Investigation on the heat transfer of energy piles with two-dimensional groundwater flow

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Abstract

The heat transfer around energy piles with groundwater flow for ground-coupled heat pump has been reported in recent years. However, the combined model was always studied on condition that groundwater flows in a one-dimensional manner. This paper describes a novel mathematical model that explains the heat transfer of energy piles with groundwater of two-dimensional (2D) flow. The analytical solutions of the governing equations are obtained based on the Green function of 2D advection. The characteristics involved in the heat transfer of energy pile when 2D groundwater plays role are investigated. Afterwards, the progressiveness of the new model is highlighted.

Keywords: energy pile; groundwater flow; heat transfer; two-dimensional; advection

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1 INTRODUCTION

Energy pile is a novel technology of ground source heat pumps (GSHPs) and has gained more and more attentions in recent years. The spiral heat transfer coils with a certain pitch are usually installed in the pile foundation of buildings to produce a new type of ground heat exchanger (GHE) with remarkable advantages [1, 2]. First, the heat transfer coefficient of spiral coils is higher than that of U-tubes, because the interior space of the pile is fully used to set heat exchange tubes with circulating liquid [3]. Secondly, the land area employed for distributing borehole GHEs is saved as the energy piles can be responsible for a certain proportion of air-conditioning load. Thirdly, the initial cost of the whole system is reduced while the expense of drilling boreholes is dropped [1, 4]. Therefore, energy piles are worthy of application and promotion because of improved performance compared with original GHEs. A schematic diagram of spiral coils that are usually installed into piles is shown in Figure 1.

Studies on energy piles have made continuous progress, especially in terms of pure conduction theory. Some simulation or models such as line source, solid cylindrical source, spiral coils source and finite element modeling were proposed, that is, both analytical models and numerical simulation methods were employed to analyze the heat transfer between the energy pile and the surrounding underground medium. In addition, the thermal and mechanical properties of pile foundations were investigated while heat conducts with time [5, 6]. However, the underground structure and medium are complicated and groundwater seepage often exists, and the investigation about the role of groundwater on the heat transfer performance of energy piles is very limited because seepage phenomenon is usually ignored. The combined heat transfer including conduction and groundwater advection was explored for energy piles by means of analytical models, but the groundwater is always assumed to flow along the positive direction of x-axis, thus only one-dimensional (1D) groundwater flow has been taken into account. For real situation, the hydraulic gradient is usually two-dimensional (2D), and therefore, the groundwater flows along both x and y directions. In view of this, existing mathematical models cannot explain the combined heat transfer of energy pile accurately.

Groundwater velocity is a vector including value and orientation. The orientation is uni-directional in the existing models [7–9]; therefore, the advection role and the corresponding temperature field cannot be expressed reasonably. This paper describes a new analytical model for energy piles when groundwater has a 2D flow, and the solutions containing value and orientation of groundwater can then be obtained. Afterwards, the characteristics embodied in the new models are analyzed.

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2 SIMULATION MODELS

2.1 The models and analytical solutions

Considering that an energy pile continuously emits heat to the surrounding underground medium and heat transfer rate is constant, the whole GHE can be regarded as a solid cylindrical heat source existing in a homogenous medium. Spiral heat exchange coils with a certain pitch are set inside the pile and the pitch is always small, the application of solid cylindrical model is acceptable [10]. The diagram of the model is shown in Figure 2.

Figure 1. *The spiral coils installed in energy piles.*

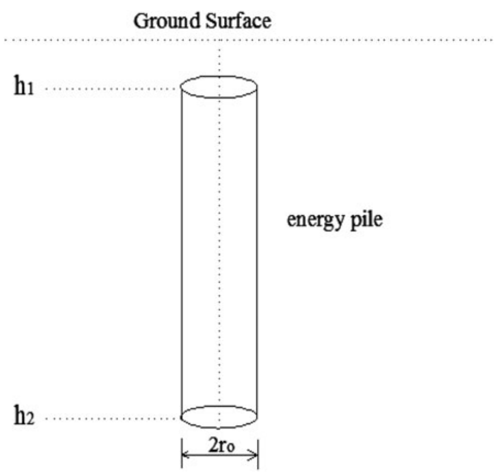

Figure 2. *A solid cylindrical heat source model of an energy pile.*

The pile with radius r_0 normally locates below the ground surface and has the starting and ending points along the depth direction, and the radius of the coils is set equally to that of the pile. Groundwater flows through the energy pile in a 2D way and the heat transfer style consists of conduction and advection [11, 12]. Previous publications usually regarded the seepage phenomenon as a 1D groundwater flow. The sketch about both 1D and 2D flow is given in Figure 3. The 1D flow only takes x-direction's advection into account, but the 2D case regards the groundwater velocity as a vector of x and y directions.

To obtain the analytical solutions of temperature response when groundwater passes the energy pile, the Green function with advection should be employed to describe the heat transfer phenomenon. This function indicates the temperature response to an instantaneous point heat source in an infinite homogeneous medium with groundwater flow, and the corresponding expression is given below:

$$G_F(x,y,z,\tau;x',y',z',\tau') = \frac{1}{8[\pi a(\tau-\tau')]^{3/2}} \exp\left\{-\frac{[x-x'-U\cos\varphi(\tau-\tau')]^2 + [y-y'-U\cos\varphi(\tau-\tau')]^2 + (z-z')^2}{4a(\tau-\tau')}\right\} \quad (1)$$

where (x, y, z) and (x', y', z') are the coordinates of any point except heat source and the point heat source emitting heat from time τ' to τ, respectively [13].

First, the temperature response induced by the infinite solid cylindrical model with groundwater flow can be realized according to the Green function. There is no limitation on the length of the energy pile and thus the ground boundary is not

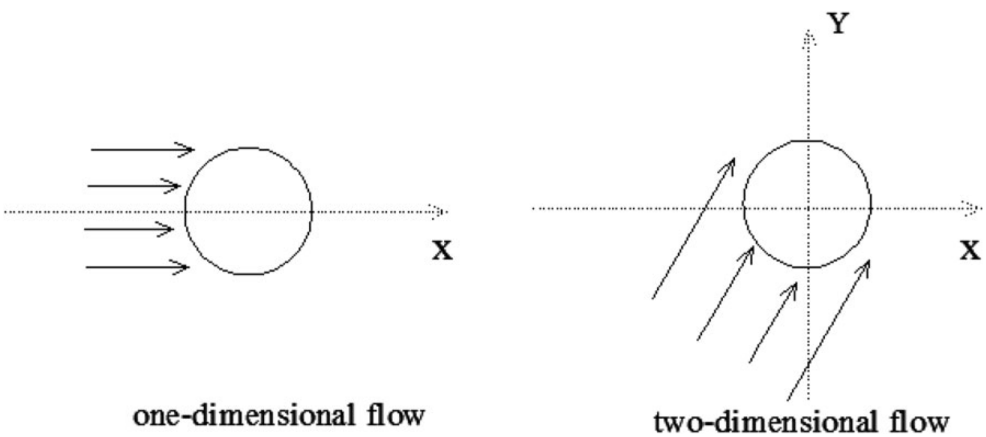

Figure 3. *The 1D flow and 2D flow of groundwater.*

considered [14]. The solution is found as:

$$\theta_i = \frac{q_l}{2\pi\rho c}\int_0^{2\pi} d\phi' \int_{-\infty}^{\infty} dz' \int_0^{\tau} \frac{1}{8[\pi a(\tau-\tau')]^{3/2}} \times \exp\left[-\frac{[x-x'-U\cos\varphi(\tau-\tau')]^2 + [y-y'-U\sin\varphi(\tau-\tau')]^2 + (z-z')^2}{4a(\tau-\tau')}\right] d\tau' \quad (2)$$

The analytical solution of the infinite model lays a firm foundation for studying the heat transfer of the finite model. The infinite model means that only heat transfer along X and Y axes are taken into consideration, which means the influence of ground boundary is ignored. Only the expression of 2D heat transfer is obtained, the study on three directional heat transfer can be achieved.

Secondly, any energy pile has finite depth and the ground boundary has influence on the temperature response induced by energy pile with advection. The temperature of the boundary is constant in the process of heat transfer which is an important precondition. A virtual source method is used to solve the heat transfer problem of the finite heat source [15]. The virtual source method means there must be a point heat sink if there is a point heat source. Accordingly, both the finite cylindrical source and the sink exist and they are symmetrical with the ground boundary as the in-between plane. The temperature response at any point except the heat source in the underground medium is the total contribution of the cylindrical heat source and the sink. The expression of the temperature response is shown below:

$$\theta_f = \frac{q_l}{16\pi^2 k}\int_0^{2\pi} d\phi' \int_0^{\tau} \frac{d\tau'}{(\tau-\tau')} \times \exp\left\{-\frac{[x-r_0\cos\phi'-U\cos\varphi(\tau-\tau')]^2 + [x-r_0\sin\phi'-U\sin\varphi(\tau-\tau')]^2}{4a(\tau-\tau')}\right\} \left[\operatorname{erfc}\left(\frac{z-h_2}{2\sqrt{a(\tau-\tau')}}\right) - \operatorname{erfc}\left(\frac{z-h_1}{2\sqrt{a(\tau-\tau')}}\right) - \operatorname{erfc}\left(\frac{z+h_1}{2\sqrt{a(\tau-\tau')}}\right) + \operatorname{erfc}\left(\frac{z+h_2}{2\sqrt{a(\tau-\tau')}}\right)\right] \quad (3)$$

The finite model can be employed to analyze the temperature response because the depth of any pile is limited. It is noticeable that there are a number of parameters in Equation (3) and the non-dimensional parameters should be adopted to reduce the number of actual parameters, i.e.

$$\Theta_f = \frac{k\theta_f}{q_l},\quad X = \frac{x}{r_0},\quad Y = \frac{y}{r_0},\quad Z = \frac{z}{r_0},\quad S = \frac{Ur_0}{a},$$
$$H_1 = \frac{h_1}{r_0},\quad H_2 = \frac{h_2}{r_0} \text{ and } Fo = \frac{a\tau}{r_0}$$

Therefore, the non-dimensional expression of Equation (3) can be changed into:

$$\Theta_f = \frac{1}{16\pi^2}\int_0^{2\pi} d\phi' \int_0^{Fo} \frac{dFo'}{(Fo-Fo')} \times \exp\left\{-\frac{[X-\cos\phi'-S\cos\varphi(Fo-Fo')]^2 + [Y-\sin\phi'-S\sin\varphi(Fo-Fo')]^2}{4(Fo-Fo')}\right\} \left[\operatorname{erfc}\left(\frac{Z-H_2}{2\sqrt{Fo-Fo'}}\right) - \operatorname{erfc}\left(\frac{Z-H_1}{2\sqrt{Fo-Fo'}}\right) - \operatorname{erfc}\left(\frac{Z+H_1}{2\sqrt{Fo-Fo'}}\right) + \operatorname{erfc}\left(\frac{Z+H_2}{2\sqrt{Fo-Fo'}}\right)\right] \quad (4)$$

2.2 The temperature field around a single energy pile

Figure 3 describes the difference between the 1D and 2D groundwater flows. In the 1D case, the included angle from velocity to the x-axis can be regarded as 0° or 180°, but the 2D groundwater velocity keeps a certain included angle which excludes 0° and 180°; the range of 2D groundwater velocity direction is from 0° to 360°. The temperature field around the energy pile passed by groundwater can be shown and the distribution of isothermals means that the 2D groundwater flow leads to more realistic temperature field, as shown in Figure 4.

The temperature distribution of an energy pile's pure conduction is symmetrical around x-axis or y-axis because there is no interference of advection, but a certain deviation occurs for the isothermal when groundwater flows through energy pile. It is evident that the asymmetry appears not only along x-axis but also y-axis.

2.3 The temperature field of a group of energy piles

High-rise buildings are usually supported by a number of piles, and therefore, groundwater flows through a pile group rather than a single pile. Assuming that each energy pile is a solid cylindrical heat source continuously releasing heat, the temperature response at any point except the heat source is the total role of all energy piles if there is no groundwater advection. All the conduction sources and groundwater flow contribute strength together for inducing temperature response while groundwater is imported [16]. Figure 5 describes that groundwater flows through the pile group. The groundwater has a certain included angle with x and y axes, and the angle is not 0° or 180°.

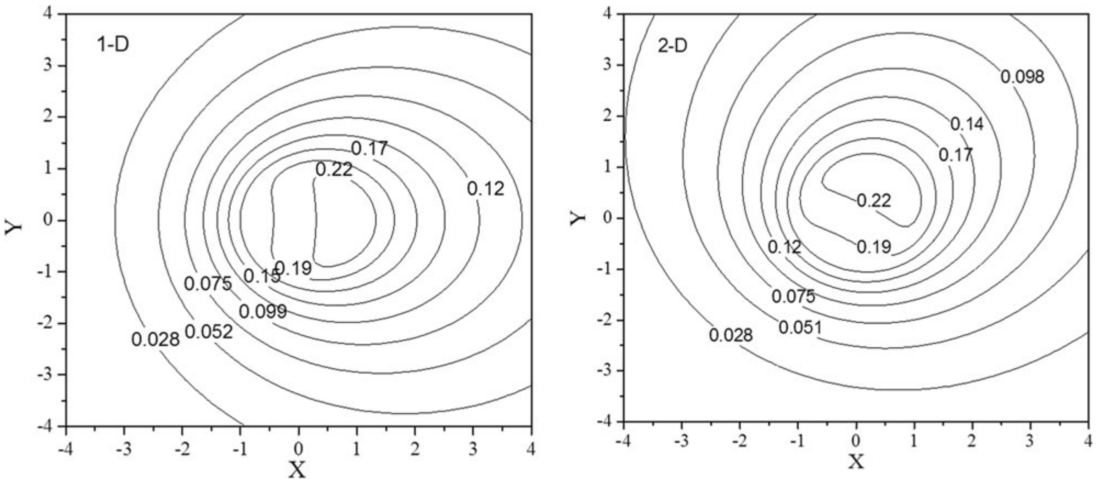

Figure 4. *The temperature fields of 1D and 2D groundwater flow.*

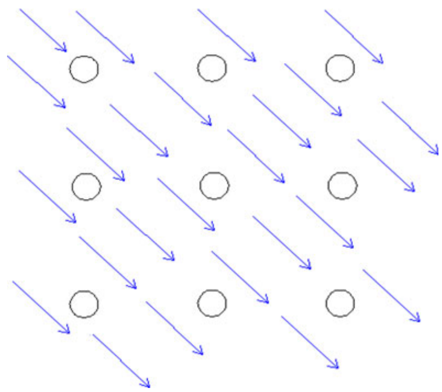

Figure 5. *Groundwater flows through a group of energy piles.*

There are nine energy piles arrayed in the form of matrix, and the separation distance of its rows and columns is equal. Equation (4) illustrates the role of a single energy pile with 2D groundwater flow, and the strength of a group of energy piles with groundwater advection is recognized in Equation (5). Therefore, Equation (5) shows the temperature response induced by many energy piles when groundwater flows through them.

$$\Theta_{f,g} = \sum_{i=1}^{n} \frac{1}{16\pi^2} \int_0^{2\pi} \mathrm{d}\phi' \int_0^{Fo} \frac{\mathrm{d}Fo'}{(Fo - Fo')} \exp\left\{ - \frac{ \begin{array}{c} [X - (X_i + \cos\phi') - S\cos\varphi(Fo - Fo')]^2 + \\ [Y - (Y_i + \sin\phi') - S\sin\varphi(Fo - Fo')]^2 \end{array} }{4(Fo - Fo')} \right\} \left[\mathrm{erfc}\left(\frac{Z - H_2}{2\sqrt{Fo - Fo'}}\right) - \mathrm{erfc}\left(\frac{Z - H_1}{2\sqrt{Fo - Fo'}}\right) - \mathrm{erfc}\left(\frac{Z + H_1}{2\sqrt{Fo - Fo'}}\right) + \mathrm{erfc}\left(\frac{Z + H_2}{2\sqrt{Fo - Fo'}}\right) \right] \quad (5)$$

where $(X_{\mathrm{i}}, Y_{\mathrm{i}})$ is the coordinate of a pile's center point, and Equation (5) can be employed to investigate the temperature field of a group of piles. The detailed information is shown in Figure 6 and the distribution of isothermals becomes more complicated. The degree of temperature response decreases gradually from the energy pile to the far locations, and the groundwater advection leads to the fact that the heat accumulated around a pile is alleviated. The superposition of each pile's temperature field is made when the pile group is considered, and the analysis on the pile group with groundwater flow is used to study the advection's influence on all the piles acted as the bearing structure of buildings. The temperature responses caused by the pile group with groundwater advection are calculated according to Equation (5).

3 CHARACTERISTICS OF 2D GROUNDWATER FLOW

3.1 Comparison of two heat transfer modes

The heat transfer mode is converted from pure conduction to combined case if groundwater seepage is considered [17]. The value of velocity is zero in Equation (4) if there is no groundwater flow. Thus, the temperature responses of the two modes can be obtained according to Equation (4), and the temperature responses with time are displayed in Figure 7 when the velocity is constant.

Figure 7 only displays the difference of the temperature responses intuitively and the role of groundwater velocity will be given in the following section.

3.2 Influence of flow orientation

3.2.1 *Temperature responses at typical points*

Groundwater flows through the boundary of an energy pile and thus the temperature responses at the boundary locations are related with seepage orientation. The points that can first contact with groundwater are exerted more forceful impact, and

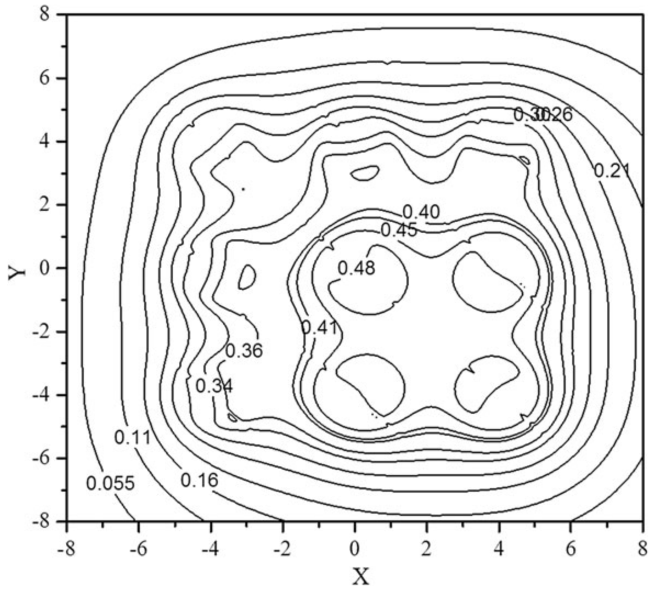

Figure 6. *Temperature field of a group of energy pile with 2D groundwater flow.*

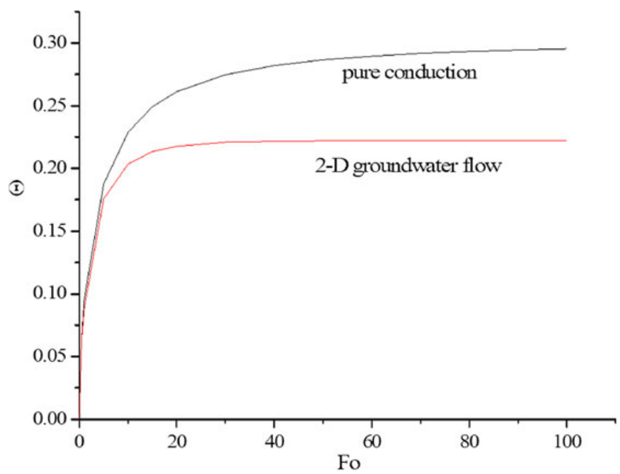

Figure 7. *Temperature response comparison between the combined case and pure conduction case.*

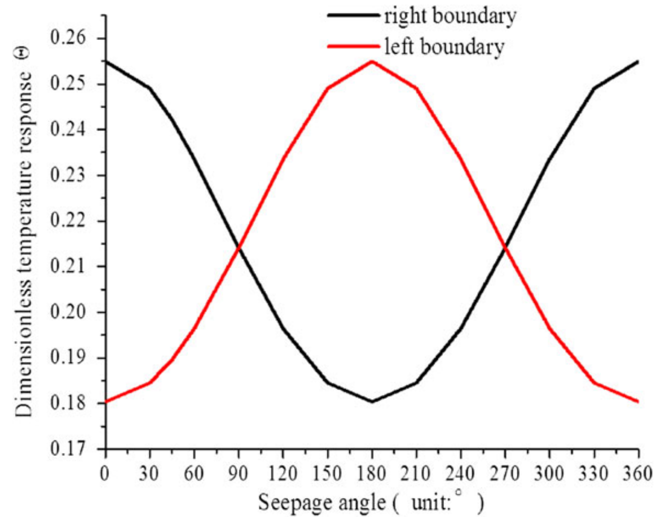

Figure 8. *Two typical points' temperature responses with the groundwater orientation.*

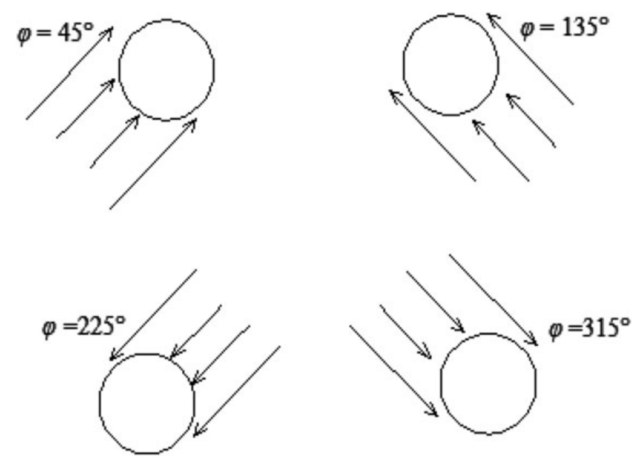

Figure 9. *Groundwater flows through an energy pile with different orientations.*

the heat emitted from the energy pile can be alleviated evidently. The orientation of groundwater determines the pile boundary points' contiguous sequence with groundwater. The points first accepting groundwater seepage show smaller temperature responses. The heat is conveyed by groundwater along the orientation direction, then the points lastly meeting groundwater are unfavorably affected by the heat coming from the front area. The left and the right boundaries are selected when energy pile is available; two points of the pile's mid-depth are employed and they, respectively, belong to the left and the right boundaries. The two points' temperature responses are not constant even though the time and the groundwater velocity's value are not changed, which means the orientation of groundwater flow has non-ignorable contribution to different points' temperature responses.

According to Equation (4), when both the time and groundwater value are constant, the two points' temperature responses are obtained with the change of groundwater orientation. Then, the results are given in Figure 8. It is observed that the two temperature responses rise and drop alternatively and their relative comparisons fluctuate continuously. In the existing references, the groundwater is assumed to flow along positive direction of *X*-axis in the process of 1D seepage, and the temperature response of the right boundary is always larger than that on the left. The 2D flow explains the importance of the orientation for the role of groundwater advection.

3.2.2 *The distribution of temperature field*

The groundwater advection makes an assault on the temperature distribution induced by energy pile's pure conduction, and the orientation is a significant factor that leads to the variation of isothermals. Figure 9 shows that the groundwater with different orientations flows through an energy pile.

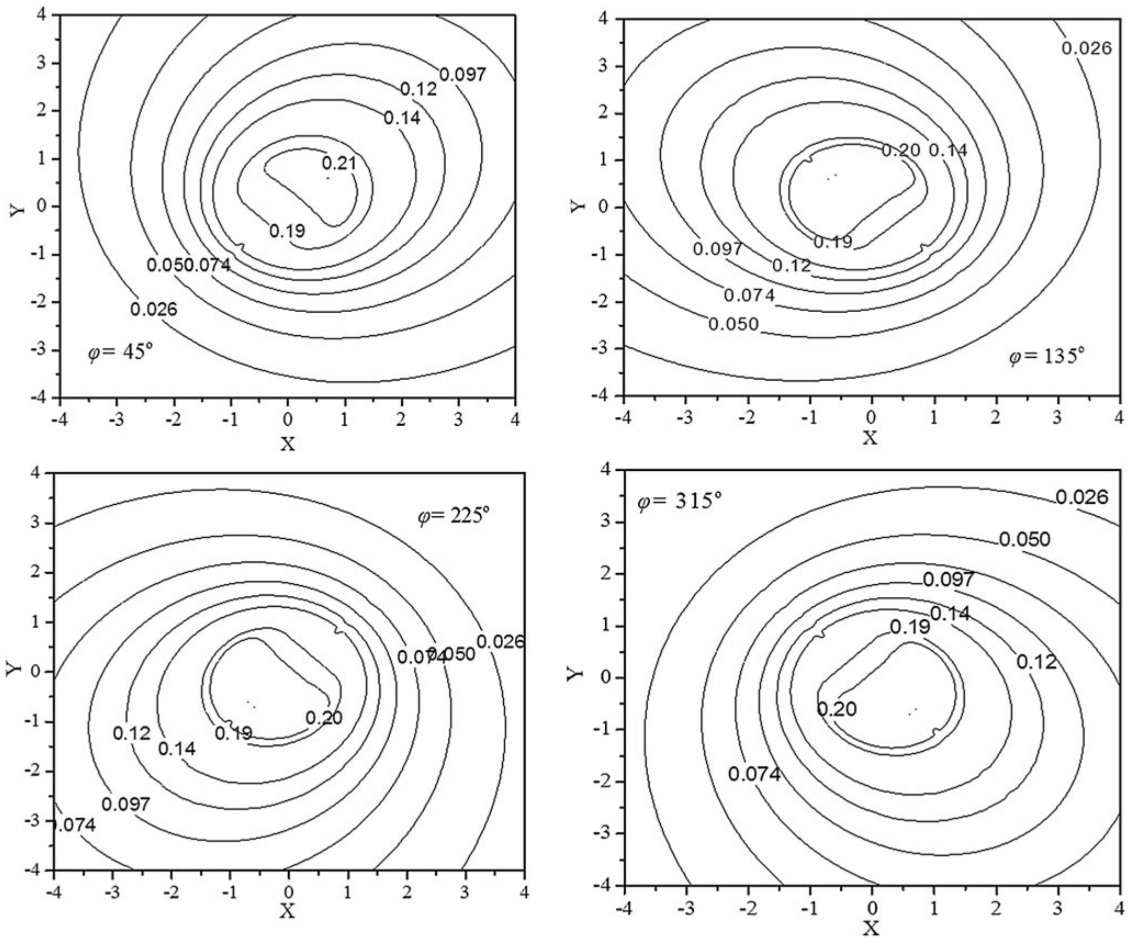

Figure 10. *Temperature field of an energy pile with groundwater flow for different orientations.*

Four orientations of 45°, 135°, 225° and 315° are adopted to exhibit the variability of flow direction, and the orientation signifies that the alleviation of accumulated heat can be done along different paths. The temperature fields with groundwater flow through an energy pile at different included angles are shown in Figure 10. The change of orientation produces various temperature distributions and thereby the importance of orientation is proven again. Consequently, any area or point around an energy pile can be controlled to accurately calculate the temperature response.

3.2.3 *The ratio of advection role along different axes*

Groundwater velocity is a vector which can be decomposed along *X*-axis and *Y*-axis, and the orientation controls the advection role's ratio of the two directions. It is assumed that the included angle between groundwater velocity and positive direction of *X*-axis is a basic parameter, which ranges from 0° and 360°. The ratio shows obvious changes when the included angle varies, which means the orientation plays a significant role on the contribution distribution along different axes, and the corresponding diagram is listed in Figure 11.

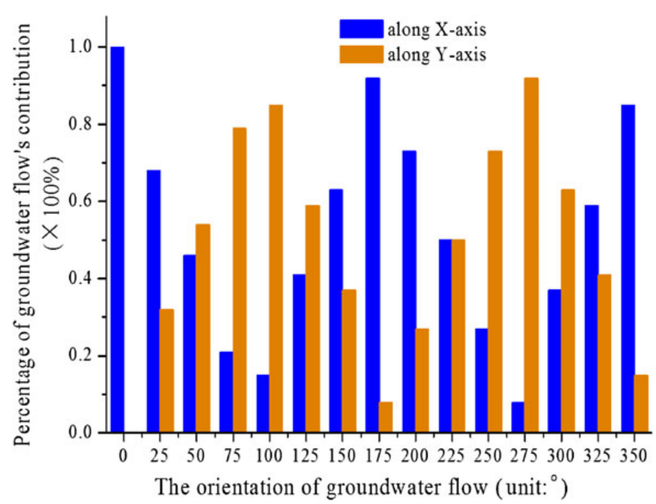

Figure 11. *The advection role's distribution along different axes with orientation.*

The contribution ratios are the distribution of advection role along different directions, and this is helpful to emphasize the necessity of groundwater orientation.

3.3 Influence of velocity strength

The value and the orientation of advection are two important parameters which constitute the elements of groundwater velocity. The influence of velocity on the heat transfer should be explained according to the equations mentioned above. The heat transfer mode consists of conduction and advection when groundwater seepage exists. The advection effectively restrains the rise of temperature response and the heat accumulated around the pile can be alleviated. The average non-dimensional temperature response should be obtained to show the mean value while Z employs different values along Z direction. The formula can be developed based on Equation (4) and the new expression is listed in Equation (6), and the additional integral toward the coordinate of the pile's boundary is added.

$$\overline{\Theta_f} = \frac{1}{2\pi}\frac{1}{16\pi^2}\int_0^{2\pi} d\beta \int_0^{2\pi} d\phi' \int_0^{Fo} \frac{dFo'}{(Fo - Fo')} \exp\left\{ -\frac{\begin{array}{c}[\cos\beta - \cos\phi' - S\cos\varphi(Fo - Fo')]^2 + \\ [\sin\beta - \sin\phi' - S\sin\varphi(Fo - Fo')]^2\end{array}}{4(Fo - Fo')} \right\} \left[\mathrm{erfc}\left(\frac{Z - H_2}{2\sqrt{Fo - Fo'}}\right) - \mathrm{erfc}\left(\frac{Z - H_1}{2\sqrt{Fo - Fo'}}\right) - \mathrm{erfc}\left(\frac{Z + H_1}{2\sqrt{Fo - Fo'}}\right) + \mathrm{erfc}\left(\frac{Z + H_2}{2\sqrt{Fo - Fo'}}\right)\right] \quad (6)$$

where β is the angle coordinate of any point at the boundary, and the mid-depth of the energy pile is selected to study the temperature response with the velocity value. Figure 12 describes that the temperature response reduces obviously with the increase in the velocity value and therefore the temperature response is depressed.

The temperature response is getting weak with the velocity value and the heat can be alleviated effectively, which is favorable for further heat transfer between an energy pile and the surrounding underground medium. Thereby, the heat transfer performance of an energy pile is improved while groundwater advection plays a role. For this reason, the heat transfer rate is increased clearly supposing that the velocity value becomes increasingly large. The heat transfer rates' ratio of the combined mode compared with the pure conduction is given in Figure 13, and the role of velocity value is so noticeable that an energy pile can reject more energy for a central air-conditioning system.

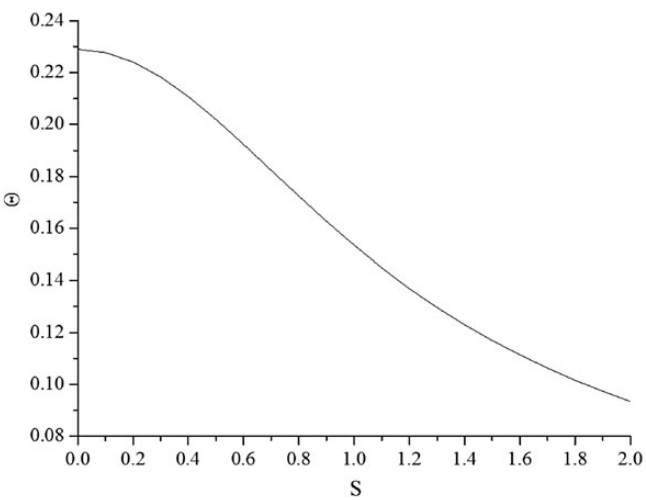

Figure 12. *Temperature response change with different flow velocities.*

3.4 Temperature response along Z-axis

The average temperature response of a pile's boundary at any horizontal plane has been found according to Equation (6). It is necessary to give the temperature responses along the depth direction of a pile, and the constant temperature of the ground boundary imposes an impact on the temperature variation [18, 19]. As shown in Figure 14, the Z coordinate of an energy

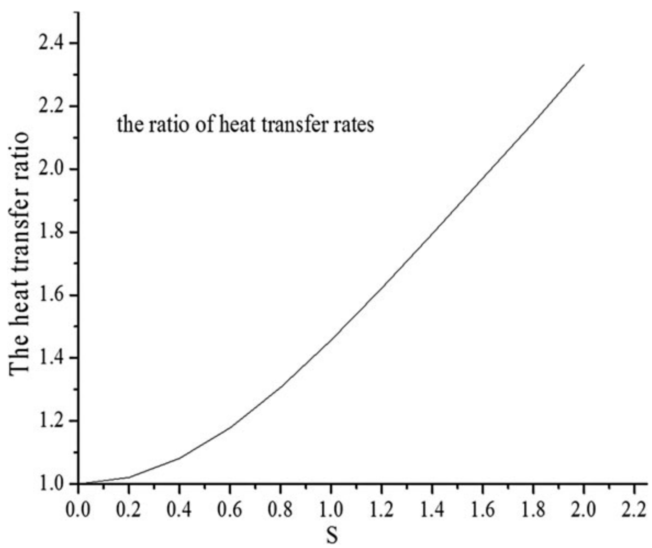

Figure 13. *The heat transfer rates' ratio with the groundwater velocity value.*

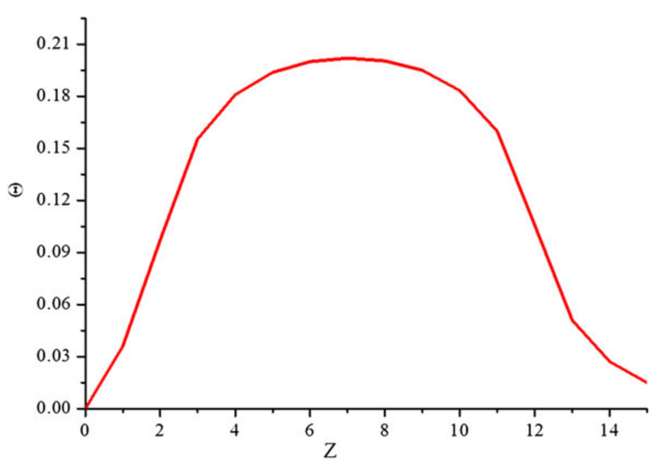

Figure 14. *The temperature response change along Z-axis.*

pile is from 2.0 to 12.0, the temperature response increases first along Z-axis and then decreases to nearly zero, this is because the ground boundary keeps zero temperature in the whole process and it exerts influence on the temperature response, and the temperature response of the infinitely distant position along Z-axis is zero. The middle area behaves stronger, which attributes to the ground boundary to a certain degree. As a result, it is easy to find the temperature response if the middle area is chosen as the research object, which can verify the reasonability of selecting the mid-depth of energy pile in the above research.

4 CONCLUSIONS

The paper proposed a new simulation model for describing the heat transfer of energy piles with groundwater flow in 2D manner. The solid cylindrical heat source model with 2D advection is investigated, and then its detailed analytical solutions of temperature responses are obtained. The temperature fields of 1D and 2D advections are analyzed to show the difference of them, and the latter is more reasonable to study the combined heat transfer including conduction and advection. The bearing structure of every building is composed of a number of piles; the impact that 2D flow exerts on the pile group is explored and the temperature distribution is explained. After making a comparison between pure conduction and combined heat transfer, the role of groundwater advection can be shown. The influences of orientation and value of groundwater are investigated, and the change of temperature response is studied. The groundwater's orientation and its value affect the temperature response at the same location when an energy pile emits heat to the surrounding medium. In addition, the temperature response along the depth direction of energy pile with 2D groundwater flow is explored. The results reveal the real situation of the heat transfer around energy piles, which is helpful to evaluate the heat transfer ability of energy pile. Accordingly, the energy pile can be in charge of more air-conditioning load and this can improve the performance of the whole GSHP system.

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