Performance of Chaos-Based Communication Systems Under the Influence of Coexisting Conventional Spread-Spectrum Systems

Francis C. M. Lau and Chi K. Tse

***Abstract*—This brief studies the performance of selected chaos-based systems which share their frequency bands with conventional spread-spectrum systems. Such a scenario may occur in normal practice when chaos-based systems are introduced while the conventional systems are still in operation. The particular chaos-based systems under study are the coherent chaos-shift-keying system and the noncoherent differential chaos-shift-keying system, and the coexisting conventional system employs the standard direct-sequence spread-spectrum modulation scheme. Analytical expressions for the bit-error rates are derived in terms of system parameters such as spreading factor, chaotic signal power, conventional spread-spectrum signal power, and noise power spectral density. Finally, computer simulations are performed to verify the analytical findings.**

***Index Terms*—Chaos communication, chaos shift keying (CSK), coexistence, differential chaos shift keying (DCSK), differential sequence spread spectrum.**

I. Introduction

Communication using chaos has attracted a great deal of attention from many researchers for more than a decade [1]–[4]. Much of the research work has focused on the basic modulation processes and the noise performance assuming ideal channel conditions. Both analog [1], [5] and digital [6]–[9] modulation schemes have been proposed, and it has been found that digital schemes are comparatively more robust than analog schemes in the presence of noise and thus represent a more practical form of systems for implementation. Direct application of chaos to conventional direct-sequence spread-spectrum (DSSS) systems was also reported on the code level [10], [11]. The basic principle is to replace the conventional binary spreading sequences, such as m sequences or Gold sequences [12], by the chaotic sequences generated by a discrete-time nonlinear map. The advantages of using chaotic spreading sequences are that an infinite number of spreading sequences exist and that the spread signal is less vulnerable to interception. Instead of applying analog chaotic sequences to spread the data symbols, Mazzini *et al.* proposed quantizing and periodically repeating a slice of a chaotic time series for spreading. It was also reported that systems using the periodic quantized sequences have larger capacities and lower bit-error rates (BERs) than those using m sequences and Gold sequences in a multiple-access environment [13], [14].

Among the various digital chaos-based communication schemes, coherent chaos-shift-keying (CSK) and noncoherent differential chaos-shift-keying (DCSK) schemes have been most thoroughly analyzed [15]–[18]. Compared with chaotic-sequence spread-spectrum modulation, CSK and DCSK modulation schemes make use of analog chaotic wideband waveforms directly to represent the binary symbols. No spreading as in traditional DSSS systems is required. Recently, the DCSK scheme has also been considered for practical implementation [19]. For coherent systems (e.g., coherent CSK system), the receiver is required to reproduce the chaotic carriers through a process called "chaos synchronization," and detection is normally achieved by correlating the incoming signal with the reproduced carriers. Although practical robust synchronization methods for chaotic signals are still not available, the study of CSK systems is important in that it can provide benchmark performance for comparison with other chaos-based communication systems. Despite the fact that their performance is inherently inferior than coherent systems, noncoherent systems present more practical forms of systems because they do not require the reproduction of chaotic carriers at the receiving end.

Being spread-spectrum, chaos-based communication systems are expected to perform well even in the presence of other wideband signals sharing the same bandwidth. Such a scenario may occur in normal practice, for example, when chaos-based systems are introduced while the conventional systems are still in operation. This aspect of performance is important, though it has rarely been addressed in the literature. It is therefore of interest to probe into the error performance of chaos-based systems in channels where other wideband communication systems coexist. Furthermore, it is useful to compare the relative performances of coherent and noncoherent chaos-based systems and the extent to which coherent systems excel in the presence of other coexisting wideband systems. In this brief, we investigate the performance of selected chaos-based digital systems when their bandwidths are co-occupied by a conventional spread-spectrum signal. The chaos-based systems under study are the coherent CSK and the noncoherent DCSK systems, and the coexisting system is a standard DSSS system. Analytical expressions for the BERs are derived in terms of system parameters such as spreading factor, chaotic signal power, conventional spread-spectrum signal power, and noise power spectral density. Finally, computer simulations are performed to verify the analytical findings.

II. System Overview

The basic problem we wish to investigate in this brief is the performance of chaos-based digital communication systems when the channel is under the influence of

- additive white Gaussian noise;
- wideband signal generated from a coexisting conventional spread-spectrum communication system, which shares the same frequency band as the chaos-based system under study.

Fig. 1 shows a block diagram of the system under study. In this system, two independent data streams, denoted by α_l and γ_l, respectively, are assumed to be sent at the same data rate. At time t, denote the output of the chaos transmitter by $s(t)$ and the conventional DSSS signal by $u(t)$. Assuming that noise $\eta(t)$ is added to the transmitted signals, the received signal consists of three components, namely, chaotic, conventional spread-spectrum, and additive noise. The receivers of the chaos-based system and the conventional system will attempt to recover their respective data streams. Coherent or noncoherent detection schemes may be applied in the chaos-based system receiver, depending upon the modulation method used.

In Section III, we focus on the coherent CSK system and the noncoherent DCSK system [3], [8]. The coexisting conventional system is a DSSS system. Our analysis will be carried out in a discrete-time fashion, and we will develop analytical expressions for the BERs of the recovered data streams for each of the chaos-based communication schemes under the afore-described environment.

III. Analysis of Bit–Error Performance

A. Coherent CSK System

We consider a discrete-time binary CSK communication system combined with a DSSS communication system, as shown in Fig. 2.

Manuscript received December 5, 2001; revised December 31, 2002. This work was supported by the Hong Kong Polytechnic University under the Young Professors Earmarked Research Grant 1-ZE03 and RGC Direct Allocation A-PD65. This paper was recommended by Associate Editor M. Di Bernardo.

The authors are with the Department of Electronic and Information Engineering, The Hong Kong Polytechnic University, Hong Kong (email: encmlau@polyu.edu.hk; encktse@polyu.edu.hk).

Digital Object Identifier 10.1109/TCSI.2003.818619

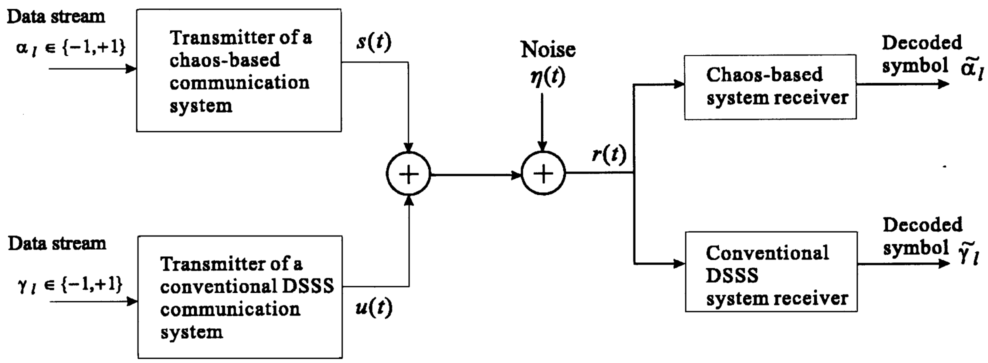

Fig. 1. Block diagram of a combined chaos-based and conventional DSSS digital communication.

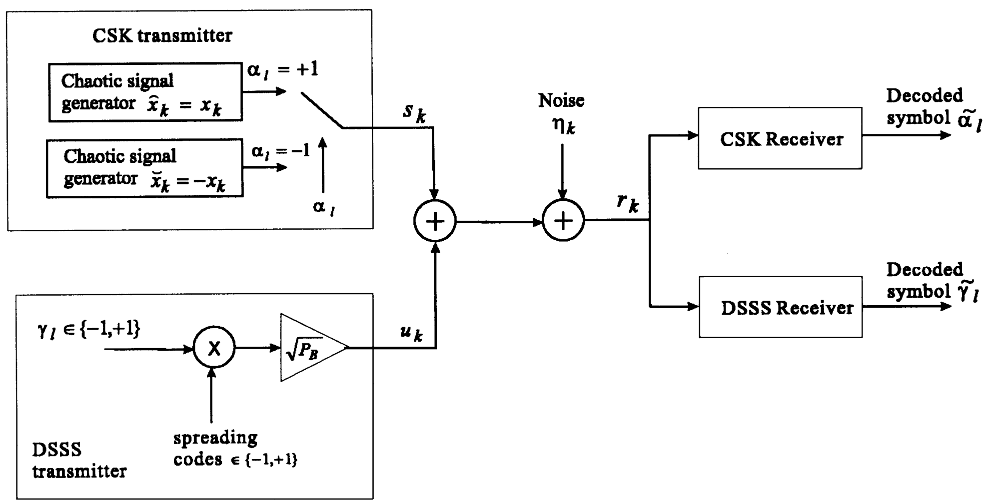

Fig. 2. Block diagram of a combined CSK-DSSS communication system.

Also, we assume that the sampling rate equals the spreading code rate of the DSSS system. In general, two sets of chaotic signal samples, denoted by $\{\hat{x}_k\}$ and $\{\check{x}_k\}$, are produced in the CSK transmitter by two chaos generators. If the symbol "$+1$" is sent, $\{\hat{x}_k\}$ is transmitted during a bit period, and if "-1" is sent, $\{\check{x}_k\}$ is transmitted. Further, we assume that "-1" and "$+1$" occur with equal probabilities. For simplicity, we consider a CSK system in which one chaos generator is used to produce chaotic signal samples $\{x_k\}$ for $k = 1, 2, \ldots$. The two possible transmitted signal samples are $\{\hat{x}_k = x_k\}$ and $\{\check{x}_k = -x_k\}$.

Suppose $\alpha_l \in \{-1, +1\}$ is the symbol to be sent during the lth bit period. Define the spreading factor 2β as the number of chaotic samples used to transmit one binary symbol. During the lth bit duration, i.e., for $k = 2\beta(l-1)+1, 2\beta(l-1)+2, \ldots, 2\beta l$, the output of the CSK transmitter is

$$s_k = \alpha_l x_k. \tag{1}$$

For the DSSS system, we assume that the period of the pseudo-random spreading sequence is very long. We denote the output power of the system by P_B. Essentially, we can model the output of the DSSS system as a random binary spreading code $b_k \in \{-1, +1\}$ multiplied by $\sqrt{P_B}$. Also, "-1" and "$+1$" occur with equal probabilities. Thus, for time k, the transmitted signal of the DSSS system is represented by

$$u_k = \sqrt{P_B} b_k. \tag{2}$$

The CSK and DSSS signals are added and corrupted by an additive white Gaussian noise before arriving at the receiving end. The received signal, denoted by r_k, is thus given by

$$r_k = s_k + \sqrt{P_B} b_k + \eta_k \tag{3}$$

where η_k is a Gaussian noise sample of zero mean and variance (power spectral density) $N_0/2$.

For the CSK system, we assume that a correlator-type receiver is employed. As shown in Fig. 3, the correlator output for the lth bit, y_l, is given by

$$\begin{aligned} y_l &= \sum_{k=2\beta(l-1)+1}^{2\beta l} r_k x_k \\ &= \underbrace{\alpha_l \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2}_{\text{required signal}} + \underbrace{\sqrt{P_B} \sum_{k=2\beta(l-1)+1}^{2\beta l} b_k x_k}_{\text{interfering DSSS signal}} \\ &\quad + \underbrace{\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k}_{\text{noise}}. \end{aligned} \tag{4}$$

Suppose a "$+1$" is transmitted in the CSK system during the lth symbol duration, i.e., $\alpha_l = +1$. For simplicity, we write $y_l|(\alpha_l = +1)$ as

$$y_l|(\alpha_l = +1) = A + B + C \tag{5}$$

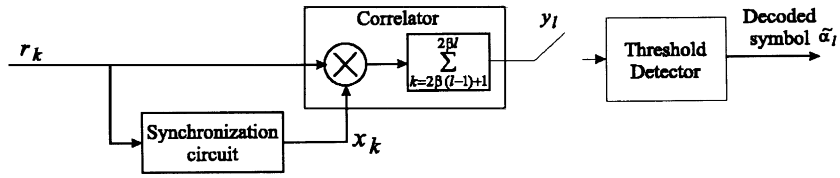

Fig. 3. Block diagram of a coherent CSK receiver.

where A, B, and C are the required signal, interfering DSSS signal, and noise, respectively, and are defined as

$$A = \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2 \tag{6}$$

$$B = \sqrt{P_B} \sum_{k=2\beta(l-1)+1}^{2\beta l} b_k x_k \tag{7}$$

$$C = \sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k. \tag{8}$$

The mean of $y_l|(\alpha_l = +1)$ is

$$\begin{aligned} E\left[y_l|(\alpha_l = +1)\right] &= \sum_{k=2\beta(l-1)+1}^{2\beta l} E\left[x_k^2\right] \\ &\quad + \sqrt{P_B} \sum_{k=2\beta(l-1)+1}^{2\beta l} E[b_k]E[x_k] \\ &\quad + \sum_{k=2\beta(l-1)+1}^{2\beta l} E[\eta_k]E[x_k] \\ &= 2\beta P_s \end{aligned} \tag{9}$$

where $P_s = E[x_k^2]$ denotes the average power of the chaotic signal. The last equality holds because $E[b_k] = 0$ and $E[\eta_k] = 0$. The mean value and the average power of the chaotic signal can be computed by numerical simulation, or by numerical integration if the invariant distribution function of $\{x_k\}$ is available. The variance of $y_l|(\alpha_l = +1)$ is

$$\begin{aligned} \mathrm{var}\left[y_l|(\alpha_l = +1)\right] &= \mathrm{var}[A] + \mathrm{var}[B] + \mathrm{var}[C] \\ &\quad + 2\mathrm{cov}[A, B] + 2\mathrm{cov}[B, C] + 2\mathrm{cov}[A, C] \end{aligned} \tag{10}$$

where $\mathrm{cov}[X, Y]$ is the covariance of X and Y defined as

$$\mathrm{cov}[X, Y] = E[XY] - E[X]E[Y]. \tag{11}$$

It can be shown that all the covariance terms are zero and the variance terms are given by

$$\mathrm{var}[A] = 2\beta\Lambda \tag{12}$$

$$\mathrm{var}[B] = 2\beta P_B P_s \tag{13}$$

$$\mathrm{var}[C] = \beta N_0 P_s \tag{14}$$

where Λ is the variance of $\{x_k^2\}$, i.e.,

$$\Lambda = \mathrm{var}\left[x_k^2\right]. \tag{15}$$

In the derivation of $\mathrm{var}[A]$, it has been assumed that the autovariance of $\{x_k^2\}$ vanishes, i.e.,

$$\mathrm{cov}\left[x_j^2, x_k^2\right] = E\left[x_j^2 x_k^2\right] - E\left[x_j^2\right] E\left[x_k^2\right] = 0 \quad \text{for } j \neq k. \tag{16}$$

Using (12) and (14), and because all covariance terms in (10) are zero, we may write (10) as

$$\mathrm{var}\left[y_l|(\alpha_l = +1)\right] = 2\beta\Lambda + 2\beta P_B P_s + \beta N_0 P_s. \tag{17}$$

For the lth symbol, an error occurs when $y_l \leq 0|(\alpha_l = +1)$. Since $y_l|(\alpha_l = +1)$ is the sum of a large number of random variables, we may assume that it follows a normal distribution when 2β is large. The error probability is thus given by

$$\begin{aligned} &\mathrm{Prob}\left(y_l \leq 0|(\alpha_l = +1)\right) \\ &\quad = \frac{1}{2}\mathrm{erfc}\left(\frac{E\left[y_l|(\alpha_l = +1)\right]}{\sqrt{2\mathrm{var}\left[y_l|(\alpha_l = +1)\right]}}\right) \\ &\quad = \frac{1}{2}\mathrm{erfc}\left(\frac{2\beta P_s}{\sqrt{4\beta\Lambda + 4\beta P_B P_s + 2\beta N_0 P_s)}}\right) \end{aligned} \tag{18}$$

where erfc(.) is the complementary error function, which is defined as

$$\mathrm{erfc}(\psi) \equiv \frac{2}{\sqrt{\pi}} \int_\psi^\infty e^{-\lambda^2} d\lambda. \tag{19}$$

Similarly, when $\alpha_l = -1$, the corresponding error probability can be shown equal to

$$\begin{aligned} &\mathrm{Prob}\left(y_l > 0|(\alpha_l = -1)\right) \\ &\quad = \frac{1}{2}\mathrm{erfc}\left(\frac{-E\left[y_l|(\alpha_l = -1)\right]}{\sqrt{2\mathrm{var}\left[y_l|(\alpha_l = -1)\right]}}\right) \\ &\quad = \frac{1}{2}\mathrm{erfc}\left(\frac{2\beta P_s}{\sqrt{4\beta\Lambda + 4\beta P_B P_s + 2\beta N_0 P_s)}}\right). \end{aligned} \tag{20}$$

Hence, the overall error probability of the lth transmitted symbol is

$$\begin{aligned} \mathrm{BER}_{\mathrm{CSK}}^{(l)} &= \mathrm{Prob}(\alpha_l = +1) \times \mathrm{Prob}\left(y_l \leq 0|(\alpha_l = +1)\right) \\ &\quad + \mathrm{Prob}(\alpha_l = -1) \times \mathrm{Prob}\left(y_l \leq 0|(\alpha_l = -1)\right) \\ &= \frac{1}{2}\mathrm{erfc}\left(\frac{2\beta P_s}{\sqrt{4\beta\Lambda + 4\beta P_B P_s + 2\beta N_0 P_s)}}\right). \end{aligned} \tag{21}$$

It can be seen from (21) that $\mathrm{BER}_{\mathrm{CSK}}^{(l)}$ is independent of l. Thus, the error probability of the lth transmitted symbol is the same as the BER of the system. The BER of the CSK system, denoted by $\mathrm{BER}_{\mathrm{CSK}}$, is therefore

$$\begin{aligned} \mathrm{BER}_{\mathrm{CSK}} &= \frac{1}{2}\mathrm{erfc}\left(\frac{2\beta P_s}{\sqrt{4\beta\Lambda + 4\beta P_B P_s + 2\beta N_0 P_s)}}\right) \\ &= \frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sqrt{\frac{\Lambda}{\beta P_s^2} + \frac{P_B}{\beta P_s} + \frac{N_0}{2\beta P_s}}}\right) \end{aligned} \tag{22}$$

$$= \frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sqrt{\left(\frac{E_b^2}{4\beta\Lambda}\right)^{-1} + \left(\frac{E_b}{2P_B}\right)^{-1} + \left(\frac{E_b}{N_0}\right)^{-1}}}\right) \tag{23}$$

where

$$E_b = 2\beta P_s \tag{24}$$

denotes the average bit energy of the CSK system. The expression given in (22) or (23) is thus the analytical BER for the noisy coherent CSK system under the interference of a DSSS signal. Note that for

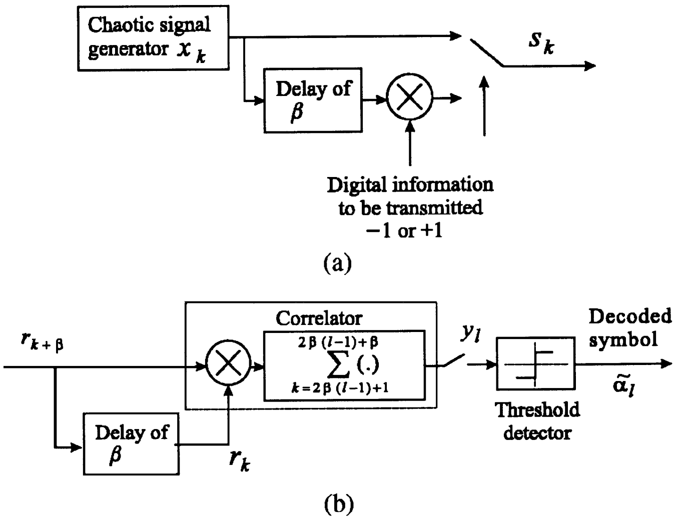

Fig. 4. Block diagram of a noncoherent DCSK system. (a) Transmitter. (b) Receiver.

fixed DSSS signal power P_B and noise power spectral density $N_0/2$, the BER can be improved by making one or more of the following adjustments.

1) Reduce the variance of $\{x_k^2\}$.
2) Increase the spreading factor 2β.
3) Increase the CSK signal power P_s.

Example: Consider the case where a logistic map is used for chaos generation. The form of the map is

$$x_{k+1} = g(x_k) = 1 - 2x_k^2 \quad \text{where} \quad x_k \in (-1, +1). \tag{25}$$

Given that the invariant distribution function of $\{x_k\}$ equals [20]

$$\rho(x_k) = \begin{cases} \frac{1}{\pi\sqrt{1-x_k^2}}, & \text{if } |x_k| < 1 \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

we obtain

$$P_s = E\left[x_k^2\right] = \int_{-\infty}^{\infty} x^2 \rho(x)dx = \int_{-1}^{1} x^2\rho(x)dx = \frac{1}{2} \tag{27}$$

$$\Lambda = \text{var}\left[x_k^2\right] = E\left[x_k^4\right] - E^2\left[x_k^2\right] = \int_{-1}^{1} x^4\rho(x)dx - \frac{1}{4} = \frac{1}{8}. \tag{28}$$

For the case where the logistic map is used to generate the chaotic samples, we substitute (27) and (28) into (22) to obtain the BER, i.e.,

$$\text{BER}_{\text{CSK}} = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{2\beta}{1 + 4P_B + 2N_0}}\right). \tag{29}$$

B. Noncoherent DCSK System

In this section, we consider the noncoherent DCSK system. For this system, the basic modulation process involves dividing the bit period into two equal slots. The first slot carries a reference chaotic signal, and the second slot bears the information. For a binary system, the second slot is the same copy or an inverted copy of the first slot depending upon the symbol sent being "+1" or "−1." Essentially, the detection of a DCSK signal can be accomplished by correlating the first and the second slots of the same symbol and comparing the correlator output with a threshold. Fig. 4 shows the block diagrams of a DCSK transmitter and receiver pair.

As in the CSK case, we assume that the DCSK signal is interfered by the DSSS signal and corrupted by additive white Gaussian noise. Using the same notations as defined in Section III-A, during the lth bit duration of the DCSK system, the transmitted DCSK signal can be written as

$$s_k = \begin{cases} x_k, & \text{for } k = 2\beta(l-1)+1, 2\beta(l-1) \\ & \quad +2, \ldots, 2\beta(l-1)+\beta \\ \alpha_l x_{k-\beta}, & \text{for } k = 2\beta(l-1)+\beta+1, 2\beta(l-1) \\ & \quad +\beta+2, \ldots, 2\beta l \end{cases} \tag{30}$$

whereas the kth transmitted signal for the DSSS system is given by

$$u_k = \sqrt{P_B}b_k. \tag{31}$$

The received noisy signal r_k is given by

$$r_k = s_k + u_k + \eta_k \tag{32}$$

where the symbol η_k is as defined previously in Section III-A.

At the DCSK receiver, the detector essentially calculates the correlation of the corrupted reference and data slots of the same symbol. We consider the output of the correlator for the lth received bit, y_l, which is given by

$$\begin{aligned} y_l &= \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} r_k r_{k+\beta} \\ &= \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} \Big[\alpha_l x_k^2 + \sqrt{P_B}b_{k+\beta}x_k + \alpha_l\sqrt{P_B}b_k x_k \\ &\qquad + P_B b_k b_{k+\beta} + \eta_{k+\beta}x_k + \alpha_l\eta_k x_k \\ &\qquad + \sqrt{P_B}\eta_{k+\beta}b_k + \sqrt{P_B}\eta_k b_{k+\beta} + \eta_k\eta_{k+\beta}\Big] \\ &= \underbrace{\alpha_l D}_{\text{required signal}} + \underbrace{\sqrt{P_B}F + \alpha_l\sqrt{P_B}G + P_B H}_{\text{interfering DSSS signal}} \\ &\quad + \underbrace{J + \alpha_l K + \sqrt{P_B}L + \sqrt{P_B}M + N}_{\text{noise}} \end{aligned} \tag{33}$$

where

$$D = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k^2 \tag{34}$$

$$F = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} b_{k+\beta}x_k \tag{35}$$

$$G = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} b_k x_k \tag{36}$$

$$H = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} b_k b_{k+\beta} \tag{37}$$

$$J = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} \eta_{k+\beta}x_k \tag{38}$$

$$K = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} \eta_k x_k \tag{39}$$

$$L = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} \eta_{k+\beta}b_k \tag{40}$$

$$M = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} \eta_k b_{k+\beta} \tag{41}$$

$$N = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} \eta_k\eta_{k+\beta}. \tag{42}$$

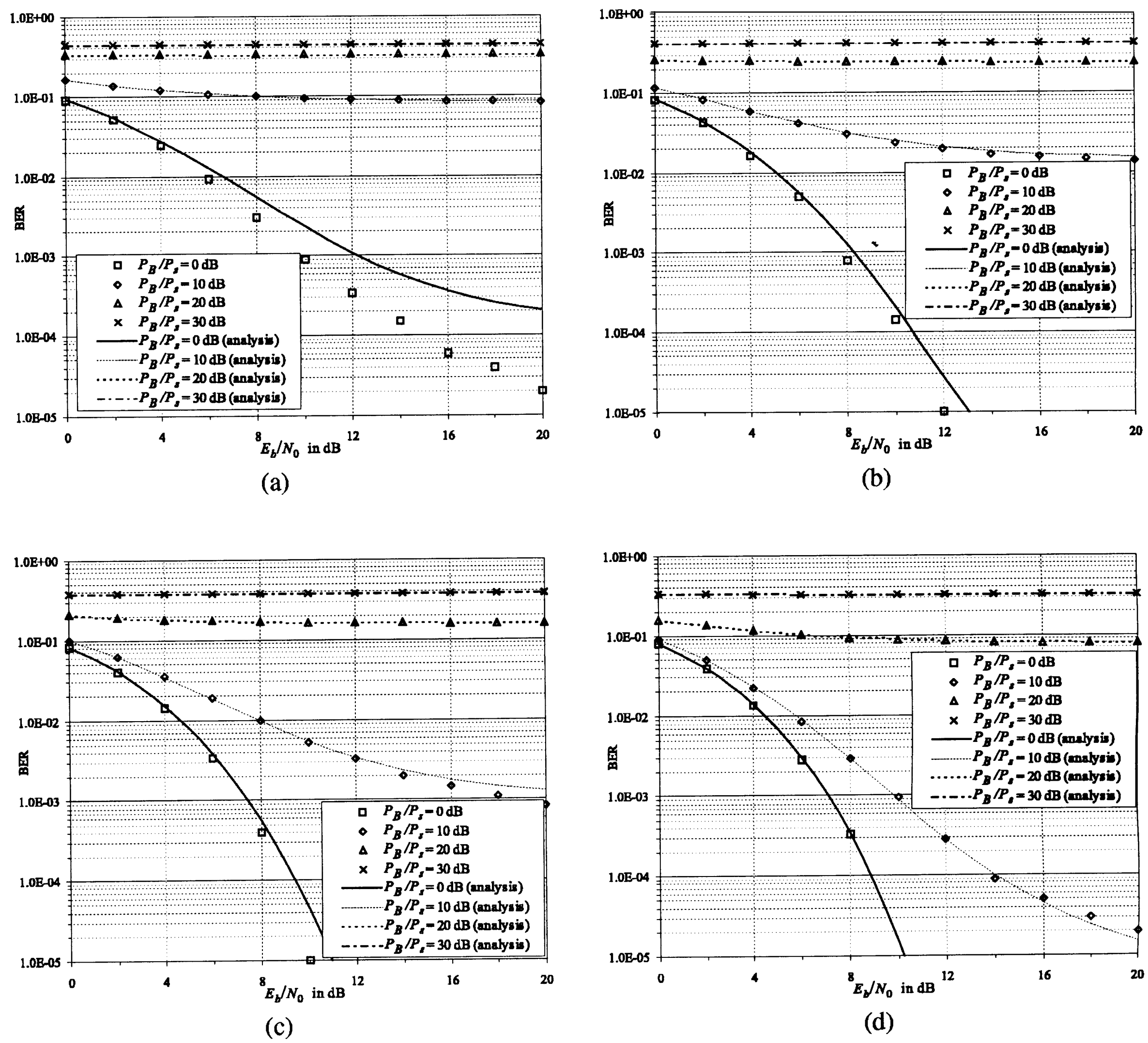

Fig. 5. BERs versus E_b/N_0 of the coherent CSK system under the interference of a DSSS signal. Simulated BERs are plotted as points and analytical BERs plotted as lines. (a) Spreading factor is 20. (b) Spreading factor is 50. (c) Spreading factor is 100. (d) Spreading factor is 200.

The means and variances of the variables D to N can be shown equal to

$$\begin{aligned} &E[D] = \beta E[x_k^2] \equiv \beta P_s && \mathrm{var}[D] = \beta \mathrm{var}\left[x_k^2\right] \equiv \beta\Lambda \\ &E[F] = 0 && \mathrm{var}[F] = \beta P_s \\ &E[G] = 0 && \mathrm{var}[G] = \beta P_s \\ &E[H] = 0 && \mathrm{var}[H] = \beta \\ &E[J] = 0 && \mathrm{var}[J] = \tfrac{\beta P_s N_0}{2} \\ &E[K] = 0 && \mathrm{var}[K] = \tfrac{\beta P_s N_0}{2} \\ &E[L] = 0 && \mathrm{var}[L] = \tfrac{\beta N_0}{2} \\ &E[M] = 0 && \mathrm{var}[M] = \tfrac{\beta N_0}{2} \\ &E[N] = 0 && \mathrm{var}[N] = \tfrac{\beta N_0^2}{4} \end{aligned} \tag{43}$$

where in the derivation of var$[D]$, it has been assumed that the auto-variance of $\{x_k^2\}$ vanishes. Further, it can be readily shown that

$$\mathrm{cov}[\mu,\nu] = 0 \qquad \forall \mu,\nu \in \{D,F,G,H,J,K,L,M,N : \mu \neq \nu\}. \tag{44}$$

Using a likewise procedure as in Section III-A, the means and variances of $y_l|(\alpha_l = +1)$ and $y_l|(\alpha_l = -1)$ can be shown equal to

$$E\left[y_l|(\alpha_l = +1)\right] = -E\left[y_l|(\alpha_l = -1)\right] = \beta P_s \tag{45}$$

$$\begin{aligned} \mathrm{var}\left[y_l|(\alpha_l = +1)\right] &= \mathrm{var}\left[y_l|(\alpha_l = -1)\right] \\ &= \mathrm{var}[D] + P_B \mathrm{var}[F] + P_B \mathrm{var}[G] \\ &\quad + P_B^2 \mathrm{var}[H] + \mathrm{var}[J] + \mathrm{var}[K] \\ &\quad + P_B \mathrm{var}[L] + P_B \mathrm{var}[M] + \mathrm{var}[N] \\ &= \beta\Lambda + 2\beta P_B P_s + \beta P_B^2 + \beta P_s N_0 \\ &\quad + \beta P_B N_0 + \frac{\beta N_0^2}{4}. \end{aligned} \tag{46}$$

Since all terms in (45) and (46) are independent of l, the BER of the DCSK system under the interference of a DSSS signal, denoted by $\mathrm{BER}_{\mathrm{DCSK}}$, equals the overall error probability of the lth transmitted symbol ($\mathrm{BER}_{\mathrm{DCSK}}^{(l)}$), i.e.,

$$\begin{aligned} \mathrm{BER}_{\mathrm{DCSK}} &= \mathrm{BER}_{\mathrm{DCSK}}^{(l)} \\ &= \mathrm{Prob}(\alpha_l = +1) \times \mathrm{Prob}\left(y_l \leq 0|(\alpha_l = +1)\right) \\ &\quad + \mathrm{Prob}(\alpha_l = -1)\mathrm{Prob}\left(y_l > 0|(\alpha_l = -1)\right) \\ &= \frac{1}{2}\mathrm{erfc}\left(\frac{\beta P_s}{\sqrt{2\beta\Lambda + 4\beta P_B P_s + 2\beta P_B^2 + 2\beta P_s N_0 + 2\beta P_B N_0 + \frac{\beta N_0^2}{2}}}\right) \\ &= \frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sqrt{\frac{4\Lambda + 4P_B^2 + 4P_B N_0 + N_0^2}{2\beta P_s^2} + \frac{4P_B + 2N_0}{\beta P_s}}}\right) \end{aligned} \tag{47}$$

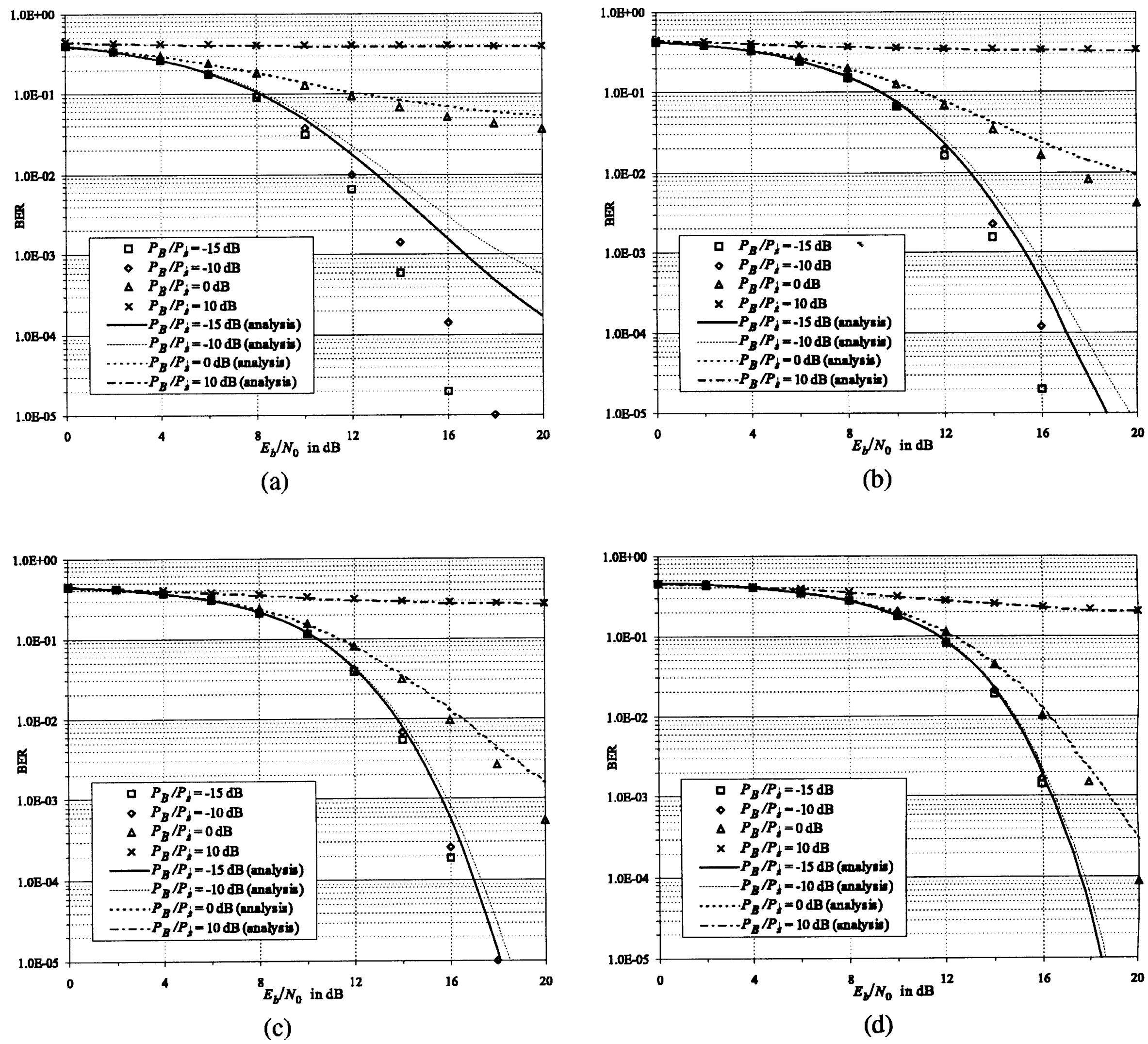

Fig. 6. BERs versus $\boldsymbol{E}_b/\boldsymbol{N}_0$ of the noncoherent DCSK system under the interference of a DSSS signal. Simulated BERs are plotted as points and analytical BERs plotted as lines. (a) Spreading factor is 20. (b) Spreading factor is 50. (c) Spreading factor is 100. (d) Spreading factor is 200.

$$=\frac{1}{2}\operatorname{erfc}\left(\frac{1}{\sqrt{\frac{8\beta\Lambda+8\beta P_B^2+8\beta P_B N_0+2\beta N_0^2}{E_b^2}+\frac{8P_B+4N_0}{E_b}}}\right) \tag{48}$$

where E_b is defined as in (24). The expression given in (47) or (48) is then the analytical BER for the noisy DCSK signal under the influence of a DSSS signal. Note that for fixed DSSS signal power P_B and noise power spectral density $N_0/2$, the BER can be reduced by making similar adjustments as suggested in Section III-A.

Example: Consider the case where the logistic map is used for generating the chaotic signal samples. We substitute (27) and (28) into (47) to obtain the BER of the DCSK system, i.e.,

$$\mathrm{BER}_{\mathrm{DCSK}}=\frac{1}{2}\operatorname{erfc}\left(\frac{1}{\sqrt{\frac{1+8P_B^2+8P_B N_0+2N_0^2}{\beta}+\frac{8P_B+4N_0}{\beta}}}\right). \tag{49}$$

IV. Computer Simulations and Discussions

In this section, the performance of the chaos-based digital communication systems under the influence of a DSSS signal is studied by computer simulations. The logistic map described in Section III-A has been used to generate the chaotic signal samples. For comparison, we also plot in each case, the analytical BERs obtained from the expressions derived in Sections III-A and B. The relevant simulated BERs for the CSK system and DCSK systems are shown in Figs. 5 and 6.

From these figures, the simulated performance is found to be better than that from the analysis. The discrepancy is due to the limited validity of the assumption of a normal distribution for the correlator output in the analysis [21]. For large spreading factors (e.g., $2\beta = 100$ and 200), where the assumption of normal distribution of the conditional correlator output holds better, we clearly see that the analytical and simulated BERs are in very good agreement.

Also, at low spreading factors, the BER performance is worse. This can be attributed to the larger variation of bit energy sent for each symbol. As a general observation, the coherent CSK system consistently performs better than the noncoherent DCSK system under the influence of a coexisting conventional DSSS signal. As shown in Fig. 7, for the CSK system, at $E_b/N_0 = 7$ dB and a spreading factor of 100, the BER degrades from about 10^{-3} to 10^{-2} when the conventional-to-chaotic-signal-power ratio (P_B/P_s) increases from 0 dB to 10 dB. For the DCSK system, with the same increase in P_B/P_s, the BER now degrades from about 10^{-3} to 0.3 at $E_b/N_0 = 20$ dB. Thus, the CSK system is more tolerant to wideband interfering signals compared with the DCSK system.

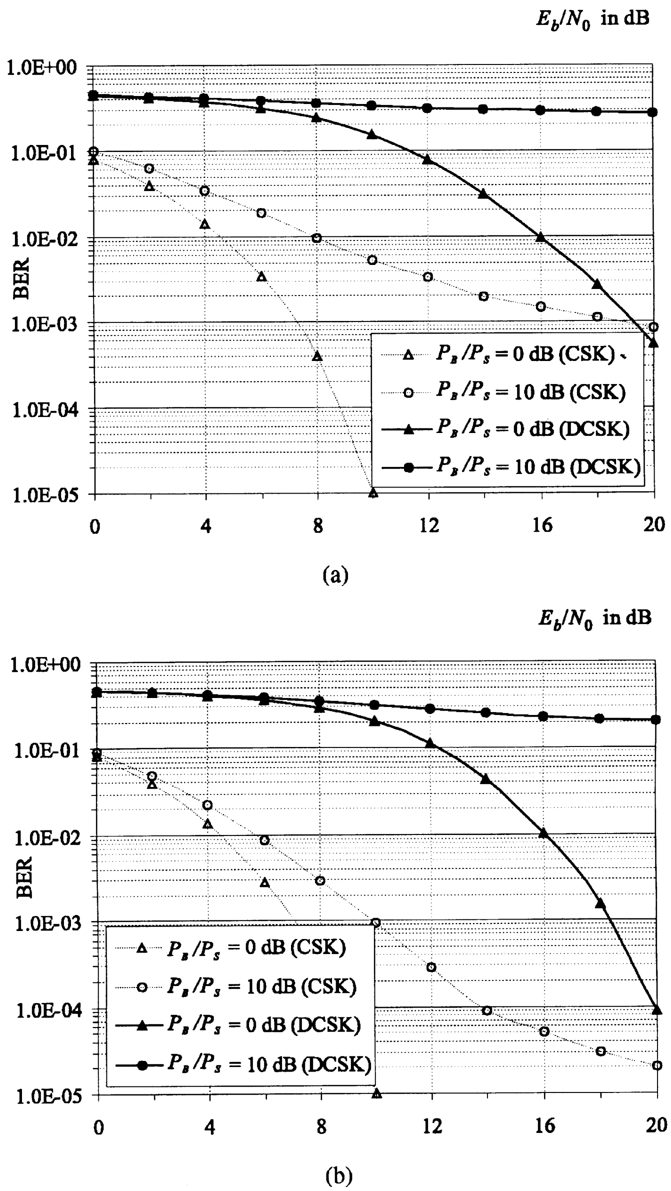

Fig. 7. Simulated BERs versus E_b/N_0 for the coherent CSK and noncoherent DCSK systems under the interference of a DSSS signal. (a) Spreading factor is 100. (b) Spreading factor is 200.

V. Conclusion

In this brief, the performance of chaos-based communication systems under the influence of a wideband signal generated from a coexisting conventional spread-spectrum system is investigated. The problem is important technically since chaos-based systems are spread-spectrum systems which are expected to resist interfering and the kind of interference considered here, namely one being generated from another conventional spread-spectrum system such as a DSSS system, represents a realistic (future) practical concern when chaos-based systems need to “cooperate” with existing systems. For the coexisting CSK-DSSS system, coherent correlation CSK receiver has been assumed. Although robust chaos synchronization techniques are still not available, the results represent *benchmark* performance that a coexisting CSK-DSSS system can achieve. A more practical scenario in which a noncoherent DCSK system coexists with a DSSS system is also investigated. The performance data presented in this study will be useful in designing coherent CSK and noncoherent DCSK systems when they are required to operate in channels already occupied by conventional wideband DSSS systems.

References

[1] L. Kocarev, K. S. Halle, K. Eckert, L. O. Chua, and U. Parlitz, “Experimental demonstration of secure communications via chaotic synchronization,” *Int. J. Bifurcation Chaos*, vol. 2, pp. 709–713, 1992.

[2] U. Parlitz, L. O. Chua, L. Kocarev, K. S. Halle, and A. Shang, “Transmission of digital signals by chaotic synchronization,” *Int. J. Bifurcation Chaos*, vol. 2, pp. 973–977, 1992.

[3] H. Dedieu, M. P. Kennedy, and M. Hasler, “Chaos shift keying: modulation and demodulation of a chaotic carrier using self-synchronizing Chua’s circuit,” *IEEE Trans. Circuits Syst. II*, vol. 40, pp. 634–642, Oct. 1993.

[4] C. W. Wu and L. O. Chua, “Transmission of digital digital signals by chaotic synchronization,” *Int. J. Bifurcation Chaos*, vol. 3, pp. 1619–1627, 1993.

[5] M. Itoh and H. Murakami, “New communication systems via chaotic synchronizations and modulation,” *IEICE Trans. Fund.*, vol. E78-A, no. (3), pp. 285–290, 1995.

[6] G. Kolumbán, M. P. Kennedy, and L. O. Chua, “The role of synchronization in digital communication using chaos—part I: fundamentals of digital communications,” *IEEE Trans. Circuits Syst. I*, vol. 44, pp. 927–936, Oct. 1997.

[7] ——, “The role of synchronization in digital communications using chaos—part II: chaotic modulation and chaotic synchronization,” *IEEE Trans. Circuits Syst. I*, vol. 45, pp. 1129–1140, Nov. 1998.

[8] G. Kolumbán, G. Kis, Z. Jáko, and M. P. Kennedy, “A robust modulation scheme for chaotic communications,” *IEICE Trans. Fund.*, vol. E81-A, no. 9, pp. 1798–1802, 1998.

[9] G. Kolumbán and M. P. Kennedy, “The role of synchronization in digital communications using chaos—Part III: performance bounds for correlation receivers,” *IEEE Trans. Circuits Syst. I*, vol. 47, pp. 1673–1683, Dec. 2000.

[10] G. Heidari-Bateni and C. D. McGillem, “Chaotic sequences for spread spectrum: an alternative to PN-sequences,” in *Proc. IEEE Int. Conf. Selected Topics in Wireless Communications*, Vancouver, BC, Canada, 1992, pp. 437–440.

[11] ——, “A chaotic direct-sequence spread-spectrum communication system,” *IEEE Trans. Commun.*, vol. 42, pp. 1524–1527, Feb. 1994.

[12] R. L. Peterson, R. E. Ziemer, and D. F. Borth, *Introduction to Spread Spectrum Communications*. Englewood Cliffs, NJ: Prentice-Hall, 1995.

[13] G. Mazzini, G. Setti, and R. Rovatti, “Chaotic complex spreading sequences for asynchronous DS-CDMA part I: system modeling and results,” *IEEE Trans. Circuits Syst. I*, vol. 44, pp. 937–947, Oct. 1997.

[14] ——, “Chaotic complex spreading sequences for asynchronous DS-CDMA part II: some theoretical performance bounds,” *IEEE Trans. Circuits Syst. I*, vol. 45, pp. 496–506, Apr. 1998.

[15] M. Hasler and T. Schimming, “Chaos communication over noisy channels,” *Int. J. Bifurcation Chaos*, vol. 10, pp. 719–735, 2000.

[16] C. C. Chen and K. Yao, “Stochastic-calculus-based numerical evaluation and performance analysis of chaotic communication systems,” *IEEE Trans. Circuits Syst. I*, vol. 47, pp. 1663–1672, Dec. 2000.

[17] T. Schimming and M. Hasler, “Optimal detection of differential chaos shift keying,” *IEEE Trans. Circuits Syst. I*, vol. 47, pp. 1712–1719, Dec. 2000.

[18] F. C. M. Lau and C. K. Tse, *Chaos-Based Digital Communication Systems: Operation, Analysis and Evaluation*. Heidelberg, Germany: Springer-Verlag, 2003.

[19] Frequency-Modulated Differential Chaos Shift Keying(FM-DCSK): From Theory to Implementation, G. Kolumbán *et al.*. (2001). [Online]. Available: http://www.mit.bme.hu/research/chaos/inspect/index.html

[20] T. Kohda and A. Tsuneda, “Even-and odd-correlation functions of chaotic Chebychev bit sequences for CDMA,” in *Proc. IEEE Int. Symp. Spread Spectrum Techniques Applications*, Oulu, Finland, 1994, pp. 391–395.

[21] A. J. Lawrance and G.Gan Ohama, “Exact and analytically approximate bit error rates in chaos communication,” in *Proc. Int. Symp. Nonlinear Theory Applications*, Xi’an, China, 2002, pp. 99–102.