Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2011, Article ID 497014, 17 pages
doi:10.1155/2011/497014

Research Article

Robust Optimization-Based Generation Self-Scheduling under Uncertain Price

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Received 19 September 2010; Revised 20 November 2010; Accepted 24 January 2011

Academic Editor: J. Rodellar

This paper considers generation self-scheduling in electricity markets under uncertain price. Based on the robust optimization (denoted as RO) methodology, a new self-scheduling model, which has a complicated max-min optimization structure, is set up. By using optimal dual theory, the proposed model is reformulated to an ordinary quadratic and quadratic cone programming problems in the cases of box and ellipsoidal uncertainty, respectively. IEEE 30-bus system is used to test the new model. Some comparisons with other methods are done, and the sensitivity with respect to the uncertain set is analyzed. Comparing with the existed uncertain self-scheduling approaches, the new method has twofold characteristics. First, it does not need a prediction of distribution of random variables and just requires an estimated value and the uncertain set of power price. Second, the counterpart of RO corresponding to the self-scheduling is a simple quadratic or quadratic cone programming. This indicates that the reformulated problem can be solved by many ordinary optimization algorithms.

1. Introduction

Electricity market is a system which organizes, manages, and coordinates the power system by means of laws and economic tools under the principle of openness, competition, and fairness. The aim of electricity market is to improve the efficiency of power industry, to lower electric price, and to ensure the security of power system at the same time. The operation process of electricity market is as follows: firstly, the market participants submit their bids to the independent system operator (ISO), considering their own profit maximization; secondly, under the power system security limits, the ISO decides the dispatch schedule such as the generated energy, load power, and spot price; thirdly, the market participants submit their bids of next period bids to ISO again. This process shows that for the generation company, it is important to make an available self-scheduling in the competitive environment so that the ISO can accept their bids. Finally, the aim of our research is to set up a suitable self-schedular model and to find an effective solution approach.

In power system analysis, the generation self-scheduling is based on optimization approach. Two types of models are used for such problem. One is the determinate optimization approach, and the other is the uncertainty optimization approach with uncertain parameters. Since the later can describe the market operation action better, we will pay our attention to it in this paper. In the literature, the uncertain self-scheduling approach has been studied extensively from the model and algorithm aspects (see [1–9]). There are two available approaches to handle the uncertainty in competitive electricity markets: probabilistic approach and fuzzy approach. By using the probabilistic method, Conejo et al. studied the self-scheduling models of multiperiod schedule where the generation company is as price-takers [2] and a price-makers [6] in a pool-based electricity market, respectively. Then a correspondent mixed-integer quadratic programming and a mixed-integer linear programming are set up. In their research, the profit and risk are considered simultaneously. Furthermore, as an example, the hydro producer's self-scheduling is considered with start-up costs [1], and an 0/1 mixed-integer linear programming model is established. Yamin studied the self-scheduling model by using the fuzzy method [8, 9]. A comprehensive fuzzy approach for self-scheduling problem is set up [9], with the uncertainty of the demand, spinning and nonspinning reserves, price, and so forth. Recently, due to the nice mathematical property of conditional Value-at-Risk (CVaR) (see [10, 11]), Jabr [3] combined the CVaR method into generation self-scheduling, under the known random distribution of power price. Then a second-order cone program is established and the polynomial interior-point approach is adopted. Furthermore, considering the case that the mean vector and covariance matrix of probability distribution may be known partially, Jabr [4, 5] developed the methodology [3] and presented a worst-case robust profit model for the generation self-scheduling. The new model is reformulated as a symmetric cone optimization under special box uncertainty. For the multiperiod consideration, Tseng and zhu [7] studied the self-scheduling and bidding strategy of a thermal generation with the ramp constraints. All of these make a meaningful contribution for the ordinal market operation and provide a guideline of bidding strategy of generation companies.

We also note that the probabilistic approach is based on the known (or partial known) distribution of random variable, and a fuzzy approach is depend on the so-called membership function. However, in the complicated electricity market, the distribution prediction of random variable is difficult, which may result an inaccurate forecast of random variables and yields a bad generation self-scheduling. This motivates us to find other method for the self-scheduling approach.

Mathematically, there is another typical approach for the uncertainty optimization problems, called robust optimization (RO). The RO method has been studied and applied various aspects recently (see [11–15] and references therein). The main characteristic of RO approach solves the optimal problems based on a uncertain set of parameters, not the distribution of them. Such method can avoid the prediction of uncertain parameters in power markets and provide a motivation for the study of the generation self-scheduling.

Under the uncertain price, this paper addresses the self-scheduling via the RO approach. A min-max self-scheduling model is set up where the security constraints of the system are considered. In order to facilitate the model, we use the dual theory of optimization and obtain the correspondent counterpart. The reformulation is a typical quadratic optimization programming and can be solved easily. The characteristics of such research are twofold. First, the self-scheduling model does not need a distribution forecast of electricity price, but just need the possible set of the price, for example, a box region set. This is the main characteristics of RO approach and the difference with [3–5]. Second, the counterpart of generation self-scheduling is ordinary convex quadratic and quadratic cone programming. This is very helpful from the viewpoint of computation.

The paper is organized as follows. Section 2 introduces the RO problem and its counterpart under a linear uncertain set. Section 3 sets up the RO-based self-scheduling model and facilitates the max-min optimization model with the cases of box and ellipsoidal uncertain sets. In Section 4, a numerical example of IEEE-30 system is tested, and some comparison with CVaR approach is also done. Last section addresses some conclusions.

2. Robust Optimization and Its Counterpart

This section presents the mathematical analysis for RO problem. The main objective is to transfer the uncertain optimization problem into a determinate optimization.

2.1. Robust Optimization

A general mathematical programming is of the form

$$\begin{aligned} \min_{x\in R^n}\quad & f_0(x,\xi) \\ \text{s.t.}\quad & f_i(x,\xi)\le 0,\quad \forall \xi\in\mathcal{U}\ (i=1,\ldots,m), \end{aligned} \tag{2.1}$$

where x is the design (decided) vector, the functions $f_0 : R^n \to R$ (the objective function) and $f_i : R^n \to R$ $(i = 1,\ldots,m)$ are structural elements of the problem (the constrained functions), and $\xi \in \mathcal{U} \subset R^l$ stands for the *dada* (or called parameters).

We have the following observation for problem (2.1):

(i) if there is no parameter vector ξ or the vector ξ is fixed (i.e., $\mathcal{U}$ has finite points), the problem reduces to ordinary nonlinear programming problems;

(ii) if $\xi \in \mathcal{U}$ with infinite elements, that is, the parameter vector ξ belongs to some set, then (2.1) is a uncertainty optimization.

The major challenges associated with above uncertain optimization are

(i) when and how can we reformulate or approximate (2.1) as a "computationally tractable" optimization problem?

(ii) How to specify reasonable uncertainty set $\mathcal{U}$ in practical applications?

Typically, a min-max model is used to handle the model (2.1), called robust (or worst-case) version, as follows:

$$\begin{aligned} \min_{x\in R^n}\quad & \sup_{\xi\in\mathcal{U}} f_0(x,\xi) \\ \text{s.t.}\quad & f_i(x,\xi)\le 0,\quad \forall \xi\in\mathcal{U}\ (i=1,\ldots,m) \end{aligned} \tag{2.2}$$

or equivalently

$$\begin{aligned} &\min_{x\in R^n} \quad \sup_{\xi\in\mathcal{U}} f_0(x,\xi) \\ &\text{s.t.} \quad \sup_{\xi\in\mathcal{U}} f_i(x,\xi) \le 0, \quad (i=1,\ldots,m). \end{aligned} \tag{2.3}$$

We call (2.2) and (2.3) robust optimization (RO) models. In this paper, we use (2.3) as a studied model. In what follows, we analyze its counterpart under some special uncertain set $\mathcal{U}$.

2.2. Counterpart of RO under Linear Uncertain Set

We assume $\mathcal{U}$ to be a linear version

$$\mathcal{U} = \mathcal{U}_\tau \equiv \left\{\hat{\xi} + \tau D\delta : \|\delta\|_p \le 1\right\}, \tag{2.4}$$

where $\hat{\xi}$ is the estimate value of ξ, $\tau \in R$ and $D \in R^{l\times l_\delta}$ are the given constant and correlative matrix with respect to parameter vector ξ. The norm $\|\cdot\|_p$ is chosen as $\|\cdot\|_1, \|\cdot\|_2, \|\cdot\|_\infty$ norms.

Suppose that the functions f_0, f_i $(i=1,\ldots,m)$ in (2.3) are continuously differentiable with respect to x and ξ. Then we can facilitate the min-max model (2.3).

(i) Objective function f_0

We make an approximation for the objective function at $\xi \in \mathcal{U}_\tau$ as follows:

$$\begin{aligned} f_0\left(x,\hat{\xi}+\tau D\delta\right) &\approx f_0\left(x,\hat{\xi}\right) + \tau(\nabla_\xi f_0, D\delta)\big|_{\xi=\hat{\xi}} \\ &= f_0\left(x,\hat{\xi}\right) + \tau\left(D^T\nabla_\xi f_0, \delta\right)\Big|_{\xi=\hat{\xi}}. \end{aligned} \tag{2.5}$$

Here (a,b) means the internal product of vector. From the bounded property of $\mathcal{U}_\tau$ and the formula $(a,b) \le \|a\|_p\|b\|_q$, we have the following derivation:

$$\begin{aligned} \sup_{\xi\in\mathcal{U}_\tau} f_0(x,\xi) &= \max_{\xi\in\mathcal{U}_\tau} f_0(x,\xi) \approx f\left(x,\hat{\xi}\right) + \tau \max_{\|\delta\|_p\le 1}\left(D^T(\nabla_\xi f_0,\delta)\right)\Big|_{\xi=\hat{\xi}} \\ &= f_0\left(x,\hat{\xi}\right) + \tau\left\|\left(D^T\nabla_\xi f_0\right)_{\xi=\hat{\xi}}\right\|_q = f_0\left(x,\hat{\xi}\right) + \tau\left\|\left((\partial_\xi f_0)D\right)_{\xi=\hat{\xi}}\right\|_q, \end{aligned} \tag{2.6}$$

where q satisfies $1/p+1/q=1$, D^T is the adjoint of D, and $\nabla f = (\partial f)^T$.

(ii) Constraint function f_i $(i = 1, \ldots, m)$

Similarly, for the constraint: $\sup_{u\in\mathcal{U}_\tau} f_i \le 0$, we have that

$$\max_{\xi\in\mathcal{U}_\tau} f_i(x,\xi) \approx f_i\left(x,\hat{\xi}\right) + \tau\left\|\left((\partial_\xi f_i)D\right)_{\xi=\hat{\xi}}\right\|_q. \tag{2.7}$$

From (2.6) and (2.7), we obtain the approximation problem of (2.3) under $U = U_\tau$:

$$\begin{aligned} \min_u \quad & f_0\left(x,\hat{\xi}\right) + \tau\left\|\left(\partial_\xi f_0\left(x,\hat{\xi}\right)\right)D\right\|_q \\ \text{s.t.} \quad & f_i\left(x,\hat{\xi}\right) + \tau\left\|\left(\partial_\xi f_i\left(x,\hat{\xi}\right)\right)D\right\|_q \le 0, \quad (i = 1, \ldots, m). \end{aligned} \tag{2.8}$$

We call (2.8) *the counterpart* of RO (2.3).

Remark 2.1. (i) The optimization problem of (2.8) is an ordinary optimization with the known estimate point $\hat{\xi}$.

(ii) If f_i $(i = 0, 1, \ldots, m)$ is a linear function with respect to the parameter vector ξ, then the derivation is accurate, that is, the approximate equality becomes equality.

(iii) The RO approach can be extended for solving the following general optimization problem:

$$\begin{aligned} \min_{(x,u)} \quad & f(x,u,\xi) \\ \text{s.t.} \quad & h(x,u,\xi) = 0, \\ & g(x,u,\xi) \le 0, \end{aligned} \tag{2.9}$$

where the variables $x \in R^{n_x}$ and $u \in R^{n_u}$ represent state variable and control variable, respectively. $\xi \in R^l$ is the system parameter $\xi \in \mathcal{U}_\tau$ defined in (2.4). $h : R^{(n_x+n_u+n_l)} \longrightarrow R^{n_x}$.

Similar to Theorem 3.1 in [16], we have the following feasibility with respect to the original uncertain optimization, which shows that the feasibility is controlled by τ defined in set $\mathcal{U}$.

Theorem 2.2. *Let $\hat{x}$ be strictly feasible to* (2.8) *at point $\hat{\xi}$ with $\tau > 0$. Assume that in the set $\mathcal{U}_\tau$, $\nabla_\xi f_i(\hat{x},\xi)$ is Lipschitz continuous with modulo L. Then it holds that*

$$f_i(\hat{x},\xi) \le \left(\frac{L}{2}\right)\|D\|^2\tau^2, \quad (i = 1, \ldots, m),\ \forall\xi \in \mathcal{U}_\tau. \tag{2.10}$$

3. RO-Based Generation Self-Scheduling under Price Uncertainty

In this section, by using the RO method, we will set up the self-scheduling model under uncertain price. We call it *RO-based generation self-scheduling* throughout this paper.

3.1. Self-Scheduling Problem in Power Systems

A generation self-scheduling can be defined by the following nonlinear programming:

$$\max_{P_G \in \Pi} f(P_G, \lambda), \tag{3.1}$$

where the decision variable is the generation output P_G, and λ is the power price. The objective and constraints (feasible region) are defined as follows.

(i) Objective function

$f(P_G, \lambda)$ represents the profit (return) of generating company

$$f(P_G, \lambda) = \lambda^T P_G - \sum_{i=1}^{N_G} C_i(P_{Gi}). \tag{3.2}$$

Here $C_i(P_{Gi})$ is the generator cost function, which is defined by a quadratic function

$$C_i(P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad \text{for } i = 1, \dots, N_G. \tag{3.3}$$

(ii) Feasible region Π

The feasible region Π of P_G consists of power generation limits, dc network model constraint, intact network line flow constraints, and security constraints (see [3, 17] for the definition of Π):

(1) power generation limits

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max}, \tag{3.4}$$

(2) DC network model

$$0 \le P_i + \sum_{j \in k(i)} a_{ij} \left(\delta_i - \delta_j\right) \le P_{Di}, \tag{3.5}$$

(3) intact network line flow constraints

$$-T_{ij}^{\max} \le T_{ij} = -a_{ij} \left(\delta_i - \delta_j\right) \le T_{ij}^{\max}, \tag{3.6}$$

(4) security constraints following the outage of lines $m_1 k_1$ to $m_r k_r$ in terms of flows in the intact network

$$-\widehat{T}_{ij}^{\max} \le \widehat{T}_{ij} + \sum_{l=1}^{r} \omega_l T_{m_l k_l} \le \widehat{T}_{ij}^{\max}, \tag{3.7}$$

where the variables and parameters have the following meaning.

$k(i)$ is the set of nodes connected to node i; P_i is power injection at node $i(= 0$ or $P_{Gi})$; $P_{Gi}^{\max}$ and $P_{Gi}^{\min}$ represent the maximum and minimum limit of P_{Gi}; P_{Di} is the forecasted power demand at node i; T_{ij} expresses the intact power flow on line ij; $\widehat{T}_{ij}$ is the contingent power flow on line ij; $T_{ij}^{\max}$: and $\widehat{T}_{ij}^{\max}$ are the prefault and postfault (emergency) rating of line ij; V_i denotes the voltage magnitude at node i; a_{ij} is electrical susceptance; δ_i expresses the voltage angle at node $i, \delta_1 = 0$; ω_l is the lth element of the row vector of load transfer coefficients.

(iii) Power price

λ is the vector of locational marginal prices (LMPs). Here we assume that the price is uncertain parameter with the following version:

$$\lambda = \overline{\lambda} + \zeta \tag{3.8}$$

or

$$\lambda = \overline{\lambda} + A\zeta, \tag{3.9}$$

where price $\overline{\lambda}$ is the given power price as an estimated value and ζ is a price fluctuation. The matrix A is an associated matrix of node price. The uncertain parameter of price is specialized by ζ.

3.2. RO-Based Generation Self-Scheduling Model

According to the uncertain generation self-scheduling problem (3.1), we will consider the robust (or called worst-case) version under some special set of $\zeta \in \mathcal{U}$. Note that in such model, the uncertain parameter (i.e., price) is just in the objective with a linear version. We transfer the uncertain model (3.1) to a deterministic optimization by

$$\begin{aligned} \max_{P_G} \quad & \inf_{\zeta \in \mathcal{U}} f(P_G, \lambda) \\ \text{s.t.} \quad & P_G \in \Pi. \end{aligned} \tag{3.10}$$

Since the RO-based model (3.10) has a complicated min-max structure of optimization, we will facilitate the model and obtain the correspondent *counterpart* of RO problem. To this end, we consider two special cases of uncertain price set $\mathcal{U}$ in (3.8) and (3.9) as

$$\begin{aligned} \mathcal{U} &= \left\{\zeta \mid e^T\zeta = 0,\ \underline{\zeta} \le \zeta \le \overline{\zeta}\right\}, \\ \mathcal{U} &= \left\{\zeta \mid e^T A\zeta = 0,\ \|\zeta\| \le 1\right\}, \end{aligned} \tag{3.11}$$

where e denotes the vector of ones and $\underline{\zeta}$ and $\overline{\zeta}$ are given constant vectors. The above two sets are called *box uncertainty* and *ellipsoidal uncertainty*, respectively.

Remark 3.1. Note that for solving the same generation self-scheduling, the main difference between our method and ones in [3–5] is twofold. First, our method is based on the RO approach and without the prediction of random variables. This is easily done in practical application, whereas the method in [3–5] depends on the distribution of uncertain power price, which needs a forecast of uncertain price. Second, two methods have a different focus on the problem. Our approach considers optimization under the worst-case, and [3–5] solved the problem under probability level of risk measure.

In the remainder of two subsections, our aim is to reformulate the optimization problem (3.10) to an ordinary optimization for cases of uncertain cases (3.11).

3.3. Counterpart of RO-Based Self-Scheduling with Box Uncertainty

Since the set $\mathcal{U}$ is bounded, the objective function of (3.10) can be rewritten as

$$\max_{P_G\in\Pi}\inf_{\zeta\in\mathcal{U}} f(P_G,\lambda) = \max_{P_G\in\Pi}\min_{\zeta\in\mathcal{U}}\left[\left(\overline{\lambda}+\zeta\right)^T P_G - \sum_{i=1}^{N_G}\left(a_i + b_i P_{Gi} + c_i P_{Gi}^2\right)\right]. \tag{3.12}$$

Computing directly, we have

$$\begin{aligned} F(P_G) &\equiv \min_{\zeta\in\mathcal{U}}\left[\left(\overline{\lambda}+\zeta\right)^T P_G - \sum_{i=1}^{N_G}\left(a_i + b_i P_{Gi} + c_i P_{Gi}^2\right)\right] \\ &= \left(\overline{\lambda}\right)^T P_G - \sum_{i=1}^{N_G}\left(a_i + b_i P_{Gi} + c_i P_{Gi}^2\right) + \min_{\zeta\in\mathcal{U}}\zeta^T P_G. \end{aligned} \tag{3.13}$$

We will use the duality theorem of linear programming to analyze the term $\min_{\zeta\in\mathcal{U}}\zeta^T P_G$. Define the corresponding Lagrangian function as

$$L(\zeta,z,\delta,\gamma) = \zeta^T P_G - z e^T\zeta + \delta^T\left(\underline{\zeta}-\zeta\right) + \gamma^T\left(\zeta-\overline{\zeta}\right). \tag{3.14}$$

It holds that

$$\nabla_\zeta L(\zeta,z,\delta,\gamma) = P_G - ze - \delta + \gamma = 0, \tag{3.15}$$

which follows

$$\max_{(z,\delta,\gamma)}\min_{\zeta} L(\zeta,z,\delta,\gamma) = \max_{(z,\delta,\gamma)}\left\{\delta^T\underline{\zeta} - \gamma^T\overline{\zeta} : P_G - ze - \delta + \gamma = 0,\ \delta\ge 0,\ \gamma\ge 0\right\}. \tag{3.16}$$

From the duality theorem of linear programming, we have

$$\min_{\zeta\in\mathcal{U}}\zeta^T P_G = \max_{(z,\delta,\gamma)}\min_{\zeta} L(\zeta,z,\delta,\gamma). \tag{3.17}$$

Then (3.13) can be reformulated as

$$F(P_G) = \max_{(z,\delta,\gamma)} \left\{ \left(\overline{\lambda}\right)^T P_G - \sum_{i=1}^{N_G} \left(a_i + b_i P_{Gi} + c_i P_{Gi}^2\right) + \left(\delta^T \underline{\zeta} - \gamma^T \overline{\zeta}\right) : P_G - ze - \delta + \gamma = 0,\ \delta \geq 0,\ \gamma \geq 0 \right\}. \tag{3.18}$$

Finally, problem (3.10) with a box uncertainty can be reformulated as

$$\begin{aligned} \max_{P_G} \quad & \inf_{\zeta \in \mathcal{U}} f(P_G, \lambda) = \max_{(P_G,z,\delta,\gamma)} \left[\left(\overline{\lambda}\right)^T P_G - \sum_{i=1}^{N_G} \left(a_i + b_i P_{Gi} + c_i P_{Gi}^2\right) + \left(\delta^T \underline{\zeta} - \gamma^T \overline{\zeta}\right) \right] \\ \text{s.t.} \quad & P_G \in \Pi, \\ & P_G - ze - \delta + \gamma = 0, \quad \delta \geq 0,\ \gamma \geq 0. \end{aligned} \tag{3.19}$$

Remark 3.2. The reformulation (3.19) is an ordinary *quadratic nonlinear programming* if Π is linear with respect to P_G, which can be solved easily by many solution methods.

3.4. Counterpart of RO-Based Self-Scheduling with Ellipsoidal Uncertainty

Similarly to the box uncertainty, we define a function as

$$\begin{aligned} F(P_G) &\equiv \min_{\zeta \in \mathcal{U}} \left[\left(\overline{\lambda} + A\zeta\right)^T P_G - \sum_{i=1}^{N_G} \left(a_i + b_i P_{Gi} + c_i P_{Gi}^2\right) \right] \\ &= \left(\overline{\lambda}\right)^T P_G - \sum_{i=1}^{N_G} \left(a_i + b_i P_{Gi} + c_i P_{Gi}^2\right) + \min_{\zeta \in \mathcal{U}} \zeta^T A^T P_G. \end{aligned} \tag{3.20}$$

Consider the term

$$\min_{\zeta \in \mathcal{U}} \zeta^T A^T P_G \tag{3.21}$$

with $\mathcal{U} = \{\zeta \mid e^T A\zeta = 0,\ \|\zeta\| \leq 1\}$. The correspondent Lagrangian function of (3.21) is

$$L(\zeta, \mu, \nu) = \zeta^T A^T P_G - \mu\left(e^T A\zeta\right) + \nu\left(\xi^T \xi - 1\right) \quad \text{with } \nu \geq 0. \tag{3.22}$$

Note that we use a relation of $\|\zeta\| \leq 1 \Leftrightarrow \zeta^T \zeta \leq 1$. From the optimal condition, we have

$$\nabla_\zeta L(\zeta, \mu, \nu) = A^T P_G - \mu A^T e + \nu\zeta = 0, \quad e^T A\zeta = 0, \quad \|\zeta\| \leq 1,\ \nu \geq 0. \tag{3.23}$$

This indicates that at the optimal point (ζ, μ, ν), it holds that

$$\min_{\zeta\in\mathcal{U}} L(\zeta, \mu, \nu) = \left(A^T P_G - \mu A^T e + \nu\zeta\right)^T \zeta - \nu = -\nu. \tag{3.24}$$

Define an auxiliary variable σ and let $\sigma \equiv \nu\zeta$; it holds that

$$\nabla_\zeta L(\zeta, \mu, \nu) = A^T P_G - \mu A^T e + \sigma = 0. \tag{3.25}$$

On the other hand, from $\|\zeta\| \leq 1$ we have

$$\nu^2\|\zeta\|^2 = \|\sigma\|^2 \leq \nu^2 \Longrightarrow \|\sigma\| \leq \nu. \tag{3.26}$$

Note that above relationship involves $\nu \geq 0$.

Combining with (3.23)–(3.26) and the duality theorem of convex programming, we obtain

$$\min_{\zeta\in\mathcal{U}} \zeta^T P_G = \max_{(\mu,\nu)} \min_{\zeta\in\mathcal{U}} L(\zeta, \mu, \nu) = \max_{(\mu,\nu,\sigma)} \left\{-\nu : A^T P_G - \mu A^T e + \sigma = 0,\ \|\sigma\| \leq \nu\right\}. \tag{3.27}$$

Therefore, the RO-based self-scheduling with ellipsoidal uncertainty is reformulated as

$$\begin{aligned}
\max_{P_G} \quad & \inf_{\zeta\in\mathcal{U}} f(P_G, \lambda) = \max_{(P_G,\mu,\nu,\sigma)} \left[\left(\overline{\lambda}\right)^T P_G - \sum_{i=1}^{N_G}\left(a_i + b_i P_{Gi} + c_i P_{Gi}^2\right) - \nu\right] \\
\text{s.t.} \quad & P_G \in \Pi, \\
& A^T P_G - \mu A^T e + \sigma = 0, \quad \|\sigma\| \leq \nu.
\end{aligned} \tag{3.28}$$

Remark 3.3. The reformulation (3.28) is a *quadratic cone programming* if the feasible region Π is linear with respect to variable P_G.

Two reformulations (3.19) and (3.28) are typical determinate optimization problems, which can be solved by many effective algorithms (see [18]). Furthermore, except the uncertainty with respect to price λ, other data can be chosen as uncertain variable, such as (i) the cost coefficients of generators (a_i, b_i, c_i), (ii) the forecasted power demand at buses P_{Di}, (iii) the bound of variables in constraints.

4. Numerical Examples for Self-Scheduling

In order to validate the RO-based self-scheduling approach, this section provides numerical examples. Some comparing approach with paper [3] are also done, and the sensitivity with respect to the uncertainty set is analyzed.

Table 1: Given data and estimate LMPs.

Bus no.	$P_G^{\min}$ [MW]	$P_G^{\max}$ [MW]	a [\$/h]	b [\$/MWh]	c [\$/MW^2h]	λ^n [\$/MWh]
1	50	200	0	2	0.00375	3
2	20	80	0	1.75	0.00175	3
5	15	50	0	1	0.0625	3
8	10	35	0	3.25	0.00834	5
11	10	40	0	3	0.025	5
13	12	40	0	3	0.025	5

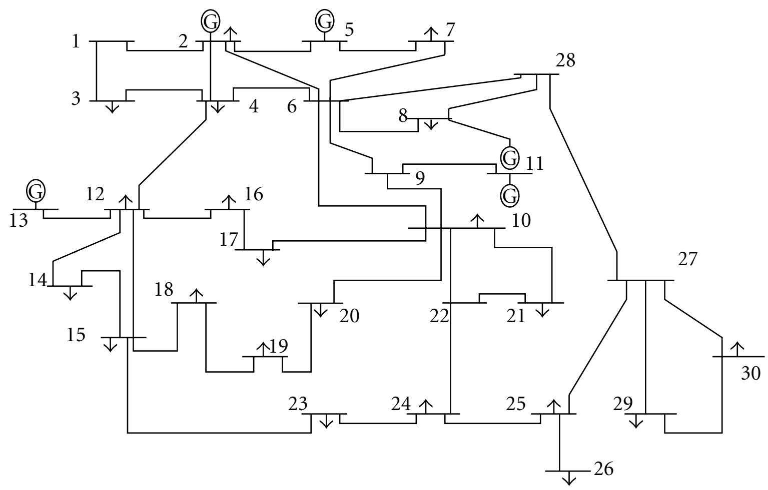

Figure 1: The IEEE 30-bus system.

4.1. Tested System

We choose the same example in [3] as the tested system, that is, IEEE-30 system with six generator buses, which are bus-1,2,5,8,11,13, respectively, see Figure 1.

Consider the mathematical model (3.1)–(3.6). Here we omit the security (3.7). The network, load, and generator data for this system are given in [17]. The coefficients in the cost function and the bound of generation outputs are specified in Table 1, together with the values of forecast (nominal) LMPs.

The given constants P_{Di} and $T_{ij}^{\max}$ in the DC network model constraints (3.5) and in the intact network line flow constraints (3.6) are reported in Tables 2 and 3, respectively.

4.2. Uncertain Set and Algorithm

In the numerical test, we consider the case of uncertain box set, that is,

$$\mathcal{U} = \left\{ \zeta \mid e^T \zeta = 0,\ \underline{\zeta} \le \zeta \le \overline{\zeta} \right\}. \tag{4.1}$$

Table 2: DC network model bounds.

Bus no.	1	2	3	4	5	6	7	8	9	10
P_{Di}	0	21.7	2.4	7.6	94.2	0	22.8	30	0	5.8
Bus-no	11	12	13	14	15	16	17	18	19	20
P_{Di}	0	11.2	0	6.2	8.2	3.5	9	3.2	19.5	2.2
Bus-no	21	22	23	24	25	26	27	28	29	30
P_{Di}	17.5	0	3.2	8.7	0	3.5	0	0	2.4	10.6

Table 3: Intact network line flow.

fbus	1	1	2	3	2	2	4	5	6	6	6
tbus	2	3	4	4	5	6	6	7	7	8	9
$T_{ij}^{\max}$	130	130	65	130	130	65	90	70	130	32	65
fbus	6	9	9	4	12	12	12	12	14	16	15
tbus	10	11	10	12	13	14	15	16	15	17	18
$T_{ij}^{\max}$	32	65	65	65	65	32	32	32	16	16	16
fbus	18	19	10	10	10	10	21	15	22	23	24
tbus	19	20	20	17	21	22	22	23	24	24	25
$T_{ij}^{\max}$	16	32	32	32	32	32	32	16	16	16	16
fbus	25	25	28	27	27	29	8	6			
tbus	26	27	27	29	30	30	28	28			
$T_{ij}^{\max}$	16	16	65	16	16	16	32	32			

The fluctuation bound of price is set as a ratio of the estimated price $\overline{\lambda}$, that is, let

$$\overline{\zeta} = n\%\overline{\lambda}, \qquad \underline{\zeta} = -\overline{\zeta}. \tag{4.2}$$

Here n is the ratio constant. For example, $n = 5$ means that the fluctuation bound of price is 5% of $\overline{\lambda}$. In the following tests, we will calculate the RO-based self-schedular by different n values.

The RO-based self-scheduling with uncertain price is the model (3.19), which is a typical quadratic programming. Then we test the system by Quadprog file in MATLAB toolbox.

4.3. Computational Results

(1) Optimal profit and output of generations

For case $n = 5$, we solve the counterpart of RO and obtain the output of generations and the optimal profit as follows:

$$\begin{gathered} P_{G1} = 128.09MW, \qquad P_{G2} = 35.00MW, \qquad P_{G5} = 18.09MW, \\ P_{G8} = 35.00MW, \qquad P_{G11} = 30.00MW, \qquad P_{G13} = 37.22MW, \\ f_{\max}(P_G) = 241.4\$/h. \end{gathered} \tag{4.3}$$

Table 4: Computing results of two methods.

CRP		RO	
β	$f_{\max}$	n	$f_{\max}$
0.9	216.34	5	214.4
0.95	210.40	6	210.71
0.99	207.43	7	207.05

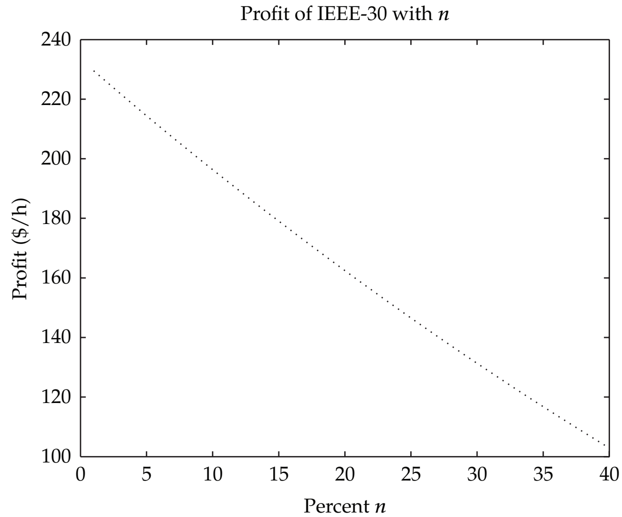

Figure 2: Relation of optimal profit and n.

With the different n values, we obtain the different profit. The results are reported in Figure 2. The curve indicates that the good estimate value of price (i.e., the small fluctuation n) will result in high profit. For the big n, the obtained profit is conservative.We also make some comparison with paper [3] and find that when n is chosen the value between 5.0–7.0, the computing values, are closed for two methods, see Table 4. Here CRP represents the results in [3], and RO indicates the results in this paper.

(2) Optimal output of generators with different n

For the different choice n, we obtain the different output of generators. The optimal self-scheduling of three cases is reported in Table 5. Comparing with the computing results in [3] (see Table 3 in [3]), we find that the value of RO method is conservative. This is identical to the theory analysis since the RO-based approach is set up in the worst-case.

(3) Sensitivity analysis of optimal output of generators

In order to test the effect of turbulence value n, we repeat to solve the RO model by using the different n. The computing results for six generators are shown in Figures 3, 4, 5, 6, 7, and 8.

From the results, we can see that for each generator, they have different sensitivity.

(i) The output P_{G1} of the slack bus-1 is decreasing when the value n is increasing. Especially, it has a big decrease for $n \geq 10$.

Table 5: Optimal output of generators.

P_G at bus-bar [MW]	$n = 5$	$n = 10$	$n = 15$
1	128.09	127.77	125.66
2	35.00	35.00	35.00
5	18.09	20.47	22.74
8	35.00	35.00	35.00
11	30.00	30.00	30.00
13	37.22	35.17	35.00

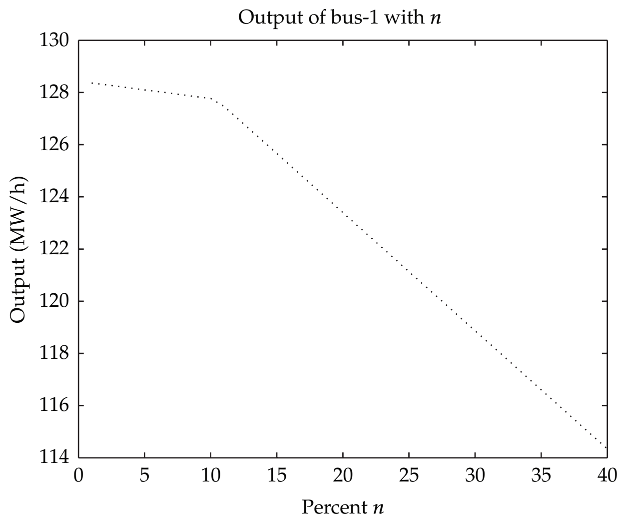

Figure 3: Optimal output of bus-1 with different n.

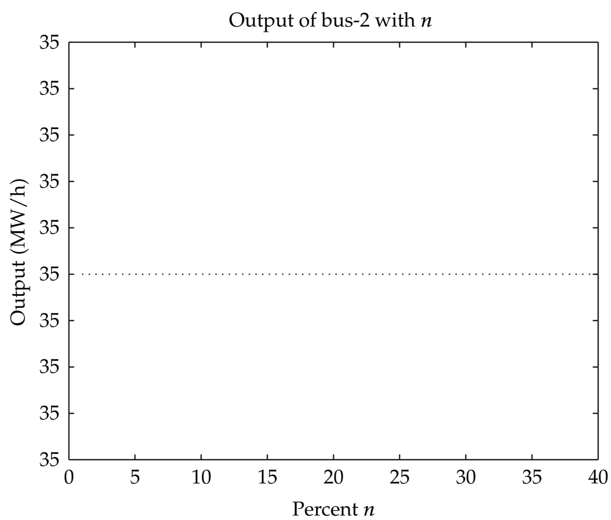

Figure 4: Optimal output of bus-2 with different n.

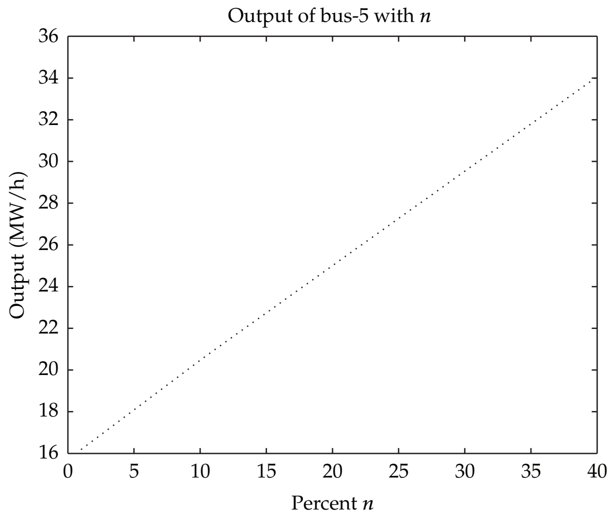

Figure 5: Optimal output of bus-5 with different n.

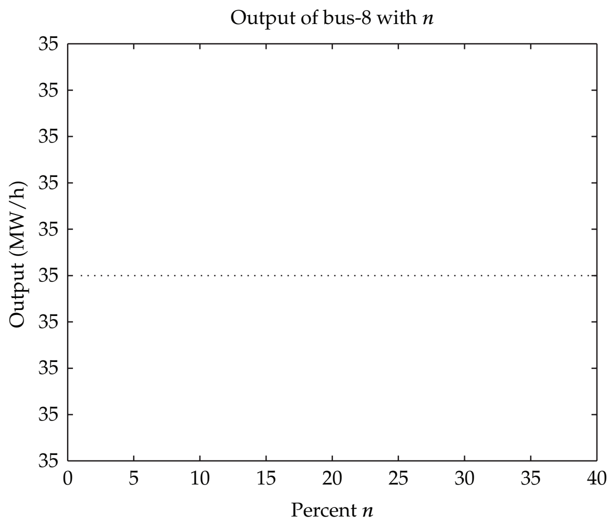

Figure 6: Optimal output of bus-8 with different n.

(ii) The outputs P_{G2}, P_{G8}, P_{G11} at bus-2, bus-8, and bus-11 are not sensitive with respect to the variety of n. This is favorable for the persistent output of the generator, and reduces the times of on-off generators.

(iii) The output P_{G5} at bus-5 is increasing in a linear version with respect to n, which means that the bus-5 is sensitive.

(iv) The output P_{G13} at bus-13 is decreasing sharply when n from 1 to 10. It almost takes on a stability state when $n \geq 10$.

The above analysis can provide some message to generation company for the self-scheduling and then guides the bidding action of generation company.

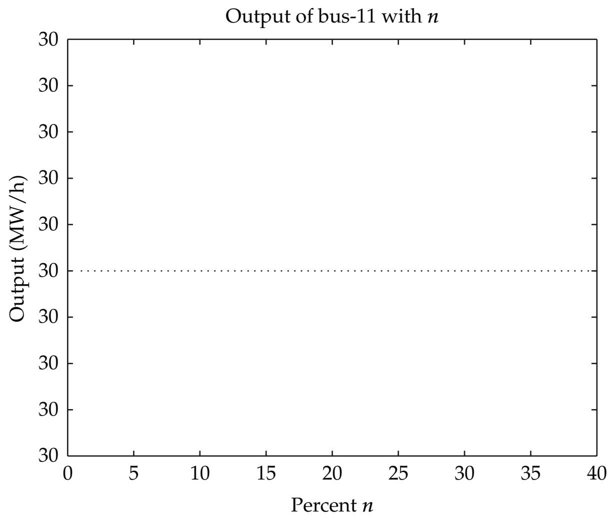

Figure 7: Optimal output of bus-11 with different n.

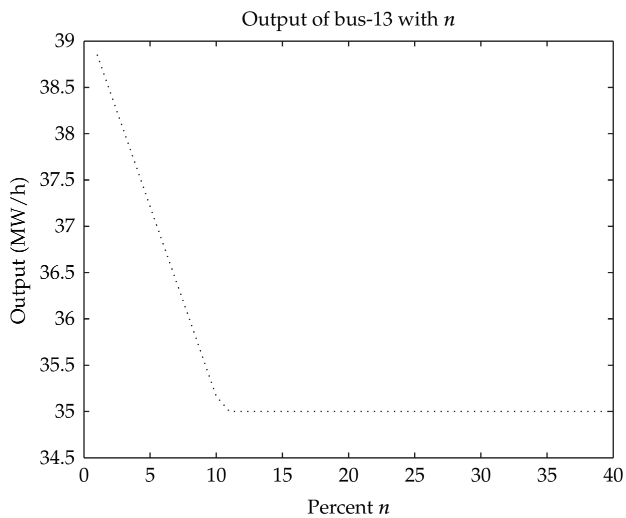

Figure 8: Optimal output value of bus-13 with different n.

5. Conclusion

This paper presents a new methodology to study the generation of self-scheduling in power market. Based on robust optimization (RO), a new self-scheduling model is established under uncertain price. The counterpart of the model is a quadratic-type programming, which can be solved by many optimization algorithms. IEEE-30 system is chosen as a tested system. The computing results show that the new method is promising. Comparing our method with other stochastic methods (e.g., CVaR approach), the computing result is conservative. From viewpoint of practical applications, the new approach is very suitable for case where the prediction of random variables is difficult. On the other hand, the robust consideration can ensure the security requirements of the systems.

We just consider the uncertain price with respect to the price-taker in this study. In fact, the proposed method can be extended to other uncertain cases for self-scheduling, such as the price-maker schedular problem and the uncertainty for cost parameters or load demand. Moreover, other related optimization problems in electricity market can be adopted RO-based approach. For example, the bidding analysis and the optimal power flow (OPF) with new energy source. These are worthy problems of our further research.

Acknowledgments

This work is supported by Natural Science Foundation of China (10871031, 10926189) and the Natural Science united Foundation of Hunan-Hengyang (10JJ8008) and the Grant of Hunan Education Department (10A015).

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