Modified Kastner Formula for Cylindrical Cavity Contraction in Mohr-Coulomb Medium for Circular Tunnel in Isotropic Medium

Y. M. Cheng

Journal of Mechanics / Volume null / Issue 01 / March 2012, pp 163 - 169
DOI: 10.1017/jmech.2012.17, Published online: 22 March 2012

MODIFIED KASTNER FORMULA FOR CYLINDRICAL CAVITY CONTRACTION IN MOHR-COULOMB MEDIUM FOR CIRCULAR TUNNEL IN ISOTROPIC MEDIUM

Y. M. Cheng *

Department of Civil and Structural Engineering
Hong Kong Polytechnic University
Kowloon, Hong Kong

ABSTRACT

The Kastner formula for cavity contraction is one of the commonly used formulae for the cavity contraction problem in tunnel excavation, and it is based on the assumption of small displacement around the cavity with no volume change in the plastic zone. In this paper, the errors arising from these assumptions are discussed, and the volume change of the plastic zone is considered. It can be seen that this assumption is reasonable for normal situations, but for rock with weak shear strength, the Kastner formula should be used with care, especially in situations where there will be relatively large volume changes in the plastic zone.

Keywords: Cavity contraction, Cavity expansion, Kastner formula.

1. INTRODUCTION

Cavity expansion theory was first studied by Bishop *et al*. [1] for the metal indentation problem and was later applied to geotechnical problems by Gibson and Anderson [2], Ladanyi [3], Palmer [4], Vesic [5], Hughes *et al*. [6], Randolph and Worth [7], Randolph *et al*. [8], Houlsby and Withers [9]. The asymptotic value, which is also known as the cavitation pressure, was determined by Hill [10], Durban and Baruch [11] and Durban [12,13]. The problem of cavity expansion and cavity contraction has attracted much attention in geotechnical problems with application to the bearing capacity of deep foundations, interpretation of pressure-meter tests, breakout resistance of anchors, pile driving, wellbore instability, underground excavation and blasting fracturing by explosives. Yu [14-16] has summarised and presented the major developments in the field of cavity expansion/contraction theory and its geotechnical applications.

The stability of tunnel surrounding rock is controlled by the stresses and the strength of the surrounding rock. If the surrounding rock is in a plastic/fracture state after the tunnel is driven, the surrounding rock will be unstable. The thickness of the plastic zone can be taken as a comprehensive index to evaluate the stability of the surrounding rock of a deep tunnel. Kastner's solution [17] is often used in elastic-plastic analysis for the surrounding rock of a circular tunnel. It is well-known that the Kastner formula is based on the ideal elastic-plastic model, and the Kastner solution may deviate from the actual situation in the surrounding rock. Kastner [18] obtained an improved solution for the extent of the plastic zone of a circular opening in elasto-perfectly plastic material, and subsequently Salencon (1969) presented a more systematic elasto-plastic analysis to the same problem. Different constitutive models have been used to obtain cavity expansion solutions that can consider the frictional, cohesive and dilatant behaviour of geomaterials (Carter *et al*. [19]; Yu and Houlsby [20]; Yu [21]; Collins *et al*. [22]; Salgado *et al*. [23]; Ladanyi and Foriero [24]).

The use of the method by Kastner [17] for the analysis of yield zone radius and tunnel boundary displacement is very popular in China and some other countries. Because there are currently many tunnel and mining works in the world (and particularly in Asia), the validity of the method by Kastner [17] becomes an important issue. Wilson [25] also derived the formulas for calculating the yield zone radius and tunnel boundary displacement for the cavity contraction problem. Because some of the assumptions in the formulae by Kastner [17] and Wilson [25] may not be satisfactory, revised formulae will be derived in this paper without these assumptions. The differences between these new results and the original results will be compared to examine the validity of the popular Kastner [17] and Wilson formulae [25].

* ceymchen@polyu.edu.hk

DOI : 10.1017/jmech.2012.17

2. UNIFIED DERIVATION OF CAVITY EXPANSION AND CAVITY CONTRACTION

Suppose that the original primary stress in the soil or rock is p_0 (at rest pressure coefficient $K_o = 1$) and that a cylindrical cavity is excavated with a uniformly distributed internal pressure p_u acting on the cavity surface. When $p_0 < p_u$, it is the case of cavity expansion (Fig. 1), and for $p_0 > p_u$, it is the case of cavity contraction (Fig. 2). For the cavity expansion problem of pressure-meter test, p_u refers to the pressure arising from the expansion of the pressure-meter membrane, and for the cavity contraction problem of tunnel excavation, p_u refers to the supporting pressure.

For both problems, the initial radius of the cavity is denoted as R_0, and the ultimate radius is denotes as R_u. A cylindrical zone around the cavity will pass into the state of plastic equilibrium if p_u is much larger than p_0 for cavity expansion, or p_u is much less than p_0 for cavity contraction. Suppose the plastic zone around the cavity will extend to a radius R_p. Beyond that radius, the rest of the mass remains in a state of elastic equilibrium. It is assumed that the soil or rock in the plastic zone behaves as Mohr-Coulomb material with shear strength parameters c and ϕ. Beyond the plastic zone, the soil or rock is assumed to behave as a linearly deformable, isotropic solid defined by a modulus of deformation E and a Poisson's ratio ν.

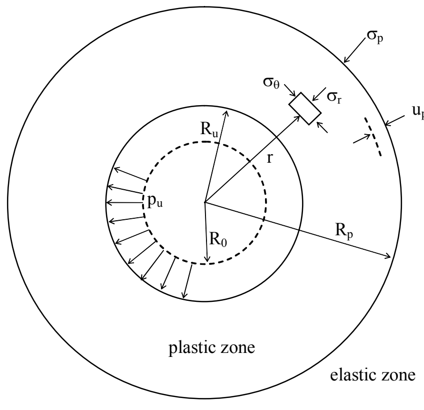


Fig. 1 Expansion of cavity

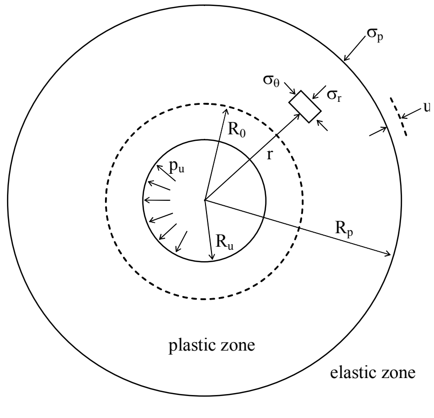


Fig. 2 Contraction of cavity

3. ELASTIC RANGE

According to theory of elasticity, we can take the stress function ϕ as

$$\phi = 0.5Ar^2 + B\ln r \tag{1}$$

where A and B are constants. The radial and circumferential stresses are then given by

$$\sigma_r = \frac{1}{r}\frac{d\phi}{dr} = A + \frac{B}{r^2} \tag{2}$$

$$\sigma_\theta = \frac{d^2\phi}{dr^2} = A - \frac{B}{r^2} \tag{3}$$

From the far field boundary condition, when $r \to \infty$, both σ_r and σ_θ equal p_0, so A equals p_0. Then,

$$\sigma_r = \frac{1}{r}\frac{d\phi}{dr} = p_0 + \frac{B}{r^2} \tag{4}$$

$$\sigma_\theta = \frac{d^2\phi}{dr^2} = p_0 - \frac{B}{r^2} \tag{5}$$

4. PLASTIC RANGE

The equilibrium differential equation is given by

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{6}$$

For cavity expansion, $\sigma_r > \sigma_\theta$, according to Mohr-Coulomb rule, σ_r is given by

$$\sigma_r = \sigma_\theta \frac{1+\sin\phi}{1-\sin\phi} + \frac{2c\cos\phi}{1-\sin\phi} \tag{7}$$

Introducing Eq. (7) into (6),

$$\frac{d\sigma_r}{dr} + \frac{2\sin\phi}{1+\sin\phi}\frac{\sigma_r}{r} + \frac{2c\cos\phi}{1+\sin\phi}\frac{1}{r} = 0 \tag{8}$$

Solving Eq. (8) gives

$$\sigma_r = Dr^{\frac{-2\sin\phi}{1+\sin\phi}} - c\cot\phi \tag{9}$$

where D is a constant. When $\sigma_r = p_u$, $r = R_u$ (radius of hole at ultimate), so

$$D = (p_u + c\cot\phi)R_u^{\frac{2\sin\phi}{1+\sin\phi}} \tag{10}$$

Introducing Eq. (10) into (9),

$$\sigma_r = (p_u + c\cot\phi)\left(\frac{R_u}{r}\right)^{\frac{2\sin\phi}{1+\sin\phi}} - c\cot\phi \tag{11}$$

Introducing Eq. (11) into (7),

$$\sigma_\theta = (p_u + c\cot\phi)\left(\frac{1-\sin\phi}{1+\sin\phi}\right)\left(\frac{R_u}{r}\right)^{\frac{2\sin\phi}{1+\sin\phi}} - c\cot\phi \tag{12}$$

At the boundary $r = R_p$, the stresses of the elastic and the plastic zones must coincide, so from Eqs. (4), (5), (11) and (12), we obtain

$$p_0 + \frac{B}{R_p^2} = (p_u + c\cot\phi)\left(\frac{R_u}{R_p}\right)^{\frac{2\sin\phi}{1+\sin\phi}} - c\cot\phi \tag{13}$$

$$p_0 - \frac{B}{R_p^2} = (p_u + c\cot\phi)\left(\frac{1-\sin\phi}{1+\sin\phi}\right)\left(\frac{R_u}{R_p}\right)^{\frac{2\sin\phi}{1+\sin\phi}} - c\cot\phi \tag{14}$$

From Eqs. (13) and (14), R_p is given by

$$R_p = R_u\left[\frac{p_u + c\cot\phi)}{(p_0 + c\cot\phi)(1+\sin\phi)}\right]^{\frac{1+\sin\phi}{2\sin\phi}} \tag{15}$$

From Eq. (15), we can also obtain

$$p_u = (p_0 + c\cot\phi)(1+\sin\phi)\left(\frac{R_p}{R_u}\right)^{\frac{2\sin\phi}{1+\sin\phi}} - c\cot\phi \tag{16}$$

For cavity contraction, $\sigma_r < \sigma_\theta$, according to the Mohr-Coulomb rule, σ_θ, is given by

$$\sigma_\theta = \sigma_r\frac{1+\sin\phi}{1-\sin\phi} + \frac{2c\cos\phi}{1-\sin\phi} \tag{17}$$

Introducing Eq. (17) into (6),

$$\frac{d\sigma_r}{dr} - \frac{2\sin\phi}{1-\sin\phi}\frac{\sigma_r}{r} - \frac{2c\cos\phi}{1-\sin\phi}\frac{1}{r} = 0 \tag{18}$$

Solving Eq. (18),

$$\sigma_r = D_1 r^{\frac{2\sin\phi}{1-\sin\phi}} - c\cot\phi \tag{19}$$

where D_1 is a constant. When $\sigma_r = p_u$, $r = R_u$ (radius of hole at ultimate), so

$$D_1 = (p_u + c\cot\phi)R_u^{\frac{-2\sin\phi}{1-\sin\phi}} \tag{20}$$

Introducing Eq. (20) into (19) gives

$$\sigma_r = (p_u + c\cot\phi)\left(\frac{R_u}{r}\right)^{\frac{-2\sin\phi}{1-\sin\phi}} - c\cot\phi \tag{21}$$

or

$$\sigma_r = (p_u + c\cot\phi)\left(\frac{r}{R_u}\right)^{\frac{2\sin\phi}{1-\sin\phi}} - c\cot\phi \tag{22}$$

Introducing Eq. (22) into (17) gives

$$\sigma_\theta = (p_u + c\cot\phi)\left(\frac{1+\sin\phi}{1-\sin\phi}\right)\left(\frac{r}{R_u}\right)^{\frac{2\sin\phi}{1-\sin\phi}} - c\cot\phi \tag{23}$$

At the boundary $r = R_p$ the stresses of the elastic and the plastic zones must coincide, so from Eqs. (4), (5), (22) and (23), we obtain

$$p_0 + \frac{B}{R_p^2} = (p_u + c\cot\phi)\left(\frac{R_p}{R_u}\right)^{\frac{2\sin\phi}{1-\sin\phi}} - c\cot\phi \tag{24}$$

$$p_0 - \frac{B}{R_p^2} = (p_u + c\cot\phi)\left(\frac{1+\sin\phi}{1-\sin\phi}\right)\left(\frac{R_p}{R_u}\right)^{\frac{2\sin\phi}{1-\sin\phi}} - c\cot\phi \tag{25}$$

Using Eq. (24) and (25) gives

$$R_p = R_u\left[\frac{(p_0 + c\cot\phi)(1-\sin\phi)}{p_u + c\cot\phi}\right]^{\frac{1-\sin\phi}{2\sin\phi}} \tag{26}$$

From Eq. (26), we can also see that

$$p_u = (p_0 + c\cot\phi)(1-\sin\phi)\left(\frac{R_p}{R_u}\right)^{\frac{2\sin\phi}{1-\sin\phi}} - c\cot\phi \tag{27}$$

5. STRESS DISTRIBUTION FOR CAVITY EXPANSION AND CAVITY CONTRACTION

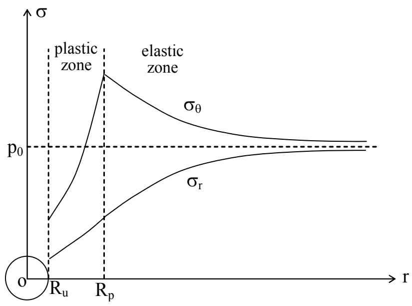
In Eq. (16), if $R_u = R_p$, then $p_u = (1 + \sin\phi)p_0 + c\cos\phi$. For cavity expansion, there is no plastic zone around the cavity when $p_u < (1 + \sin\phi)p_0 + c\cos\phi$. Similarly, in Eq. (27), if $R_u = R_p$, then $p_u = (1 - \sin\phi)p_0 - c\cos\phi$. For cavity contraction, there is no plastic zone around the cavity when $p_u > (1 - \sin\phi)p_0 - c\cos\phi$. So

$$p_u < (1-\sin\phi)p_0 - c\cos\phi:\quad R_p = R_u\left[\frac{(p_0 + c\cot\phi)(1-\sin\phi)}{p_u + c\cot\phi}\right]^{\frac{1-\sin\phi}{2\sin\phi}} \tag{28}$$

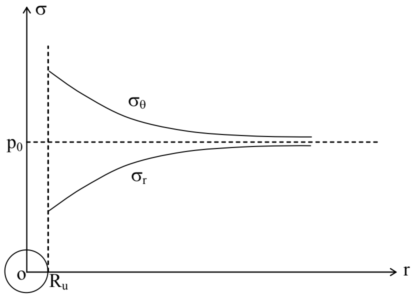
$$(1-\sin\phi)p_0 - c\cos\phi \le p_u \le (1+\sin\phi)p_0 + c\cos\phi:\quad R_p = R_u \ \text{(no plastic zone)} \tag{29}$$

$$p_u > (1+\sin\phi)p_0 + c\cos\phi:\quad R_p = R_u\left[\frac{p_u + c\cot\phi)}{(p_0 + c\cot\phi)(1+\sin\phi)}\right]^{\frac{1+\sin\phi}{2\sin\phi}} \tag{30}$$

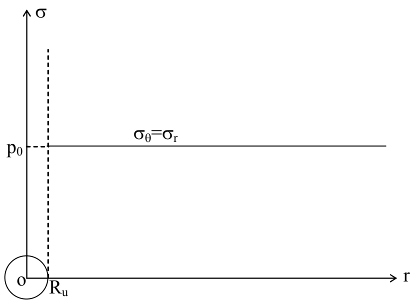
In Fig. 3, the stress distribution around the cavity is shown with the variation of pressure p_u. When $p_u < (1 - \sin\phi)p_0 - c\cos\phi$, in the plastic zone, both σ_r and σ_θ increase with the increase in the radius, while in the elastic zone, σ_θ decreases and σ_r still increases with the increase in the radius. At radius R_p, σ_θ has the largest value. When $(1 - \sin\phi)p_0 - c\cos\phi \le p_u < p_0$, there is no plastic zone around the cavity, and σ_r increases while σ_θ decreases with the increase in the radius. When $p_u = p_0$, in all the soil or rock mass around



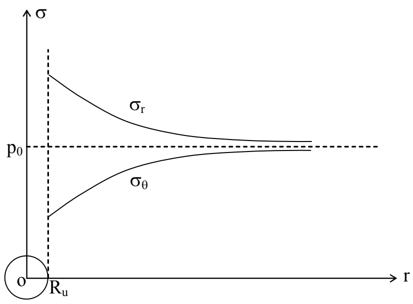
(a) $p_u < (1-\sin\phi)p_0 - c\cos\phi$



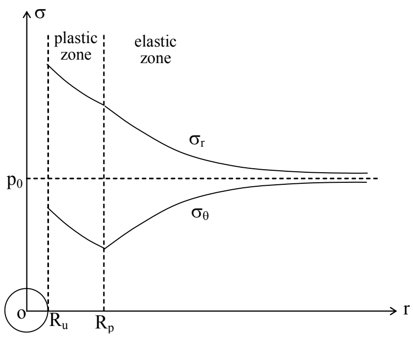
(b) $(1-\sin\phi)p_0 - c\cos\phi \le p_u < p_0$



(c) $p_u = p_0$



(d) $p_0 < p_u \le (1 + \sin\phi)p_0 + c\cos\phi$



(e) $p_u > (1 + \sin\phi)p_0 + c\cos\phi$

Fig. 3 Stress distribution around the cavity

the cavity, σ_r is always equals to σ_θ with no plastic zone. When $p_0 < p_u \le (1 + \sin\phi)p_0 + c\cos\phi$, there is also no plastic zone around the cavity, and σ_r decreases while σ_θ increases with the increase in the radius. When $p_u > (1 + \sin\phi)p_0 + c\cos\phi$, in the plastic zone, both σ_r and σ_θ decrease with the increase in the radius, while in the elastic zone, σ_θ increases while σ_r still decreases with the increase in the radius. At radius R_p, σ_θ has its lowest value.

5.1 Kastner's Formula for Cavity Contraction

For a deep circular tunnel excavation, the radius R_p of the plastic zone was derived by Kastner [17] as

$$R_p = R_0 \left[\frac{(p_0 + c\cot\phi)(1-\sin\phi)}{p_u + c\cot\phi} \right]^{\frac{1-\sin\phi}{2\sin\phi}} \tag{28}$$

Comparing Eq. (28) with Eq. (26), we can see that in the Kastner formula [17], it is implicitly assumed that $R_u = R_0$. This assumption is based on the small displacement around the cavity and no volume change in the plastic zone. Based on this assumption, the displacement of the tunnel boundary can be derived as

$$\mathrm{u}_0 = \frac{\sin\phi}{2GR_0}(p_0 + c\cot\phi)R_p^2 \tag{29a}$$

where $\mathrm{G} = \dfrac{E}{2(1+\nu)}$. Putting G and R_p into the above equation gives

$$\mathrm{u}_0 = R_0 \frac{1+\nu}{E} \sin\phi (p_0 + c\cot\phi) \left[\frac{(p_0 + c\cot\phi)(1-\sin\phi)}{p_u + c\cot\phi} \right]^{\frac{1-\sin\phi}{\sin\phi}} \tag{29b}$$

5.2 Wilson's Formula for Cavity Contraction

For a deep circular tunnel excavation, the radius of the plastic zone was derived by Wilson [25] as

$$\overline{r} = r_0 \left\{ \frac{2q - \sigma_0 + p'(k+1)}{(p+p')(k+1)} \right\}^{1/(k-1)} \tag{30a}$$

In the above equation: $\overline{r} = R_p$, $r_0 = \mathrm{R}_0$, $k = \dfrac{1+\sin\phi}{1-\sin\phi}$, $\sigma_0 = \dfrac{2c\cos\phi}{1-\sin\phi}$, $p' = c \cot\phi$, $q = p_0$, $p = p_u$. Thus, putting these items into the above equation gives

$$R_p = R_0 \left[\frac{(p_0 + c\cot\phi)(1-\sin\phi)}{p_u + c\cot\phi} \right]^{\frac{1-\sin\phi}{2\sin\phi}} \tag{30b}$$

It can be seen that this equation is the same as the one derived by Kastner [17]. However, for the derivation of the tunnel boundary displacement, the expansion in the yield zone is considered by Wilson [25], and an average "expansion factor" ε, which is linearly dependent only on the distortion, is assumed. The equation for the tunnel boundary displacement according to Wilson [25] is

$$\mathrm{u}_0 = r_0 \frac{1+\nu}{E}\left\{\frac{(k-1)q+\sigma_0}{(k+1)}\right\}\left\{\frac{2q-\sigma_0+p'(k+1)}{(p+p')(k+1)}\right\}^{(2+\varepsilon)/(k-1)} \tag{31a}$$

Using the symbols presented above, this equation can be expressed as

$$\mathrm{u}_0 = R_0 \frac{1+\nu}{E}\sin\phi(p_0+c\cot\phi)\left[\frac{(p_0+c\cot\phi)(1-\sin\phi)}{p_u+c\cot\phi}\right]^{(2+\varepsilon)\frac{1-\sin\phi}{2\sin\phi}} \tag{31b}$$

Above, when the expansion factor ε is equal to zero, Eq. (31b) can be reduced to the Kastner formula [17].

5.3 Modified Kastner Formula for Cavity Contraction

It is clear that for calculating the radius of the plastic zone, both the Kastner [17] and Wilson [25] formulae implicitly assume that R_u=R_0. For calculating the displacement of the tunnel boundary, no volume change in the plastic zone is assumed in the Kastner formula [17]. Volume change is considered by Wilson [25], but the assumption of small displacement compared to the tunnel radius is still required to simplify the derivation. The expansion factor by Wilson is different from the concept of volumetric strain in classical elasticity/plasticity theory, and is proposed by Wilson [25] without any theoretical background and no simple method can be used to determine this complex value.

In this paper, the error caused by the assumption of $R_u = R_0$ will be discussed, and the volume change of the plastic zone is considered by introducing an average volumetric strain Δ, which is similar to Vesic's derivation for cavity expansion (Vesic, [5]). The relationship of R_u and R_0 is derived as follows.

At the radius $r = R_p$, the radial displacement, u_p, can easily be computed from the elastic theory:

$$u_p = \frac{1+\nu}{E}R_p(p_0-\sigma_{r(p)}) \tag{32}$$

It is the boundary of elastic zone and plastic zone when $r = R_p$, so

$$\sigma_{r(p)}+\sigma_{\theta(p)} = 2p_0 \tag{33}$$

$$\sigma_{\theta(p)} = \frac{1+\sin\phi}{1-\sin\phi}\sigma_{r(p)}+\frac{2c\cos\phi}{1-\sin\phi} \tag{34}$$

where $\sigma_{r(p)}$ and $\sigma_{\theta(p)}$ are the radial stress and tangential stress at the boundary of the elastic zone. From Eqs. (33) and (34), it can be shown that

$$\sigma_{r(p)} = (1-\sin\phi)p_0 - c\cos\phi \tag{35}$$

Introducing Eq. (35) in Eq. (32),

$$u_p = \frac{1+\nu}{E}R_p\sin\phi(p_0+c\cot\phi) \tag{36}$$

The volume change in the plastic zone is equal to the change of volume of the elastic zone plus the change of volume of the plastic zone. It can be written as

$$\pi R_0^2-\pi R_u^2 = \pi(R_p+u_p)^2-\pi R_p^2+\left[\pi(R_p+u_p)^2-\pi R_0^2\right]\Delta \tag{37}$$

where Δ is the volume change ratio in the plastic zone and $\Delta > 0$ denotes an increase of the volume.

Defining $F = \left[\frac{(p_0+c\cot\phi)(1-\sin\phi)}{p_u+c\cot\phi}\right]^{\frac{1-\sin\phi}{2\sin\phi}}$ and $H = \frac{1+\nu}{E}\sin\phi(p_0+c\cot\phi)$, Eqs. (26) and (36) can be written as

$$R_p = FR_u \tag{38}$$

$$u_p = HFR_u \tag{39}$$

Introducing Eqs. (38) and (39) into (37) yields

$$R_u = R_0\sqrt{\frac{1+\Delta}{1+F^2\left[(1+H)^2(1+\Delta)-1\right]}} \tag{40a}$$

Defining $\xi = \sqrt{\frac{1+\Delta}{1+F^2\left[(1+H)^2(1+\Delta)-1\right]}}$,

$$R_u = \xi R_0 \tag{40b}$$

Introducing Eq. (40) into Eq. (26) gives

$$R_p = \xi R_0\left[\frac{(p_0+c\cot\phi)(1-\sin\phi)}{p_u+c\cot\phi}\right]^{\frac{1-\sin\phi}{2\sin\phi}} \tag{41a}$$

or

$$R_p = \frac{R_0\left[\frac{(p_0+c\cot\phi)(1-\sin\phi)}{p_u+c\cot\phi}\right]^{\frac{1-\sin\phi}{2\sin\phi}}}{\sqrt{\frac{1+\Delta}{1+\left[\frac{(p_0+c\cot\phi)(1-\sin\phi)}{p_u+c\cot\phi}\right]^{\frac{1-\sin\phi}{\sin\phi}}\left[1+\frac{1+\nu}{E}\sin\phi(p_0+c\cot\phi)\right]^2(1+\Delta)-1\Big\}}}} \tag{41b}$$

Obviously, the tunnel boundary displacement u_o is the difference between R_u and R_{-0}, so

$$u_0 = R_0 - R_u = (1-\xi)R_0 \quad \text{and} \quad \xi = R_u / R_0 \tag{42a}$$

or

$$u_0 = \left\{ 1 - \sqrt{\frac{1+\Delta}{1+\left[\frac{(p_0 + c\cot\phi)(1-\sin\phi)}{p_u + c\cot\phi}\right]^{\frac{1-\sin\phi}{\sin\phi}} \left\{ \left[1+\frac{1+\nu}{E}\sin\phi(p_0 + c\cot\phi)\right]^2 (1+\Delta) - 1 \right\}}} \right\} R_0 \tag{42b}$$

Thus, the modified formulae proposed in this paper are derived as Eqs. (41) and (42) for the plastic zone radius and the tunnel boundary displacement, respectively. We will compare the results from the modified formula against the results from the Kastner [17] and Wilson [28] formulae. Suppose a 3m diameter tunnel is excavated in a rock mass with a Poisson ration $\nu = 0.3$ and elastic modulus $E = 3000$MPa. The primary stress $p_0 = 7.5$MPa, and the supporting pressure $p_u = 0.1$MPa. For the first example, the cohesion c is equal to 2MPa, and the friction angle ϕ is equal to 30°. The ξ values for different volume expansion ratios, Δ, are presented in Table 1. It can be seen for this normal situation that the ratio of R_u to R_0 is nearly equivalent to 1, and the difference between R_u and R_0 is not larger than 2%. It is hence reasonable to assume $R_u = R_0$ in the Kastner [17] and Wilson formulae [25] for normal situations. If the friction angle is changed to 20° and the cohesion is reduced to 0.4MPa (Table 2), there are great differences between R_u and R_0. For weak rock, one should be careful when using the Kastner [17] and Wilson formulae [25] to calculate the plastic zone radius, especially in situation where a relatively large volume change in the plastic zone may occur.

For the comparison of the tunnel boundary displacement, though the volume change in the yield zone is considered in both the modified formula derived in this paper and the Wilson formula [25], different volume change parameters are used by Wilson and the authors without any direct relation between the parameters, so it is difficult to directly compare the results of these two formulas. Therefore, the comparison of tunnel displacement between Kastner [17] and the modified formula was conducted first (shown in Fig. 4), and then the comparison between Kastner [17] and Wilson formula [25] was conducted (shown in Fig. 5). If there is no volume change in the yield zone ($\Delta = 0$, $\varepsilon = 0$), the difference between the modified formula and Kastner formula [17] is not large (not exceeding 4% in these examples). The results using the Kastner formula [17] and the Wilson formula [25] are the same when ε is equal to zero because the Wilson formula [25] is reduced to the Kastner formula [17] in this case. If volume change is considered, with the increase in the volume change parameter, the result using the modified formula increases much faster than the Wilson formula [25].

Table 1 The R_u to R_0 ratio for a competent rock situation ($\xi = R_u / R_0$)

c (MPa)	ϕ(°)	Δ	ξ
2	30	0	0.9964
2	30	0.01	0.9937
2	30	0.04	0.9863

Table 2 The R_u to R_0 ratio for weak rock ($\xi = R_u / R_0$)

c (MPa)	ϕ(°)	Δ	ξ
0.4	20	0	0.9757
0.4	20	0.01	0.8993
0.4	20	0.04	0.751

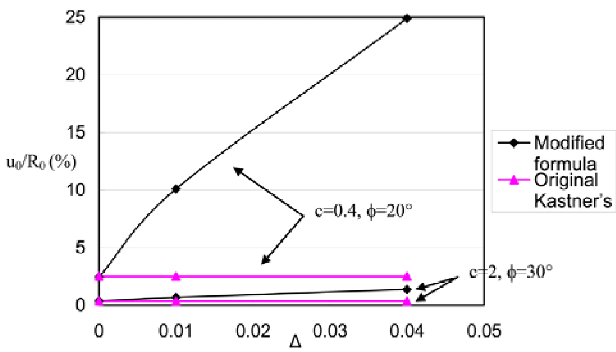


Fig. 4 Comparison of tunnel displacement between Kastner [17] and modified formula in this study

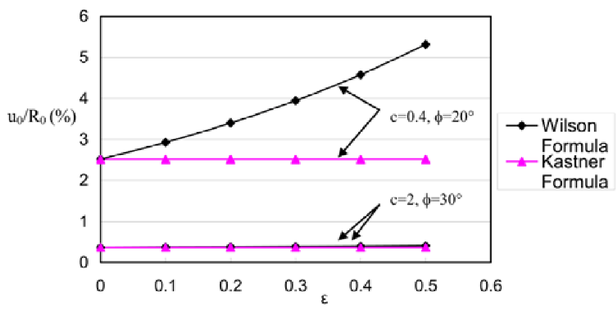


Fig. 5 Comparison of tunnel displacement (mm) between Kastner [17] and Wilson [25] formulae

6. CONCLUSIONS

The Kastner [17] and the Wilson [25] formulae are commonly used for the analysis of circular tunnels by engineers. For the calculation of the yield zone radius, it is assumed that $R_u = R_0$ in both the Kastner [17] and the Wilson [25] formulae, and this assumption is based on small displacement around the cavity. For the tunnel boundary displacement, the Kastner formula [17] requires the assumption of no volume change in the plastic zone, which is not required in the Wilson formula [25], but the assumption of small displacement is still required in the method by Wilson [25]. In this paper, these limitations are removed, and the modified formulae are compared with the more popular formulae by Kastner [17] and Wilson [25]. From the comparisons, it can be seen that the assumption of $R_u = R_0$ is reasonable for normal situations, but for rock with weak shear strength, great differences can occur between R_u and R_0. Engineers should be careful when using the Kastner [17] or the Wilson [25] formula, especially in situations where a relatively large volume change ratio of plastic zone may occur.

ACKNOWLEDGEMENTS

The present project is funded by Research Grants Council through the project PolyU 513808.

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(Manuscript received January 28, 2011, accepted for publication July 11, 2011.)