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Review Article

Strong and Weak Convergence Theorems for an Infinite Family of Lipschitzian Pseudocontraction Mappings in Banach Spaces

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The purpose of this paper is to study the strong and weak convergence theorems of the implicit iteration processes for an infinite family of Lipschitzian pseudocontractive mappings in Banach spaces.

1. Introduction and Preliminaries

Throughout this paper, we assume that E is a real Banach space, E^* is the dual space of E, C is a nonempty closed convex subset of E, $\mathcal{R}^+$ is the set of nonnegative real numbers, and $J : E \to 2^{E^*}$ is the normalized duality mapping defined by

$$J(x) = \{ f \in E^* : \langle x, f \rangle = \|x\| \cdot \|f\|, \ \|x\| = \|f\| \}, \quad x \in E. \tag{1.1}$$

Let $T : C \to C$ be a mapping. We use $F(T)$ to denote the set of fixed points of T. We also use "$\to$" to stand for strong convergence and "$\rightharpoonup$" for weak convergence. For a given sequence $\{x_n\} \subset C$, let $W_\omega(x_n)$ denote the *weak ω-limit set*, that is,

$$W_\omega(x_n) = \{ z \in C : \text{there exists a subsequence } \{x_{n_i}\} \subset \{x_n\} \text{ such that } x_{n_i} \rightharpoonup z \}. \tag{1.2}$$

Definition 1.1. (1) A mapping $T : C \to C$ is said to be *pseudocontraction* [1], if for any $x, y \in C$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y) \rangle \leq \|x-y\|^2. \tag{1.3}$$

It is well known that [1] the condition (1.3) is equivalent to the following:

$$\|x-y\| \leq \|x-y+s[(I-Tx)-(I-Ty)]\|, \tag{1.4}$$

for all $s > 0$ and all $x, y \in C$.

(2) $T : C \to C$ is said to be *strongly pseudocontractive*, if there exists $k \in (0,1)$ such that

$$\langle Tx - Ty, j(x-y) \rangle \leq k\|x-y\|^2, \tag{1.5}$$

for each $x, y \in C$ and for some $j(x-y) \in J(x-y)$.

(3) $T : C \to C$ is said to be *strictly pseudocontractive in the terminology of Browder and Petryshyn [1]*, if there exists $\lambda > 0$ such that

$$\langle Tx - Ty, j(x-y) \rangle \leq \|x-y\|^2 - \lambda\|(I-T)x-(I-T)y\|^2, \tag{1.6}$$

for every $x, y \in C$ and for some $j(x-y) \in J(x-y)$.

In this case, we say T is a *λ-strictly pseudocontractive mapping*.

(4) $T : C \to C$ is said to be *L-Lipschitzian, if there exists $L > 0$ such that*

$$\|Tx - Ty\| \leq L\|x-y\|, \quad \forall x, y \in C. \tag{1.7}$$

Remark 1.2. It is easy to see that if $T : C \to C$ is a λ-strictly pseudocontractive mapping, then it is a $(1+\lambda)/\lambda$-Lipschitzian mapping.

In fact, it follows from (1.6) that for any $x, y \in C$,

$$\begin{aligned} \lambda\|(I-T)x-(I-T)y\|^2 &\leq \langle (I-T)x-(I-T)y, j(x-y) \rangle \\ &\leq \|(I-T)x-(I-T)y\|\|x-y\|. \end{aligned} \tag{1.8}$$

Simplifying it, we have

$$\|(I-T)x-(I-T)y\| \leq \frac{1}{\lambda}\|x-y\|, \tag{1.9}$$

that is,

$$\|Tx - Ty\| \leq \frac{1+\lambda}{\lambda}\|x-y\|, \quad \forall x, y \in C. \tag{1.10}$$

Lemma 1.3 (see [2, Theorem 13.1] or [3]). *Let E be a real Banach space, C be a nonempty closed convex subset of E, and $T : C \to C$ be a continuous strongly pseudocontractive mapping. Then T has a unique fixed point in C.*

Remark 1.4. Let E be a real Banach space, C be a nonempty closed convex subset of E and $T : C \to C$ be a Lipschitzian pseudocontraction mapping. For every given $u \in C$ and $s \in (0,1)$, define a mapping $U_s : C \to C$ by

$$U_s x = su + (1-s)Tx, \quad x \in C. \tag{1.11}$$

It is easy to see that U_s is a continuous strongly pseudocontraction mapping. By using Lemma 1.3, there exists a unique fixed point $x_s \in C$ of U_s such that

$$x_s = su + (1-s)Tx_s. \tag{1.12}$$

The concept of pseudocontractive mappings is closely related to accretive operators. It is known that T is pseudocontractive if and only if $I - T$ is accretive, where I is the identity mapping. The importance of accretive mappings is from their connection with theory of solutions for nonlinear evolution equations in Banach spaces. Many kinds of equations, for example, Heat, wave, or Schrödinger equations can be modeled in terms of an initial value problem:

$$\frac{du}{dt} = Tu - u, \quad u(0) = u_0, \tag{1.13}$$

where T is a pseudocontractive mapping in an appropriate Banach space.

In order to approximate a fixed point of Lipschitzian pseudocontractive mapping, in 1974, Ishikawa introduced a new iteration (it is called *Ishikawa iteration*). Since then, a question of whether or not the Ishikawa iteration can be replaced by the simpler Mann iteration has remained open. Recently Chidume and Mutangadura [4] solved this problem by constructing an example of a Lipschitzian pseudocontractive mapping with a unique fixed point for which every Mann-type iteration fails to converge.

Inspired by the implicit iteration introduced by Xu and Ori [5] for a finite family of nonexpansive mappings in a Hilbert space, Osilike [6], Chen et al. [7], Zhou [8] and Boonchari and Saejung [9] proposed and studied convergence theorems for an implicit iteration process for a finite or infinite family of continuous pseudocontractive mappings.

The purpose of this paper is to study the strong and weak convergence problems of the implicit iteration processes for an infinite family of Lipschitzian pseudocontractive mappings in Banach spaces. The results presented in this paper extend and improve some recent results of Xu and Ori [5], Osilike [6], Chen et al. [7], Zhou [8] and Boonchari and Saejung [9].

For this purpose, we first recall some concepts and conclusions.

A Banach space E is said to be uniformly convex, if for each $\varepsilon > 0$, there exists a $\delta > 0$ such that for any $x, y \in E$ with $\|x\|, \|y\| \le 1$ and $\|x - y\| \ge \varepsilon$, $\|x + y\| \le 2(1-\delta)$ holds. The modulus of convexity of E is defined by

$$\delta_E(\varepsilon) = \inf\left\{1 - \left\|\frac{x+y}{2}\right\| : \|x\|,\ \|y\| \le 1,\ \|x - y\| \ge \varepsilon\right\}, \quad \forall \varepsilon \in [0,2]. \tag{1.14}$$

Concerning the modulus of convexity of E, Goebel and Kirk [10] proved the following result.

Lemma 1.5 (see [10, Lemma 10.1]). *Let E be a uniformly convex Banach space with a modulus of convexity δ_E. Then $\delta_E : [0,2] \to [0,1]$ is continuous, increasing, $\delta_E(0) = 0$, $\delta_E(t) > 0$ for $t \in (0,2]$ and*

$$\|cu + (1-c)v\| \leq 1 - 2\min\{c, 1-c\}\delta_E(\|u - v\|), \tag{1.15}$$

for all $c \in [0,1]$, and $u, v \in E$ with $\|u\|, \|v\| \leq 1$.

A Banach space E is said to satisfy the Opial condition, if for any sequence $\{x_n\} \subset E$ with $x_n \rightharpoonup x$, then the following inequality holds:

$$\limsup_{n\to\infty}\|x_n - x\| < \limsup_{n\to\infty}\|x_n - y\|, \tag{1.16}$$

for any $y \in E$ with $y \neq x$.

Lemma 1.6 (Zhou [8]). *Let E be a real reflexive Banach space with Opial condition. Let C be a nonempty closed convex subset of E and $T : C \to C$ be a continuous pseudocontractive mapping. Then $I - T$ is demiclosed at zero, that is, for any sequence $\{x_n\} \subset E$, if $x_n \rightharpoonup y$ and $\|(I - T)x_n\| \to 0$, then $(I - T)y = 0$.*

Lemma 1.7 (Chang [11]). *Let $J : E \to 2^{E^*}$ be the normalized duality mapping, then for any $x, y \in E$,*

$$\|x + y\|^2 \leq \|x\|^2 + 2\langle y, j(x + y)\rangle, \quad \forall j(x + y) \in J(x + y). \tag{1.17}$$

Definition 1.8 (see [12]). Let $\{T_n\} : C \to E$ be a family of mappings with $\bigcap_{n=1}^{\infty} F(T_n) \neq \emptyset$. We say $\{T_n\}$ satisfies the *AKTT-condition*, if for each bounded subset B of C the following holds:

$$\sum_{n=1}^{\infty}\sup_{z\in B}\|T_{n+1}z - T_n z\| < \infty. \tag{1.18}$$

Lemma 1.9 (see [12]). *Suppose that the family of mappings $\{T_n\} : C \to C$ satisfies the AKTT-condition. Then for each $y \in C$, $\{T_n y\}$ converges strongly to a point in C. Moreover, let $T : C \to C$ be the mapping defined by*

$$Ty = \lim_{n\to\infty} T_n y, \quad \forall y \in C. \tag{1.19}$$

Then, for each bounded subset $B \subset C$, $\lim_{n\to\infty}\sup_{z\in B}\|Tz - T_n z\| = 0$.

2. Main Results

Theorem 2.1. *Let E be a uniformly convex Banach space with a modulus of convexity δ_E, and C be a nonempty closed convex subset of E. Let $\{T_n\} : C \to C$ be a family of L_n-Lipschitzian and* *pseudocontractive mappings with* $L := \sup_{n\geq 1} L_n < \infty$ *and* $\mathcal{F} := \bigcap_{n\geq 1} F(T_n) \neq \emptyset$. *Let* $\{x_n\}$ *be the sequence defined by*

$$\begin{aligned} &x_1 \in C, \\ &x_n = \alpha_n x_{n-1} + (1-\alpha_n) T_n x_n, \quad n \geq 1, \end{aligned} \tag{2.1}$$

where $\{\alpha_n\}$ *is a sequence in* $[0,1]$. *If the following conditions are satisfied:*

(i) $\limsup_{n\to\infty} \alpha_n < 1$;

(ii) *there exists a compact subset* $K \subset E$ *such that* $\bigcup_{n=1}^{\infty} T_n(C) \subset K$;

(iii) $\{T_n\}$ *satisfies the AKTT-condition, and* $F(T) \subset \mathcal{F}$, *where* $T : C \to C$ *is the mapping defined by* (1.19).

Then x_n *converges strongly to some point* $p \in \mathcal{F}$

Proof. First, we note that, by Remark 1.4, the method is well defined. So, we can divide the proof in three steps.

(I) For each $p \in \mathcal{F}$ the limit $\lim_{n\to\infty}\|x_n - p\|$ exists.

In fact, since $\{T_n\}$ is pseudocontractive, for each $p \in \mathcal{F}$, we have

$$\begin{aligned} \|x_n - p\|^2 &= \langle x_n - p, j(x_n - p)\rangle \\ &= \alpha_n \langle x_{n-1} - p, j(x_n - p)\rangle + (1-\alpha_n)\langle T_n x_n - p, j(x_n - p)\rangle \\ &\leq \alpha_n \|x_{n-1} - p\| \|x_n - p\| + (1-\alpha_n)\|x_n - p\|^2, \quad \forall n \geq 1. \end{aligned} \tag{2.2}$$

Simplifying, we have that

$$\|x_n - p\| \leq \|x_{n-1} - p\|, \quad \forall n \geq 1. \tag{2.3}$$

Consequently, the limit $\lim_{n\to\infty}\|x_n - p\|$ exists, and so the sequence $\{x_n\}$ is bounded.

(II) Now, we prove that $\lim_{n\to\infty}\|x_n - T_n x_n\| = 0$.

In fact, by virtue of (2.1) and (1.4), we have

$$\begin{aligned} \|x_n - p\| &\leq \left\| x_n - p + \frac{1-\alpha_n}{2\alpha_n}(x_n - T_n x_n) \right\| \\ &= \left\| x_n - p + \frac{1-\alpha_n}{2}(x_{n-1} - T_n x_n) \right\| \\ &= \left\| \alpha_n x_{n-1} + (1-\alpha_n) T_n x_n - p + \frac{1-\alpha_n}{2}(x_{n-1} - T_n x_n) \right\| \\ &= \left\| \frac{x_{n-1} + x_n}{2} - p \right\| \\ &= \|x_{n-1} - p\| \cdot \left\| \frac{x_{n-1} - p}{2\|x_{n-1} - p\|} + \frac{x_n - p}{2\|x_{n-1} - p\|} \right\|. \end{aligned} \tag{2.4}$$

Letting $u = (x_{n-1} - p)/\|x_{n-1} - p\|$ and $v = (x_n - p)/\|x_{n-1} - p\|$, from (2.3), we know that $\|u\| = 1$, $\|v\| \le 1$. It follows from (2.4) and Lemma 1.5 that

$$\|x_n - p\| \le \|x_{n-1} - p\| \left\{ 1 - \delta_E \left(\frac{\|x_{n-1} - x_n\|}{\|x_{n-1} - p\|} \right) \right\}. \tag{2.5}$$

Simplifying, we have that

$$\|x_{n-1} - p\| \left\{ \delta_E \left(\frac{\|x_{n-1} - x_n\|}{\|x_{n-1} - p\|} \right) \right\} \le \|x_{n-1} - p\| - \|x_n - p\|. \tag{2.6}$$

This implies that

$$\sum_{n=1}^{\infty} \|x_{n-1} - p\| \left\{ \delta_E \left(\frac{\|x_{n-1} - x_n\|}{\|x_{n-1} - p\|} \right) \right\} \le \|x_0 - p\|. \tag{2.7}$$

Letting $\lim_{n\to\infty}\|x_n - p\| = r$, if $r = 0$, the conclusion of Theorem 2.1 is proved. If $r > 0$, it follows from the property of modulus of convexity δ_E that $\|x_{n-1} - x_n\| \to 0 \ \ (n \to \infty)$. Therefore, from (2.1) and the condition (i), we have that

$$\|x_{n-1} - T_n x_n\| = \frac{1}{1 - \alpha_n} \|x_n - x_{n-1}\| \longrightarrow 0 \quad (\text{as } n \longrightarrow \infty). \tag{2.8}$$

In view of (2.1) and (2.8), we have

$$\|x_n - T_n x_n\| = \alpha_n \|x_{n-1} - T_n x_n\| \longrightarrow 0 \quad (\text{as } n \longrightarrow \infty). \tag{2.9}$$

(III) Now, we prove that $\{x_n\}$ converges strongly to some point in $\mathcal{F}$.

In fact, it follows from (2.9) and condition (ii) that there exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $\|x_{n_i} - T_{n_i} x_{n_i}\| \to 0$ (as $n_i \to \infty$), $T_{n_i} x_{n_i} \to p$ and $x_{n_i} \to p$ (some point in C). Furthermore, by Lemma 1.9, we have $T_{n_i} p \to Tp$. consequently, we have

$$\begin{aligned} \|p - Tp\| &\le \|p - x_{n_i}\| + \|x_{n_i} - T_{n_i} p\| + \|T_{n_i} p - Tp\| \\ &\le \|p - x_{n_i}\| + \|x_{n_i} - T_{n_i} x_{n_i}\| + \|T_{n_i} x_{n_i} - T_{n_i} p\| + \|T_{n_i} p - Tp\| \\ &\le (1 + L)\|p - x_{n_i}\| + \|x_{n_i} - T_{n_i} x_{n_i}\| + \|T_{n_i} p - Tp\| \longrightarrow 0. \end{aligned} \tag{2.10}$$

This implies that $p = Tp$, that is, $p \in F(T) \subset \mathcal{F}$. Since $x_{n_i} \to p$ and the limit $\lim_{n\to\infty}\|x_n - p\|$ exists, we have $x_n \to p$.

This completes the proof of Theorem 2.1. □

Theorem 2.2. *Let E be a uniformly convex Banach space satisfying the Opial condition. Let C be a nonempty closed convex subset of E and $\{T_n\} : C \to C$ be a family of L_n-Lipschitzian* *pseudocontractive mappings with* $L := \sup_{n\geq 1} L_n < \infty$ *and* $\mathcal{F} := \bigcap_{n\geq 1} F(T_n) \neq \emptyset$. *Let* $\{x_n\}$ *be the sequence defined by* (2.1) *and* $\{\alpha_n\}$ *be a sequence in (0, 1). If the following conditions are satisfied:*

(i) $\limsup_{n\to\infty} \alpha_n < 1$,

(ii) *for any bounded subset* B *of* C

$$\lim_{n\to\infty} \sup_{z\in B} \|T_m T_n z - T_n z\| = 0, \quad \textit{for each } m \geq 1. \tag{2.11}$$

Then the sequence $\{x_n\}$ *converges weakly to some point* $u \in \mathcal{F}$.

Proof. By the same method as given in the proof of Theorem 2.1, we can prove that the sequence $\{x_n\}$ is bounded and

$$\begin{gathered} \lim_{n\to\infty} \|x_n - p\| \text{ exists}, \quad \text{for each } p \in \mathcal{F}; \\ \lim_{n\to\infty} \|x_n - T_n x_n\| = 0. \end{gathered} \tag{2.12}$$

Now, we prove that

$$\lim_{n\to\infty} \|T_m x_n - x_n\| = 0, \quad \text{for each } m \geq 1. \tag{2.13}$$

Indeed, for each $m \geq 1$, we have

$$\begin{aligned} \|T_m x_n - x_n\| &\leq \|T_m x_n - T_m T_n x_n\| + \|T_m T_n x_n - T_n x_n\| + \|T_n x_n - x_n\| \\ &\leq (1+L)\|T_n x_n - x_n\| + \|T_m T_n x_n - T_n x_n\| \\ &\leq (1+L)\|T_n x_n - x_n\| + \sup_{z\in\{x_n\}} \|T_m T_n z - T_n z\|. \end{aligned} \tag{2.14}$$

By (2.12) and condition (ii), we have

$$\lim_{n\to\infty} \|T_m x_n - x_n\| = 0, \quad \text{for each } m \geq 1. \tag{2.15}$$

The conclusion of (2.13) is proved.

Finally, we prove that $\{x_n\}$ converges weakly to some point $u \in \mathcal{F}$.

In fact, since E is uniformly convex, and so it is reflexive. Again since $\{x_n\} \subset C$ is bounded, there exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $x_{n_i} \rightharpoonup u$. Hence from (2.13), for any $m > 1$, we have

$$\|T_m x_{n_i} - x_{n_i}\| \longrightarrow 0 \quad (\text{as } n_i \longrightarrow \infty). \tag{2.16}$$

By virtue of Lemma 1.6, $u \in F(T_m)$, for all $m \geq 1$. This implies that

$$u \in \mathcal{F} := \bigcap_{n\geq} F(T_n) \cap W_\omega(x_n). \tag{2.17}$$

Next, we prove that $W_\omega(x_n)$ is a singleton. Let us suppose, to the contrary, that if there exists a subsequence $\{x_{n_j}\} \subset \{x_n\}$ such that $x_{n_j} \rightharpoonup q \in W_\omega(x_n)$ and $q \neq u$. By the same method as given above we can also prove that $q \in \mathcal{F} := \bigcap_{n\geq 1} F(T_n) \cap W_\omega(x_n)$. Taking $p = u$ and $p = q$ in (2.12). We know that the following limits

$$\lim_{n\to\infty} \|x_n - u\|, \qquad \lim_{n\to\infty} \left\|x_n - q\right\| \tag{2.18}$$

exist. Since E satisfies the Opial condition, we have

$$\begin{aligned}
\lim_{n\to\infty} \|x_n - u\| &= \limsup_{n_i\to\infty} \|x_{n_i} - u\| < \limsup_{n_i\to\infty} \left\|x_{n_i} - q\right\| \\
&= \lim_{n\to\infty} \left\|x_n - q\right\| = \limsup_{n_j\to\infty} \left\|x_{n_j} - q\right\| \\
&< \limsup_{n_j\to\infty} \left\|x_{n_j} - u\right\| = \lim_{n\to\infty} \|x_n - u\|.
\end{aligned} \tag{2.19}$$

This is a contradiction, which shows that $q = u$. Hence,

$$W_\omega(x_n) = \{u\} \subset \mathcal{F} := \bigcap_{n\geq 1} F(T_n). \tag{2.20}$$

This implies that $x_n \rightharpoonup u$.

This completes the proof of Theorem 2.2. □

In the next lemma, we propose a sequence of mappings that satisfy condition (iii) in Theorem 2.1. Moreover, we apply this lemma to obtain a corollary of our main Theorem 2.1.

Let E be a Banach space and C be a nonempty closed convex subset of E. From Definition 1.1(3), we know that if $T : C \to C$ is a λ-strictly pseudocontractive mapping, then it is a $((1+\lambda)/\lambda)$-Lipschitzian pseudocontractive mapping.

On the other hand, by the same proof as given in [12] we can prove the following result.

Lemma 2.3 (see [12] or [9]). *Let E be a smooth Banach space, C be a closed convex subset of E. Let $\{S_n\} : C \to C$ be a family of λ_n-strictly pseudocontractive mappings with $\mathcal{F} := \bigcap_{n=1}^{\infty} F(S_n) \neq \emptyset$ and $\lambda := \inf_{n\geq 1}\lambda_n > 0$. For each $n \geq 1$ define a mapping $T_n : C \to C$ by:*

$$T_n x = \sum_{k=1}^{n} \beta_n^k S_k x, \quad x \in C, \ n \geq 1, \tag{2.21}$$

where $\{\beta_n^k\}$ is sequence of nonnegative real numbers satisfying the following conditions:

(i) $\sum_{k=1}^{n} \beta_n^k = 1$, *for all $n \geq 1$;*

(ii) $\beta^k := \lim_{n\to\infty}\beta_n^k > 0$, *for all $k \geq 1$;*

(iii) $\sum_{n=1}^{\infty} \sum_{k=1}^{n} |\beta_{n+1}^k - \beta_n^k| < \infty$.

Then,

(1) *each* T_n, $n \geq 1$ *is a* λ*-strictly pseudocontractive mapping;*

(2) $\{T_n\}$ *satisfies the AKTT-condition;*

(3) *if* $T : C \to C$ *is the mapping defined by*

$$Tx = \sum_{k=1}^{\infty} \beta^k S_k x, \quad x \in C. \tag{2.22}$$

Then $Tx = \lim_{n\to\infty} T_n x$ *and* $F(T) = \bigcap_{k=1}^{\infty} F(T_n) = \mathcal{F} := \bigcap_{n=1}^{\infty} F(S_n)$.

The following result can be obtained from Theorem 2.1 and Lemma 2.3 immediately.

Theorem 2.4. *Let* E *be a uniformly convex Banach space,* C *be a nonempty closed convex subset of* E. *Let* $\{S_n\} : C \to C$ *be a family of* λ_n*-strictly pseudocontractive mappings with* $\mathcal{F} := \bigcap_{n=1}^{\infty} F(S_n) \neq \emptyset$ *and* $\lambda := \inf_{n\geq 1} \lambda_n > 0$. *For each* $n \geq 1$ *define a mapping* $T_n : C \to C$ *by*

$$T_n x = \sum_{k=1}^{n} \beta_n^k S_k x, \quad x \in C, \ n \geq 1, \tag{2.23}$$

where $\{\beta_n^k\}$ *is a sequence of nonnegative real numbers satisfying the following conditions:*

(i) $\sum_{k=1}^{n} \beta_n^k = 1$, *for all* $n \geq 1$;

(ii) $\beta^k := \lim_{n\to\infty} \beta_n^k > 0$, *for all* $k \geq 1$;

(iii) $\sum_{n=1}^{\infty} \sum_{k=1}^{n} |\beta_{n+1}^k - \beta_n^k| < \infty$.

Let $\{x_n\}$ *be the sequence defined by*

$$\begin{gathered} x_1 \in C, \\ x_n = \alpha_n x_{n-1} + (1 - \alpha_n) T_n x_n, \quad n \geq 1, \end{gathered} \tag{2.24}$$

where $\{\alpha_n\}$ *is a sequence in* $[0, 1]$. *If the following conditions are satisfied:*

(i) $\limsup_{n\to\infty} \alpha_n < 1$;

(ii) *there exists a compact subset* $K \subset E$ *such that* $\bigcup_{n=1}^{\infty} S_n(C) \subset K$. *Then,* $\{x_n\}$ *converges strongly to some point* $p \in \mathcal{F}$.

Proof. Since $\{S_n\} : C \to C$ is a family of λ_n-strictly pseudocontractive mappings with $\lambda := \inf_{n\geq 1} \lambda_n > 0$. Therefore, $\{S_n\}$ is a family of λ-strictly pseudocontractive mappings. By Remark 1.2, $\{S_n\}$ is a family of $(1 + \lambda)/\lambda$-Lipschitzian and strictly pseudocontractive mappings. Hence, by Lemma 2.3, $\{T_n\}$ defined by (2.21) is a family of $(1 + \lambda)/\lambda$-Lipschitzian, strictly pseudocontractive mappings with $\bigcap_{n=1}^{\infty} F(T_n) = \mathcal{F} := \bigcap_{n=1}^{\infty} F(S_n) \neq \emptyset$ and it has also the following properties:

(1) $\{T_n\}$ satisfies the AKTT-condition;

(2) if $T : C \to C$ is the mapping defined by (2.22), then $Tx = \lim_{n\to\infty} T_n x, x \in C$ and $F(T) = \mathcal{F} := \bigcap_{k=1}^{\infty} F(S_n) = \bigcap_{n=1}^{\infty} F(T_n)$. Hence, by Definition 1.1, $\{T_n\}$ is also a family of $(1+\lambda)/\lambda$-Lipschitzian and pseudocontractive mappings having the properties (1) and (2) and $\mathcal{F} := \bigcap_{n=1}^{\infty} F(T_n) \neq \emptyset$. Therefore, $\{T_n\}$ satisfies all the conditions in Theorem 2.1. By Theorem 2.1, the sequence $\{x_n\}$ converges strongly to some point $p \in \mathcal{F} : \bigcap_{k=1}^{\infty} F(S_n) = \bigcap_{n=1}^{\infty} F(T_n)$.

This completes the proof of Theorem 2.4. □

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