

Dynamic Assessment of Shear Connection Conditions in Slab-Girder Bridges by Kullback-Leibler Distance

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Abstract: Shear connectors are widely used in composite bridges that provide composite action. Their damage will reduce the load-carrying capacity of the structure. In this study, a novel method based on Kullback-Leibler distance (KLD) was developed to assess the integrity of the shear connectors. A bridge model was constructed in the laboratory and some removable anchors were specially designed and fabricated to link the beams and slab that were cast separately. Each anchor consists of a threaded bar that penetrates through the soffit of the beam and ties up into an embedded nut cap to simulate a shear connector in the real bridges. Different damage scenarios were introduced by pulling out some connectors. Vibration tests were carried out in each damage scenario. Various damage detection methods have been applied and results show that the method was able to detect all the assumed damage scenarios successfully and consistently.

Key words: connection model, damage detection, vibration, slab-girder bridges, Kullback-Leibler measure.

1. INTRODUCTION

Composite slabs are widely used in bridges that consist of a reinforced-concrete slab supported on steel or concrete girders. Their behaviour is highly affected by the level of interaction that occurs between the reinforced concrete slab and the steel or concrete girders, which is usually jointed together by shear connectors. Damage or failure of the shear connectors will affect the composite action of the bridge girders and slab, and therefore reduce the bridge load-carrying capacity and the horizontal shear resistance. Some of these bridges may not satisfy current load requirements and require retrofitting or strengthening. Recent studies showed that the blind bolts could be utilised to retrofit/strengthen existing composite bridges (Mirza *et al.* 2011). Before retrofitting/strengthening these existing structures, one may be required to detect the integrity of the existing shear connectors. The inaccessibility of the connection system makes direct

inspection difficult. On the other hand, the huge number of shear connectors prevents any local non-destructive testing methods to access the connectors one by one. It is of practical importance to develop a new non-destructive assessment technique to detect the integrity of shear connectors.

Damage identification techniques can be classified into either local or global methods. Most currently used techniques, such as visual, acoustics, magnetic field, eddy current etc., are effective but local in nature. They require that the vicinity of the damage is known a priori and the position of the structure being inspected is readily assessable. The global methods quantify the healthiness of a structure by examining changes in its vibrational characteristics or the static behaviour under load. The core of this group of methods is to seek some damage indices that are sensitive to structural damage. Doebling *et al.* (1998) and Brownjohn (2007) presented a literature review on the damage assessment

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methodologies based on the parameters including the natural frequencies, mode shapes, mode shape curvature, the flexibility matrix and stiffness matrix. Morassi and Dilena (2003) carried out an experimental study on damage-induced changes in modal parameters of steel-concrete composite beams subjected to small vibration. Damage was induced by removing concrete around some elements connecting the steel beam and the reinforced concrete slab and consequently causing a lack of structural solidarity between the two beams. Dilena and Morassi (2003) presented a method to identify the damage using the frequency changes. A 1:3 scaled bridge model was built in the laboratory by Xia *et al.* (2007). The laboratory study showed that the dynamic response of the bridge deck is insensitive to damage in the connector system. This is one of the main difficulties associated with the use of the vibration methods for damage detection, especially for shear connectors. The local method that compared the vertical responses of the girders and slab could be used to identify damage in shear connectors. A damage index based on the norm of Frequency Response Function (FRF) differences of vibrations measured simultaneously on slab and concrete girders was developed to detect shear link damage. The method was proven yielded reliable prediction of shear link damage. The sensitivity range of the method was also quantified based on both numerical and laboratory test data. The method has been successfully applied to the full slab-girder bridges (Xia *et al.* 2008). A damage indicator based on the local modal curvature and the wavelet transform modulus maxima is used from damage in shear connectors (Liu and De Roeck 2009).

Most of the vibration-based damage assessment methods require modal properties that are from traditional Fourier transform (FT). However, when the damage is very small, the damage-induced changes of physical structural properties are always too insignificant to disclose the damage using the FT-based method. In addition, the measured vibration signals are often contaminated with noise. The Wavelet Transform (WT) based method for vibration signal analysis is gradually adopted in many areas due to its good time-frequency localization. Hou *et al.* (2000) used a simple structural model with multiple breakable springs subjected to harmonic excitation to show that the wavelet transform can successfully be used to identify both abrupt and cumulative damage. The wavelet packet transform (WPT) is an extension of the WT that provides complete level-by-level decomposition. The WPT enables the extraction of features from signals that combine stationary and non-stationary characteristics with arbitrary time-frequency resolution. Sun and Chang (2002) concluded that the WPT-based

component energy was a sensitive condition index for structural damage assessment. This index is sensitive to changes of structural rigidity and insensitive to measurement noise. The WPT component energy combined with well-trained neural network models is used to identify the location and the severity of damage. Yam *et al.* (2003) also extracted the structural damage feature based on energy variation of structural vibration responses decomposed using wavelet packet, and the neural network is used to establish the mapping between the structural damage feature and damage status. This method needs accurate model information for both the healthy and damaged conditions to train the neural network model, which is difficult and challenging in practice, especially for complex structures. Law *et al.* (2005) developed a method to identify damage in structures using wavelet packet sensitivity. The sensitivity of wavelet packet transform component energy with respect to local change in the system parameters is derived analytically basing on the dynamic response sensitivity. The sensitivity-based method is then used for damage detection of structures. The relative wavelet entropy based on vibration signals from the intact and damaged structures is used for damage detection by Ren and Sun (2008). Zhu and Hao (2008) presented a method to identify the damage in the shear connection system using wavelet based Kullback-Leibler distance (KLD).

In this study, a new method based on KLD was developed to assess the integrity of the shear connectors. Two damage indicators, the Kullback-Leibler spectral distance and wavelet-based KLD (WKLD), were investigated using the experimental study. A scaled bridge model was constructed in the laboratory. Some removable anchors were specially designed and fabricated to link the beams and slab that cast separately. Each anchor consists of a threaded bar that penetrates through the soffit of the beam and ties up into an embedded nut cap to simulate a shear connector in the real bridges. Different damage scenarios were introduced by pulling out some connectors. Vibration tests were carried out in each damage scenario. Experimental results show that the method is reliable and effective to indicate the damage location.

2. DAMAGE IDENTIFICATION USING WAVELET BASED KULLBACK-LEIBLER DISTANCE

2.1. Kullback-Leibler Distance (KLD)

The Kullback-Leibler distance (KLD) is a natural distance function between two probability densities, i.e., a measure of discrimination between two probability density functions, $P(x)$ and $Q(x)$. According to the

definition (Veldhuis and Klabbers 2003; Zhou *et al.* 2005), the KLD is as follows

$$D(P \parallel Q) = \int P(x) \log \left\{ \frac{P(x)}{Q(x)} \right\} dx \quad (1)$$

Owing to the concavity of the logarithmic function, the KLD is positive when P and Q are different and is zero if P equal to Q (Mackay 2003). It is this property which has allowed the KLD to be widely used for image classification.

Let $P(f)$ and $Q(f)$ denote two power spectral densities, i.e.

$$\int_0^{f_{\max}} P(f) df = \int_0^{f_{\max}} Q(f) df = 1 \quad (2)$$

where f_{\max} is the maximum frequency of the signal. The KLD between $P(f)$ and $Q(f)$ is defined as

$$D_{KL}(P, Q) = \int_0^{f_{\max}} P(f) \log \frac{P(f)}{Q(f)} df \quad (3)$$

and the symmetrical KLD as

$$D_{SKL}(P, Q) = (D_{KL}(P, Q) + D_{KL}(Q, P)) / 2 = \frac{1}{2} \int_0^{f_{\max}} (P(f) - Q(f)) \log \frac{P(f)}{Q(f)} df \quad (4)$$

2.2. Wavelet Based Kullback-Leibler Distance (WKLD)

A signal can be expressed by the Discrete Wavelet Transform (DWT) in terms of local basis functions (Daubechies 1988). We employ Daubechies wavelets in the following studies as they satisfy the two crucial requirements: the orthogonality of local basis functions and second-order accuracy or higher, depending on the dilation expression adopted.

A function $f(t)$ can therefore be approximated in terms of its DWT as

$$f(t) = f_0^{DWT} \varphi(t) + f_1^{DWT} \psi(t) + f_2^{DWT} \psi(2t) + \dots + f_{2^j+k}^{DWT} \psi(2^j t - k) \quad (5)$$

where both j and k are in the integer domain. $\varphi(t)$ and $\psi(t)$ are the scaling function and the mother wavelet function respectively, and they satisfy the following relations,

$$\begin{aligned} \varphi(t) &= \sqrt{2} \sum_{k=-\infty}^{+\infty} h(k) \varphi(2t - k); \\ \psi(t) &= \sqrt{2} \sum_{k=-\infty}^{+\infty} g(k) \varphi(2t - k); \\ \psi_{j,k}(t) &= 2^{-\frac{j}{2}} \psi(2^{-j} t - k) \end{aligned} \quad (6)$$

where $h(k)$ and $g(k)$ are the low-pass and high-pass analysis filters respectively which are all constants. $f_{2^j+k}^{DWT}$ is the wavelet transform coefficients. Because of the orthogonality on both translation and scale of the Daubechies wavelets, we have,

$$\int \psi_{j,k}(t) \psi_{m,n}(t) dt = \delta_{j,m} \delta_{k,n}. \quad (7)$$

We have the following with real wavelets from Eqns 5 and 7,

$$f_0^{DWT} = \int f(t) \varphi(t) dt; \quad f_{2^j+k}^{DWT} = \int f(t) \psi_{j,k}(t) dt \quad (8)$$

The idea of separating the signal into packets is to obtain an adaptive partitioning of the time-frequency plane depending on the signal of interest. The discrimination of a wavelet packet subband can be defined as its ability to differentiate between any two signals $p(t)$ and $q(t)$ in the transformation domain. The KLD can be defined as follows

$$D_{j,k}^{WKLD}(p_{j,k}, q_{j,k}) = (D_{KL}(p_{j,k} \parallel q_{j,k}) + D_{KL}(q_{j,k} \parallel p_{j,k})) / 2 \quad (9)$$

where $p_{j,k}$, $q_{j,k}$ are the power spectral density of $p_{j,k}(t)$, $q_{j,k}(t)$, respectively. $p_{j,k}(t) = \int p(t) \psi_{j,k}(t) dt$ and $q_{j,k}(t) = \int q(t) \psi_{j,k}(t) dt$.

3. EXPERIMENTAL SETUP

3.1. Experimental Setup

A scaled bridge model of 6250 mm × 1090 mm × 50 mm was constructed in the laboratory (Xia *et al.* 2007), as shown in Figure 1. The dimension of the bridge model is as follows: a span 6000 mm in length, the girders 100 mm wide by 300 mm deep, the diaphragms is 210 mm × 300 mm, and a slab of 50 mm depth, with 475 mm spacing between two girders. There are 9 connectors at 600 mm intervals along each beam and 8 mm in diameter. The connectors are denoted as S1-S27. The bridge model rested on two steel frames, which acted as the abutments of the bridge and were fixed to the strong floor. The model

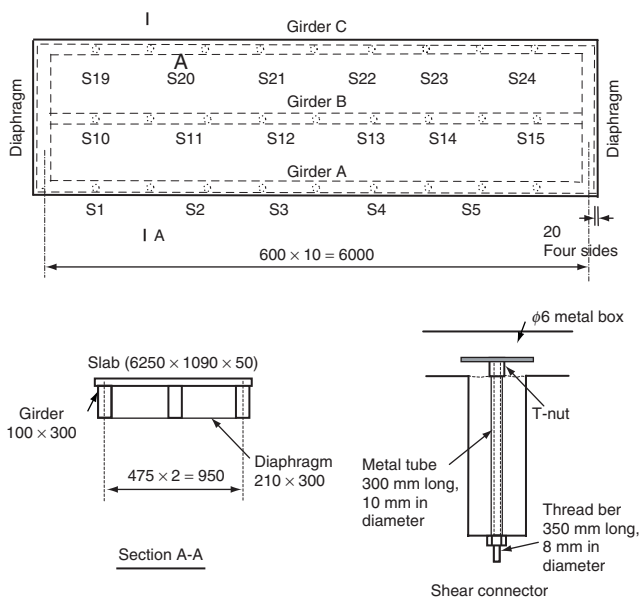


Figure 1. Plan of the model and details (unit: mm)
 ○ Shear connectors (27 in total)

was left for a period of 28 days before testing commenced in order to ensure that the specified concrete strength was achieved.

Design of the shear links incorporates the ability not only to simulate failure of particular links, but also to reset them to an undamaged state. All shear link fixity is provided by securing both ends of the shear link thread. The top end is secured by a T-nut. This is positioned at the mid depth of the slab, and provides anchorage once the slab has been poured. Anchorage between the T-nut and the slab has been achieved by welding a small horizontal metal bar($\phi 6$ mm) on top of the T-nut. After the slab pour the T-nut position is permanently fixed. In the lower part, the thread is surrounded by a metal tube which is fixed in place as a result of pouring the concrete beams. To set the shear link to an undamaged state, the thread is screwed into the T-nut, a nut and washer are then positioned and tightened at the beam soffit. To set the link to a damaged state the thread is simply unscrewed from the T-nut and completely removed.

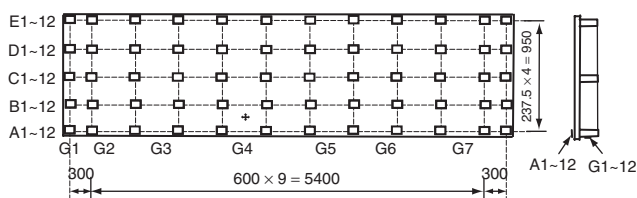


Figure 2. Sensor location (+ hammer location, □ sensor location)

Vibration tests were conducted in this study to detect the removal ('damage') of shear connectors in the bridge. The intact state and several damage states (simulated by loosening the connectors) were tested using hammer impact. The impact and sensor locations are shown in Figure 2. Three damage scenarios (denoted D1-D3) together with the intact state (D0) were investigated. D1 is to simulate the damage in the end that the anchors S8 and S9 were loosen from the girder, D2 is to simulate two damage locations in the middle and end of the structure that S4, S5, S8 and S9 were loosen, and D3 is to simulate three damage locations in the left, middle and right parts of the structure that S1, S2, S4, S5, S8 and S9 were loosen. Table 1 shows the measurements for each case. In each case, 12 accelerometers were placed on the slab (denoted as "A", "B", "C", "D", "E") and on the girder corresponded to "A" (denoted as "G"). For each impact, 4096 points of data were recorded with a sampling frequency of 500 Hz. Vertical responses were measured in all cases.

3.2. Experimental Modal Analysis

The mode shapes are extracted from measurements using Rational Fraction Polynomial method (Ewins 2000). Figure 3 shows the first four mode shapes of the undamaged state D0 and the damaged state D1. The first and third mode shapes are bending modes, and other two are torsional modes. Visual inspection on the mode shapes cannot find the changes around the damage location because the mode shape difference expands to a wider range owing to normalization process of mode shapes. Table 2 shows the natural frequencies and damping ratios of the undamaged state D0 and the

Table 1. Measurement locations in each damage case

	Intact (D0)	D1 (S8 and S9 loosen)	D2 (S4, 5, 8, 9 loosen)	D3 (S1, 2, 4, 5, 8, 9 loosen)
On the slab	A1~12 B1~12 C1~12 D1~12 E1~12	A1~12 B1~12 C1~12 D1~12 E1~12	A1~12 B1~12 C1~12 D1~12 E1~12	A1~12 — — — —
On the girder	G1~12	G1~12	G1~12	G1~12

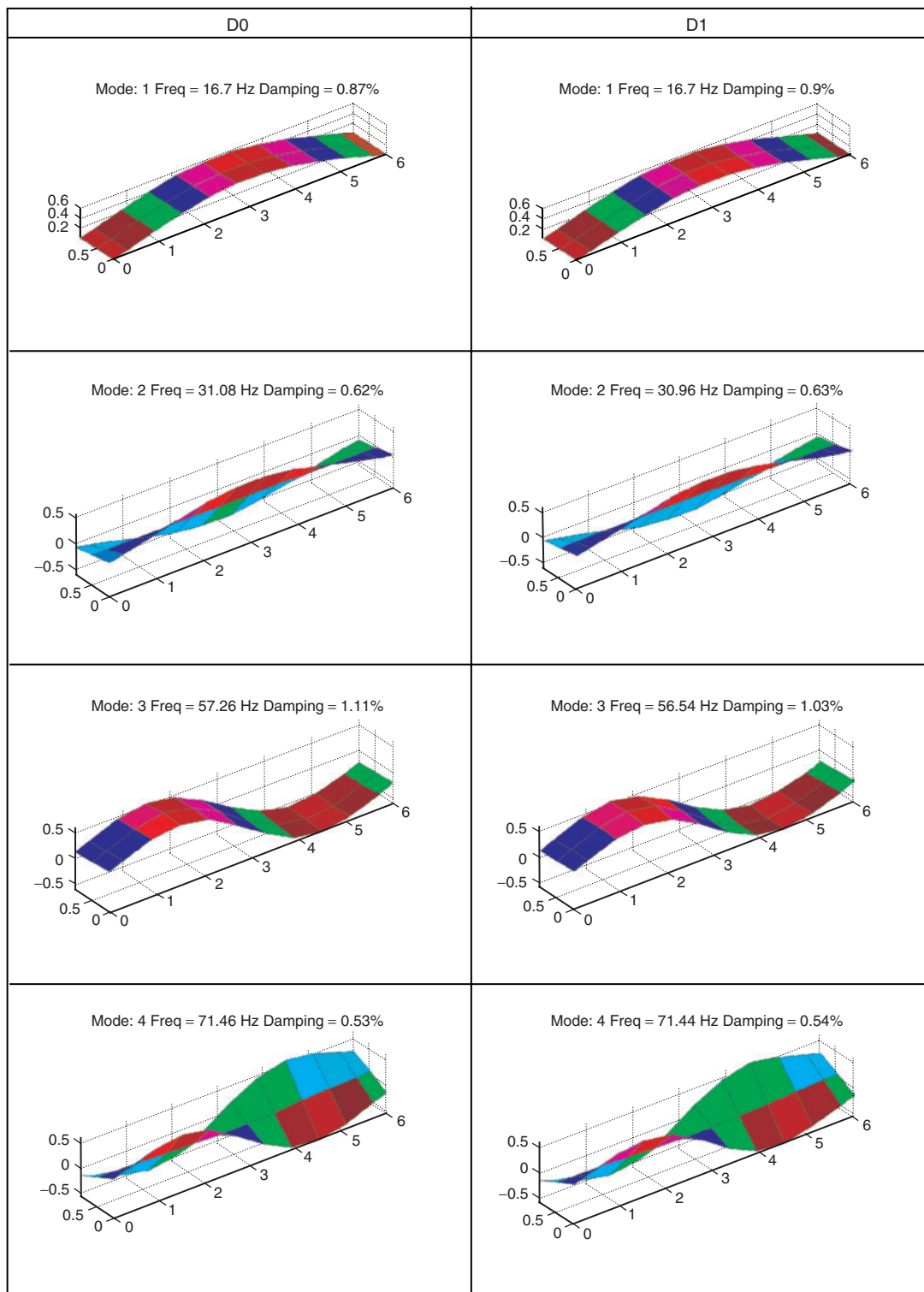


Figure 3. Mode shapes of D0 and D1

damaged states D1 and D2. From Table 2, it can be seen that frequency differences between the two states D0 and D1 are insignificant. For the first ten modes, the maximum frequency change between D0 and D1 occurs in the 5th model and it is 1.87%. The maximum

frequency change between D0 and D2 is also in the 5th mode and it is 2.66%. Compared with D1, the changes in D2 are more significant as the damage is more severe. Damping ratios generally increase slightly in the damaged states. Due to the difficulty of measuring it

Table 2. Natural frequencies and damping ratios of D0, D1 and D2

Mode	D0		D1			D2		
	Frequency (Hz)	Damping ratio (%)	Frequency (Hz)	Damping ratio (%)	Frequency change (%)	Frequency (Hz)	Damping ratio (%)	Frequency change (%)
1	16.70	0.87	16.67	0.90	0.18	16.56	1.84	0.84
2	31.08	0.62	30.96	0.63	0.39	30.99	0.71	0.29
3	57.26	1.11	56.54	1.03	1.26	56.30	1.07	1.68
4	71.46	0.53	71.44	0.54	0.03	70.90	0.56	0.78
5	84.63	0.44	83.09	0.66	1.82	82.38	0.69	2.66
6	98.50	1.35	97.68	1.33	0.83	97.42	1.42	1.10
7	118.21	0.75	116.15	1.10	1.74	116.92	0.93	1.09
8	120.42	0.69	118.29	0.61	1.77	122.79	0.57	1.97
9	123.75	0.22	121.97	0.67	1.44	125.16	0.42	1.14
10	126.80	0.30	125.70	0.61	0.87	126.05	0.62	0.59

accurately, damping is rarely used in damage detection. Basically, the measured frequencies have a noise level of about 0.7% (Xia *et al.* 2006). The effect of environments on the frequency, especially temperature, could be up to 5% over the 24-hour period (Cornwell *et al.* 1999). Because it is difficult to remove the effects of environmental factors, it will affect the condition assessment that is based on the changes in modal data before and after onset of damage. Therefore it can be concluded that using vibration frequencies and mode shapes is difficult to give confident damage detections of shear links between concrete girder and slab of bridges.

4. DAMAGE IDENTIFICATION USING KLD

4.1. Damage Identification Using KLD

In this study, two tests are conducted in the laboratory: one is to compare the measurements obtained simultaneously on the slab and on the corresponding points on the girder, and another one is to compare the measurements on the slab only for undamaged and damaged cases. Figure 4 shows the KLD values from the measurements on the slab and on the girder. Figures 5 and 6 show the results using the measurements on the slab only. The KLD value is calculated from the measurements for undamaged and damaged cases. From these results, the following observations can be obtained

- (1) In Figure 4, there is no obvious peak value for D0 and the maximum KLD value is 0.003. For D1, the maximum value is 0.110 at Sensor A11 on the slab and G11 on the girder (outside of S9). Sensors A10 and G10 are located in the middle of two connectors S8 and S9, but the KLD value is small. For D2, there are two peak values at Sensors A6 (between S4 and S5) and A11 (outside of S9). For D3, there are three peak values at Sensors A3 (between S1 and S2), A6

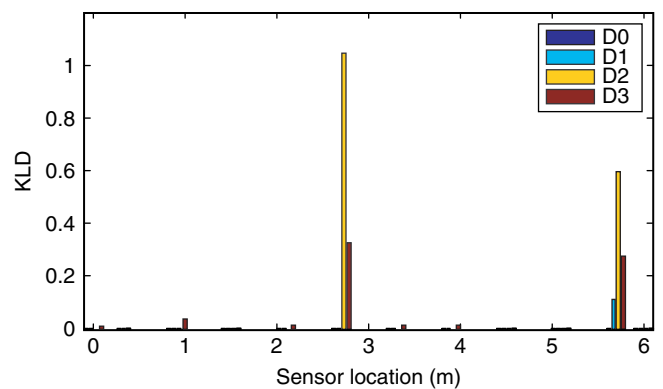


Figure 4. KLD from measurements on the slab and the girder

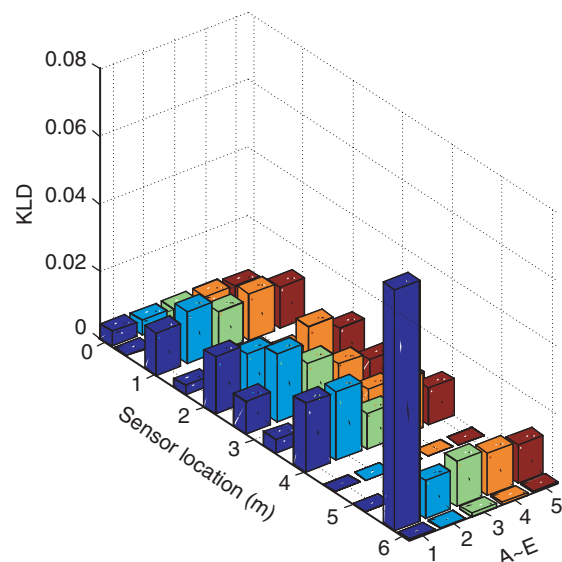


Figure 5. KLD from measurements on the slab only (D0~D1)

(between S4 and S5) and A11 (outside of S9). The peak value at Sensor A3 is much smaller than other two peak values because S1 and S2 were not fully removed. These results show the KLD value from the simultaneous measurements

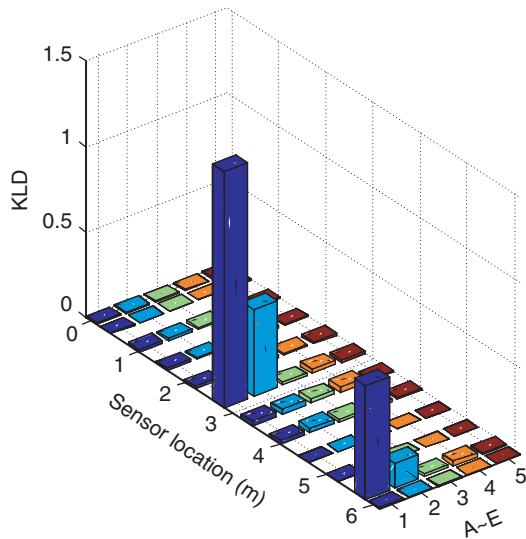


Figure 6. KLD from measurements on the slab only (D0~D2)

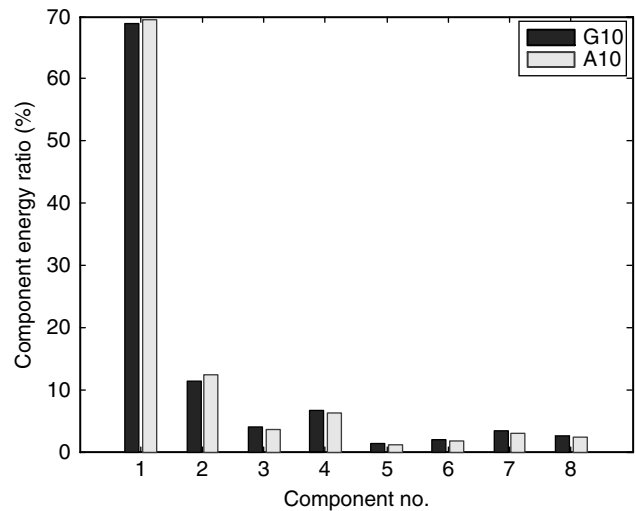
on the slab and on the girder is a good indicator of the damaged connector location. This result is similar to that defined by using FRF differences (Xia *et al.* 2007).

- (2) Figures 5 and 6 show the results by comparing the measurements on slab only for undamaged and damaged cases. The maximum value is at sensor A11 in Figure 5, and there are two peaks at Sensor A6 and A11 in Figure 6. It also shows the KLD value from the measurements on the slab only can be a good indicator of the damage location. In Figure 6, the KLD values at B6 and B11 are also large due to these measured points are close to the damage locations. This approach avoids measurement on the girders, which sometimes is difficult and dangerous. However, the draw back of this method is that it needs a reference measurement data of the intact bridge, which is often not available in practice.

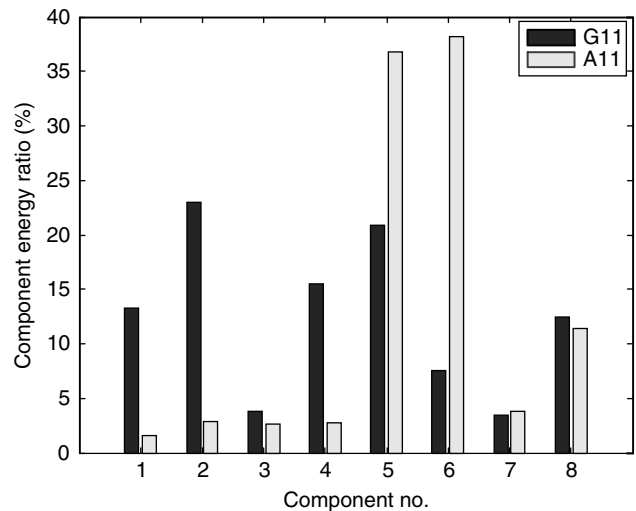
4.2. Damage Identification Using WKLD

In the above, the KLD value is obtained by comparing the power spectral densities of two measurements in the whole frequency range. The imperfection in the shear connectors is the local damage. The vibration features from measurements, such as natural frequency, mode shapes obtained from Fourier transform, are not sensitive to local damage. However, a subband signal may be sensitive to local damage. The idea of separating the signal into packets is to obtain an adaptive partitioning of the time-frequency plane depending on the signal of interest. The KLD value can then be obtained by comparing the components in a subband.

Figure 7 shows the comparison of the wavelet packet component energy ratios for the measurements on the slab and the corresponding girder. The signal is decomposed into 8 components at level 3. In Figure 7(a), there is no obvious difference of the component energy ratios from the measurements at G10 and A10. Some component energy ratios at G11 are different with that at A11, especially the 6th component. Figure 8 shows the WKLD values using the 6th component for cases D0, D1 and D2. There are no peak values for D0, two peak values at Sensors A6 and A11 for D1 and three peak values at Sensors A3, A6 and A11 for D2. The results show that the WKLD value also gives a good indication for damage locations. Similar to the KLD value, Figures 9 and 10 show the WKLD values using the measurements on the slab only. Figure 9 shows the value for comparison of D0 and D1 and that of D0 and



(a) From measurements at G10 and A10



(b) From measurements at G11 and A11

Figure 7. Component energy ratios of measurements for D1

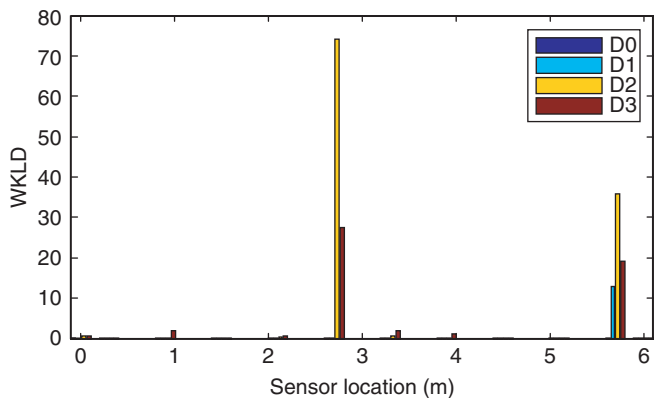


Figure 8. WKLD using the 6th wavelet packet of measurements on the slab and girder

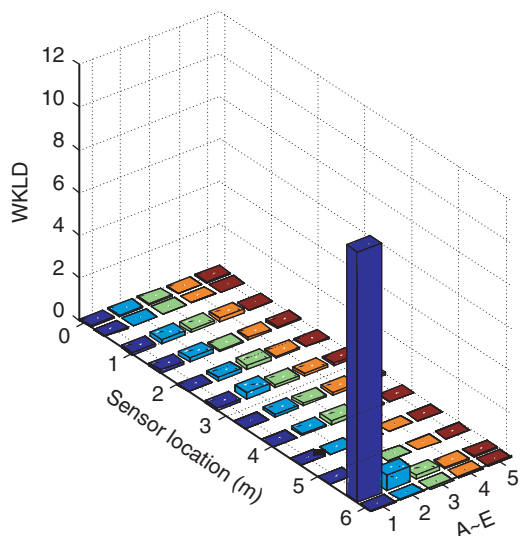


Figure 9. WKLD using the 6th wavelet packet of measurements on the slab only (D0~D1)

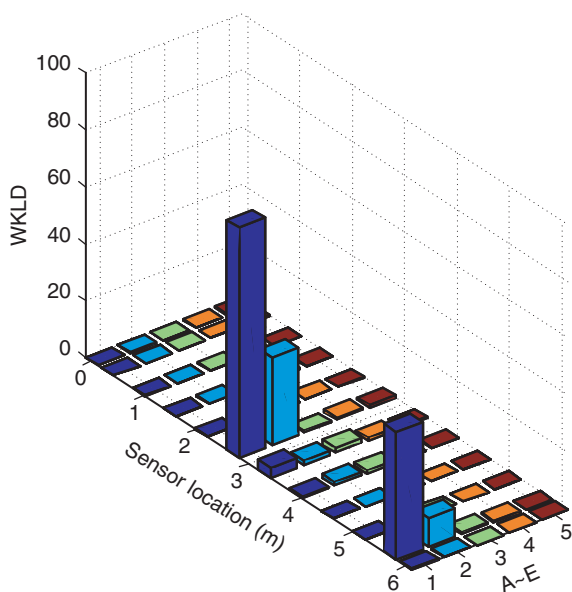


Figure 10. WKLD using the 6th wavelet packet of measurements on the slab only (D0~D2)

D2 is shown in Figure 10. Compared with the KLD value, the WKLD value is more sensitive to the damage.

5. SENSITIVE RANGE OF KLD METHODS FOR DAMAGE DETECTION

From the figures presented above it is clear that the damage of connectors only affects the measured vibration data near the connectors. For example in D1, removing S8 and S9 leads to a significant change in Sensor A11 (outside of S9), minor changes in Sensors A9 and A10, and almost no change in vibration data measured in other sensors. Therefore, it is necessary to determine the sensitive range of damage in shear connectors to the vibration data. To achieve this, vibration responses at points around connector S15 were measured before and after removal of the connector (damage state D4). Nine sensors were placed on the slab near the damage (Sensors B13~B21) and at corresponding points on the beam (G13~G21). The distances between these points and S15 are 25, 100, 150, 200, 300 and 400 mm, as shown in Figure 11.

Using the approach described above, the KLD values from measurements on the slab and girder are shown in Figure 12(a) for the undamaged (D0) and damaged (D4) cases and the WKLD values are shown in Figure 12(b). It can be seen that in D0 the KLD value is less than 0.002. In D4, the maximum KLD value occurs at points near S15 and the value decreases as the distance increases. At points 300 mm away, KLD values are about 0.003 and 0.006, while at points 400 mm away the difference is only about 0.001 only. Therefore the reliable damage detection range is about 300 mm. This implies that damage in a connector can cause significant change in vibration properties at points within a 300 mm radius. This conclusion is also supported by observation of Figures 4 and 8. When the sensors are placed in this range, the damage can be reliably detected, otherwise it cannot. Consequently distance between the sensors should be

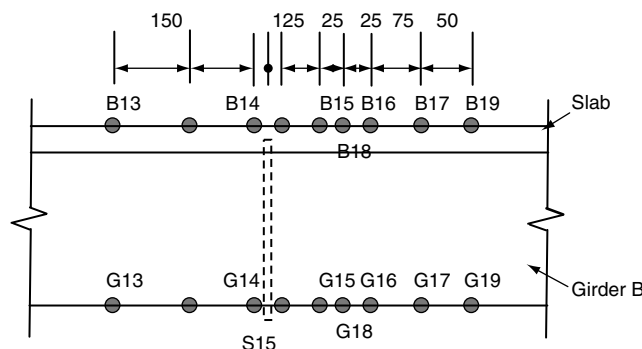
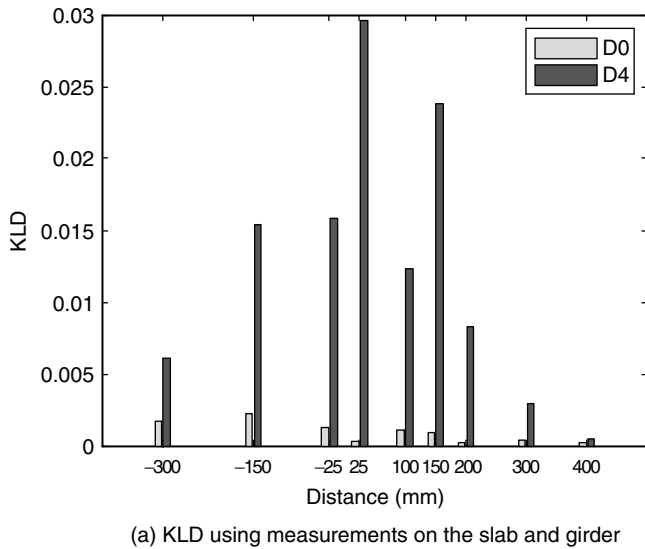
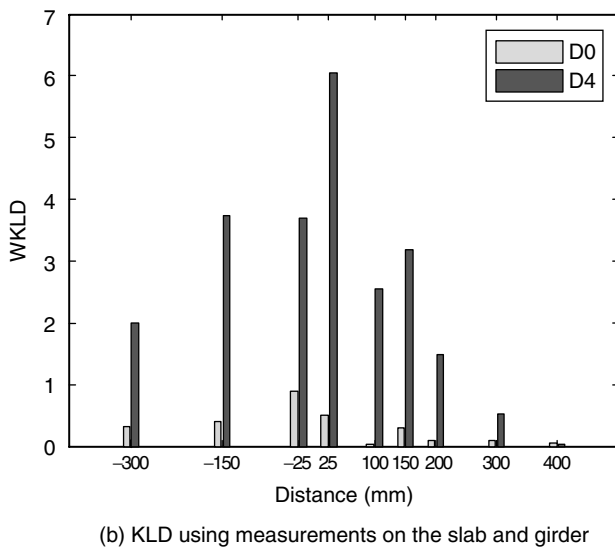


Figure 11. Sensor location around S15 (B13~21 and G13~21) to find the sensitive radius to damage (unit: mm)



(a) KLD using measurements on the slab and girder



(b) KLD using measurements on the slab and girder

Figure 12. KLD and WKLD using measurements on the slab and girder

600 mm to detect possible damage in all the connectors. The result is similar to that using the correlation of the vertical frequency response functions (COFRF) of the girder and the corresponding slab points and the relative difference of the frequency response functions (RDFRF) between the girder and the slab (Xia *et al.* 2007). In practice, it is difficult to measure the input excitation to obtain the frequency response function. The proposed method uses the output response only and it is not necessary to measure the input excitation.

6. CONCLUSIONS

A novel method based on KLD has been developed to assess the integrity of the shear connectors. Vibration tests have been carried out on the scaled bridge model and 3 damage scenarios were simulated by loosening 2 or

4 or 6 shear connectors at different locations. The following conclusions can be obtained,

- (1) Experimental results show that both KLD and WKLD values are good indicators of the damage location. As the WKLD value is obtained in a sub-band component, it is more sensitive to the damage than that of the KLD value.
- (2) The KLD and WKLD values can be obtained from measurements on the slab and girder. The reference data from undamaged states is not necessary, and thus the technique is suitable for identifying the damage in the shear connection system of existing bridges.
- (3) The sensitivity range for damage detection was studied. It was found that the damage can be detected when the sensor was placed less than 300 mm from the damage.
- (4) Compared with the CORFRF and RDFRF values, KLD and WKLD are obtained from output responses only. It is not necessary to measure the input excitation.

The proposed technique could be used to detect the integrity of the existing shear connectors, and then the condition of existing bridges could be assessed next step. Future work will develop the innovative blind bolts for rehabilitation in existing steel infrastructure and dynamic assessment of integrity of composite actions before and after retrofitting/strengthening.

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