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Disordered junction arrays used for Coulomb blockade thermometry

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Abstract. A possible application of nanometer-sized junction arrays is to Coulomb blockade thermometry (CBT), although only highly disordered arrays can be fabricated at present. In this paper, the characteristics of CBT device with disordered arrays will be studied. Similar to what is observed for uniform arrays, there is a dip at zero bias voltage in the differential conductance of disordered arrays. However, the half-width $V_{1/2}$ of the dip for one-dimensional disordered arrays is largely dispersed. This study suggests that better devices can be developed by connecting a number of one-dimensional arrays in parallel to form an array group. The dispersion of half-width is quite small with values of $V_{1/2}$ close to a constant. Further, the effects of electromagnetic environment and low temperature on the half-width are investigated. Results are agreed with those observed experimentally, that for the effect of the environment is negligible for large arrays. The half-width of a disordered array may be bigger or smaller than the ideal value, depending on the extend of disorder.

PACS. 73.23.Hk Coulomb blockade; single-electron tunneling – 07.20.Dt Thermometers

1 Introduction

A primary thermometry method known as the Coulomb blockade thermometry (CBT) has been proposed [1–10]. This method is based on the temperature properties of the Coulomb blockade effect in one- or two-dimensional arrays of nano-sized tunnel junctions at temperatures where the charging energy E_C ($E_C = e^2/2C_{\text{eff}}$, where C_{eff} is the effective capacitance of the tunnel junctions) is less than the thermal fluctuation energy k_BT [7]. The major advantages of the CBT method are that the associated electrical measurement is simple, and the devices are not sensitive to magnetic field [4,11]. In fact, the CBT instruments for cryogenic temperature have already been developed and tested [4].

In the CBT method, the differential conductance-voltage characteristics $G - V$ curve is measured. The temperature can then be extracted by using only natural constants and a calculable prefactor from the properties of this curve. There is a dip in the $G - V$ curve due to the Coulomb blockade effect. In high temperature limit [1,5], the depth of this dip is inversely proportional to temperature, and the half-width $V_{1/2}$ (the full width at half minimum of the conductance dip) is directly proportional to temperature. The half-width is useful for primary thermometry since it depends only on the temperature T through some constants. Following the equation for $G - V$ curve [2,7,8], an array of larger junction number should give a larger $V_{1/2}$ compared to a double-junction system, and then should be sensitive to temperature. However, the arrays are not ordered in general. Junction parameters, including tunnel resistance R_T and capacitance C, were largely dispersed [12]. On the other hand, a one-dimensional (1D) array of tunnel junctions will only work when every junction is functioning properly. If one junction is damaged, the array cannot be used. A parallel-connected array group (PAG), or a two-dimensional (2D) array where the metal islands are connected with tunnel junctions both in parallel and in series, have been proposed [8] to solve this problem. To apply the realistic junction arrays to the CBT, the differential conductance of disordered arrays will be investigated in this paper. The $G - V$ curves and half-width for disordered model of the PAG, in which several disordered 1D arrays are connected in parallel, are calculated. The results for a number of the disordered arrays show that for those PAGs containing large number of disordered 1D arrays, the values of $V_{1/2}$ are always deviated negatively from the values of the uniform 1D array. However, the deviation dispersion among the different PAGs is considerably reduced in comparison with the case of individual disordered 1D arrays. It is possible that PAG containing a very large number of disordered 1D arrays may be used as an ideal uniform 1D array. In addition, the effect of electromagnetic environment and low temperature are investigated,

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and correction to the value of $V_{1/2}$ from electromagnetic environment and low temperature are presented.

2 Model and key equations

A schematic diagram of the model for the disordered PAG structure is shown in Figure 1. All 1D arrays are connected in parallel and have random distributions of junction parameters. To simplify the calculations, the thickness of the barriers between the metal islands is assumed to be uniform [2]. In this way, $R_{T,i}C_i$ can be taken as constants for all junctions. The values of the junction resistance are allowed to be distributed randomly in the region: $R_0 - \Delta R$ to $R_0 + \Delta R$.

For a single arbitrary 1D array of N-junctions, in the high temperature limit the normalized conductance can be expressed as [2]:

$$\frac{G(V)}{G_T} = 1 - 2\sum_{i=1}^{N} \frac{R_{T,\,i}\Delta_i}{R_\Sigma k_B T} g\left(\frac{R_{T,\,i}}{R_\Sigma} eV/k_B T\right), \quad (1)$$

where $R_\Sigma = \sum_{i=1}^{N} R_{T,\,i}$ is the total tunnel resistance; Δ_i describes the Coulomb threshold for the ith junction, and is given by the inverse capacitance matrix $\mathbf{C}^{-1}$ of the junction array, *i.e.*, $\Delta_i = [(\mathbf{C}^{-1})_{i-1,i-1} + (\mathbf{C}^{-1})_{i,i} - 2(\mathbf{C}^{-1})_{i,i-1}]e^2/2$. The function g was introduced in reference [1], and is defined as

$$g(x) = [x\sinh(x) - 4\sinh^2(x/2)]/8\sinh^4(x/2).$$

The elements of capacitance matrix, C_{ij}, are defined as

$$Q_i = \sum_j C_{ij}V_j + \sum_\alpha C_{i\alpha}V_\alpha \quad (2)$$

where Q_i and V_j are the charge and electrostatic potential of the ith metal island between the junctions i and $i+1$ respectively; and V_α is the voltage of the αth outer electrode (lead). If only the nearest capacitive coupling is considered in the 1D junction array, the capacitance matrix can be written as

$$C_{ij} = \begin{cases} C_i + C_{i+1} & \text{if } j = i \\ -C_i & \text{if } j = i-1, \; i,j = 1,2\cdots N-1. \\ -C_{i+1} & \text{if } j = i+1 \end{cases} \quad (3)$$

For a $M \times N$ PAG structure of M arrays and N junctions in each 1D array, as shown in Figure 1, the normalized conductance is

$$\frac{G(V)}{G_T} = 1 - \frac{2}{G_T}\sum_{i=1}^{M}\sum_{j=1}^{N} \frac{R_{T,\,ij}\Delta_{ij}}{R_{\Sigma,i}^2 k_B T} g\left(\frac{R_{T,\,ij}}{R_{\Sigma,i}} eV/k_B T\right), \quad (4)$$

where $G_T = \sum_i (R_{\Sigma,i})^{-1}$, $R_{\Sigma,i} = \sum_j R_{T,\,ij}$, and $R_{T,\,ij}$ is the resistance of the jth junction in the ith array. $\Delta_{ij} = [(\mathbf{C}_i^{-1})_{j-1,j-1} + (\mathbf{C}_i^{-1})_{j,j} - 2(\mathbf{C}_i^{-1})_{j,j-1}]e^2/2$, and $\mathbf{C}_i^{-1}$ is the inverse capacitance matrix of the ith junction array.

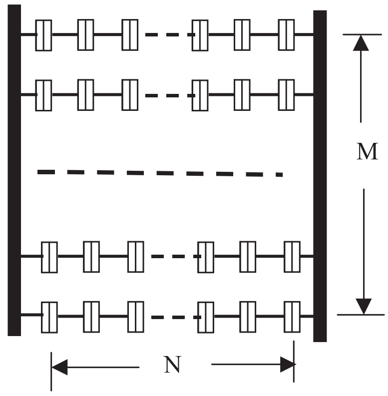

Fig. 1. Schematic presentation of the parallel-connected group structure.

For the disordered PAG, from equation (4),

$$\frac{\Delta G(0)}{G_T} = 1 - \alpha/k_B T, \quad (5)$$

and

$$V_{1/2} = N\beta k_B T/e \quad (6)$$

where

$$\alpha = \frac{1}{3G_T}\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{R_{T,\,ij}\Delta_{ij}}{R_{\Sigma,i}^2},$$

and β can be obtained from the following equation

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{R_{T,\,ij}\Delta_{ij}}{R_{\Sigma,i}^2}\left\{g\left(\frac{NR_{T,\,ij}}{2R_{\Sigma,i}}\beta\right) - \frac{1}{12}\right\} = 0. \quad (7)$$

Using result of reference [13], Δ_{ij} can be read as

$$\Delta_{ij} = \frac{1}{C_{ij}} - \frac{1}{C_{ij}^2\sum_j 1/C_{ij}} \quad (8)$$

and then equation (7) becomes

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{R_{T,\,ij}^2(R_{\Sigma,i} - R_{T,\,ij})}{R_{\Sigma,i}^3}\left\{g\left(\frac{NR_{T,\,ij}}{2R_{\Sigma,i}}\beta\right) - \frac{1}{12}\right\} = 0 \quad (9)$$

where the assumption $R_{T,ij}C_{ij}$ =constant has been used. In general, α and β depend on the junction parameters including the resistance R_{ij} and capacitance C_{ij}. The values of them have to be calibrated by a standard thermometer, so the instrument can be used as secondary thermometers only. But for ideal uniform arrays, β takes a constant of 5.439, which is independent of the junction parameters R_T and C. Therefore, it is worthwhile to study the deviation of the value of β from the constant in the disordered case.

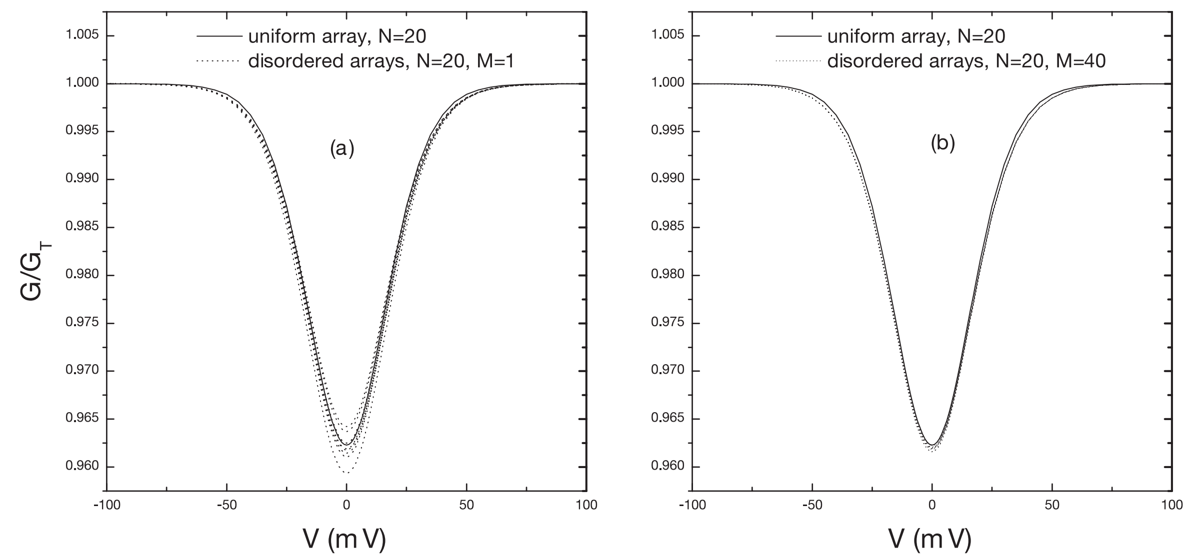

Fig. 2. Normalized conductance G/G_T *vs.* bias voltage V for simulated samples of disordered junction arrays (dashed line) and uniform arrays (solid lines). (a) The results for nine simulated samples of disordered 1D arrays, and (b) the results for nine simulated samples of disorded PAG ($M = 40$).

3 Numerical results and discussion

The conductance G/G_T is calculated from equation (4) as a function of bias voltage V for disordered junction arrays and uniform array respectively, where temperature $T = 4.2$ K. The calculated conductances for nine simulated samples of disordered 1D arrays and nine samples of disordered PAG are shown in Figures 2a and 2b. These disordered arrays have random values of junction resistance $R_{T,ij}$, lying between $(1-20\%)R_0$ and $(1+20\%)R_0$; and the capacitance $C_{ij} = R_0C_0/R_{T,ij}$, where $R_0 = 170$ kΩ and $C_0 = 1e^2/\mathrm{K}$ (where K is used as a energy unit, *i.e.* e^2/C_0 is equal to k_BT for $T = 1$ K). In the calculation, the resistance of each junction is produced by $(1 + 0.2r)R_0$, where r is a random number and $-1 < r < 1$. The curve for a uniform array is also plotted for $R_{ij} = R_0$ and $C_{ij} = C_0$ in Figures 2a and 2b. Similar to the uniform arrays, the conductance curves of the disordered arrays show a dip at zero bias. Dispersion of the junction parameters R_T and C results in the differences between the curves [see Figs. 2a and 2b]. However, the differences between the different PAGs are quite small in comparison to those between different 1D arrays.

The values of β for different simulated samples of disordered junction arrays are calculated according to equation (7). The results for 20 samples of single 1D arrays (solid circles) and 20 samples of PAGs (open circles and triangles) are shown in Figure 3. These disordered arrays are produced in the same way as mentioned above, and the values of junction resistances are taken randomly between $R_0(1 - 20\%)$ and $(1 + 20\%)R_0$. But R_0 and C_0 can be arbitrary constants because the values of β are independent of R_0 and C_0 as given by equation (7). It is obvious that for the case of PAGs, the fluctuation of β is remarkably reduced in comparison with the case of individual 1D arrays, and decreases with the chain number M. For sufficiently large M, the fluctuation of β among the different disordered PAG samples would be very small with its value approaching to a constant. Therefore, a large number of 1D arrays connected in parallel may be used as an uniform 1D array. For the 2D "aligned" arrays, an analytical expression for conductance like equation (4) is not yet available. Since the metal islands of the neighboring chain in the 2D array are connected to each other by tunneling junctions, the junction chain in the 2D array structure cannot be taken as an independent 1D array. The fluctuation of β values among the 2D disordered arrays is not expected to be reduced with the increase of M.

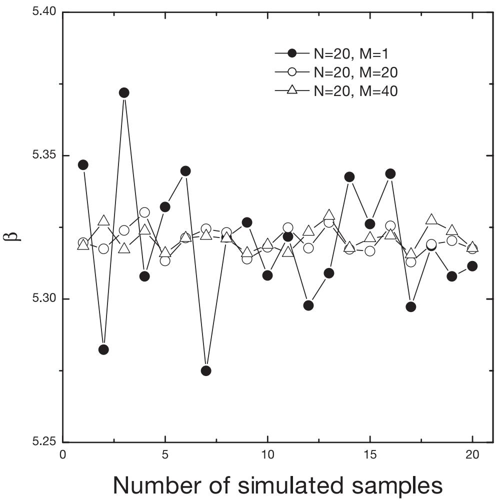

Fig. 3. The values of β for different simulated samples of disordered junction arrays.

On the other hand, β is always smaller than the constant 5.439 for both 1D arrays and PAGs, agreeing with others [2,7]. The experimental values [10] of half-width

$V_{1/2}$ for 1D PAG deviated slightly from the ideal value $V_{1/2,0}$, and little dispersed among different samples. The values of $V_{1/2}$ for 2D array structure deviate largely and are more dispersed. However, the experiments showed that for both 1D PAG and 2D array structures, the half-width for both cases deviated positively from the ideal value. The origin of this phenomenon [10] was ascribed to the impedance of the environment. In fact, the conductance expression equation (1) or (3) is exact only for environmental impedance free and high temperature limit ($k_BT >> \hbar/\pi R_TC$) [5]. In case of $k_BT \sim \hbar/\pi R_TC$, the simple linear relation equation (6) is not exact. So, the correction to $V_{1/2}$ has two origins: first come from the higher order correction of $\hbar/\pi k_BTR_TC$, and second from the impedance of the environment which would vanishes for large number of junctions N. To consider the low-temperature correction to $V_{1/2}$, the half-width can be expressed by

$$V_{1/2} = N\beta_T(T)k_BT/e. \tag{10}$$

Using the phase correlation theory [5,14] and assuming $R_{T,ij}C_{ij}$ =constant, β_T for the PAG can be calculated by the following

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{R_{T,\,ij}^2}{R_{\Sigma,i}^2}\left\{\frac{R_{\Sigma,i}-R_{T,\,ij}}{R_{\Sigma,i}}\right.$$
$$\times\left[h\left(\frac{NR_{T,\,ij}}{2R_{\Sigma,i}}\beta_T,\ u\right)-\frac{1}{2}h\left(0,\ u\right)\right]$$
$$+\frac{R_{T,\,ij}}{R_{\Sigma,i}(1+R_{\Sigma,i}/R_{e,i})}$$
$$\times\left[h\left(\frac{NR_{T,\,ij}}{2R_{\Sigma,i}}\beta_T,\ u(1+R_{\Sigma,i}/R_{e,i})\right)\right.$$
$$\left.\left.-\frac{1}{2}h\left(0,\ u(1+R_{\Sigma,i}/R_{e,i})\right)\right]\right\}=0. \tag{11}$$

Here, $u = \hbar/\pi k_BTR_0C_0$, $R_{e,i}$ is environmental resistance in the ith chain, and

$$h(v,u)=\frac{1}{\pi^2}\int_0^\infty\frac{x(1-e^{-ux})\cos(vx/\pi)}{\sinh^2(x)}\mathrm{d}x. \tag{12}$$

For zero environmental resistance and high temperature limit ($u << 1$), equation (11) becomes equation (9). This can be shown by taking $R_{e,i} \to 0$ and then expanding $h(v,u)$ in u to first order.

Values of β_T are plotted against the junction number N in Figure 4, for $R_{e,i} = R_e = 0.014R_0$ and different r and u, where for disordered junction arrays β_T represents the mean values among 20 simulated samples. Results of a uniform array for $R_e = 0.001R_0$ and $u = 1/20$ are also plotted in the same figure (see open diamonds). Similar to experimental results reported in reference [10], the correction to half-width is always positive and increase with the decrease of N. The rapid increase of half-width for the small junction number N is due to the effect of the environmental resistance. But for larger N, the environmental resistance has no effect on the half-width. The low-temperature correction exists for any N. As shown above,

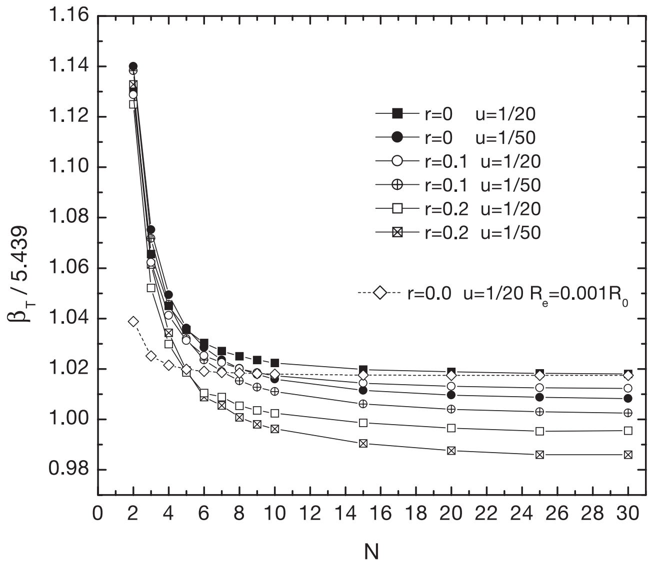

Fig. 4. The values of β_T as the function of junction number N in each chain of the PSG. For disordered junction arrays β_T represents the mean values among 20 simulated samples.

the deviation due to the disorderliness of junction is negative. Therefore, the experimental values of half-width $V_{1/2}$ for realistic junction arrays may be either bigger or smaller than the ideal value of $5.439Nk_BT/e$ [2,10].

4 Conclusion

To make better use of disordered junction arrays for high quality CBT, the differential conductance of disordered junction array groups has been studied. The junction array groups are composed of many disordered 1D junction arrays. Similar to uniform arrays, the differential conductance of disordered arrays shows up a dip at zero bias voltage. The dispersion of half-width $V_{1/2}$ (or the conductance dip depth $\Delta G/G_T$) for the individual 1D disordered arrays is very large. However, the dispersion for parallel-connected disordered array groups composed of many 1D arrays is quite small. $V_{1/2}$ are close to a constant for large junction number N, and deviate negatively from the ideal value $5.439Nk_BT/e$. Therefore, it can be concluded that PAG containing a very large number of disordered 1D arrays may be used as an uniform 1D array for temperature measurement. Furthermore, the effects of the environmental impedance and low temperature on the half-with were investigated. The correction to the value of the $V_{1/2}$ from effects is positive. The calculated results showed that for sufficient large junction number N, the effect of the environment vanishes as demonstrated by experiments. Combining this effect and non-uniformity of the junction parameters, it can be explained that the half-width $V_{1/2}$ for realistic junction arrays may be either larger or smaller than the ideal value $5.439Nk_BT/e$, depending on the extend of disorder.

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