

Geo-rough Space

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KEY WORDS rough set; rough spatial entity; rough topological relationship; rough symbols

ABSTRACT Rough set is a new approach to uncertainties in spatial analysis. In this paper, rough set symbols are simplified and standardized in terms of rough interpretation and specialized indication. Rough spatial entities and their topological relationships are also proposed in rough space, thus a universal intersected equation is developed, and rough membership function is further extended with the gray scale in our case study. We complete three works. First, a set of simplified rough symbols is advanced on the basis of existing rough symbols. Second, rough spatial entity is put forward to study the real world as it is, without forcing uncertainties into crisp set. Third, rough spatial topological relationships are studied by using rough matrix and their figures. The relationships are divided into three types: crisp entity and crisp entity (CC), rough entity and crisp entity (RC), and rough entity and rough entity (RR). A universal intersected equation is further proposed. Finally, the maximum and minimum maps of river thematic classification are generated via rough membership function and rough relationships in our case study.

1 Introduction

By using the GIS, people try to abstract, generalize and analyze a spatial entity in a computerized information system. And the ideal spatial entity is defined and described crisply. However, the spatial entity is often complex and varying at scales of time and space. People have to select its most important spatial aspects. First the exact object model is used in GIS, and then cartographic convention enhances it^[1]. But the procedure may lose details in one or more dimensions. Furthermore, some attribute values of the spatial entity are inaccessible, inexact or vague. The above facts make it indiscernible to associate a spatial element (e.g. pixel) with a given entity.

It is fundamental for GIS to determine whether the spatial element belongs to a predefined entity. The classification determination is performed according to the accessible attribute values. In order to improve the exact object model, some theories and techniques, for example, continuous field model, error band, epsilon band, "S" band, fuzzy set, decision theory, cloud theory, have been further put forward and applied. As an extension of set theory for the study of spatial entity characterized by incomplete and inexact information^[2], rough set is further developed on spatial description in this paper.

Rough set specifies a spatial entity by giving an upper and a lower approximation. The lower approximation is the set of spatial elements that surely belong to the spatial entity, while the upper approximation is the set of spatial elements that possibly belong to it. Since rough set was introduced, it has been

applied in medicine system, language processing, pattern recognition, etc. Recently, rough set has also been applied in GIS. Stell and Worboys used rough set to handle imprecision due to finite spatial or semantic resolution. Ahlqvist, et al. thought that rough set was a feasible alternative to GIS via rough classification and accuracy assessment^[3-6].

However, in the process of rough set applications and developments, various descriptive symbols came into being, and it has been proved to be difficult to study rough set further^[2-6]. Simultaneously, the lower approximation is the subset of the upper approximation in rough set, and it is always computed twice. This wastes a great deal of resource since the certain lower approximation occupies the most part of a spatial entity.

2 Rough set and its improvements

Pawlak^[2] originally considered a rough set as a family of sets with the same lower and upper approximations. On the basis of Pawlak's work, Iwinski^[3] regarded a rough set as a pair of composed sets. Then Pawlak^[3] gave another way to describe a rough set by rough membership function. According to whether statistical information is used, the existing rough set models may be grouped into two major classes: algebraic and probabilistic mod-

els^[4].

2.1 Trial to standardize rough set symbols

There exist various rough set models to be unified. With the applications of the rough set, different types of symbols on the rough set concepts are developed in different fields and intents, even though the rough set inventor, Zdzislaw Pawlak, often gave different symbols in his papers^[2-6]. In order to understand a paper, readers have to compare the new symbols with old ones. Thus it is difficult to further communicate with each other in different application fields of the rough set. The more widely rough set is applied, the worse this situation will be. In the sequel, the further development of the rough set will be impeded. "In view of many generalizations and extensions of rough set theory, some kind of unification of the basic theory seems to be badly needed."^[3] So it becomes very necessary to standardize various symbols. As a trial to unify rough set symbols, a set of simplified genetic rough symbols is proposed on the basis of the existing different rough symbols^[2-6], mainly Pawlak symbols. The new symbols are in terms of rough interpretation and specialized indication (Table 1). For example, "Lr", "X" or "Lr(X)" respectively denote "Lower approximation in rough set", "spatial entity X" in GIS.

Table 1 List of rough symbols and their definitions

Proposed symbols	Existing symbols	Presentations
U	U	Discourse universe that is a finite and non-empty set.
R	R	Equivalence relation on U , $R \subseteq U \times U$. (U, R) formalizes an approximate space.
X	X	Arbitrary set $X \subseteq U$
$\sim X$	$\sim X, \sim X$	The complement set of X , $X \cup (\sim X) = U$
U/R	U/R	Equivalence class set composed of disjoint subsets of U partitioned by R .
$[x]_R$	$[x]_{IND(R)}, [x]_R, R(x), r(x)$	Equivalence class of R including element x . It may also interpreted as all rough-related elements of x , or a neighborhood of x .
$Lr(X)$	$\underline{APR}(X), \underline{A}(X), \underline{AX}, X, Int(X), R_-(X), R^*$	Lower approximation (interior set) of X on U . $Lr(X) = \{x \in U \mid [x]_R \subseteq X\}$
$Ur(X)$	$\overline{APR}(X), \overline{A}(X), \overline{AX}, \underline{X}, Cl(X), R^+(X), R^*$	Upper approximation (closure set) of X on U . $Ur(X) = \{x \in U \mid [x]_R \cap X \neq \Phi\}$
$Pos(X)$	$POS(X)$	Positive region. $Pos(X) = Lr(X)$
$Neg(X)$	$NEG(X)$	Negative region. $Neg(X) = U - Ur(X)$
$Bnd(X)$	$BND(X), Bn(X), Bd(X), Boundary(X)$	Boundary region. $Bnd(X) = Ur(X) - Lr(X)$

2.2 *Brief rough set*

Rough set characterizes both certainties and uncertainties. In Table 1 context, $Lr(X)$ is certain “Yes”, $Neg(X)$ is surely “No”, while both $Ur(X)$ and $Bnd(X)$ are uncertain “Yes or no”. That is to say, with respect to an element $x \in U$, it is sure that $x \in Pos(X)$ belongs to X in terms of its features, but $x \in Neg(X)$ does not belong to X ; while $x \in Bnd(X)$ cannot be ensured by means of available information whether it belongs to X . So it can be seen that $Lr(X) \subseteq X \subseteq Ur(X) \subseteq U$, $U = Pos(X) \cup Bnd(X) \cup Neg(X)$, and $Ur(X) = Pos(X) \cup Bnd(X)$. X is defined *iif* $Lr(X) = Ur(X)$, while X is rough with respect to $Bnd(X)$ *iif* $Lr(X) \neq Ur(X)$. A subset $X \subseteq U$ defined with the lower approximation and upper approximation is called rough set. Rough degree is $R_d(X) = R_{card}(Ur(X) - Lr(X)) / R_{card}(X) \times 100\%$, where $R_{card}(X)$ denotes the cardinality of set X . X is crisp when $R_d(X) = 0$. For instance, regard U as an image, the rectangle becomes a pixel.

2.3 *Rough membership function*

Probabilistic rough set is with respect to rough membership function. Rough set can also be defined with a rough membership function $\mu_X(x)$, $\mu_X(x) \in [0, 1]^{[2]}$ (Eq. 1).

$$\mu_X(x) = \frac{R_{card}(X \cap [x]_R)}{R_{card}([x]_R)} = \begin{cases} 1 & x \in Pos(x) \\ (0, 1) & x \in Bnd(x) \\ 0 & x \in Neg(x) \\ 1 - \mu_{\sim X}(x) & x \in \sim X \end{cases} \quad (1)$$

The rough membership value may be regarded as the probability of $x \in X$ given that x belongs to an equivalence class. That is, it is taken for a conditional probability to illustrate a certain degree of x belonging to X , $\mu_X(x) + \mu_{\sim X}(x) = 1$. Let $P(X|[x]_R) = \mu_X(x)$ and $\alpha \in [0, 1]$, a probabilistic rough set in α context is defined as Eq. 2. In this sense, $\mu_X(x)$ gives a probabilistic rough space of X via a pair of upper approximation and lower approximation.

$$\begin{aligned} Lr_\alpha(X) &= \{x \mid P(X|[x]_R) \geq 1 - \alpha\}, \\ Ur_\alpha(X) &= \{x \mid P(X|[x]_R) > \alpha\} \end{aligned} \quad (2)$$

2.4 *Differences between rough set and other methods*

There are relationships between rough set and other theories, e. g. fuzzy set, cloud theory, evidence theory. All of them can deal with uncertainties, for example, characterizing indeterminate phenomena via mathematical syntax and semantics. However, we may still distinguish the rough set from other theories in some aspects. In the following, it should be noted that x is a spatial parameter, and $\mu(x)$ is its corresponding membership value to a class X .

1) Rough set gives an interval of $[\mu_{min}(x), \mu_{max}(x)]$ with respect to x . In other words, an element has many corresponding values, one to many. The determination is that the element “is”, “is not” or “is maybe” in a given class. These values formalize the interval. The data in $Bnd(X)$ between the lower approximation and upper approximation are rough for set X . But it is not sure that they belong to the set X . As an extension to the classical (traditional, sharp or crisp) set, rough set focuses on the uncertainties caused by incomplete, insufficient or inaccessible information. Compared with other methods, rough set can close describe the spatial entities as they are in the real world, including both certainties and uncertainties.

2) Fuzzy set gives a value $\mu(x)$, $\mu(x) \in [0, 1]$, via a fuzzy membership function, with respected to x , namely, one parameter to one functional value. Fuzzy set is also an extensive set of the classical set, and may perform an uncertain classification. But fuzzy set pays more attention to the uncertainties caused by vague, dim or indistinct information, and it is either difficult or rather arbitrary to determine the fuzzy membership functions. Moreover, fuzzy set depends on human experience, and it loses uncertain properties once the fuzzy membership degree $\mu(x)$ is given.

3) Cloud theory, which has three numerical characteristics, specifies a discrete data point with the value $\mu(x)$ in x context. The tuples $(x, \mu(x))$ are called cloud drops. The discrete degree is determined by the membership $\mu(x)$. But the range and interval of $\mu(x)$ is unsure. Cloud model is also

the uncertainty transition between a linguistic term of a qualitative concept and its numerical representation.

4) Evidence theory is also named as Dempster-Shafer theory, Dempster-Shafer theory of evidence, or Dempster-Shafer theory of belief. It has belief function and plausibility function. These two functions of evidence theory are similar to the upper and lower approximations of rough set. And this similarity has promoted the work on the relationships between rough set and evidence theory. The overall crispness measure can be interpreted as a belief value in the sense of Dempster-Shafer logic. However, the belief function depends on experience.

3 Rough spatial entity

Both spatial entities and spatial relationships formalize an approximate space. A spatial entity may be interpreted as spatial phenomena, natural objects with geometric feature of point, line, area, volume, cases, states, processes, observations and so on. As an alternative, rough set is proposed to characterize spatial entities in GIS. U is composed of spatial entities with attributes (features, variables, etc.), and R is the spatial relationship among the spatial entities. Both of them formalize an approximate space (U, R) . Point, line and area in vector space, pixel and grid in raster space, unit cube in a multi-dimensional space are considered as equivalence class of rough spatial entity. In rough set context, point, line, area and volume have size and shape. Attributes and a pair of approximations describe a point, and a series of such points linked together are lines. The lines called boundaries bound areas, and volumes are bounded by smooth area.

A pair of upper approximation and lower approximation specifies a rough spatial entity. If a spatial entity $X \subseteq U$ is given, X may not be represented precisely for the available information is insufficient. The observed value of an attribute is usually unequal to its true value. When an attribute has been observed for many times, the observed values may formalize an uncertain observed zone

around the true value, namely a pair of approximations. As to a spatial element $x \in U$, lower approximation $Lr(X)$ is the set of x that surely belongs to the true X , while upper approximation $Ur(X)$ is the set of x that possibly belongs to X . And uncertain region of X is $Bnd(X)$ (Fig. 1).

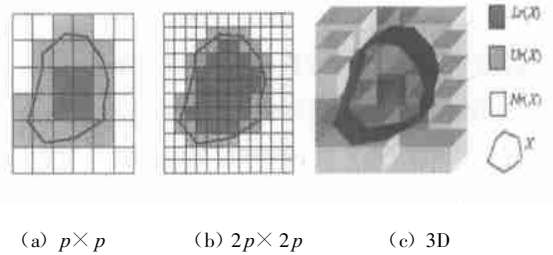


Fig. 1 Rough spatial entity and its illustrations of low resolution high resolution and 3D

Thus, during the spatial analysis based on GIS, rough set can more totally propagate the spatial entity properties (both certain and uncertain) for most spatial true values are not known exactly. As an alternative mathematical interpretation in the sense of rough set, object model is $Lr(X) = Ur(X)$, field model, error band, epsilon band, and “S” band are $Lr(X) \neq Ur(X)$. And for rough degree $Rd(X)$, field model $>$ error band $>$ epsilon band $>$ “S” band. Each of them may be taken as the special condition of rough space. Since vector data and raster data are main original data in GIS, rough vector space and rough raster space will be mainly studied in this section.

3.1 Rough vector space

The object model represents spatial entities via crisply delineated point, line, area and volume in a defined absolute reference system. Their attributes that characterize the space at the points, along the lines or within the area or volumes are assumed to be constant over the whole object extent. It is implemented by GIS vector structure. For example, lines are linked by a defined topology to form networks, which, if opened, can represent rivers or if closed, the abstract or defined boundaries of polygons in turn represent land parcels, soil units or administrative areas. The object model is assumed $Lr(X) = Ur(X)$ without roughness. In fact, $Lr(X) \neq Ur(X)$ when reality is described by the object model in a computerized GIS. Spatial vector

objects often have an extension around them for errors and uncertainties due to unavailable information (Fig. 2(a), 2(b)). Given uncertain positive parameters $\hat{\delta}_1$, $\hat{\delta}_2$ in rough set context, X can be represented by $X = Lr(X) + \hat{\delta}_1$ or $X = Ur(X) - \hat{\delta}_2$. In the sense of $\hat{\delta}_1$ and $\hat{\delta}_2$, $Bnd(X) = \hat{\delta}_1 + \hat{\delta}_2$, $\sim X = U - X = U - Lr(X) + \hat{\delta}_1 = U - Ur(X) + \hat{\delta}_2$. Error ellipse may be used as their depicted mathematical model. Burrough (1996) argued that the object model was suitable for a spatial entity that could be mapped on external features of the landscape, while the field model adapted to a spatial entity when its single quantitative attributes were measured and mapped.

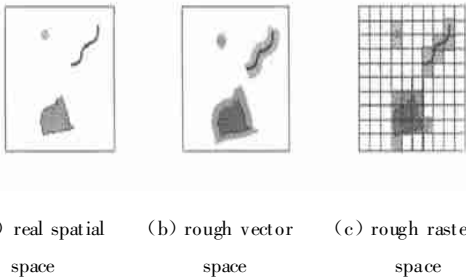


Fig. 2 Rough spatial point, line and area

3.2 Rough raster space

Rough raster space brings approximations into the shapes and forms of a spatial entity. Raster data is for the field model opposed to the object model. Rough spatial point, line and area in the raster space are essential when the real world is put into a computerized GIS. They are illustrated in Fig. 2 (a), 2(c). As Fig. 2 reveals, $Lr(X)$ of the point and line are both empty. $Lr(X)$ of the area has only two equivalence classes. All $Ur(X)$ are relatively bigger than X and $Lr(X)$. So spatial uncertainties (positional and attribute uncertainties) in GIS really exists. Cartographic generalization is a changeable processing of the lower approximation of spatial objects and their upper approximation. However, the pair of approximations of various spatial entities changes in different directions. One becomes bigger, while the other smaller.

Rough set gives a new interpretation on image resolution. Spatial raster data becomes important because many images are rasters. A raster is regarded as a spatial equivalence class in the rough

raster space. The spatial entities are defined with the raster data approximately, e. g. boundaries, and a piece of spatial image is discretized to a regular grid, i. e. an image pixel at a predetermined resolution. The image resolution decides the pixel size. The higher the image resolution is, the less rough degree $Rd(X)$ of the spatial raster entity X is. When the resolution is high enough, or the raster is small enough, the pair of lower and upper approximations of an entity are equal, $Lr(X) = Ur(X)$, namely, the entity is not rough. However, bigger computation storage is also demanded. Therefore, rough set gives another new interpretation on remote sensing image changing with resolution.

Rough multi-dimensional space is composed of a series of unit spatial cubic objects. Spatial object is composed of many blocks. It seems a spatial object like building is built up with toy's blocks. Blocks belonging to the lower approximation are included in the spatial object, while the skin of the objects crosses blocks belonging to the upper approximation but not belonging to the lower approximation. In other words, two "balls" with the same center represent a spatial entity in the multi-dimensional rough raster space. One with a smaller radius is composed of the lower approximation, while the other with a bigger radius is the upper approximation.

3.3 Studying objects as they are

Mathematically, point has no size, line has length but no size, and area no thickness. The attributes of a spatial entity are assumed to vary continuously and smoothly, and they can be described with a smooth mathematical function. However, this model is too abstract. Thus, uncertainties are unavoidable when an abstract mathematical object is used in the study of a complex real object. It is ideal to study a spatial entity as it is. Rough set tries its best to maintain the original characters of the real world via a pair of lower and upper approximations. True value is the lower approximation, while the observed extension is the upper approximation. When a spatial entity has been observed for several times, observed values formalize an extension around the true value because of insufficient infor-

mation. The incomplete information may be from instruments, human being or mathematical functions. Rough set can keep and propagate the uncertain information until final decisions. We argue that superfluous information is better than the lack of information before a decision is made.

4 Rough spatial relationships

Rough spatial topological relationship R is essential in a rough space (U, R) . Before rough topology is advanced, it is necessary to briefly review the development of topological relationships. Munkres defined the meaning of standard topology. Original spatial topological relationships were for simple point (0-dimensional), line (one-dimensional) and area (two-dimensional), with four-intersection model on interior X^o and boundary ∂X . When

$$R_{r9}(A, B) = \begin{pmatrix} \text{Pos}(A) \cap \text{Pos}(B) & \text{Pos}(A) \cap \text{Bnd}(B) & \text{Pos}(A) \cap \text{Neg}(B) \\ \text{Bnd}(A) \cap \text{Pos}(B) & \text{Bnd}(A) \cap \text{Bnd}(B) & \text{Bnd}(A) \cap \text{Neg}(B) \\ \text{Neg}(A) \cap \text{Pos}(B) & \text{Neg}(A) \cap \text{Bnd}(B) & \text{Neg}(A) \cap \text{Neg}(B) \end{pmatrix} \quad (3)$$

Eq. 3 is surely able to tell and propagate certainties ($\text{Pos}(X)$, $\text{Neg}(X)$) and uncertainties ($\text{Bnd}(X)$). one (non-empty) and zero (empty) values are employed for GIS to be computerized. Note that $\text{Neg}(X)$ is different from $\sim X$, the complement of X for $\text{Neg}(X) = U - Ur(X)$, while $\sim X = U - X = U - Ur(X) + \partial X$. So rough spatial relationships give richer information that includes certain and uncertain data, and this may improve the quality of image interpretation. In this sense, Eq. 3 is universal whenever different thematic maps are overlapped. In the rough space of the same image map, it is sure that $\text{Pos}(A) \cap \text{Pos}(B) = 0$.

The rough relationships may be divided into three kinds, i. e. CC (rough relationships between crisp entities and crisp entities), RC (rough relationships between rough entities and crisp entities) and RR (rough relationships between rough entities and rough entities). Here, rough area-area topological relationships in two-dimensional space are discussed mainly. Because area is from line, and line is from point, area is studied as a case. The topologies of point-point, point-line, point-area, line-line and line-area may be regarded as the special cases of area-area. Fig. 3 illustrates the intersection rela-

their limitations appeared, it was extended to 9-intersection model on interior, boundary and exterior X . Then Clementini and Felice introduced areas with broad boundaries composed of an inner boundary and an outer boundary, and transformed the 29 topological matrices to the 44 matrices with 0 and 1 values. Chen et al. proposed a Voronoi-based 9-intersection model via replacing the exterior X^- of an entity with its Voronoi region X^v with \emptyset (empty) and $\neq \emptyset$ (non-empty) values^[5,6]. However, it is difficult to ensure their interior X^o , exterior X^- , or X^v exactly because of insufficient information. In the sequel, boundary ∂X is also unsure, for example, it is a true case that uncertainties exist and is unavoidable in GIS. As an alternative, we propose rough topology via respectively replacing the interior, boundary and exterior with positive region, boundary region and negative region as Eq. (3).

tionships between two rough spatial entities, where $Lr(A)$, $Lr(B)$ are respectively the lower approximations of rough entities A , B ; $Ur(A)$, $Ur(B)$ are respectively the upper approximations; $\text{Bnd}(AB)$ is a rough region between A and B , which is the most uncertain part. Because the indeterminate region often happens in the boundary, it is unable for an uncertainty to take place between the lower approximation A and B . So the intersection relationship often exists at the indeterminate transition zone in image classification, which is composed of two neighboring upper approximations. In the rough space, the set of topological relationships are {disjoint, touch/meet, overlap, equal, covers, covered by, contains, contained by / inside} which are studied by using rough matrices and their figures (Fig. 3). Excluding spatial entities that contain roughness, there are also crisp spatial entities (e. g. administrative boundary) in rough space. According to the above-mentioned, a crisp spatial entity X is a special rough entity where $Lr(X) = Ur(X)$. So rough spatial relationships in the same rough space are divided into three types, CC (Fig. 3 (a)), RC (Fig. 3 (b)) and RR (Fig. 3(c)).

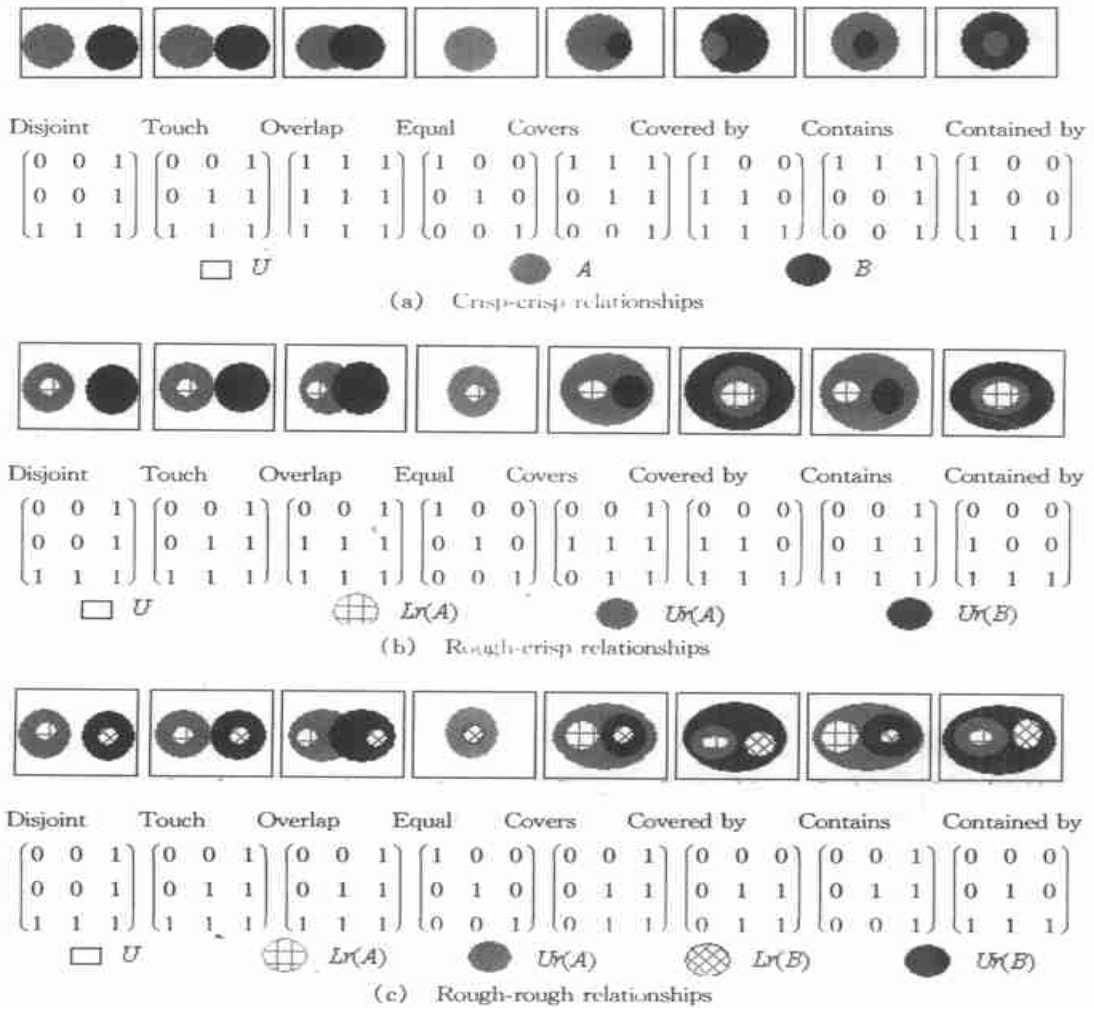


Fig. 3 Rough spatial topological relationships and their corresponding rough matrices

Moreover, a universal equation can be deduced from Eq. 3 to represent the intersected rough regions. When more than two rough spatial entities are intersected, rough regions among them will be described with $Bnd(A_1, \dots, A_i, \dots, A_n)$ (Eq. 4).

$$R_{r9}(A_1, \dots, A_i, \dots, A_n) =$$

$$\left. \begin{matrix} R_{r9}(A_1, A_1) & \dots & R_{r9}(A_1, A_i) & \dots & R_{r9}(A_1, A_n) \\ \dots & \dots & \dots & \dots & \dots \\ R_{r9}(A_i, A_1) & \dots & R_{r9}(A_i, A_i) & \dots & R_{r9}(A_i, A_n) \\ \dots & \dots & \dots & \dots & \dots \\ R_{r9}(A_n, A_1) & \dots & R_{r9}(A_n, A_i) & \dots & R_{r9}(A_n, A_n) \end{matrix} \right\} \quad (4)$$

Here, we take $n = 3$ as an example to interpret the equation. Supposed there are three rough spatial entities A, B and C , which intersect with each other. Besides the two intersected regions, $Bnd(A, B), Bnd(A, C)$ and $Bnd(B, C)$, a new rough region $Bnd(ABC)$ also appears (Fig. 4).

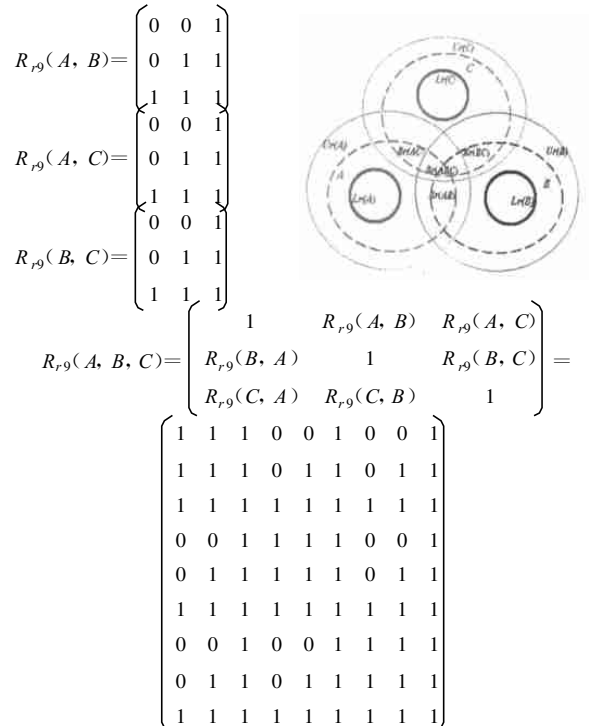


Fig. 4 Rough topological relationships among A, B and C and their rough matrices

5 Case study

As a case study, the method of rough classification based on rough spatial description is proposed and is used to extract river information from a remote sensing image. On the basis of a pair of lower and upper approximations, maximum and minimum maps of river thematic classification are generated via integrating the reduction of superfluous attributes, rough membership function and rough relationships. The original image (Fig. 5(a)) is a piece of remote sensing TM image. There are many conditional attributes, such as image gray scale, the satellite parameters, air refraction, affecting the decisional attributes, i. e. image classification. After other conditional attributes are reduced, gray scale is selected to extract the river classification from the image. Let G_x be the gray scale of a pixel x and G_X be the gray scale of river pixel. Then the rough membership function (Eq. 5) can be extended from Eq. 1.

$$\mu_X(x) = \frac{G_x}{G_X} = \begin{cases} 1 & x \in Lr(X) \\ (0, 1] & x \in Ur(X) \\ 0 & x \in Neg(X) \end{cases} \quad (5)$$

As Fig. 5 (b) and Fig. 5 (c) reveal, the lower approximation $Lr(X)$ is the minimum water map with certainties, while the upper approximation $Ur(X)$ is the maximum water map with uncertainties. Here, $R_d(X) = R_{card}(Ur(X) - Lr(X)) / R_{card}(X) \times 100\% = 10.37\%$. Compared with the crisp classification with only one result, the rough classification not only includes both certainties and uncertainties, but also tells the certainties from the uncertainties.

Furthermore, the results are compared with those from the maximum likelihood classification and the fuzzy classification. The comparison indicates that the rough classification based on rough set contains more information and is of high precision. We get the maximum possible river, minimum certain river, rough confidence degree, possible error, etc. The maximum possible river approaches the river in floodtime, while the minimum certain river comes near the river in low water. Moreover, the precision is improved by 7% than the maximum likelihood classification or by 2% than the fuzzy classifica-

tion.

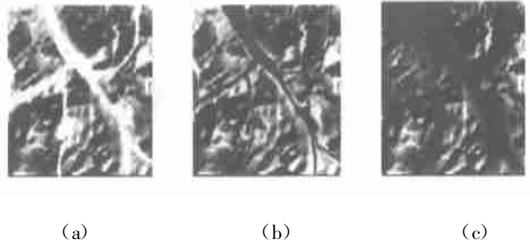


Fig. 5 Rough water thematic maps (continuums)

6 Conclusions

We have proposed a set of rough set symbols in terms of rough interpretation and specialized indication. And the differences between rough set and other methodologies show that rough set can close describe the spatial entity in the real world. In rough set context, we also propose rough spatial entities and their topological relationships. A universal intersected equation and rough membership function with gray scale are further developed. Three kinds of rough spatial topological relationships i. e. CC, RC and RR, are studied by using rough matrices and their figures. The result of the case study not only included more information but also was confidential and practical. This has indicated the method of rough spatial description is an valuable approach to geomatics.

References

- 1 Burrough P A, Frank A U (1996) Geographic objects with indeterminate boundaries. London: Taylor & Francis.
- 2 Pawlak Z (1991) Rough sets: theoretical aspects of reasoning about data. London: Kluwer Academic Publishers.
- 3 Polkowski L, Skowron A (1998) Rough sets in knowledge discovery 1: methodologies and applications. Heidelberg: Physica-Verlag.
- 4 Polkowski L, Skowron A (1998) Rough sets in knowledge discovery 2: applications, case studies and software systems. Heidelberg: Physica-Verlag.
- 5 Wang S L, Li D R, Shi W Z, et al. (2002) Theory and application of geo-rough space. *Geomatics and Information Science of Wuhan University*, 27(3): 274-282 (in Chinese)
- 6 Polkowski L, Tsumoto S, Lin T Y (2000) Rough methods and applications: new developments in knowledge discovery in information systems. Berlin: Physica-Verlag.