Geo-spatial Information Science 10(3):208-212
DOI 10.1007/s11806-007-0065-5

Volume 10, Issue 3
September 2007

Article ID: 1009-5020(2007)03-208-05

Document code: A

Constrained Edge Dynamic Deleting in CD-TIN Based on Influence Domain Retriangulation of Virtual Point

WANG Yanbing WU Lixin SHI Wenzhong

Abstract Constrained Delaunay triangulated irregular network is one kind of dynamic data structures used in geosciences. The research on point and edges insertion in CD-TIN is the basis of its application. Comparing with the algorithms of points and constrained edge insertion, there are very a few researches on constrained edge deletion in CD-TIN. Based on the analysis of the polymorphism of constrained edge, virtual points are used to describe the intersection of constrained edges. A new algorithm is presented, called as influence domain retriangulating for virtual point (IDRVP), to delete constrained edges with virtual points. The algorithm is complete in topology. Finally, the algorithm is tested by some applications cases.

Keywords CD-TIN; data updating; constrained edge deletion; virtual point; influence domain retriangulation
CLC number P208

Introduction

Delaunay triangular irregular networks (D-TIN) and constrained Delaunay triangular irregular networks (CD-TIN), as two basic concepts, have been well known in computational geometry for many years. They have practical applications in several fields: geographic information systems (GIS), finite element methods (FEM), computer graphics and 3D reconstruction[1-6].

Many researchers studied the problem consisting of, given a set of points to obtain its D-TIN, or given a set of points and constrained edges to obtain its CD-TIN[7-9]. And many different algorithms for insertion of D-TIN or CD-TIN were proposed[7-9]. Based on these, some researchers studied on the point deletion for D-TIN or regular triangulations (RT)[2,4,7, 9-11]. J. D. Boissonnat, et al. presented an algorithm for point deletion of CD-TIN called integral ear elimination (IEE) that improved on the EE algorithm[12]. The algorithm IEE can be used for the deletion of constrained point and non-constrained point of CD-TIN.

However, constrained edges deletion is not mentioned by above algorithms. Considering the polymorphism of constraints, this paper presented virtual point to describe the intersection of constrained edges. Using virtual point, any of two constrained segments intersected is divided into two sub-constrained segments, and the virtual point is inserted in CD-TIN. And IDRVP is presented for constrained edge deletion operation.

1 CD-TIN and polymorphism of constrained edges

A CD-TIN can be seen as the triangulation closest to the D-TIN that respects given constrained edges.

Received on June 4, 2007.

WANG Yanbing, Institute of 3D Information Acquisition and Application, Capital Normal University, 105 North Xisanhuan Road, Beijing 100037, China.
E-mail: cnuwyb@yahoo.com.cn; wybing@cumtb.edu.cn

CD-TIN must follow the constrained empty circle during insertion and deletion operations. If there are no constrained edges, CD-TIN is degraded to be a D-TIN.

In GIS and computer geometry, constrained edges are considered to be any set of broken lines, such as polygon edges, broken lines and simple segments. That is to say, a constrained edge is composed of one or more constrained segments, and it is allowed to be any form: open or close, intersection or self-intersection. Such character of constrained edges is called polymorphism. Using above algorithms to construct CD-TIN, the constraints do not fulfill the property of constrained empty circle and visibility any more. Therefore, the paper presented virtual point to describe the intersection of constrained edges.

2 Definition of virtual point

2.1 Virtual point

If a new constrained segment intersected with another one when it inserted in CD-TIN, the point of intersection would be added in both of the two constrained segments as a new point for constructing CD-TIN. We call the point intersected virtual point. Compared to the endpoints of constrained segment, virtual point would be deleted while delete either constrained edge, which just is an interpolated inner point of constrained edges.

Broken lines are composed of several segments, and constrained polygons of constrained segments end to end. Broken lines and polygons insertion into CD-TIN can be seen as segments insertion in order. Note that the first point and end point of polygon are the same point.

2.2 Influence domain retriangulation

Influence domain retriangulation is an algorithm that presented by M. Vigo and N. Pla for constrained segment insertion[13]. But it could not be used for intersection of constrained edges without considering their polymorphism. The author presents a new algorithm for the constrained edges inserting considering their polymorphism.

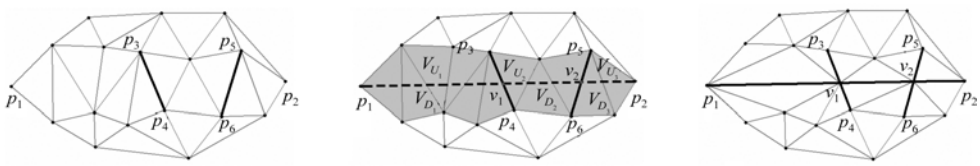
Let p_1, p_2 be the two points which are the endpoints of the constrained segment $p_1p_2\,(p_1, p_2 \in V)$, the routine of p_1p_2 insertion into CD-TIN can be summarized as following steps.

1) Insert a constrained segment p_1p_2 in a CD- TIN, all edges $e_i\,(i=1,2,\cdots,n)$ which are crossed by p_1p_2 are saved in an edge stack. If an edge e_i is a constrained segment, then e_i is marked e_i' and the virtual point intersected between e_i and p_1p_2 is inserted.

2) Delete all the non-constrained edges e_i crossed by p_1p_2, so that a polygon that we called influence domain of constrained edge p_1p_2 without triangulation is left. Clearly, the influence domain contains some constrained edges intersected with p_1p_2.

3) Recursively retriangulate the upper and lower influence domain $(V_{U_i}, V_{D_i}, i=1,2,\cdots,n)$ that is cut by constrained segment p_1p_2 and e_i' following constrained empty circle property.

Fig.1 shows the process for insertion of a constrained segment p_1p_2 within CD-TIN. Because of the intersection of constrained segment p_1p_2, p_3p_4 and p_5p_6, the point of intersect v_1 and v_2 interpolate into p_1p_2 as virtual point so that p_1p_2 is composed of three constrained segments p_1v_1, v_1v_2 and v_2p_2 .Search all the edges crossed by p_1p_2 and remove the non-constrained edges to form an influence domain polygon. Retriangulate the upper and lower polygons $(V_{U_i}, V_{D_i}, i=1,2,\cdots,6)$ that cut by p_1p_2, p_3p_4 and p_5p_6. In each recursive retriangulation, empty circle criterion with respect to the bounding points of the polygon is tested, and the constrained



(a) CD-TIN before inserting constrained edge p_1p_2 (b) Influence domain of p_1p_2 and intersect point between p_1p_2 and other constrained edges (c) CD-TIN after inserting constrained edge p_1p_2

Fig.1 Virtual point insertion and re-triangulation

segments $p_1v_1, v_1v_2, v_2p_2, p_3v_1, v_1p_4, p_5v_2, v_2p_6$ is visible. This is the fundamental point where the proposed algorithm differs from Reference [6]. Anyone of the polygons is retriangulated according to the constrained empty circle property.

The interior of the influence domain V_{U_i} and V_{D_i} is retriangulated using a recursive algorithm (Fig.2). The following is a description of retriangulation for the case of V_{U_1}. Firstly, all the triangulation of V_{U_1} must contain a triangle with constrained segment of p_1v_1 as one of its edges. So the recursive algorithm searches a point p'_0 in the bounding of V_{U_1} that forms a triangle of $T(p_1, v_1, p'_0)$ with vertices p_1, v_1. The triangle fulfills the constrained empty circle property, that is to say, the circumcircle of $T(p_1, v_1, p'_0)$ cannot contain any point of V_{U_I} except its three vertices. After inserting $T(p_1, v_1, p'_0)$, the V_{U_1} is divided into two sub-domains: $F_E(p_1, p'_1, \cdots p'_0)$ and $F_U(p'_0, p'_i, \cdots v_1)$. Recursively apply the algorithm to retriangulate the sub-domains F_E and F_U until the entire domain triangulated.

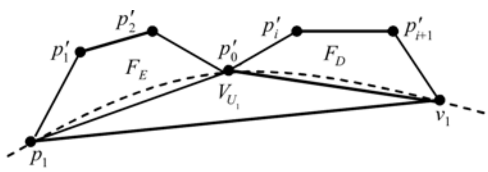


Fig.2 Reconstruct of influence domain

3 Constrained edge deletion in CD-TIN

Constrained edge deletion in CD-TIN is an inverse operation of edge insertion. Firstly, a constrained edge may be a polygon, broken line or segment, to delete it is to remove all the segments of this edge one by one. Secondly, constrained edges may be intersected one another so as to use virtual point to represent the point of intersection during insert a constrained edge. Therefore, the operation of deleting a constrained edge with virtual point, this operation must satisfy the constrained empty circle property. Considering topological completeness of CD-TIN, another constrained edge crossed virtual point cannot be deleted although virtual point will be deleted. Therefore, we define constrained edge deletion in two kinds of conditions that to do with virtual point or without virtual point.

If there is no virtual point in constrained edge, we can delete constrained point of this edge one by one using IEE algorithm[12]. Else if there is virtual point in the edge, we use a new algorithm IDRVP.

3.1 Influence domain retriangulation for virtual point（IDRVP）

IDRVP is presented to delete constrained edges with virtual point. This operation must satisfy the constrained empty circle property. Considering topological completeness of CD-TIN, another constrained edge crossed virtual point cannot be deleted although virtual point will be deleted. Such as the Fig.3 show, constrained edge p_1p_2 is deleted, but $p'_1p'_2$ and $p'_3p'_4$ could not be deleted.

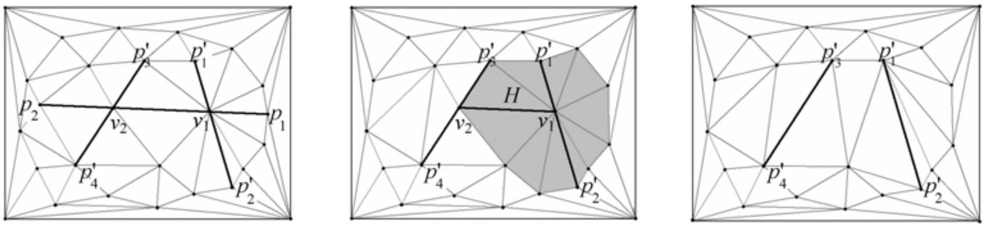
Given a CD-TIN graph $G = \{V, A \cup A'\}$, delete a constrained edge C with virtual point v. Firstly, the influence domain of v is searched, which is a polygon H of the boundary of all the triangles connected to v (virtual point influence domain). Secondly, H is divided into two parts H_L and H_R by the other constrained edge C_i crossed v. Thirdly, H_L and H_R are retriangulated individually according to the constrained empty circle property. Finally, virtual point v is deleted.

As Fig.3 shown, p_1p_2 is a constrained edge that will be deleted, v_1 and v_2 are two points that $p'_1p'_2$ and $p'_3p'_4$ intersected with p_1p_2. Then p_1p_2 is composed of three constrained segments p_1v_1, v_1v_2 and v_2p_2. When deleting p_1p_2, v_1 and v_2 will be deleted, but $p'_1p'_2$ and $p'_3p'_4$ are left in CD-TIN. The detail operation is: ① delete constrained point p_1 and p_2 using IEE, and constrained segments p_1p_2 and v_2p_2 will be deleted simultaneously; ② search influence domain H of v_1 and divide it into two parts H_L and H_R by $p'_1p'_2$, then H_L and H_R are retriangulated individually and delete v_1; 3) repeat Step 2) to delete v_2. Finally, we can get a new CD-TIN that constrained edge p_1p_2 is deleted.

3.2 Constrained edge deletion algorithm

According to above analysis, constrained edges deletion goes as follows (Fig.4).

The pseudo codes of constrained edge deletion in CD-TIN:

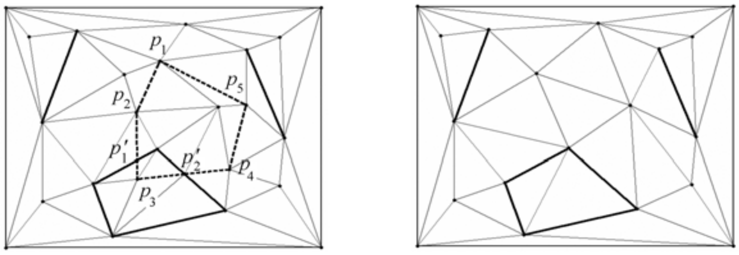


(a) Original CD-TIN, constrained edge p_1p_2 with virtual points v_1,v_2

(b) CD-TIN deleted constrained points p_1, p_2

(c) CD-TIN after deleting virtual points v_1,v_2

Fig.3 Constrained edges deleting with virtual point



(a) Constrained polygon ($p_1 p_2 ... p_5$) is deleted with virtual points p_1' and p_2'

(b) CD-TIN after delete polygon($p_1 p_2 ... p_5$)

Fig.4 Constrained polygon deleting

void DeleteCD-TINEdge(CD-TIN *T*, CEdge *ab*, CVertex *a*, CVertex *b*)

{ Edge_List1= e_i；//all constrained segments of *ab*

for (all constrained segments e_i in Edge_List1)

{ Del_Vertex_List1=p_i; //p_i are constrained points of constrained edge *ab*;

Del_Vertex_List2= p'_j;

// p'_j are virtual points intersected between other constrained edges Edge$_i$ and *ab*;

}

for(all constrained points p_i in Del_Vertex_ List1)

DeleteCD-TINPoint(*T*, p_i); //using IEE

for (all virtual points p'_j in Del_Vertex_List2)

{ store Edge$_i$ and its nodes *a'*, *b'*；

DeleteVirtualPoint(*T*, p'_j); //using IDRVP

}

}

4 Applications

According to above algorithms, the authors have developed a prototype system and tested the usefulness of IDRVP algorithm with applications in several fields. Fig.5 tests that the dynamical construction of CD-TIN with different types constrained edges such as broken lines, polygons and segments. And this example illustrates the correctness of polymorphism of constrained edges with virtual point and IDRVP.

CD-TIN is an important data structure for geographical information representation in GIS. It is widely used in DEM, 3D City model, urban planning and design, etc. For example, the dynamical adding and deleting buildings in digital terrain model can be

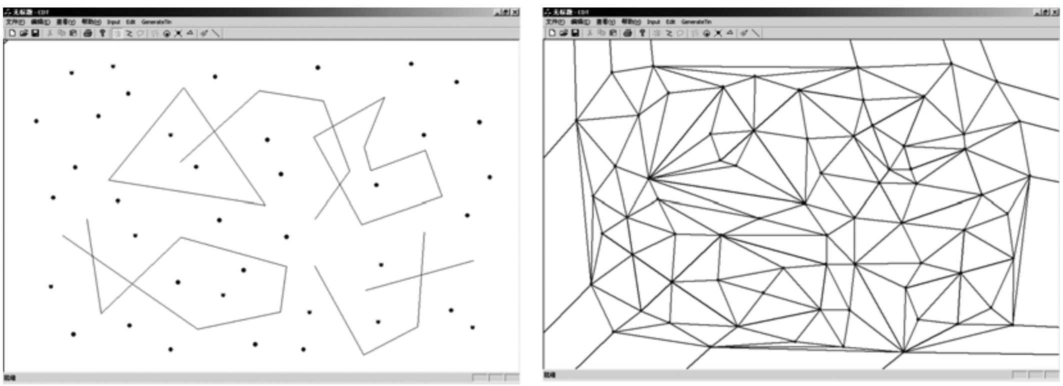
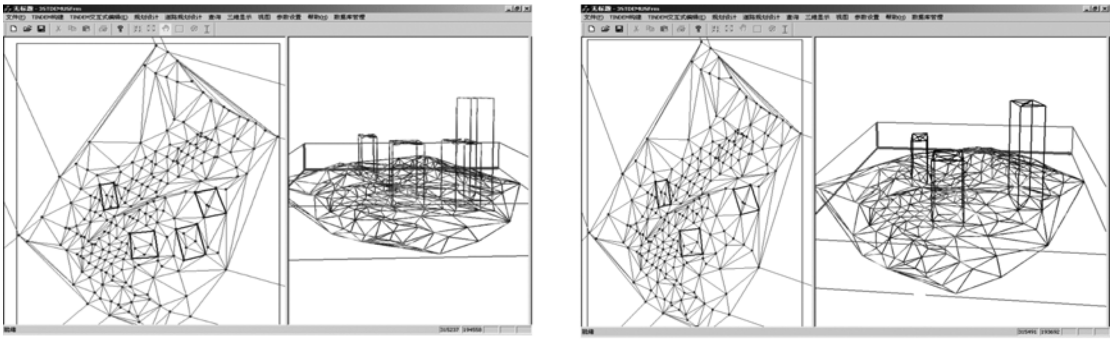


Fig.5 A CD-TIN with different kinds of constrained edges



(a) Adding buildings in DEM (b) Deleting buildings in DEM

Fig.6 Application of CD-TIN in 3D integration of building and terrain model

used in the integration of 3D city model. Usually, TIN is a basic model for representing DTM, buildings can be integrated into the terrain TIN as some constrained edges, and be deleted using IDRVP algorithm (Fig.6).

5 Conclusions

Constrained edges deletion is important to the dynamic update of CD-TIN. Considering the polymorphism of constrained edges, virtual point is introduced to represent the intersection form. IDRVP is presented for deletion of constrained edge with virtual point. The algorithm has such advantages: ① it is more easily to calculate for that the insertion and deletion of constrained edges in a local domain; ② the implementation of constrained edges deletion is realized by two steps of constrained point deletion and virtual point deletion, all the new triangular fulfill the property of constrained empty circle, so the algorithm is complete in topology.

IDRVP is exemplified its use by means of the application of city dynamic construction based on DEM in 3DGIS, and tested that the algorithm is robust by adding and deleting buildings on DEM. As future work, the algorithm is also useful for city planning, city sight design and 3D digital city.

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