# MODELING AND ANALYSIS OF FORWARD CONTRACT WITH BILATERAL OPTIONS IN ELECTRICITY MARKET

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This paper presents the modeling and analysis of a new forward contract with bilateral options in electricity Abstract market. This new contract enables both the seller and the buyer to take advantage of flexibility in generation and consumption to obtain a monetary benefit while simultaneously removing the risk of market price fluctuations. Theoretical model for pricing this type of forward contract is developed and analyzed. Some distinguishing features are revealed. Numerical examples are used to demonstrate the validity of the proposed model.

forward contracts; electricity market; risk management; option; pricing model Key words: CLC number: TM 73; F123. 9

# **0** Introduction

Restructuring and deregulation of electric power industry has given rise to vigorous, increasingly diverse and competitive market place. Electricity market prices are bound to be volatile as a consequence of the unique physical attributes of electricity. An increasing number of market participants are recognizing the importance and necessity of risk management, especially after observing the market anomalies in California. It is widely appreciated that contractual arrangements, physically or financially, play an important role as means for risk management in electricity markets. Forward contracts or contractual arrangements similar to option and futures on electricity are being employed around the world  $[1 \sim 3]$ .

To meet market participants' increasing demand for appropriate risk management tools and methodology in order to establish their risk management programs in one way or another, a variety of research work has been conducted on topics of electricity market risk management with contractual instruments. This paper, however, has a more limited focus. It deals with the design of forward contracts bundled with financial options. In a competitive electricity market, participants wishing to ensure a fixed electricity price while taking advantage of their flexibility and willingness to curtail load or supply can do so by using a forward contract bundled with financial options that provides a hedge against price risk and reflects the real choices available to the participants. A forward contract bundled with financial options, or optional forward contracts, gives the option holder a right, but not an obligation, to purchase or sell the contracted energy at the delivery time for a given price,

called the strike price, thus enables the option holder to hedge against the risk of profit loss under unfavorable situation, while retaining the ability to participate in a favorable market position. Hence the optional forward contract has much more flexibility that is of interest to market participants.

Relating to optional electricity forward contract design and modeling, several papers have been published in the past. For examples the equivalence between interruptible load services and forward contracts bundled with a call option, called a callable forward, was described in reference [4]. The supplyside analog, i.e. forward contracts bundled with a put option, called puttable forward, was introduced in reference [5]. A double call option was introduced in reference [6] to account for the effect of early notification of curtailment, which allows the consumer to secure the benefit of its option to curtail load. Theoretical framework for modeling risk in optional forward contracts between host utilities and independent producers was presented in reference [7]. However, the optional forward contracts given in reference  $[4 \sim 6]$  involve unilaterally giving utilities the options to curtail the consumers' contracted energy or reject the contracted energy supplied by independent producers at the delivery time. These arrangements make the consumers lose the ability to take advantage of falling prices and the producers lose the ability to take advantage of rising prices. While the optional forward contract proposed in reference [7] does give both the contracting parties the options, a reconciliation procedure is needed to make the contract workable because the contract prices and penalties, derived separately from respective rational behavioral models of the utility and independent producer, cannot be assured to agree with each other all the time.

收稿日期: 2002-12-13; 修回日期: 2003-02-21。

contract referred to as a forward contract with bilateral options, in which the seller of the contract holds a right to curtail the contracted energy when the spot price is high while the buyer has a choice to decline the contracted energy when the spot price is low. The option theory is employed to formulate the contract price. The strike prices of options are derived from solving an equilibrium model in which both the buyer and the seller are inclined to maximize their individual profit. The features of this kind of optional forward contract are explained and numerical examples are presented to illustrate its validity.

# 1 Electricity Forward Contracts with Bilateral Options

In this paper, we assume that there is an electricity spot market to which both sides of forward contracts have free access. The participants of forward transactions include independent generators large consumers and suppliers (who produce or consume physical quantities of electrical energy), and norphysical traders (marketers). Both the contract sellers and the buyers are supposed to be commercially rational and flexible in "consumption" and "production" of electricity.

Consider a contractual arrangement between a seller and a buyer for trading one unit of electrical energy at some specified time Tin the future. We assume that there are some informative market coordinators or arbitrators that serves to facilitate the forw ard contract transactions and at time t ( $t \le T$ ) when the forw ard contract is made, both the seller and the buyer can understand from certain market coordinator or arbitrator that the spot market price  $p_T$  at time T has a given probability distribution  $Q_{p_T}(x)$ . It is natural to assume that  $Q_{p_T}(x)=0$  if  $x \le 0$  for any distribution function which is to be relevant to this problem. It is also typical to assume that once the contract is set the buyer is required to pay the seller according to the contracted price and contracted quantity at some time before the delivery time T.

It is agreed in the contractual arrangement that the seller holds an option to interrupt the supply of the contracted energy to the buyer at time T. The buyer, on the other hand, holds an option to reject the supply of the contracted energy from the seller at time T. Let  $k_{\rm C}$  denote the monetary compensation per unit of energy to be paid by the seller to the buyer if the seller declines to supply the contracted energy at time T, and  $k_{\rm P}$  the monetary compensation per unit of energy to be paid by the seller to the buyer if the buyer declines to accept the contracted energy at time T. For a commercially rational seller, the optimal decision at time T is to curtail whenever  $p_T > k_{\rm C}$ because if  $p_T > k_{\rm C}$ , the seller can sell the curtailed energy on (1994-2017) china Academic Journal Electronic Put the spot market for price  $p_T$ , yielding a positive profit of  $p_T$ — $k_c$ . Similarly, the optimal decision for a commercially rational buyer at time T is to reject whenever  $p_T < k_P$ , because if  $p_T < t_P$ , the buyer can purchase the required one unit of energy from the spot market at a price of  $p_T$ , yielding a profit of  $k_P - p_T$ . It should be noted that the case of  $k_P < k_c$  describes the normal situation that is of practical significance.

In terms of option theory<sup>[8]</sup>, the aforementioned contractual arrangement we propose is a type of optional forward contract, which we call a forward contract with bilateral options. A forward contract with bilateral options is a bundle of three contracts The first of these is a forward contract, which is owned by the buyer and which guarantees that the seller will deliver to the buyer one unit of energy at time T. The second contract is a call option on the same unit of energy. The call option, which is sold by the buyer back to the seller, confers the right, but not the obligation, to purchase the unit of energy at time T for a given price, called the strike price  $k_{\rm C}$ . We describe the resulting obligation by saying that the buyer is short a call while the seller owns a call. The third one is a put option on the same contracted energy. The put option, which is sold by the seller to the buyer, is the right, but not the obligation, to sell the unit of energy at time T for a given price, called the strike price  $k_{\rm P}$ . Similarly, we describe the resulting obligation by saying that the buyer owns a put while the seller is short a put.

Thus a buyer who owns a forward contract with bilateral options is guaranteed to receive from the seller at time T the strike price  $k_{\rm C}$  (and no energy); or one unit of energy, at the option of the seller (exercise the call or not) when the spot price  $p_T$  turns out to be greater than  $k_{\rm P}$ ; or receive a profit of  $k_{\rm P}-p_T$  plus one unit of energy purchased from the open spot market when  $p_T$  turns out to be so low that the buyer exercises the put option. Figure 1 illustrates the contractual obligations payments and choices in a forward contract with bilateral options.

	Contract price $f_0$	
Seller:	<b></b>	Buyer:
Short 1 forward	If $p_T > k_C$ , pay strike price $k_C$	Owns 1 forward
Owns 1 call with strike price $k_c$	If $k_{\rm P} < p_T < k_{\rm C}$ , supply 1 unit energy	Short 1 call with strike price $k_{C}$
Short 1 put with strike price $k_p$	If $p_T < k_p$ , pay strike price $k_p$	Owns I put with strike price $k_p$

Fig. 1 Contractual obligations payments and choices for a forward contract with bilateral options

# 2 Theoretical Model

Let us further assume that both the seller and the buyer are risk blish neutral (i.e. the perceived benefit equals the expected monetary benefit) and hence aim to maximize their expected monetary benefits. A theoretical model for linking the degree of future uncertainties in spot market price  $p_T$  with the contract price  $f_0$ and strike prices  $k_C$ ,  $k_P$  is developed as follows.

#### 2.1 Derivation of the Contract Price

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Given that the condition  $k_P < p_T < k_C$  holds, i.e. both the seller and the buyer will not exercise the options stipulated in the forward contract with bilateral options then the contract price  $f_0$  should be equated as the expected value of the spot market price at time T. In fact, either the seller or the buyer will execute his/her option to yield a positive profit if  $p_T > k_C$  or  $p_T < k_P$  respectively. Accordingly, the expected payoffs of these options should be incorporated in the contract price.

The payoff for the seller's call option is max  $\{0, p_T - k_C\}$ . Note that if  $p_T > k_C$  the seller can exercise his/her option to obtain 1 unit of energy at the cost of  $k_C$ , which can then be sold on the open spot market for price  $p_T$ , yielding a profit of  $p_T < k_C$ . If  $p_T < k_C$  using the option in this way would yield a loss of  $k_C - p_T$ , so the rational seller will not exercise the option, so its payoff is 0. Hence the expected payoff of the seller's call option on a unit energy at time T can be expressed as

$$P_{c} = E[\max\{0, p_{T} - k_{c}\} + H_{d}] = \int_{k_{c}}^{\infty} (x - k_{c}) q_{p_{T}}(x) dx = \bar{p}_{T} - k_{c} + \int_{0}^{k_{c}} Q_{p_{T}}(x) dx$$
(1)

where:  $H_t$  represents the best information available at time t;  $q_{p_T}(x)$  is the probability density function of  $p_T$  estimated at time t;  $p_T$  is the mean value of  $p_T$ .

A similar analysis can be conducted for the buyer's put option. If  $p_T < k_P$ , the buyer can exercise his' her option to obtain strike price  $k_P$ , and purchase 1 unit of energy on the open spot market at price  $p_T$ , yielding a profit of  $k_P - p_T$ . If  $p_T > k_P$ , using the option in this way would yield a loss of  $p_T - k_P$ . Therefore the payoff for the buyer's put option is max $\{0, k_P - p_T\}$ . Thus the expected payoff of the buyer's put option on a unit energy at time T can be given by

$$E_{P_{p}} = E[\max\{0 | k_{P} - p_{T}\} | H_{d}] = \int_{0}^{k_{p}} (k_{P} - x) q_{p_{T}}(x) dx = \int_{0}^{k_{p}} Q_{p_{T}}(x) dx \qquad (2)$$

Under the assumption of expected value pricing, the contracted price  $f_0$  should be equal to the expected spot price at time T, less the expected payoff of the seller's call option, and plus the expected payoff of the buyer's put option. Using equation (1) and equation (2),  $f_0$  can be derived as follows:

$$f_{0} = \bar{p}_{T} - E_{P_{c}} + E_{P_{p}} = k_{c} - \int_{0}^{k_{c}} Q_{p_{T}}(x) dx + \int_{0}^{k_{p}} Q_{p_{T}}(x) dx = k_{c} - \int_{k_{p}}^{k_{c}} Q_{p_{T}}(x) dx$$
(3)

$$\begin{cases} \frac{\partial f_0}{\partial k_{\rm C}} = 1 - \mathcal{Q}_{p_T}(k_{\rm C}) \ge 0 \\ \frac{\partial^2 f_0}{\partial k_{\rm C}^2} = - q_{p_T}(k_{\rm C}) \le 0 \\ \begin{cases} \frac{\partial f_0}{\partial k_{\rm P}} = \mathcal{Q}_{p_T}(k_{\rm P}) \ge 0 \\ \frac{\partial^2 f_0}{\partial k_{\rm P}^2} = q_{p_T}(k_{\rm P}) \ge 0 \end{cases}$$

$$(5)$$

Hence the contract price  $f_{0}$  is non-decreasing, and concave in  $k_{\rm C}$  and convex in  $k_{\rm P}$ . Figure 2 illustrates the typical variation of contract price  $f_0$  with strike prices  $k_{\rm C}$  and  $k_{\rm P}$ . Note that  $k_{\rm P} \leq k_{\rm C}$  is the case that describes the practical situation.

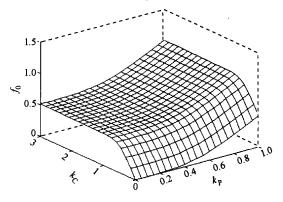


Fig. 2 Contract price  $f_0$  versus strike prices  $k_C$  and  $k_P$ 

In the deterministic case, i. e. the spot price  $p_T$  is known for certain at time t, then it can be shown from equation (3) that ① if  $p_T > k_C$ , then  $f_0 = k_C$ ; ② if  $k_P < p_T < k_C$ , then  $f_0 = p_T$ ; ③ if  $p_T < k_P$ , then  $f_0 = k_P$ . However, the deterministic case is not of practical significance, hence in general  $k_P < f_0 < k_C$ .

# 2.2 Equilibrium Selection of $k_{\rm C}$ and $k_{\rm P}$

As shown in figure 1, if a buyer purchases a unit forward contract with bilateral options having strike prices  $k_{\rm C}$  and  $k_{\rm P}$ , the resulting monetary benefit to the buyer at time T will be

$$B_{B0} = \begin{cases} k_{C} - f_{0} & p_{T} > k_{C} \\ v - f_{0} & k_{P} < p_{T} < k_{C} \\ v + k_{P} - f_{0} - p_{T} & p_{T} < k_{P} \end{cases}$$
(6)

where v is the buyer's monetary benefit from "consumption" of a unit electricity at time T. Thus the expected benefit to the buyer can be expressed as:

$$E[B_{B0} | H_{t}] = (k_{C} - f_{0})(1 - Q_{p_{T}}(k_{C})) + (v - f_{0})(Q_{p_{T}}(k_{C}) - Q_{p_{T}}(k_{P})) + \int_{0}^{k_{P}} (v + k_{P} - f_{0} - x)q_{p_{T}}(x) dx$$
(7)

Using equation (3) and integrating by parts, this reduces to:

$$E[B_{B0} \mid H_{t}] = (v - k_{C}) Q_{p_{T}}(k_{C}) + \int_{0}^{k_{C}} Q_{p_{T}}(x) dx \quad (8)$$

Similarly, if a seller sells a unit forward contract with bilateral options having strike prices  $k_{\rm C}$  and  $k_{\rm P}$ , the resulting monetary benefit to the seller at time *T* will be

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$$B_{S0} = \begin{cases} f_0 - k_C - w + p_T & p_T > k_C \\ f_0 - w & k_P < p_T < k_C \\ f_0 - k_P & p_T < k_P \end{cases}$$
(9)

where w is the seller's cost for "production" of a unit electricity at time T. The expected benefit to the seller is:

$$E[B_{S0} \mid H_{l}] = \int_{k_{c}}^{\infty} (f_{0} - k_{c} - w + x) q_{p_{T}}(x) dx + (f_{0} - w)[Q_{p_{T}}(k_{c}) - Q_{p_{T}}(k_{P})] + (f_{0} - k_{P})Q_{p_{T}}(k_{P})$$
(10)

Using equation (3) and integrating by parts yields:

$$E[B_{50} | H_{l}] = \bar{p}_{T} - w + (w - k_{P}) Q_{p_{T}}(k_{P}) + \int_{0}^{k_{P}} Q_{p_{T}}(x) dx$$
(11)

If both the buyer and the seller are risk-neutral and economically rational, then they will separately choose  $k_{\rm C}$  and  $k_{\rm P}$  to maximize their individual expected benefit. Note that, under the fact that the contract price is set by equation (3), the buyer's expected benefit is independent of  $k_{\rm P}$  as shown in equation (8) while the seller's expected benefit is independent of  $k_{\rm C}$  as given in equation (11). Therefore equilibrium solution for the buyer and the seller to select  $k_{\rm C}$  and  $k_{\rm P}$  in order to maximize their own expected benefit can be simply expressed as follows

$$\begin{cases} \frac{\partial E[B_{00} \mid H_{1}]}{\partial k_{C}} = (v - k_{C})q_{p_{T}}(k_{C}) = 0\\ \frac{\partial E[B_{00} \mid H_{1}]}{\partial k_{P}} = (w - k_{P})q_{p_{T}}(k_{P}) = 0 \end{cases}$$
(12)

In general,  $q_{p_T}(\circ) \neq 0$ , thus we have

$$k_{\rm C} = v \text{ and } k_{\rm P} = w \tag{13}$$

That is actually strike price  $k_{\rm C}$  relies on the buyer's rational selection while strike price  $k_{\rm P}$  hinges on the seller's rational selection. Under equilibrium selection given by equation (13), it can be noted from equation (6) and equation (9) that at time T the seller has a benefit of at least  $f_0 - w$  and the buyer at least  $v - f_0$ .

# **3** Analysis

The callable forward contract introduced in reference [4] involves giving a utility (the seller) a right (call option), when the spot price at the delivery time is high enough, to "buy back" the contracted energy from a consumer (the buyer) at a strike price that is set by the rational consumer to be his/ her value of v. The puttable forward contract given in reference [5] confers a choice (put option) on a utility (the buyer), when the spot price at the delivery time is low enough, to "sell back" the contracted energy to an independent producer (the seller) at a strike price that is selected by the rational producer to be his/ her production cost w.

The forward contract with bilateral options proposed in this paper, which can be viewed as a combination of the callable and puttable forward involves giving a seller a right, when the spot price at the delivery time is high enough, too "buy back" the

contracted energy from a buyer at a strike price that is actually determined by the rational buyer's v. It also gives the buyer a choice, when the spot price at the delivery time is low enough to "sell back" the contracted energy to the seller at a strike price that is in nature specified by the rational seller' w. Hence, under this proposed mechanism, the value of each unit of "consumption" can be voluntarily revealed by the buyers and the cost of each unit of " production" can also be voluntarily disclosed by the sellers. An incentive compatible result can then be achieved regarding the selection of the strike prices. In addition, the contractual arrangement makes both buyers with  $v < p_T$  and sellers with  $w > p_T$  be excluded from consuming or producing energy. These characteristics will be helpful to improve efficiency in dispatching electricity production and consumption.

Table 1 shows contract prices, payoff structures and total expected benefits to seller and buyer under different contractual arrangements. In each case, the strike prices are determined by participants' rational selection and the expected value pricing method is employed to set the contract prices.

 Table 1
 Comparisons of four types of contract models

Туре	Payoff Structure		Total Expected	
	Buyer	Seller	Benefit	
0	$B_{\rm B0} \gg v - f_0$	$B_{\rm S0} \ge f_0 - w$	$E_{B0} = v - w + E_{P_{C}} + E_{P_{P}}$	
1	$B_{\rm B1} = v - f_1$	$B_{\rm S1} \ge f_1 - w$	$E_{\rm BI} = v - w + E_{P_{\rm C}}$	
2	$B_{B2} \gg v - f_2$	$B_{S2} = f_2 - w$	$E_{B2} = v - w + E_{P_{P}}$	
3	$B_{B3} = v - f_3$	$B_{S3} = f_3 - w$	$E_{B3} = v - w$	

Contract type: 0: forward contract with bilateral options; 1: callable forward contract; 2: puttable forward contract; 3: forward contract in which both the seller and buyer has no option.

- $f_i$ : contract price for a unit forward contract of type i.
- $B_{\mathrm{B}i}$ : buyer's monetary benefit at time T from purchasing a unit forward contract of type *i*.
- $B_{Si}$ ; seller's monetary benefit at time T from selling a unit forward contract of type *i*.
- $E_{\text{B}i}$ : total expected benefit to the buyer and the seller at time T from trading a unit forward contract of type i.

Some other advantages that result from using the proposed forward contract with bilateral options are to be noted. ① The buyer can hedge the risk of profit loss when the spot price rises while retaining the ability to take advantage of falling prices. In the meantime, the seller can hedge the risk of profit loss when the spot price falls, while retaining the ability to take advantage of rising prices. Consequently, the forward contract with bilateral options presents a more equitable and reasonable payoff structure as shown in table 1. ② It is straightforward that  $E_{B0} \ge E_{B1} \ge E_{B3}$  and  $E_{B0} \ge E_{B2} \ge E_{B3}$ . That is, the forward contract with bilateral options enables the buyer and the seller to earn a larger total expected benefit than those of other contract models. ③ The expected value pricing method combined with option pricing theory, is employed to formulate the contract price. The strike prices of options are derived from solving an equilibrium model, which embodies both the seller and buyer's rational selections. As a result, there is no requirement of reconciliation procedure for agreement on prices.

# 4 Examples

Suppose in an open electricity market, flexible sellers and buyers adopt the proposed forward contracts with bilateral options for trading electrical energy at the future specified time T. The spot price  $p_T$  is known at time t to obey a trapezoid probability distribution, as shown in figure 3, where  $c^- a = b$ -d. Assuming a=0 and b=1.5, this makes the mean value  $\bar{p}_T$  equal to 0.75, in arbitrary currency units. Standard deviation  $\sigma_T$  values of 0.307 and 0.380, in the same arbitrary currency units (so do the following numerical values), are used to represent different degrees of uncertainty in  $p_T$ . Note that in the following calculation, the strike prices  $k_{\rm C}$  and  $k_{\rm Pare}$  set by rational buyers and sellers, as shown in equation (13), and the contract price can be obtained using equation (3). Two cases, i.e. case for single seller with several buyers and case for single buyer with several sellers, are examined respectively as follows.

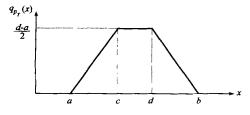


Fig. 3 Probability distribution of spot price at time T

#### 4.1 Case for Single Seller with Several Buyers

Suppose that a seller with w=0.5 has forward contracts with bilateral options with several different prospective buyers. Analysis including parametric relationships between the contract prices  $f_0$  and the buyers' v for different degrees of uncertainty in  $p_T$  is carried out. The results are in table 2.

Table 2 Test results for case of single seller with w = 0.5

	$\sigma_T = 0.307$		$\sigma_T = 0.380$	
Buyers'	$(E_{P_{\rm P}} = 0.037\ 2)$		$(E_{P_{\rm p}}=0.062\ 8)$	
V -	$E_{P_{C}}$	$f_0$	$E_{P_{C}}$	$f_0$
0.6	0.2143	0.572 9	0.247 4	0.565 4
0.8	0.1021	0.685 1	0.1397	0.673 1
1.0	0.0372	0.750 0	0.062 8	0.750 0
1.2	0.0080	0.779 2	0.016 6	0.796 2
1.4	0.0003	0.786 9	0.000 6	0.812 2

In this case, the expected payoff of the buyers' put option,  $E_{P_p}$ , remains unchanged for a given  $\sigma_T$ . It can be obtained that  $E_{P_p} = 0.0372$  when  $\sigma_T = 0.307$ , and  $E_{P_p} = 0.0628$  when  $\sigma_T$  = 0.380. The following results are observed in table 2. ① For a buyer with  $\eta = 1.0$ , the probability for the seller to express 15

the call option is the same as the probability for the buyer to exercise the put option, and we have  $E_{P_c} = E_{P_p}$  for a given  $\sigma_{T}$ , so the contract price equals to  $\overline{p}_T$ . (2) When the buyer's v goes below 1. 0, the seller will have increasing opportunities to exercise his call, which makes  $E_{P_{p}} \geq E_{P_{p}}$  for a given  $\sigma_{T}$ , and hence the contract price be less than  $\overline{p}_{T}$ . In addition, the larger the uncertainty in  $p_T$ , the lower the contract price because  $E_{P_{\alpha}}$ increases more quickly than  $E_{P_p}$  with increasing  $\sigma_T$ . In other words, if  $\sigma_T$  increases, the buyer will pay a low er contract price at the cost of increasing possibility that the seller interrupts the contracted energy. 3 For a buyer with  $v \ge 1.0$ , since the probability for the seller to exercise the call will be less than the one for the buyer to exercise the put, we have  $E_{P_c} \leq E_{P_p}$  for a given  $\sigma_T$ , and hence the contract price is greater than  $\overline{p}_T$ . Furthermore the larger the uncertainty in  $p_T$  the higher the contract price because  $E_{P_{\mathrm{D}}}$  increases more quickly than  $E_{P_{\mathrm{o}}}$  with increasing  $\sigma_T$ . In other words, when  $\sigma_T$  increases, the seller will receive a higher contract price at the cost of increasing possibility that the buyer rejects the contracted energy.

## 4.2 Case for Single Buyer with Several Sellers

Suppose that a buyer with v=1. 0 has forward contracts with bilateral options with several different prospective sellers. Analysis including parametric relationships between contract prices  $f_0$  and the sellers' w for different degrees of uncertainty in  $p_T$  is carried out. The results are tabulated in table 3.

			•••	
	$\sigma_T = 0.307$		$\sigma_T = 0.380$	
S ellers' w	$(E_{P_{\rm P}} = 0.0372)$		$(E_{P_{\rm C}} = 0.062 \ 8)$	
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	E <sub>P</sub>	$f_0$	$E_{P_{\rm P}}$	$f_0$
0.1	0.000 3	0.713 1	0.0006	0.6878
0.3	0.008 0	0.720 8	0.0166	0.703 8
0.5	0.037 2	0.750 0	0.0628	0.750 0
0.7	0.102 1	0.814 9	0.1397	0.826 9
0.9	0.214 3	0.927 1	0.2474	0.934 6

Table 3 Test results for case of single buyer with v = 1.0

Note that in this case, the expected payoff of the sellers' call option,  $E_{P_c}$ , remains unchanged for a given  $\sigma_T$ , and if  $\sigma_T = 0$ . 307 then  $E_{P_p} = 0.037$  2 and if  $\sigma_T = 0.380$  then  $E_{P_c} = 0.0628$ . The following results are revealed in table 3. ① For a seller with w=0.5, it can be seen that  $E_{P_p}=E_{P_c}$  for a given  $\sigma_T$ , so the contract price equals to  $\bar{p}_T$ . ② When the seller's w goes above 0. 5, the buyer will have more possibility to exercise his put than the seller to exercise his call, which makes  $E_{P_p} \geq E_{P_c}$ for a given  $\sigma_T$ , and hence the contract price is greater than  $\overline{p}_T$ . In addition, the larger the uncertainty in  $p_T$  the higher the contract price because  $E_{P_p}$  increases more quickly than  $E_{P_c}$  with increasing  $\sigma_T$ . In other words, when  $\sigma_T$  increases, the seller will receive a higher contract price at the cost of increasing possibility that the buyer rejects the contracted energy. ③ For a seller whose w is less than 0.5, since the buyer will have less possibility to exercise his put than the seller to exercise call, we have  $E_{P_p} \leq E_{P_c}$  for a given  $\sigma_{T}$  and hence the contract price is

by erwith y = 1.0, the probability for the seller to exercise less than  $\bar{p}_T$ . Furthermore, the larger the uncertainty in  $p_T$  the 1994-2017 China Academic Journal Electronic Publishing House. All rights reserved.

lower the contract price because  $E_{P_{c}}$  increases more quickly than  $E_{P_{p}}$  with increasing  $\sigma_{T}$ . In other words, when  $\sigma_{T}$  increases, the buyer will pay a low er contract price at the cost of increasing possibility that the seller interrupts the contracted energy.

Table 2 and table 3 can also be used to substantiate the theoretical result in table 1 that the optional forward contract proposed in this paper provides the buyer and seller a greater overall expected benefit than other contract models. Furthermore, it can be noted that the higher the uncertainty in  $p_T$  the larger is the whole expected benefit to the buyer and seller. While the results given in these numerical examples can be easily understood, the fact that reliability of these results is dependent on accuracy of the estimated probability distribution of the spot price at the delivery time should be noted. How to make a good estimation for this purpose is a difficult problem in the developing electricity markets. Although some research works have been done on this topiq there are still many problems to be addressed in this field.

## 5 Conclusions

In this paper, we model and analyze an electricity forward contract with bilateral options in an open spot market. This contractual arrangement allows both the seller and the buyer to take advantage of flexibility in production and consumption to obtain a monetary benefit, while simultaneously removing the risk of market price fluctuations. It is shown that an incentive compatible result can be achieved regarding the selection of the strike prices, and efficiency in dispatching electricity production and consumption can also be supported. In addition, this kind of optional forward contract presents a more equitable and reasonable payoff structure that allows the buyer and seller to earn a larger overall expected benefit. Numerical examples are used to demonstrate the validity of the proposed model.

## Acknow ledgements

The authors gratefully acknowledge the research funding support from the Hong Kong Polytechnic University Central Research Grant.

## References

- Herguera I. Bilateral Contract and the Spot Market for Electricity; Some Observations on the British and the NordPool Experiences. Utilities Policy, 2001, 9(2): 73~80
- 2 Mielczarski W, Michalik G. Open Electricity Markets in Australia: Contract and Spot Prices. IEEE Power Engineering Review, 1999, 19(2): 49~51
- 3 Collins R A. The Economics of Electricity Hedging and a Proposed Modification for the Futures Contract for Electricity. IEEE Trans on Power Systems, 2002, 17(1): 100~107
- 4 Gedra T W, Varaiya P P. Markets and Pricing for Interruptible Electric Power. IEEE Trans on Power Systems, 1993, 8(1): 122~ 128
- 5 Gedra T W. Optional Forward Contracts for Electric Power Markets. IEEE Trans on Power Systems, 1994, 9(4): 1766 ~ 1773
- 6 Oren S S. Integrating Real and Financial Options in Demand-side Electricity Contracts. Decision Support Systems 2001, 30(3); 279 ~ 288
- 7 David A K. Modeling Risk in Energy Contracts with Investor Owned Generation. IEE Proceedings—Generation, Transmission and Distribution, 1994, 141(1): 75~80
- 8 Hull J C. Options. Futures and Other Derivatives. 3rd ed. Englewood Cliffs (NJ); Prentice Hall Inc, 1996

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# 具有双边选择权的电力远期合同建模与分析

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**摘要:**提出并分析了一种具有双边选择权的电力市场远期合同模型。这种新的远期合同使得合同双方可利用供电和用电的灵活性来获取经济收益,同时可回避市场价格的波动风险。发展和分析了该类远期合同的 定价理论模型,并表明了其特点。所给算例表明该合同模型的有效性。

关键词:远期合同;电力市场;风险管理;选择权;定价模型