Smart Modeling of the Human-Structure Interaction with Human Intervention

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Abstract

The combined pedestrian and structural system is regarded as an integrated system for studying the structural vibration when the human body responds to the structural movements. A simple bipedal walking model with a time-variant damper is adopted at a given walking speed. A control force in a feedback manner is generated by the pedestrian to compensate for the energy dissipated with the system damping in walking and to regulate the walking performance of the pedestrian. The effects of stiffness, damping of the leg, and the landing angle of attack are investigated in the numerical studies. The results show that more energy must be input by the pedestrian to maintain the steady walking and relatively uniform dynamic behavior of the body mass of pedestrian in the process.

Key words: Biomechanics, Bipedal walking model, Human-structure interaction, feedback control, time-variant damper

1. Introduction

Human–structure interaction is an important but relatively new issue when designing slender structures occupied and dynamically excited by humans (Ellis and Ji, 1997). It has been well studied by many researchers (Ellis and Ji, 1997; Brownjohn, 1999; Ji, 2003; Sachse et al., 2003, 2004; Dougill et al, 2006; Kim et al, 2008). However, the walking footstep force is usually treated as moving force when analyzing the human-induced vibrations. This approach depends on an assumption that the forces generated by pedestrian will be the same on a flexible footbridge as those observed in tests on a rigid or very stiff base (Dougill et al, 2006). Moreover, the key parameters demonstrating walking force, such as the stiffness, damping, length and the angle of attack of leg, cannot be reflected in this model. Hence, it is difficult to study how structural vibrations can influence the forces induced by pedestrian and how pedestrian influences the dynamic properties of civil engineering structures with this moving load model. A simple human body model (ISO, 1981) having two vertical DOFs was adopted for considering the human-structure dynamic interaction (Kim et al, 2008). However, this model can only represent the vertical displacement of the center of mass (COM) rather than any physical motion of the leg. The leg stiffness corresponding to the vertical displacement of the COM is not the real leg stiffness. Nevertheless, the model can not reproduce the dynamics at standing observed in double support phase.

In biomechanics, human is not only an actuator generating walking force but also a complex dynamic system. It has been shown that a bipedal model consisting of a lump mass supported by spring limbs with damper can be tuned to simulate periodic human walking behavior (Geyer et al., 2006; Whittington and Thelen, 2009; Kim and Park, 2011). Previous studies have shown that active energy input in a feedback or feed-forward manner was required to compensate for energy loss to achieve continuous step cycle behavior (Kim and Park, 2011). However, no such feedback control was included in current bipedal walking model. Moreover, the damper of the bipedal model is assumed to be time-invariant linear component at a given walking speed (Kim and Park, 2011). It is unreasonable because if the damping remains constant, the ground reaction force generating by the pedestrian will not be zero when the leg touches down at the start of the step cycle.

The aim of the paper is to present a theory to account for human structure interaction with human responding to the structural vibration. The combined pedestrian and structural system is regarded as an integrated system for studying the structural vibration when the human body responds to structural movements. Firstly, a time-variant damper will be modeled at a given walking speed. Then, a control force in a feedback manner is generated by the pedestrian to compensate for the energy dissipated with the system damping in walking and to regulate the walking performance. Finally, the dynamic interaction of human-structure system will be studied with consideration of the effects of stiffness, damping of the leg and the structure, and the landing angle of attack of legs in the numerical simulations.

2. Human-Structure dynamic interaction system

The bipedal walking model with damped compliant leg can be tuned to simulate periodic human walking behavior and generate the walking force. Therefore, it can be adopted while considering the human-structure interaction. The pedestrian will react to the vibration of structure due to the mechanical properties of the body.

Consider a simply supported beam subjected to a pedestrian, as shown in Fig.1. The beam is assumed to be of the Euler-Bernoulli type with constant cross sections. The human body is modeled as a lumped mass m_h at the center of mass (COM) and the legs are described as two massless, linear springs of equal rest length l_0 and stiffness k_{leg} with time-variant damper neglecting the roller feet (Geyer et al., 2006; Whittington and Thelen, 2009; Kim and Park, 2011). A passive spring provides a compliant mechanism to absorb collision impacts and to generate push-off impulses, whereas the damper restrained excessive motion of the COM (Kim and Park, 2011). Both springs and damper act independently and influence the model dynamics only during standing when the spring and damper force opposes the gravitational force.

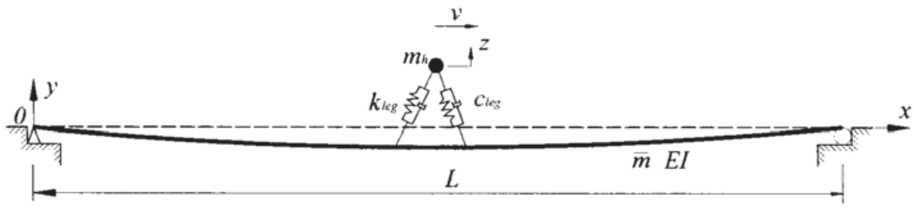

Fig.1. Pedestrian walking over a simply supported beam

A complete step, which is defined as the interval between the 'heel strike' of two feet, is divided into two periods: single support and double support. For example, the double support phase begins with 'touch down' (TD) of the leading leg and ends with 'touch off' (TO) of the trailing leg (Fig.2), where the single support phase begins. Then, the trailing leg is repositioned ahead of the body's COM at a given angle of attack and becomes the leading leg for the next step. When the trailing leg hits the ground, the single support phase is finished.

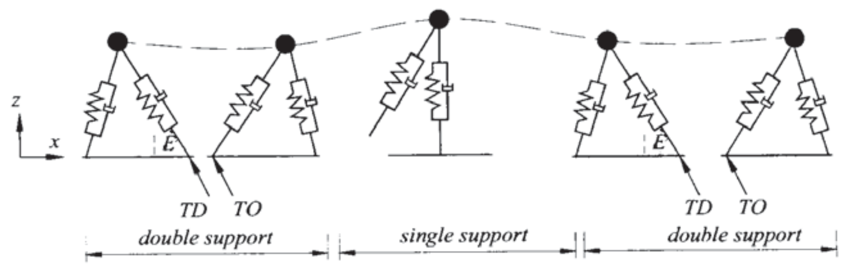

Fig.2. Schematic of the biomechanical walking model (θ_0 is angle of attack)

2.1 Equation of motion

The equations of motion governing the vibration of the bridge and pedestrian are derived from the Lagrangian equation (Appendix A). The dynamic equation of the human-structure interaction system in a matrix form can be written as follows:

$$M\ddot{U} + C\dot{U} + KU = F(t) \quad (1)$$

where M, C, K, U, $\dot{U}$, $\ddot{U}$, $F(t)$ are the mass, damping and stiffness matrices, displacement, velocity, acceleration and force vectors, respectively.

It can be seen from Eqs.(A-15) and (A-16) that ($k_{l,v}$, $k_{t,v}$), ($k_{l,h}$, $k_{t,h}$) are the effective vertical stiffness, effective horizontal stiffness of two legs, respectively. The effective vertical stiffness, ($k_{l,v}$, $k_{t,v}$), determined from a combination of the stiffness of leg spring (k_{leg}), the horizontal and vertical displacements of COM (u, z) and the displacement of structure (y), does correspond to the vertical motion of the COM rather than any physical spring in the model. Therefore, the equations of human-structure system are coupled in terms of the interacting force at a contact point. In the movement of a pedestrian, the interacting force between human and structure changes together with the contact points. Although the leg stiffness is constant during walking, the effective leg stiffness is nonlinear.

2.2. Leg damping in double support phase

We assume the sum of damping in the two legs equal to damping c_{leg} in double support phase. The damping of the leading leg changes from zero to c_{leg} in double support while remaining constant in the single support phase. Therefore, the coefficient of damping during double support phase can be represented as follows:

$$\alpha(t) = \frac{L_t(t) - L_t(0)}{l_0 - L_t(0)} \quad (2)$$

where L_t (0), L_t (t) are the length of the trailing leg at the start of double support phase and at time instant 't', respectively.

According to above assumption, the damping of the leading leg and the trailing leg in double support phase can be expressed as follows:

$$c_l = \alpha(t)c_{leg} \quad (3)$$

$$c_t = (1 - \alpha(t))c_{leg} \quad (4)$$

where c_l, c_t are the damping of the leading leg and the damping of the trailing leg, respectively. It is obviously that the coefficient, $\alpha(t)$, is nonlinear.

2.3. Feedback mechanism

A damped compliant walking model was able to reproduce a one-step gait cycle that consists of double and single support phases. This model requires an energy input mechanism to maintain the steady walking because energy has been dissipated with the system damping in the walking process. This energy expenditure was distributed throughout the gait cycle (Whittington and Thelen, 2009). One possible candidate, among many ways to provide the additional energy, is to apply a control force to the system in a feedback or feed-forward manner (Kim and Park, 2011). The control force is applied by the pedestrian to maintain a balance in the total energy of the system during walking.

Assuming the external work in the horizontal direction equals to the energy loss in the process, the control force can be obtained from the following equation.

$$F_{ctrl}(t) = \frac{E_0 - E(t)}{\Delta u(t)} \tag{5}$$

where $F(t)$, E_0, $E(t)$, $\triangle u(t)$ are the control horizontal force, initial energy input, energy of system and horizontal displacement increment at time instant 't', respectively.

The total energy of the system, E, including the potential energy and the kinetic energy is given by

$$E(t)=\frac{1}{2}m_h v_z^2 + \frac{1}{2}m_h v_x^2 + \frac{1}{2}k_{leg}(\Delta L_l)^2 + \frac{1}{2}k_{leg}(\Delta L_t)^2 + m_h g z \tag{6}$$

where v_z, v_x, ΔL_l, ΔL_t are the vertical velocity, horizontal velocity, compression of leading leg and length of trailing leg, respectively; z is the vertical displacement of the COM.

Then, the force vector in Eqs.1 can be revised as follows:

$$F(t) = [0,0,.....,-m_h g, F_{ctrl}(t)]_{n+2}^{T} \tag{7}$$

3. Dynamic Analysis

To analyze for this dynamic interaction Modified Newton-Raphson method is used to solve the equations of human-structure interaction system. Analysis is performed to investigate the dynamic behavior of both the footbridge and pedestrian.

3.1 Dynamic analyses for the footbridge under a single pedestrian

Fig.1 shows a simply supported footbridge subjected to a pedestrian. The following data is assumed: stiffness EI=9.78*10^9 N m^2, mass per unit-length m=2.84*10^3 kg/m, length of footbridge L=42 m, all damping ratios of structure ξ=0.02, leg stiffness k_{leg}=19.29 kN/m, damping ratio of human body ξ=0.08, human mass m_h=80 kg, angle of attack θ_0=66.5°, the initial energy of the pedestrian motion E_0=820J.

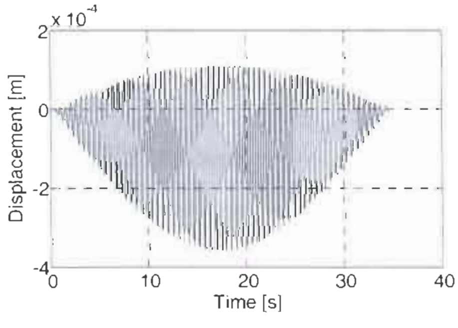

Fig. 3 Displacement at midspan of footbridge

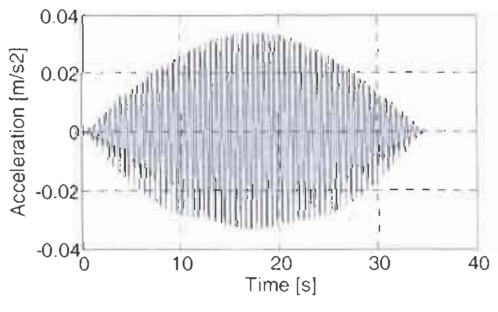

Fig. 4 Acceleration at midspan of footbridge

The angle of attack θ_0 is assumed to remain constant at a given walking speed in the process. Fig.3 and Fig.4 show that the dynamic response at midspan of footbridge is similar to the results using the moving load model. With the pedestrian moving to the center of footbridge, the dynamic response increases. The dynamic responses of the human body are steady in walking (Fig.5, Fig.6). If the horizontal control force was not applied, the pedestrian will stumble after limit step cycles.

The forces generated by the pedestrian walking across the bridge are the same as those on level rigid ground (Fig.7). The control forces in each step are approximately same (Fig.8).These show that the dynamic interaction of human-structure system is very small. The reason is that the vibration of footbridge is small.

The spectra of the displacement and acceleration at midpspan of the footbridge are shown in Fig.9 and Fig.10. The footstep frequency and its multiples can be clearly identified from these spectra. Such an observation is consistent with findings from other existing methods.

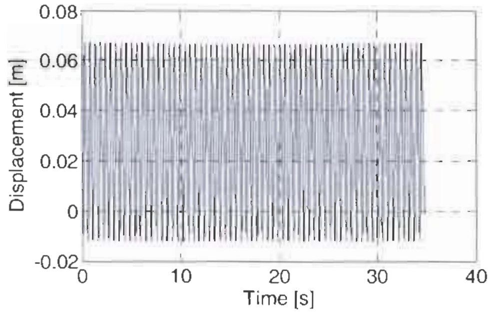

Fig. 5 Displacement of human body in walking

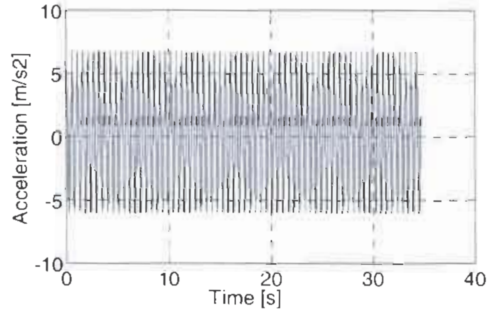

Fig. 6 Acceleration of human-body in walking

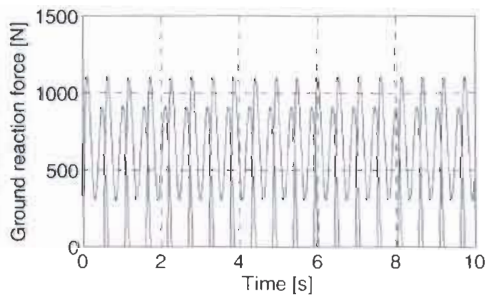

Fig. 7 Ground reaction force in walking

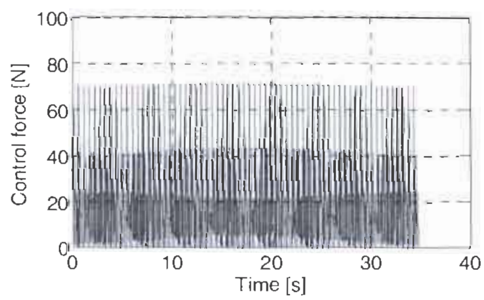

Fig. 8 Control force in walking

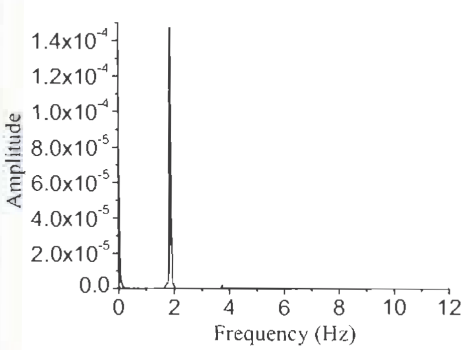

Fig.9 Spectrum of footbridge displacement response

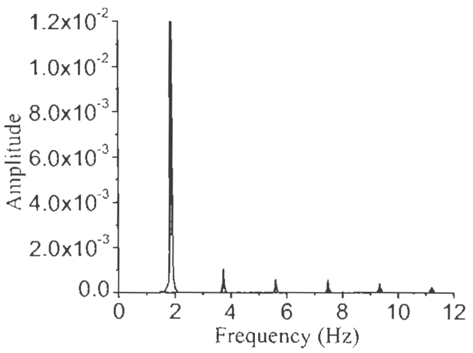

Fig.10 Spectrum of footbridge acceleration response

3.2 Leg stiffness and angle of attack effects on dynamic response

The analysis also shows that the model with no damping has only three independent parameters: angle of attack, spring stiffness and initial system energy E_0 (Geyer, 2006). To evaluate how the leg stiffness and angle of attack affect the dynamic response of bridge, the parameters mass, rest leg length, leg damping and initial system energy are fixed at m_h=80kg, l_0=1m, 0.08, E_0=820J, respectively.

The relationship between the leg stiffness k_{leg} and the angle of attack θ_0 for a steady walking (Seyfarth et al., 2002) is adopted as:

$$K_{leg}(\theta_0) = \frac{1600}{l_0(1-\sin\theta_0)} \tag{8}$$

Hence, only the angle of attack is the independent parameter in this study. The angle of attack is varied from 60° to 70° (Farley and Gonzalez, 1996).

Fig.11 and Fig.12 show that the variation in the angle of attack at a constant energy input has a substantial effect on the dynamic response of structure. When the angle of attack equal 62°, the peak displacement and acceleration response reach maximum. According to the analysis, the step frequency corresponding to the angle of attack θ_0 =62° is consistent with the natural frequency of structure f_s=1.65 Hz. Hence, this is a resonant situation and is the critical situation for the footbridge. The dynamic interaction of human-structure system in this case can be clearly observed from the control force time history in the process (Fig.13). The interaction force between human and structure increases with the pedestrian moving to the center of footbridge. The reason is that the displacement of footbridge is much larger in resonant situation (Fig.14). As a result, more external energy must be input to maintain the steady walking and to have similar dynamic behavior of the COM in the process (Fig.15).

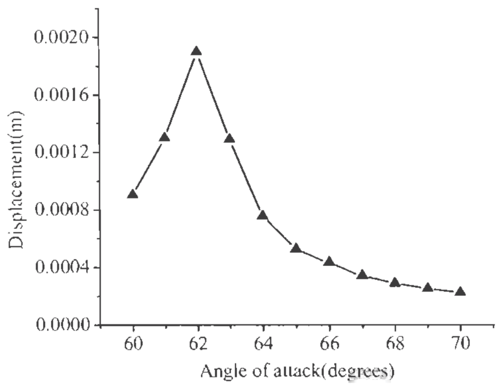

Fig.11 Effect of angle of attack on the peak displacement

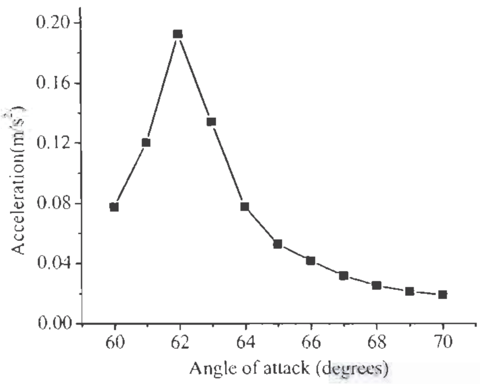

Fig.12 Effect of angle of attack on the peak acceleration

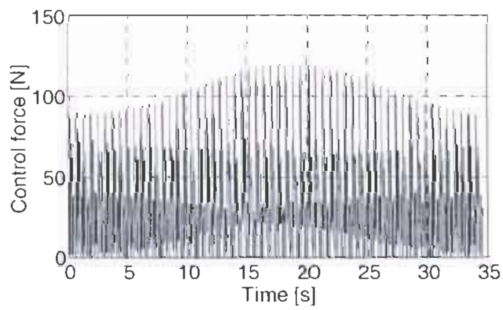

Fig.13 Control force in walking (angle of attack being 62°)

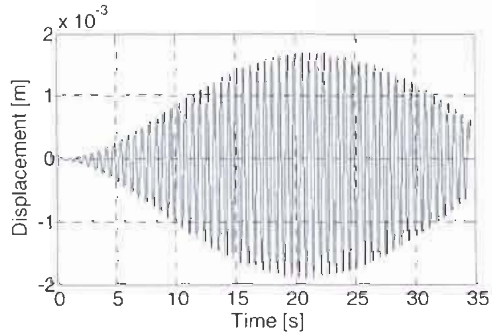

Fig.14 Displacement at midspan of footbridge (angle of attack being 62°)

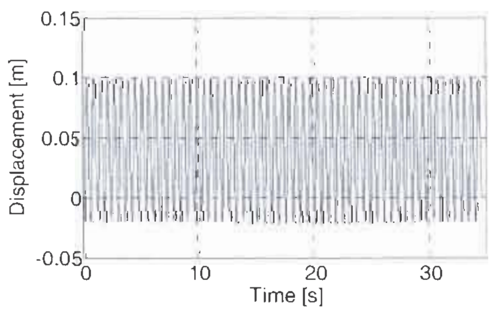

Fig.15 Displacement of COM in walking (angle of attack being 62°)

4. Discussion and Conclusions

The bipedal walking model with damped compliant leg can reproduce a one-step gait cycle that consists of double and single support phases. It can be adopted to describe the human-structure interaction in this study. An improvement to the current bipedal walking model is the assuming of damping and a control force provided by the pedestrian. The axial velocity is not zero when the leg touches down at the start of cycle. This implied that the leading leg damping must be zero at the start of the step cycle. A control force in a feedback manner is applied by the pedestrian to compensate for the energy dissipated with the system damping in walking and to regulate the walking performance of the pedestrian. The control force can reflect the dynamic interaction of the human-structure system. As mentioned previously, a control force is the only one possible candidate among many ways to provide the additional energy. Many tests have shown that the leg stiffness will be adjusted to allow similar dynamic behavior on different surfaces (Ferris and Farley, 1997; Farley et al, 1998; Grimmer et al, 2006; Mueller et al, 2010). However, the mechanism of leg stiffness adjustment is still not clear.

The relations between the dynamic response and angle of attack or leg stiffness are not linear. The analysis results imply that the vibration of structure will be larger when the step frequency closes to the natural frequency of structure, which is consistent with the findings from other existing methods. The dynamic interaction will increase when the vibration of the structure increases. More external energy must be input to maintain the steady walking and to have similar dynamic behavior of the COM in the process.

The current model provides a basic understanding of how the structural vibration can influence the walking forces induced by pedestrian and how pedestrian can influence the dynamic properties of the structure. Therefore, the suggested model could be used to carry out dynamic analyses for a footbridge under crowd-induced excitation.

Appendix A: Equations of motion of human-structure interaction system

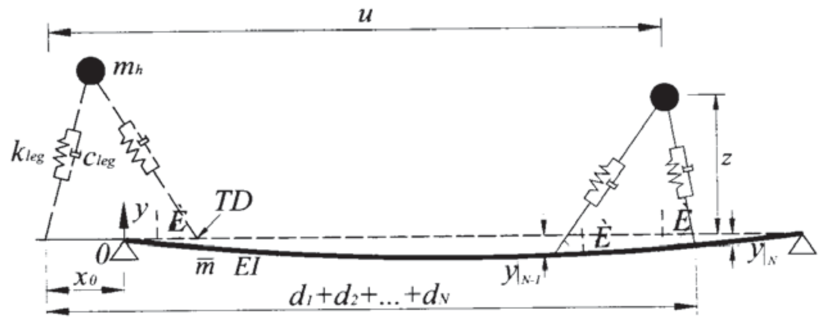

Fig.A.1 Mathematical model for the dynamic analysis of the simply supported beam

The following process is based on the mechanical properties of the system in the double support phase. In fact, the process for single support phase is same to the double support phase. At last, the equations of motion can be expressed in the same form for the two phases.

The displacement of beam can be expressed as:

$$y(x,t) = \sum_{i=1}^{\infty} Y_i(t)\phi_i(x) = \Phi(x)Y(t) \tag{A-1}$$

where $\phi_i(x)$ is the ith modal function. $Y_i(t)$ is the generalized coordinates.

The decompression of leg spring is

$$L_l = \sqrt{(\sum_{i=1}^{N} d_i - u)^2 + (z - y|_N)^2} \tag{A-2}$$

$$L_t = \sqrt{(u - \sum_{i=1}^{N-1} d_i)^2 + (z - y|_{N-1})^2} \tag{A-3}$$

where u, z, d_i, $y|_N$ are the horizontal displacement and vertical displacement of the COM, the ith step length, the displacement of beam corresponding to the nth contact point, respectively. The subscripts 'l' and 't' indicate the leading leg and the trailing leg.

The axial spring velocity can be obtained by the derivative of the decompression.

$$v_l = (\dot{z} - \dot{y}|_N)\sin\theta_l - \dot{u}\cos\theta_l \tag{A-4}$$

$$v_t = (\dot{z} - \dot{y}|_{N-1})\sin\theta_t + \dot{u}\cos\theta_t \tag{A-5}$$

where θ_l, θ_t are the leading leg angle and the trailing leg angle, respectively.

The kinetic energy of human-structure interaction system is

$$T = \frac{1}{2}m_h\ddot{z}^2 + \frac{1}{2}m_h\dot{u}^2 + \frac{1}{2}\int_0^L m\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\phi_i(x)\right)^2 dx \tag{A-6}$$

The potential energy of human-structure interaction system is

$$V = \frac{1}{2}k_{leg}(L_l - l_0)^2 + \frac{1}{2}k_{leg}(L_t - l_0)^2 + m_h gz + \frac{1}{2}\int_0^L EI\sum_{i=1}^{\infty}Y_i(t)^2\left(\frac{d^2(\phi_i(x))}{dx^2}\right)^2 dx \tag{A-7}$$

The variation of virtual work done by non-conservative force based on the assumption of the damping of leg in double support phase is:

$$\begin{aligned}\delta W &= -\alpha(t)c_{leg}v_l\delta(\Delta L_l) - (1-\alpha(t))c_{leg}v_t\delta(\Delta L_t) - \int_0^L c_s\dot{y}''\delta y'' dx \\ &= Q_1\delta q_1 + Q_2\delta q_2 + \ldots\ldots + Q_n\delta q_n + Q_{n+1}\delta z + Q_{n+2}\delta u\end{aligned} \tag{A-8}$$

where Q_i is the generalized forcing functions corresponding to the coordinate Y_i, u, z; y'' is the curvature of the beam.

The variation of displacement of two legs can be obtained as

$$\delta(\Delta L_l) = -\cos\theta_l\delta u + \sin\theta_l(\delta z - \sum_{i=1}^{N}\phi_i(N)\delta Y_i) \tag{A-9a}$$

$$\delta(\Delta L_t) = \cos\theta_t\delta u + \sin\theta_t(\delta z - \sum_{i=1}^{N}\phi_i(N-1)\delta Y_i) \tag{A-9b}$$

The variation of curvature of the beam can be obtained as

$$\delta y'' = \sum_{i=1}^{\infty}\frac{d^2(\phi_i(x)}{dx^2}\delta Y_i(t) \tag{A-10}$$

Substituting Eqs. (A-9) and (A-10) into Eq. (A-8) gives:

$$\begin{aligned}\delta W &= \sum_{i=1}^{\infty}\Big(\alpha(t)c_{leg}v_l\sin\theta_l\phi_i(N) + (1-\alpha(t))c_{leg}v_t\sin\theta_t\phi_i(N-1)\Big)\delta Y_i(t) \\ &+ \sum_{i=1}^{\infty}\left(\int_0^L c_s\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\frac{d^2(\phi_i(x)}{dx^2}\right)\frac{d^2(\phi_i(x)}{dx^2}dx\right)\delta Y_i(t) \\ &+ \Big(-\alpha(t)c_{leg}v_l\sin\theta_l - (1-\alpha(t))c_{leg}v_t\sin\theta_t\Big)\delta z + \Big(\alpha(t)c_{leg}v_l\cos\theta_l - (1-\alpha(t))c_{leg}v_t\cos\theta_t\Big)\delta u\end{aligned} \tag{A-11}$$

According to Eq.(A-8) gives:

$$\begin{aligned}Q_i &= \alpha(t)c_{leg}v_l\sin\theta_l\phi_i(N) + (1-\alpha(t))c_{leg}v_t\sin\theta_t\phi_i(N-1) \\ &-\int_0^L c_s\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\frac{d^2(\phi_i(x)}{dx^2}\right)\frac{d^2(\phi_i(x)}{dx^2}dx\end{aligned} \tag{A-12a}$$

$$Q_{n+1} = -\alpha(t)c_{leg}v_l\sin\theta_l - (1-\alpha(t))c_{leg}v_t\sin\theta_t \tag{A-12b}$$

$$Q_{n+2} = \alpha(t)c_{leg}v_l\cos\theta_l - (1-\alpha(t))c_{leg}v_t\cos\theta_t \tag{A-12c}$$

The Lagrangian equations of motion of the human-structure interaction system are given by (Clough,1993)

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Y}_i}\right) + \frac{\partial V}{\partial Y_i} = Q_i \qquad (i = 1, 2, ...n) \tag{A-13a}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}}\right) + \frac{\partial V}{\partial z} = Q_{n+1} \tag{A-13b}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}}\right) + \frac{\partial V}{\partial u} = Q_{n+2} \tag{A-13c}$$

Substituting Eqs.(A-6) and (A-7) and Eqs.(A-10) into Eqs.(A-11) results in the system equations of motion in a matrix form,

$$M\ddot{U} + C\dot{U} + KU = F \tag{A-14}$$

where,

$$M = \begin{bmatrix} M_1 & 0 & ... & 0 & 0 \\ 0 & M_2 & ... & 0 & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & M & 0 \\ 0 & 0 & ... & 0 & M \end{bmatrix}_{(n+2)\times(n+2)} \tag{A-15}$$

$$C = \begin{bmatrix} c_{11} & c_{12} & ... & c_{1,n+1} & c_{1,n+2} \\ c_{21} & c_{22} & ... & c_{2,n+1} & c_{2,n+2} \\ ... & ... & ... & ... & ... \\ c_{n+1,1} & c_{n+1,2} & ... & \alpha(t)c_1 + (1-\alpha(t))c_2 & (1-\alpha(t))c_4 - \alpha(t)c_3 \\ c_{n+2,1} & c_{n+2,2} & ... & (1-\alpha(t))c_4 - \alpha c_3 & c_{leg} - \alpha(t)c_1 - (1-\alpha(t))c_2 \end{bmatrix}_{(n+2)\times(n+2)}$$

$$K = \begin{bmatrix} \omega_1^2 M_1 & 0 & ... & -k_{t,v}\phi_1(N-1) - k_{l,v}\phi_1(N) & 0 \\ 0 & \omega_2^2 M_2 & ... & -k_{t,v}\phi_2(N-1) - k_{l,v}\phi_2(N) & 0 \\ ... & ... & ... & ... & ... \\ ... & ... & ... & k_{t,v} + k_{l,v} & 0 \\ 0 & 0 & ... & 0 & k_{t,h} - k_{l,h} \end{bmatrix}_{(n+2)\times(n+2)}$$

$$F = \left[0, 0, ..., -Mg, 0\right]_{n+2}^{T}, \qquad U = \left[Y_1, Y_2, ...Y_n, z, u\right]_{n+2}^{T}$$

in which,

$$c_{i,i} = 2\xi_i\omega_i M_i + (1-\alpha(t))c_2\Phi_{i,i}(N-1) + \alpha(t)c_1\Phi_{i,i}(N) \qquad (i = 1, 2....n)$$

$$c_{i,j} = c_{j,i} = (1-\alpha(t))c_2\Phi_{i,j}(N-1) + \alpha(t)c_1\Phi_{i,j}(N) \qquad (i \neq j \leq n)$$

$$c_{i,n+1} = -\alpha(t)c_1\phi_i(N) - (1-\alpha(t))c_2\phi_i(N-1) \qquad (i = 1, 2....n)$$

$$c_{i,n+2} = \alpha(t)c_3\phi_i(N) - (1-\alpha(t))c_4\phi_i(N-1) \qquad (i = 1, 2....n)$$

$$c_{n+1,i} = -\alpha(t)c_1\phi_i(N) - (1-\alpha(t))c_2\phi_i(N-1) \qquad (i = 1, 2....n)$$

$$c_{n+2,i} = \alpha(t)c_3\phi_i(N) - (1-\alpha(t))c_4\phi_i(N-1) \qquad (i = 1, 2....n)$$

$$c_1 = c_{leg}\sin^2\theta_l \quad , \quad c_2 = c_{leg}\sin\theta_l\cos\theta_l \,, \quad c_3 = c_{leg}\sin^2\theta_t \quad , c_4 = c_{leg}\sin\theta_t\cos\theta_t$$

$$k_{l,v} = k_{leg}(1 - l_0 / L_l)(1 - y|_N / z), \qquad k_{t,v} = k_{leg}(1 - l_0 / L_t)(1 - y|_{N-1} / z) \tag{A-16}$$

$$k_{l,h} = k_{leg}(1 - l_0 / L_l)(\sum_{i=1}^{N} d_i / u - 1), \qquad k_{t,h} = k_{leg}(1 - l_0 / L_t)(1 - \sum_{i=1}^{N-1} d_i / u)$$

$$\Phi_{i,j}(N) = \phi_i(N)\phi_j(N), \quad \phi_i(N) = \sin(i\pi(\sum_{n=2}^{N} d_n + x_0) / L)$$

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