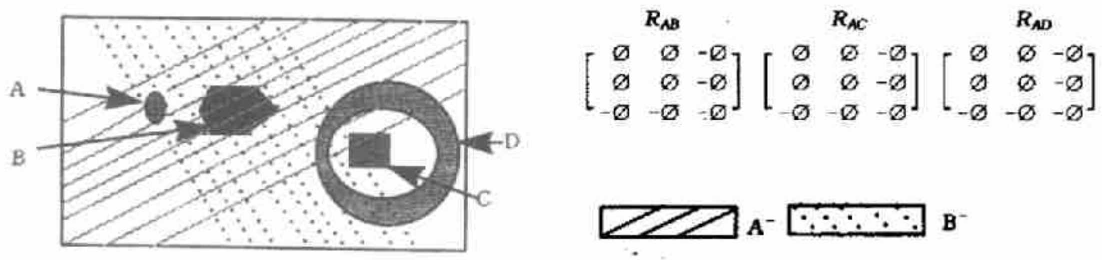


Fig. 4 Complements of any two objects are largely overlapped

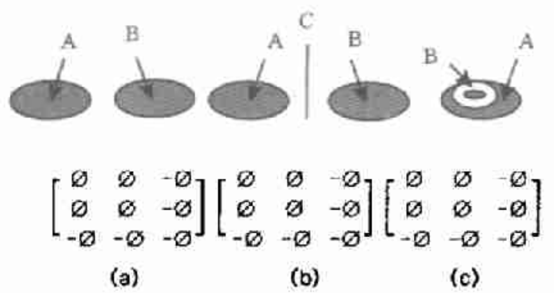


Fig. 5 Different disjoint relations

even impossible to calculate the five complement-related intersections $\partial A \cap \partial B^-$, $A^0 \cap B^-$, $A^- \cap \partial B$, $A^- \cap B^0$, and $A^- \cap B^-$ since the complement of an object is the set of all points of R^2 not contained in that object. On one hand, it is difficult to calculate the intersections of two object's components from their geometric data automatically. Qualitative analysis is required for deriving the 9-

intersection. On the other hand, it is also difficult to derive automatically those spatial objects which satisfy given 9-intersection. In other words, it is difficult to manipulate the spatial relations with the 9-intersection.

The 9-intersection model has also limitations in distinguishing relations between complex objects with holes. For the cover and covered-by relations between A and B in Fig. 6(a), two distinct 9-intersections can be found. In that case, A and B are simple homogeneously 2-dimensional, connected area objects (Egenhofer and Franzosa, 1991; Clementini *et al.*, 1993). However, the 9-intersection would be the same when A or B has a hole, as shown in Fig. 6(b). The interior and boundary of the object (i.e., A) falling in the hole do not intersect the interior of the other object (i.e., B), so $\partial A \cap \partial B^0 = \varnothing$, $A^0 \cap B^0 = \varnothing$, and $A^0 \cap \partial B = \varnothing$. Moreover, the five complement-related intersections $\partial A \cap B^-$, $A^0 \cap B^-$, $A^- \cap \partial B$, $A^- \cap B^0$ and $A^- \cap B^-$ all take non-empty values. Some other examples are illustrated in Fig. 6(c), 6(d) and 6(e).

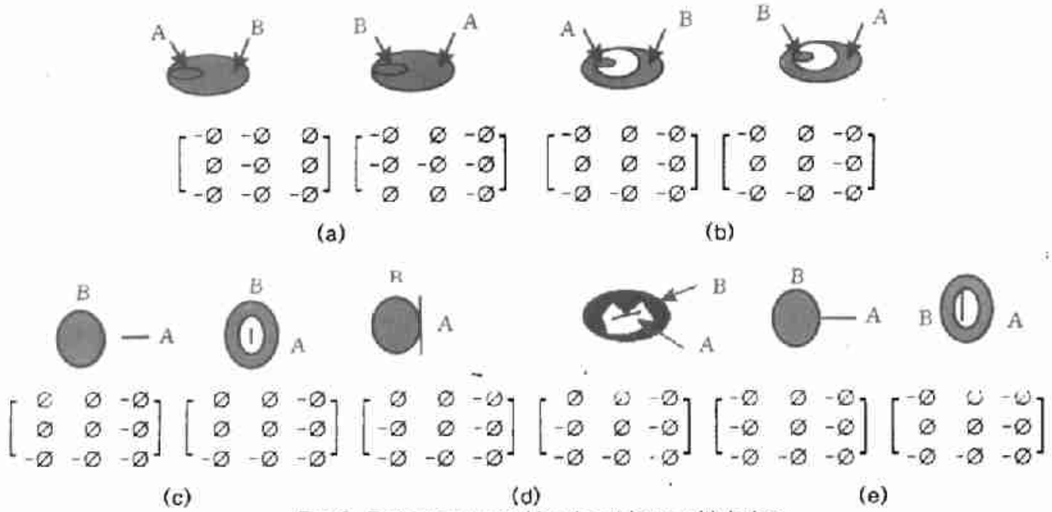


Fig. 6 Problems caused by the objects with holes

3 Redefining the exteriors by replacing A^- with A^V

We propose here to re-define the exterior of an object with the Voronoi region. The whole space is subdivided into a set of Voronoi regions (or tiles) according to the distribution of the objects in the Voronoi diagram, which is one of the most fundamental data structures in computational geometry and is a space-filling topological structure (Aurenhammer, 1991). Each spatial object has its own 'influence region' (Voronoi region) containing all locations closer to that object than to any others, as shown in Fig. 7. It was found that the Voronoi diagram is another spatial model which poses a variety of challenges to the 'usual way of doing things' in GISs (Gold, 1989, 1991, 1992; Wright and Goodchild, 1997). The application of the Voronoi diagram in GIS spatial data modeling and analysis has been investigated for the past few years (Yang and Gold, 1995; Gold *et al.*, 1996; Edwards *et al.*, 1996; Hu and Chen, 1996; Chen and Cui, 1997).

Suppose that we have a set of spatial objects in IR^2, $SO = \{O_1, \cdots, O_i, \cdots, O_n\}$ $(1 \leqslant n \leqslant \infty)$, where O_i may be a point object O_P or line object O_L or area object O_A. An area object is not necessarily convex, and may have holes in which another area may exist. The object Voronoi region of O_i (called O^V) can be defined as:

$$V(O_i) = \{p \mid ds(p, O_i) \leqslant ds(p, O_j), j \neq i, j \in \text{In}\}$$

From the properties of the Voronoi diagram, these Voronoi regions are finite if a bounded region S is defined. In addition, each Voronoi region has limited adjacent regions. By replacing the complement of an object with its Voronoi region, it is possible to overcome the limitations of the original 9-intersection model. And we have now a new Voronoi-based 9-intersection (briefly called VNI) framework as follows

$$\begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^0 & \partial A \cap B^V \\ A^0 \cap \partial B & A^0 \cap B^0 & A^0 \cap B^V \\ A^V \cap \partial B & A^V \cap B^0 & A^V \cap B^V \end{bmatrix} \quad (1)$$

where A^V is object A's Voronoi region; B^V is object B's Voronoi region. $\partial A \cap B^V, A^0 \cap B^V, A^V \cap \partial B, A^V \cap B^0$ and $A^V \cap B^V$ are the five new intersections related to the Voronoi regions.

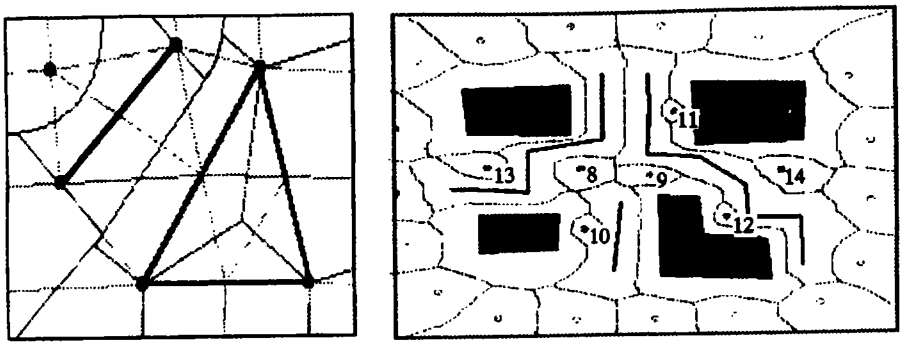
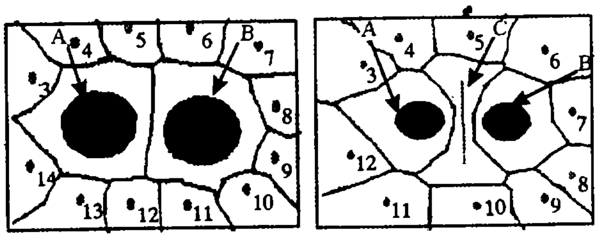


Fig. 7 Voronoi diagram of point, line and area objects

It is because the Voronoi region of A share the same boundary with that of B that $A^V \cap B^V$ would be non-empty when two objects are adjacent, such as A and B in Fig. 8(a). When there is an object C between A and B (Fig. 8(b)), their Voronoi regions are separated by that of C and $A^V \cap B^V$ is empty. It is therefore possible to distinguish the adjacent relations from other disjoint relations with the VNI. More adjacent relations could be defined and derived with the Voronoi diagram, such as immediate neighbor, nearest neighbor, second-nearest, lateral neighbor, tracing neighbor, etc. [Chen *et al.*, 1997].



$$\begin{bmatrix} \varnothing & \varnothing & -\varnothing \\ \varnothing & \varnothing & -\varnothing \\ -\varnothing & -\varnothing & -\varnothing \end{bmatrix} \begin{bmatrix} \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & -\varnothing \end{bmatrix} \qquad \begin{bmatrix} \varnothing & \varnothing & -\varnothing \\ \varnothing & \varnothing & -\varnothing \\ -\varnothing & -\varnothing & -\varnothing \end{bmatrix} \begin{bmatrix} \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing \end{bmatrix}$$

(a) (b)

Fig. 8 Distinguishing disjoint relations with VNI

When the boundary of object A meets with that of object B, their Voronoi regions would also meet with each other according to the definition of Voronoi diagram. $\partial A \cap \partial B$ and $A^V \cap B^V$ would both take non-empty values in this case as shown in Fig. 9(a). In addition, the boundary of object A meets with B^V and B's boundary meets with A^V. The Voronoi-based 9-intersection for the meet relation between area objects is therefore different from the original 9-intersetion. If object A's boundary meets the inner boundary of B which has a hole as shown in Fig. 9(b), then A's Voronoi region intersects B's inner boundary that results in $\partial A \cap \partial B = -\varnothing$, $A^V \cap \partial B = -\varnothing, A^V \cap B^V = -\varnothing$ and $\partial A \cap B^V = -\varnothing$. Moreover, when the whole body of A is contained in the hole of B, we say A's interior overlaps with the convex of B and $A^0 \cap B^V = -\varnothing$. The example shown in Fig. 9(b) has the same original 9-intersection, but has a different Voronoi-based 9-intersection than that in Fig. 9 (a). The example illustrated in Fig. 9(d) is a contained-by relation which has the same 9-intersection with the contains relation shown in Fig. 9(e). There is no intersection of boundaries and interiors between the two objects. However, the boundary and interior of the contained object intersect with the Voronoi convex of the other object. Another example is given by Fig. 9(f) where a line meets a homogeneously 2-dimensional and connected area B and the line falls into an area's hole in Fig. 9(g). It can be seen from these examples that it is possible to distinguish relations between complex objects with holes using the Voronoi-based 9-interscection model.

Both vector and raster approaches have been developed for generating the Voronoi diagram of points, lines and areas (Gold and Yang, 1995; Li, *et* *al.*, 1998). A area Voronoi diagram is constructed from its corresponding line Voronoi diagram in the vector-based Voronoi diagram (Okabe *et al.*, 1992; Gold and Yang, 1995). It is generated directly by defining "distance between objects" in the raster Voronoi diagram (Li *et al.*, 1998). Using these generating approaches, it is possible to calculate the Voronoi region of a given spatial object (point, line or area). The Voronoi boundary and interior of an area object could also be derived from the Voronoi diagram. It is therefore feasible to calculate the five complement-related intersections $\partial A \cap \partial B^-$, $A^0 \cap B^-$, $A^- \cap \partial B$, $A^- \cap B^0$ and $A^- \cap B^-$ automatically from spatial data.

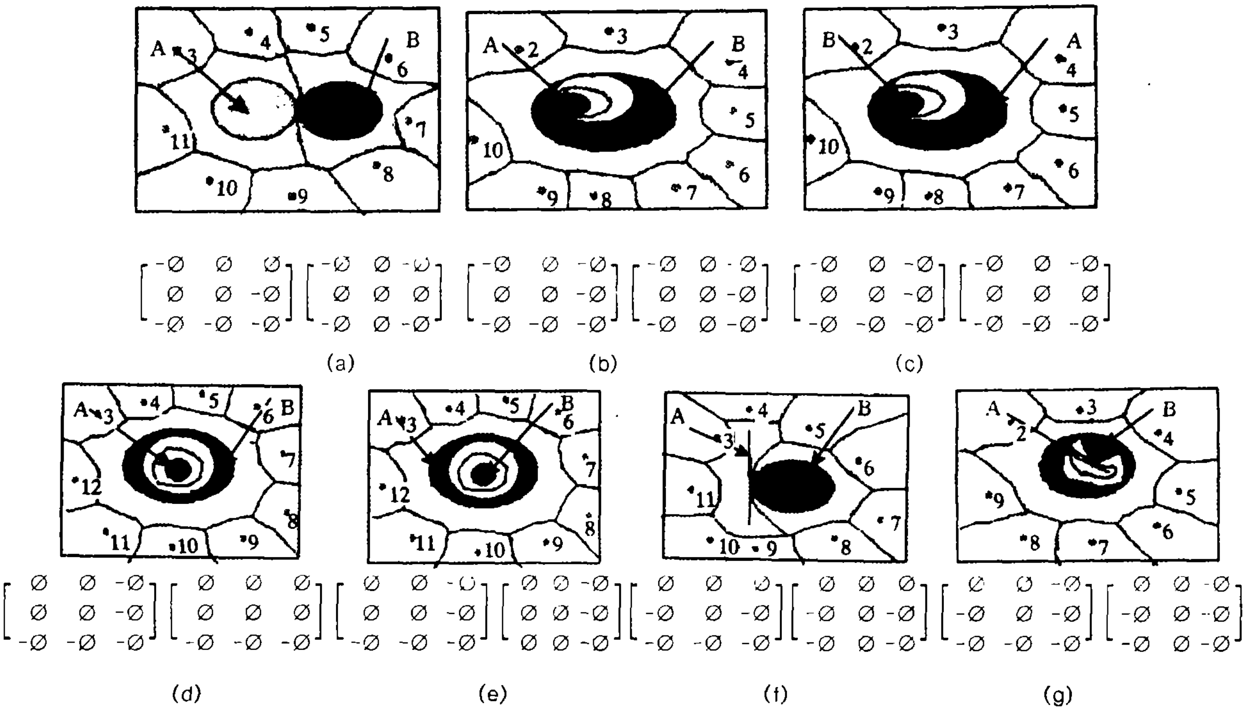


Fig. 9 Distinguishing relations with VNI

4 Formalizing topological relations with VNI

Topological relations between point, line and area objects including relations between area-area, line-line, line-area, point-piont, point-line and point-area objects have been formalized with the new VNI model. Some of the results are listed in Table 1. Among the thirteen topologically distinct relationships between two areas characterized with the VNI, seven of them could not be distinguished using the original 9-interscetion model. Some of the distinguished relations between line-line objects are shown in Fig. 10.

Abdelmoty *et al.* classified the approaches for formalizing spatial relations into two categories. (Abdelmoty and Williams, 1994). One is the intersection-based model where an object is represented in terms of its components, and relationships are the result from the combinatorial intersection of those components. The original 9-intersection model is such a model that spatial relations between objects considered are the result from the exhaustive combinatorial intersection of their components. The other is the interaction-based approach, where the body of an object is considered as a whole and is not decomposed into its components. It is interested to note that the Voronoi-based 9-intersection model proposed here has integrated the intersection and interaction approaches. The four sets $\partial A \cap \partial B$, $\partial A \cap$

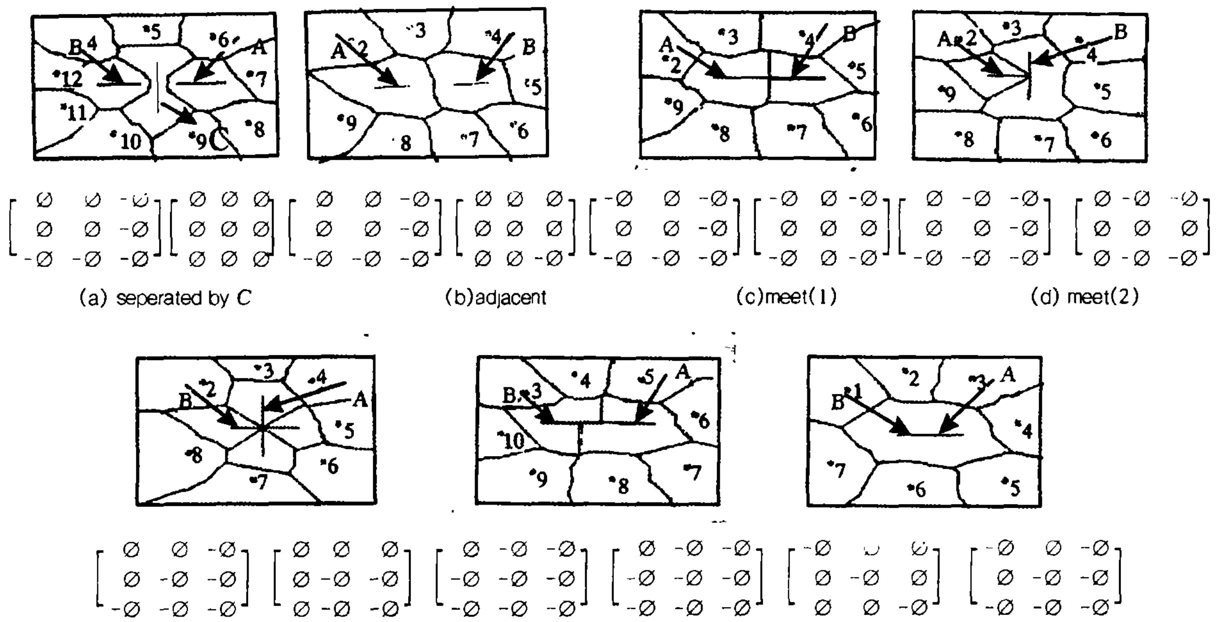
Table 1 Distinguished topological relationships using VNI

	Case	Result
AA	Area/Area	13
LL	Line/Line	8
LA	Line/Area	13
PP	Point/Point	3
PL	Point/Line	4
PA	Point/Area	5

B^0, $\partial B \cap A^0$ and $A^0 \cap B^0$ of the VNI can take the intersection between objects into account. By generating Voronoi region for the whole body of each spatial object, the interactions between adjacent objects can be distinguished with the five sets $\partial A \cap B^V$, $A^0 \cap B^V$, $A^V \cap \partial B$, $A^V \cap B^0$ and $A^V \cap B^V$.

5 Further investigations

From the above discussions, it is clear that the 9-interscetion model can be improved if the exterior of an object would be redefined by replacing its complement with its Voronoi region. One of the advantages of the new Voronoi-based 9-intersection is that it is possible to distinguish disjoint relations since each object has limited neighbors instead of having relations to all other objects. Being able to deal with objects with holes is another advantage of the VNI. Moreover, it is possible or easier to compute the five exterior-based intersections (i.e., $\partial A \cap \partial B^V$, $A^0 \cap B^V$, $A^V \cap \partial B$, $A^V \cap B^0$ and $A^V \cap B^V$) since the Voronoi regions of each object can be generated and manipulated. Thus, it is easier to query and manipulate the topological relations between two given objects with the new 9-intersection model.



(a) seperated by C (b) adjacent (c) meet(1) (d) meet(2)

(e) corss (f) overlap (g) equal

Fig. 10 Part of relations between line-line objectys with VNI

5.1 Comparison of the results of VNI with the original 9-intersection

Detailed analysis of the relations distinguished by the VNI and comparison with the results obtained by the original 9-interscetion model is under investigation. While the original 9-intersection model deals with the simple objects defined as homogeneously 2-dimensional, connected areas and lines with exactly two end points, the area objects used in the VNI may have holes. The definition needs to be extended for covering the kinds of geometric objects, such as loop objects. Moreover, the conceptual neighborhoods of topological relations, Closest-Topological-Relationship-Graph for describing changes in topological relationships, directional relationships between arbitrary shapes and flow direction relationships will be examined with the VNI.

5.2 Development of a special toolkit for manipulating spatial relations with VNI

It is very helpful if the 9-intersection and its corresponding semantics of spatial relations could be derived when two objects exist. On the other hand, when a 9-intersection is given, it is also very essential to retrieve the spatial objects which satisfy such spatial relation. A special toolkit is under development by the authors now for deriving the VNI from the geometry or spatial location of geographic entities. With this toolkit, users can then examine whether a specific spatial relation occurs with their expectations and GIS researchers can easily investigate what spatial relation exists for a given spatial data set.

5.3 Inferencing spatial relations with the VNI

Since the Voronoi diagram forms a pattern of packed convex polygons covering the whole space, all spatial objects are linked together by their Voronoi regions. This makes it possible to deduce or reason about the relations between any two spatial objects with the Voronoi tessellation. The constraints among objects under a given spatial relation can be translated into VNI templates (or primitives), i. e, a pattern of empty, non-empty or arbitrary intersections representing the constraints among interiors, boundaries and Voronoi regions. Aggregates of two or more 9 VNI templates would be used for defining the compound spatial relations. One of the applications would be deriving the contiguity, connectivity and inclusion directly from the spaghette data set.

References

1 Abdelmoty A I, Williams M H. Advanced geographic data modeling. *Spatial data modeling and query language for 2D and 3D applications*. Delft: The Netherlands, 1994

2 Franz A. Voronoi diagrams —— a survey of a fundamental geometric data structure. ACM Computing Surveying, 1991, 23(3)

3 Zheng C, Chen J. Organizing and retrieving cadasatral data based on spatio-temporal topology between father-son parcels. *Journal of Wuhan Technical University of Surveying and Mapping*, 1997, 22(3): 216 ~ 221 (in Chinese)

4 Chen J, Gold C M, Cui B L, Yong H U, *et al*. Extending ordinary planar-graph-based spatial data model with Voronoi approach. In: *Proceedings of IEAS & IWGIS '97*. Beijing, 1997. 18 ~ 20

5 Chen J, Cui B L. Adding topological functions to MapInfo with Voronoi approach. *Journal of Wuhan Technical University of Surveying and Mapping*, 1997, 22(3): 195 ~ 200 (in Chinese)

6 Clementini, Elisel, Paolino Di Felice, Peter van Oosterom. A small set of formal topological relationships suitable for end-user interaction. *Advances in Spatial Databases, Lecture Notes in Computer Science*. Springer-Verlag, 1993. 277 ~ 295

7 Edwards G, Ligozat G, Fryl A, *et al*. A Voronoi-based PIVOT representation of spatial concepts and its application to route descriptions expressed in natural language. *Spatial Data Handling*'96, 1996, 7B. 1 ~ 15

8 Egenhofer M J. Reasoning about binary topological relations. In: *Proceedings of the 2nd Symposium on Large Spatial Databases*. *Lecture Notes in Computer Science*, Springer-Verlag, 1991. 143 ~ 160

9 Egenhofer M J, Franzosa R D. Point-set topological spatial relations. *INT. J. Geographical Information Systems*, 1991, 5(2): 161 ~ 176

10 Egenhofer M J, Herring J. Categorizing binary topological relationships between regions, lines, and points in geographic databases. Technical Report, Department of Surveying Engineering, University of Maine, Orono, 1991

11 Egenhofer M J, Al-taha K K. Reasoning about gradual changes of topological relations, in Theories and Methods of Spatio-temporal Reasoning in Geographic Science. *Lecture Notes in Computer Science*, No. 639. Springer Verlag, 1992. 196 ~ 219

12 Egenhofer M J. Definition of line-line relations or geographic databases. *IEEE Data Engineering Bulletin*, 1993, 7(3): 40 ~ 45

13 Egenhofer M J, Jayant S. Topological relations between regions in IR^2 and ZZ^2. Advances in Spatial Databases. *Lecture Notes in Computer Science*, No. 692. Springer-Verlag, 1993. 316 ~ 226

14 Egenhofer M J, Clementini E, Paolino Fdi Felice. Topological relations between regions with holes. *INT. J. Geographical Information Systems*, 1994, 8(2): 129 ~ 142

15 Egenhofer M J, David M M. Modeling conceptual neighborhoods of topological line-region relations. *INT. J. Geographical Information Systems*, 1995, 9(5): 555 ~ 565

16 Florence J, Egenhofer M J. Distribution of topological relations in geographic datasets. In: *Proceedings of ASPRS/ACSM Annual Convention and Exposition*, 1996. 314 ~ 324

17 Gold C M. Spatial adjacency —— a general approach. *Auto-carto* 9, 1989, 298 ~ 311

18 Gold C M. Problems with handling spatial data —— the Voronoi approach. *CISM Journal ACSGC*, 1989, 45(1) 65 ~ 80

19 Gold C M. Dynamic spatial data structures —— the Voronoi approach. In: *Proceedings of the Canadian Conference on GIS*, Ottawa, 1992. 245 ~ 255

20 Gold C M. Review: spatial tesselations —— concepts and applications of Voronoi diagrams. *Int. J. GISs*, 1994, 8(2): 237 ~ 238

21 Gold C M, Natel J, Yang W. Outside-in: an alternative approach to forest map digitizing. *Int. J. GISs*, 1996,

10(3):291～310

22 Gold C M, Yang W. Spatial data management tools based on dynamic Voronoi data model. (VORDLL 1.1)-User's manual. Industrial Chair on Geomatics, Geomatics Research Center, University Laval, 1995

23 Guo W, Chen J. Describing 3D spatial relations with point-set topology. *Acta Geodetica et Cartographica Sinica*, 1997, 26(2): 122～127 (in Chinese)

24 Hu Y, Chen J. Representing and querying spatially nearest-neighborhood relationship based on Voronoi diagram. In: *Proceedings of the 2nd Conference of Chinese GIS Association*. Beijing, 1996

25 Li C M, Chen J, Li Z L. Raster-based methods for the generation of Voronoi diagrams for spatial objects (submitted for reviewing)

26 David M, Egenhofer M J, Shariff A R. Towards a standards for spatial relations in DSTS and GISs. In: *Proceedings of GIS/LIS'95 Annual Conference*, 1995. 686～695

27 Odabe A, Boots B, Sugihara K. Spatial tessellations —— concepts and applications of Voronoi diagram. London: John Willey & Sons, 1992

28 Dimitris P, Theodoridis Y. Spatial relations, minimum bounding rectangles, and spatial data sturctures. *INT. J. GISs*, 1997, 11(2): 111～138

29 Sun Y G, Zhang Z, Chen J. Complete topological spatial relations: the framework and the case in 2D space. In: *Proceedings of "Advances in Urban Spatial Information and Analysis"*. Wuhan: Press of Wuhan Techrical University of Surveying and Mapping, 1993. 16～25

30 Winter Egenhofer M J, Herring J R. Topological relations between discrete regions. *Advances in Spatial Database, SSD'95*, 1995. 310～327

31 Wright D J, Goodchild M F. Data from the deep: implications for the GIS community. *INT. J. GIS*, 1997, 11(6): 523～528

32 Yang W, Gold C M. Dynamic spatial object condensation based on the Voronoi diagram. In: *Proc. of Towards Three Dimensional, Temporal and Dynamic Spatial Data Modeling and Analysis*, Wuhan, 1995. 134～145