The cases of $H_{(1)}(z)$ and $H_{(2)}(z)$ are "trivial" because we can take $k_1 = 0$ in Step 4), and write

$$H_{(1)}(z) = \left[\frac{1}{(z-1)^2}\right] + \left[\frac{\frac{4}{3}}{z-1} + \frac{\frac{1}{3}}{z-\frac{1}{8}} - \frac{\frac{1}{3}}{z+\frac{1}{8}} + \frac{0.9}{z+0.729}\right]$$
$$H_{(2)}(z) = \left[\frac{1}{(z-1)^2}\right] + \left[\frac{\frac{5}{3}}{z-1} + \frac{\frac{1}{6}}{z-\frac{1}{8}} - \frac{\frac{1}{6}}{z+\frac{1}{8}} + \frac{0.81}{z+0.729}\right].$$

In both cases, positive realizations of dimension 6 can be given as follows:

$$c_1 = (1 \quad 0 \quad 1 \quad 0 \quad 1.675 \quad 0.507\,925)$$
$$A_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{64} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.729 \\ 0 & 0 & 0 & 0 & 1 & 0.271 \end{pmatrix}$$
$$b_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{1}{12} \\ \frac{4}{3} \\ 0 \end{pmatrix}$$
$$c_2 = (1 \quad 0 \quad 1 \quad 0 \quad 0.514 \quad 1.354\,294)$$
$$A_2 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{64} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.729 \\ 0 & 0 & 0 & 0 & 1 & 0.271 \end{pmatrix}$$
$$b_2 = \begin{pmatrix} 0 \\ 1 \\ \frac{1}{3} \\ 0 \\ \frac{5}{3} \\ 0 \end{pmatrix}.$$

A positive realization of the original $H(z)$ is then possible to construct as in [4] or [8].

## V. Conclusion

In this brief, we provided a general *finite step procedure* for checking the nonnegativity of the impulse-response sequence of $H(z)$, which answers an open problem raised in [1]. For primitive transfer functions a new method of positive realization was proposed by reducing the pole order of the dominant pole.

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# Analysis of Bit Error Rates for Multiple Access CSK and DCSK Communication Systems

W. M. Tam, Francis C. M. Lau, and Chi K. Tse

***Abstract*—In this paper, multiple-access techniques for use with chaos shift keying (CSK) and differential CSK under a noisy condition are analyzed using a discrete-time approach. A mixed analysis-simulation technique is developed to calculate the bit error rates (BERs). When certain assumptions are made, closed-form analytical expressions of the BERs are found.**

***Index Terms*—Bit error rate (BER), chaos communication, chaos shift keying (CSK), differential chaos shift keying (DCSK), multiple access .**

## I. Introduction

Chaos-based spread-spectrum communication techniques have emerged rapidly during the last decade, and much research effort has been devoted to the development of analog and digital chaos-based communication systems that can achieve performance comparable to the existing communication systems. Typically, in a digital chaos-based communication system, digital symbols are mapped to nonperiodic chaotic basis functions [1]–[4].

Multiple access is an important requirement for spread-spectrum communication. For chaos-based systems, only a few multiple-access schemes have been proposed. For instance, a method based on multiplexing chaotic signals has been proposed by Carroll and Pecora [5], and some chaos-based approaches for generating spreading codes have been applied to conventional CDMA systems [6]–[8]. Furthermore, multiple access using differential chaos shift keying (DCSK) has been introduced by Kennedy *et al.* [2] and [9], and the multiple-access capability of frequency-modulated DCSK (FM-DCSK) has been studied by Jáko *et al.* [10]. Recently, an alternative multiple-access technique for use with DCSK has been proposed by Lau *et al.* [11]. Only a preliminary study of the performance of this scheme under a noise-free environment has been given.

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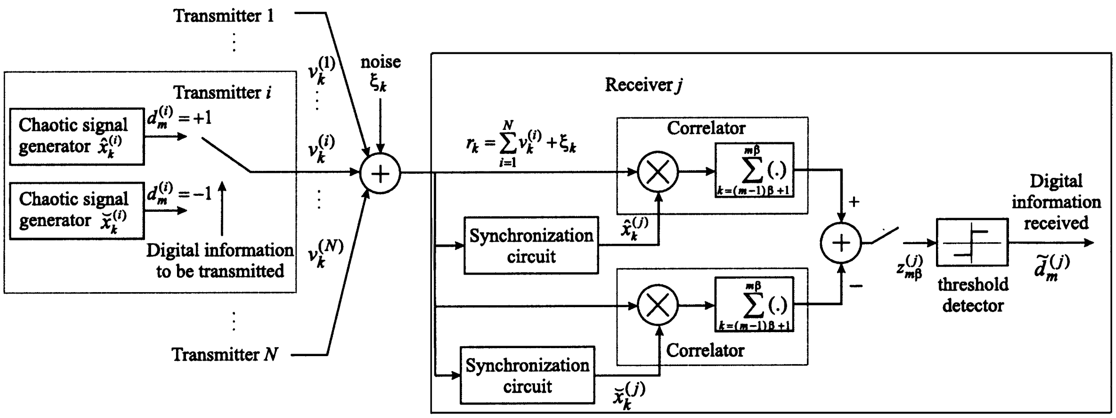


Fig. 1. MA-CSK communication system.

In this paper, we investigate in depth, using a discrete-time approach, a CSK system with multiple users, as well as the multiple-access DCSK (MA-DCSK) system proposed by Lau *et al.* [11]. It is assumed that the amplitude of the carrier is modulated by the chaotic signals (amplitude modulation) and we analyze the system using the equivalent baseband model [12], [13]. A mixed analysis-simulation (MAS) technique is developed to calculate the bit error rates (BERs). When certain assumptions hold, analytical expressions for the BERs can also be obtained. In Sections II and III, the multiple-access CSK (MA-CSK) and MA-DCSK systems are described and the corresponding BER equations are derived. Results found by the MAS method and the analytical method are compared with simulations in Section IV.

## II. Analysis of MA-CSK Systems

### A. Transmitter Structure

We consider a MA-CSK communication system with $N$ users, as shown in Fig. 1. A discrete-time approach will be adopted in the following analysis. In the transmitter of the $i$th user, a pair of chaotic sequences, denoted by $\{\hat{x}_k^{(i)}\}$ and $\{\check{x}_k^{(i)}\}$, are generated by a chaotic map with different initial conditions. We also assume that the mean value of each of the chaotic sequences is zero in order to avoid transmitting any dc component which is a waste of power. Denote the $m$th transmitted symbol for the $i$th user by $d_m^{(i)} \in \{-1, +1\}$ and assume that "$-1$" and "$+1$" occur with equal probabilities for all users. Define the spreading factor $\beta$ as the number of chaotic samples used to transmit one binary symbol. During the $m$th bit duration, i.e., for time $k = (m-1)\beta + 1, (m-1)\beta + 2, \ldots, m\beta$, the output of the transmitter of user $i$, $v_k^{(i)}$, is

$$v_k^{(i)} = \begin{cases} \hat{x}_k^{(i)} & \text{if } d_m^{(i)} = +1 \\ \check{x}_k^{(i)} & \text{if } d_m^{(i)} = -1. \end{cases} \tag{1}$$

The overall transmitted signal of the whole system at time $k$, denoted by $v_k$, is derived by summing the signals of all users, i.e., $v_k = \sum_{i=1}^{N} v_k^{(i)}$.

### B. Receiver Structure

The received signal, denoted by $r_k$, is given by $r_k = v_k + \xi_k$ where $\xi_k$ denotes the additive white Gaussian noise with zero mean and variance (power spectral density) $N_0/2$. Assume that synchronized versions of the chaotic signals $\{\hat{x}_k^{(j)}\}$ and $\{\check{x}_k^{(j)}\}$ can be reproduced at the receiver. The detection essentially involves correlating the incoming signal with the locally regenerated chaotic signals and sampling the outputs of the correlators at the end of each symbol duration. The $m$th decoded symbol for the $j$th user, denoted by $\tilde{d}_m^{(j)}$, is determined according to following rule:

$$\tilde{d}_m^{(j)} = \begin{cases} +1 & \text{if } z_{m\beta}^{(j)} = \sum\limits_{k=(m-1)\beta+1}^{m\beta} r_k \hat{x}_k^{(j)} - \sum\limits_{k=(m-1)\beta+1}^{m\beta} r_k \check{x}_k^{(j)} > 0 \\ -1 & \text{if } z_{m\beta}^{(j)} = \sum\limits_{k=(m-1)\beta+1}^{m\beta} r_k \hat{x}_k^{(j)} - \sum\limits_{k=(m-1)\beta+1}^{m\beta} r_k \check{x}_k^{(j)} \le 0. \end{cases} \tag{2}$$

### C. Derivation of BER

Consider the $j$th user. Without loss of generality, we consider the probability of error for the first symbol. For brevity, the subscripts of the variables $d_m^{(j)}$, $\tilde{d}_m^{(j)}$, $\hat{c}_{m\beta}^{(j)}$, $\check{c}_{m\beta}^{(j)}$, and $z_{m\beta}^{(j)}$ are omitted. If "$+1$" is transmitted for user $j$, i.e., $d^{(j)} = +1$ and $v_k^{(j)} = \hat{x}_k^{(j)}$, it is readily shown that the mean value of $z^{(j)} | (d^{(j)} = +1)$ equals

$$\mathrm{E}\left[z^{(j)} \middle| \left(d^{(j)} = +1\right)\right] = \beta \mathrm{E}\left[\left(\hat{x}_k^{(j)}\right)^2\right]. \tag{3}$$

Also, the variance of $z^{(j)} | (d^{(j)} = +1)$ is given by

$$\begin{aligned} &\mathrm{var}\left[z^{(j)} \middle| \left(d^{(j)} = +1\right)\right] \\ &= \beta \mathrm{var}\left[\left(\hat{x}_k^{(j)}\right)^2\right] + \sum_{\substack{k=1\\k\neq l}}^{\beta} \sum_{l=1}^{\beta} \mathrm{cov}\left[\left(\hat{x}_k^{(j)}\right)^2, \left(\hat{x}_l^{(j)}\right)^2\right] \\ &\quad + \beta \mathrm{E}^2\left[\left(\hat{x}_k^{(j)}\right)^2\right] \\ &\quad + \sum_{\substack{k=1\\k\neq l}}^{\beta} \sum_{l=1}^{\beta} \mathrm{E}^2\left[\hat{x}_k^{(j)} \hat{x}_l^{(j)}\right] \\ &\quad + 2\beta \mathrm{E}\left[\left(\hat{x}_k^{(j)}\right)^2\right] \sum_{\substack{i=1\\i\neq j}}^{N} \mathrm{E}\left[\left(\hat{x}_k^{(i)}\right)^2\right] \\ &\quad + 2 \sum_{\substack{i=1\\i\neq j}}^{N} \sum_{\substack{k=1\\k\neq l}}^{\beta} \sum_{l=1}^{\beta} \mathrm{E}\left[\hat{x}_k^{(i)} \hat{x}_l^{(i)}\right] \\ &\quad \times \mathrm{E}\left[\hat{x}_k^{(j)} \hat{x}_l^{(j)}\right] + \beta N_0 \mathrm{E}\left[\left(\hat{x}_k^{(j)}\right)^2\right]. \end{aligned} \tag{4}$$

Likewise, we can find $z^{(j)}|(d^{(j)}=-1)$ and confirm that

$$\mathrm{E}\left[z^{(j)}\left|\left(d^{(j)}=-1\right)\right.\right] = -\,\mathrm{E}\left[z^{(j)}\left|\left(d^{(j)}=+1\right)\right.\right] \tag{5}$$

$$\mathrm{var}\left[z^{(j)}\left|\left(d^{(j)}=-1\right)\right.\right] = \mathrm{var}\left[z^{(j)}\left|\left(d^{(j)}=+1\right)\right.\right]. \tag{6}$$

Since $z^{(j)}|(d^{(j)}=+1)$ and $z^{(j)}|(d^{(j)}=-1)$ are the sum of a large number of random variables, we assume that both of them are normally distributed. The BER for user $j$ can thus be computed from

$$\begin{aligned}\mathrm{BER}^{(j)} &= \frac{1}{2}\mathrm{Prob}\left(z^{(j)}\le 0\left|\left(d^{(j)}=+1\right)\right.\right)\\ &\quad+\frac{1}{2}\mathrm{Prob}\left(z^{(j)}>0\left|\left(d^{(j)}=-1\right)\right.\right)\\ &=\frac{1}{2}\mathrm{erfc}\left(\frac{\mathrm{E}\left[z^{(j)}\left|\left(d^{(j)}=+1\right)\right.\right]}{\sqrt{2\mathrm{var}\left[z^{(j)}\left|(d^{(j)}=+1)\right.\right]}}\right)\end{aligned} \tag{7}$$

where erfc$(.)$ is the complementary error function [13].

*1) MAS Technique:* To find the BER, we can first attain the values of the terms (expected values, variances and covariances) on the right-hand side of (3) and (3) by numerical simulations. Then, the values of $\mathrm{E}[z^{(j)}|(d^{(j)}=+1)]$ and $\mathrm{var}[z^{(j)}|(d^{(j)}=+1)]$ can be computed and substituted into (7) to get the BER. Since both numerical simulation and analytical method are involved in obtaining the BER, we refer to it as the MAS technique.

*2) Analytical BERs:* To ensure that all users are treated equally, we assume that all users transmit with equal average power $P_s$. Further, if both the autovariance of $\{(\hat{x}_k^{(j)})^2\}$ and the autocorrelation of $\{\hat{x}_k^{(j)}\}$ vanish, i.e.,

$$\mathrm{cov}\left[\left(\hat{x}_k^{(j)}\right)^2,\left(\hat{x}_l^{(j)}\right)^2\right]=0, \qquad k\neq l \tag{8}$$

$$\mathrm{E}\left[\hat{x}_k^{(j)}\hat{x}_l^{(j)}\right]=0, \qquad k\neq l \tag{9}$$

the analytical BER for user $j$ can be reduced to

$$\mathrm{BER}^{(j)}_{\text{CSK-I}}=\frac{1}{2}\mathrm{erfc}\left(\left[\frac{2\Psi^{(j)}}{\beta}+\frac{2(2N-1)}{\beta}+2\left(\frac{E_b}{N_0}\right)^{-1}\right]^{-\frac{1}{2}}\right) \tag{10}$$

where $\Psi^{(j)}=\mathrm{var}[(\hat{x}_k^{(j)})^2]/P_s^2=\mathrm{var}[(\hat{x}_k^{(j)})^2]/\mathrm{E}^2[(\hat{x}_k^{(j)})^2]$, and $E_b=\beta P_s$ represents the average bit energy.

Finally, if all users employ the same map that generate sequences satisfying (8) and (9), the values $\Psi^{(j)}$ will become identical and all users will have the same BER.

## III. Analysis of MA-DCSK Systems

### A. Frame Structure of the Transmitted Signal

In a MA-DCSK system, to avoid excessive interference, and hence, misdetection, the separation between the reference and data samples must be different for different users. A multiple-access scheme has been proposed by Lau *et al.* [11] where the separation between the reference and data samples differs for different users, as illustrated in Fig. 2.

Suppose there are $N$ users in the system. Define a superframe as the minimum number of slots within which all $N$ users transmit an integral number of frames. Within one superframe, denote the number of frames sent by user $i$ by $f_N^{(i)}$. From now on, we restrict our discussion to one superframe which is assumed to contain $2N_s$ slots, where $2N_s=Nf_N^{(N)}=(N-1)f_N^{(N-1)}=\ldots=2f_N^{(2)}=f_N^{(1)}$.

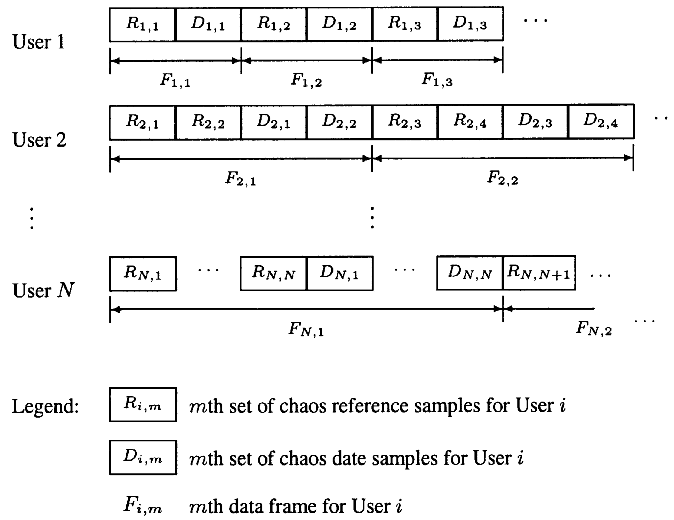


Fig. 2. Transmission scheme for the MA-DCSK communication system.

Fig. 3 shows an MA-DCSK communication system in a discrete-time mode. In the transmitter of the $i$th user, a chaotic map is used to generate a chaotic sequence $\{x_k^{(i)}\}$ with zero mean. The chaotic maps for different users are different in general. Assume that $\alpha$ chaotic samples are sent in each slot (spreading factor $=2\alpha$). Consider the transmitted signal of user $i$ during the $m$th time slot. Denote the output of the transmitter by $y_k^{(i)}$. If the slot is a reference sample slot, $y_k^{(i)}=x_k^{(i)}$. If the slot corresponds to a data sample slot sending a binary symbol "$+1$", $y_k^{(i)}=x_{k-i\alpha}^{(i)}$. Otherwise, if the slot corresponds to a data sample slot sending a binary symbol "$-1$", $y_k^{(i)}=-x_{k-i\alpha}^{(i)}$. Thus, for the $m$th time slot of user $i$, we define $a_m^{(i)}$ as

$$a_m^{(i)}=\begin{cases}+1, & \text{if } y_k^{(i)}=x_{k-l}^{(i)}\\ -1, & \text{if } y_k^{(i)}=-x_{k-l}^{(i)}\end{cases} \tag{11}$$

where $l=0$ for reference samples, and $l=i\alpha$ for data samples. Therefore, the transmitted signal of user $i$ during the $m$th time slot can be represented by

$$y_k^{(i)}=a_m^{(i)}x_{k-l}^{(i)};\quad k=(m-1)\alpha+1,(m-1)\alpha+2,\ldots,m\alpha. \tag{12}$$

The overall transmitted signal at time $k$, denoted by $y_k$, equals $y_k=\sum_{i=1}^{N}y_k^{(i)}$.

### B. Receiver Structure

At time $k$, the received signal, $r_k$, is given by $r_k=y_k+\xi_k$. For each user, the signal received during a reference sample slot will correlate with the signal at the corresponding data sample slot. Depending on whether the output is larger or smaller than the threshold, a "$+1$" or "$-1$" is decoded. In Fig. 3, a block diagram of a DCSK correlator receiver is shown.

Define the $u$th sub-state of the system at the $m$th slot by $\tilde{A}_m^{(u)}$, i.e.,

$$\tilde{A}_m^{(u)}=\left[a_m^{(1)}\;a_m^{(2)}\;\ldots\;a_m^{(u-1)}\;a_m^{(u+1)}\;\ldots\;a_m^{(N)}\right]^T. \tag{13}$$

Consider the $j$th user and the received signal during the $m$th time slot. Suppose the slot corresponds to a reference sample slot for user $j$, i.e., $a_m^{(j)}=+1$. These reference samples will correlate with the received

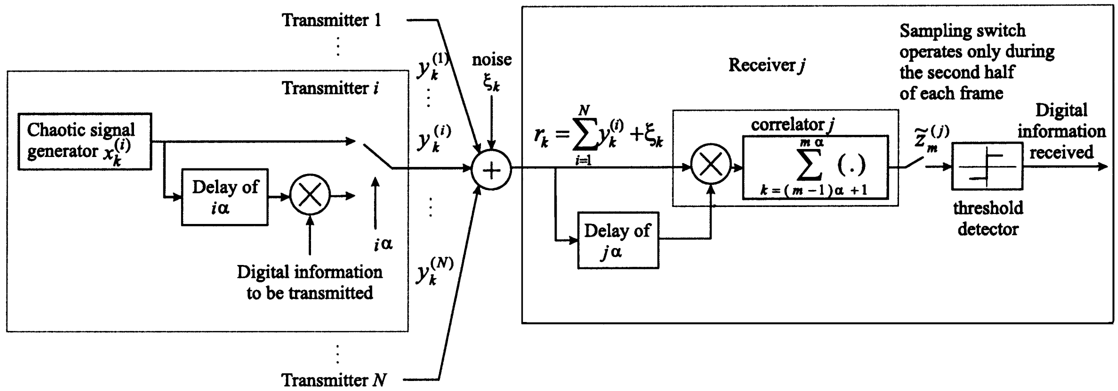


Fig. 3. MA-DCSK communication system.

samples $j$ slots later, i.e., the $(m+j)$th slot. The output of the correlator, denoted by $z_m^{(j)}|(\tilde{A}_m^{(j)}, \tilde{A}_{m+j}^{(j)}, a_m^{(j)} = +1, a_{m+j}^{(j)})$, is given by

$$\begin{aligned} z_m^{(j)} &\left| \left(\tilde{A}_m^{(j)}, \tilde{A}_{m+j}^{(j)}, a_m^{(j)} = +1, a_{m+j}^{(j)}\right) \right. \\ &= \sum_{k=(m-1)\alpha+1}^{m\alpha} \left(\sum_{u=1}^{N} y_k^{(u)} + \xi_k\right)\left(\sum_{v=1}^{N} y_{k+\alpha j}^{(v)} + \xi_{k+\alpha j}\right) \\ &= \sum_{u=1}^{N}\sum_{v=1}^{N}\sum_{k=(m-1)\alpha+1}^{m\alpha} a_m^{(u)} a_{m+j}^{(v)} x_{k-l_u}^{(u)} x_{k+\alpha j-l_v}^{(v)} \\ &\quad + \sum_{u=1}^{N}\sum_{k=(m-1)\alpha+1}^{m\alpha} a_m^{(u)} x_{k-l_u}^{(u)} \xi_{k+\alpha j} \\ &\quad + \sum_{v=1}^{N}\sum_{k=(m-1)\alpha+1}^{m\alpha} a_{m+j}^{(v)} x_{k+\alpha j-l_v}^{(v)} \xi_k \\ &\quad + \sum_{k=(m-1)\alpha+1}^{m\alpha} \xi_k \xi_{k+\alpha j} \end{aligned} \tag{14}$$

where $l_i = 0$ $(i = 1, 2, \ldots, N)$ for reference samples, and $l_i = i\alpha$ for data samples. The decoded symbol corresponding to this pair of time slots, denoted by $\tilde{d}_m^{(j)}$, is determined according to the following rule:

$$\tilde{d}_m^{(j)} = \begin{cases} +1, & \text{if } z_m^{(j)} \left|\left(\tilde{A}_m^{(j)}, \tilde{A}_{m+j}^{(j)}, a_m^{(j)} = +1, a_{m+j}^{(j)}\right) > 0\right. \\ -1, & \text{if } z_m^{(j)} \left|\left(\tilde{A}_m^{(j)}, \tilde{A}_{m+j}^{(j)}, a_m^{(j)} = +1, a_{m+j}^{(j)}\right) \le 0.\right. \end{cases} \tag{15}$$

### C. Derivation of BER

Denote the binary symbol transmitted by the $j$th user in the $(m+j)$th slot by $d_m^{(j)}$, i.e., $a_{m+j}^{(j)} = d_m^{(j)}$. Using a likewise procedure as in Section II, it is readily shown that the average BER for user $j$, $\text{BER}^{(j)}$, is (for details, refer to [14])

$$\begin{aligned} \text{BER}^{(j)} = \frac{1}{2N_s} &\sum_{m\in S^{(j)}} \sum_{\tilde{A}_m^{(j)}\in U^{(N-1)}} \sum_{\tilde{A}_{m+j}^{(j)}\in U^{(N-1)}} \\ &\times \left[\frac{1}{2}\text{erfc}\left(\frac{\text{E}\left[\tilde{z}_m^{(j)} \left| d_m^{(j)} = +1\right.\right]}{\sqrt{2\text{var}\left[\tilde{z}_m^{(j)} \left| d_m^{(j)} = +1\right.\right]}}\right) + \frac{1}{2}\text{erfc}\left(\frac{-\text{E}\left[\tilde{z}_m^{(j)} \left| d_m^{(j)} = -1\right.\right]}{\sqrt{2\text{var}\left[\tilde{z}_m^{(j)} \left| d_m^{(j)} = -1\right.\right]}}\right)\right] \\ &\times \text{Prob}\left(\tilde{A}_m^{(j)}, \tilde{A}_{m+j}^{(j)}\right) \end{aligned} \tag{16}$$

where $U \in \{-1, +1\}$ and $S^{(j)}$ is the set of slot numbers corresponding to the reference slots of user $j$ over one superframe. Define $B(\tilde{A}_m^{(j)}, \tilde{A}_{m+j}^{(j)})$ as the average probability of occurrence of $(\tilde{A}_m^{(j)}, \tilde{A}_{m+j}^{(j)})$ over all reference slots, i.e.,

$$B\left(\tilde{A}_m^{(j)}, \tilde{A}_{m+j}^{(j)}\right) = \frac{1}{N_s} \sum_{m\in S^{(j)}} \text{Prob}\left(\tilde{A}_m^{(j)}, \tilde{A}_{m+j}^{(j)}\right). \tag{17}$$

Thus, (16) can be rewritten as

$$\begin{aligned} \text{BER}^{(j)} = \frac{1}{2} &\sum_{\tilde{A}_m^{(j)}\in U^{(N-1)}} \sum_{\tilde{A}_{m+j}^{(j)}\in U^{(N-1)}} \\ &\times \left[\frac{1}{2}\text{erfc}\left(\frac{\text{E}\left[\tilde{z}_m^{(j)} \left| d_m^{(j)} = +1\right.\right]}{\sqrt{2\text{var}\left[\tilde{z}_m^{(j)} \left| d_m^{(j)} = +1\right.\right]}}\right) + \frac{1}{2}\text{erfc}\left(\frac{-\text{E}\left[\tilde{z}_m^{(j)} \left| d_m^{(j)} = -1\right.\right]}{\sqrt{2\text{var}\left[\tilde{z}_m^{(j)} \left| d_m^{(j)} = -1\right.\right]}}\right)\right] \\ &\times B\left(\tilde{A}_m^{(j)}, \tilde{A}_{m+j}^{(j)}\right). \end{aligned} \tag{18}$$

*1) MAS Technique:* As in the MA-CSK case, we find out the required expected values, variances and covariances by numerical simulations. The values of $\text{E}[\tilde{z}_m^{(j)}|d_m^{(j)} = +1]$, $\text{E}[\tilde{z}_m^{(j)}|d_m^{(j)} = -1]$, $\text{var}[\tilde{z}_m^{(j)}|d_m^{(j)} = +1]$ and $\text{var}[\tilde{z}_m^{(j)}|d_m^{(j)} = -1]$ are then computed and put into (18) to obtain the BER.

*2) Analytical BER:* As in Section II-C2, we assume that all users transmit with equal average power $P_s$. Assume that the chaotic sequence generated by user $i$, $\{x_k^{(i)}\}$, satisfies the following conditions:

$$\begin{aligned} \text{cov}\left[\left(x_k^{(i)}\right)^2, \left(x_n^{(i)}\right)^2\right] &= \text{E}\left[\left(x_k^{(i)}\right)^2 \left(x_n^{(i)}\right)^2\right] \\ &\quad - \text{E}\left[\left(x_k^{(i)}\right)^2\right] \text{E}\left[\left(x_n^{(i)}\right)^2\right] \\ &= 0, \qquad k \neq n \end{aligned} \tag{19}$$

$$\text{E}\left[x_k^{(i)} x_n^{(i)}\right] = 0, \qquad k \neq n \tag{20}$$

$$\text{E}\left[x_k^{(i)} x_{k+\kappa}^{(i)} x_{k+\mu}^{(i)} x_{k+\theta}^{(i)}\right] = 0 \qquad \forall \kappa, \mu, \theta \in I^+ \text{ and } 0 < \kappa < \mu < \theta. \tag{21}$$

TABLE I
STATISTICAL PROPERTIES OF THE NORMALIZED CHAOTIC SEQUENCES

| | Logistic map | Cubic map | Bernoulli-shift map |
|---|---|---|---|
| $\mathrm{E}[x_k]$ | 0 | 0 | 0 |
| $\mathrm{E}[x_k^2]$ | 1 | 1 | 1 |
| $\mathrm{E}[x_k^3]$ | 0 | 0 | 0 |
| $\mathrm{var}[x_k]$ | 1 | 1 | 1 |
| $\mathrm{var}[x_k^2]$ | 0.5 | 0.5 | 0.9874 |

Then, (18) can be readily simplified to

$$\mathrm{BER}_{\mathrm{DCSK-I}}^{(j)} = \frac{1}{2}\mathrm{erfc}\left(\left[\frac{2\Psi^{(j)}}{\alpha} + \frac{2(N^2-1)}{\alpha} + 4N\left(\frac{E_b}{N_0}\right)^{-1} + 2\alpha\left(\frac{E_b}{N_0}\right)^{-2}\right]^{-\frac{1}{2}}\right) \tag{22}$$

where $\Psi^{(j)} = \mathrm{var}[(x_k^{(j)})^2]/P_s^2 = \mathrm{var}[(x_k^{(j)})^2]/\mathrm{E}^2[(x_k^{(j)})^2]$ and $E_b = 2\alpha P_s$ denotes the average bit energy.

In addition to the statistical properties given in (19) to (21), if all users employ the same map, the values of $\Psi^{(j)}$ are now identical and all users will have the same bit error performance.

## IV. RESULTS AND DISCUSSIONS

The following maps are used in our simulations.

5) Logistic map: $x_{k+1} = 2x_k^2 - 1$
6) Cubic map: $x_{k+1} = 4x_k^3 - 3x_k$
7) Bernoulli-shift map: $x_{k+1} = \begin{cases} 1.2x_k + 1, & \text{when } x_k < 0 \\ 1.2x_k - 1, & \text{when } x_k > 0. \end{cases}$

In the simulations, the chaotic sequences are normalized before transmission. The statistical properties of the normalized sequences are shown in Table I. Based on the invariant distribution of the logistic map and the cubic map [15], it is readily shown that the chaotic sequences generated from these two maps satisfy the statistical properties assumed in (8), (9), and (19) to (21). Also, due to the shortage of space, we only present the results in which all chaotic signals are generated by the same map but with different initial conditions. (For other related results, please refer to Lau and Tse [14].)

### A. MA-CSK System

In the first simulation, the cubic map is used with a spreading factor of 100. In Fig. 4, the BERs are plotted against $E_b/N_0$ for a three-user system. It can be seen that both the MAS and the analytical results are in good agreement with the brute-force (BF) simulated results. In Fig. 5, we present the BF simulated BERs against $E_b/N_0$ for the cases corresponding to different choices of chaotic maps. It is found that the chaotic sequences generated by the Bernoulli-shift map produce a higher BER, while the BERs of the users using cubic map and logistic map are the same. One reason is that the value of $\mathrm{var}[x_k^2]$ is larger for the normalized chaotic sequences generated from the Bernoulli-shift map compared with that of the cubic map and logistic map (see Table I).

### B. MA-DCSK System

We consider the results for a three-user MA-DCSK system in which all chaotic sequences are generated from the cubic map with a spreading factor of 200 $(\alpha = 100)$. As shown in Fig. 6, all methods give consistent results. Next, we compare the BF simulated BERs for different choices of chaotic maps. Fig. 7 shows the results when

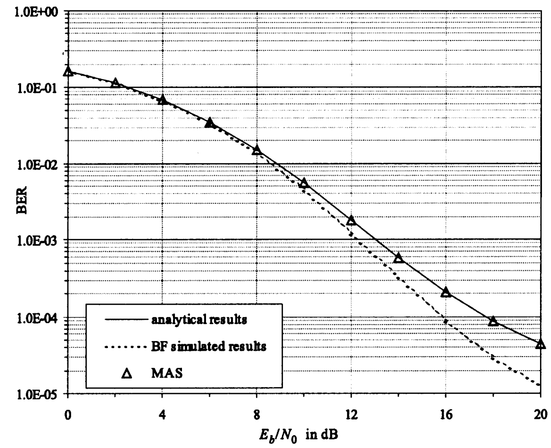


Fig. 4. BER versus $E_b/N_0$ in a three-user MA-CSK system. Cubic map is used and $\beta = 100$.

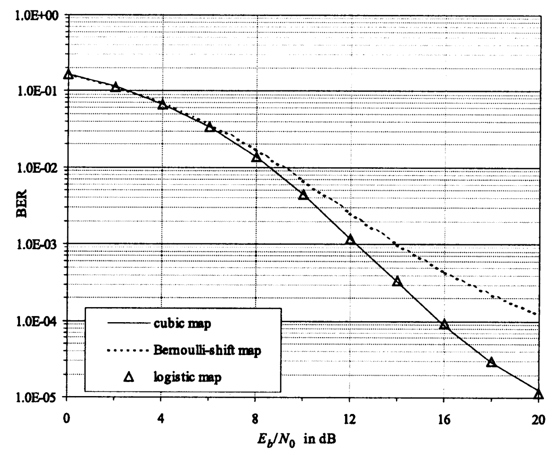


Fig. 5. Simulated BER versus $E_b/N_0$ in a three-user MA-CSK system. Cubic map, Bernoulli-shift map and logistic map are used, respectively. $\beta = 100$.

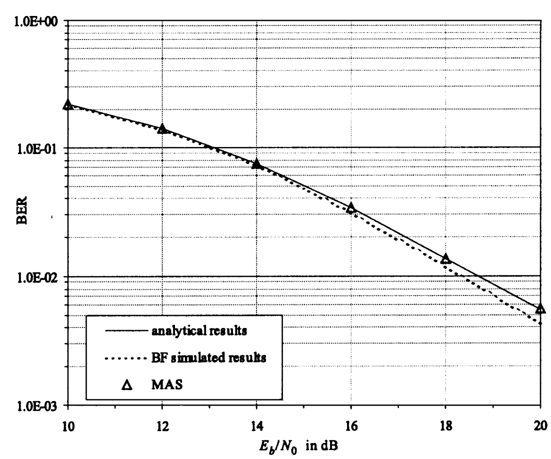


Fig. 6. BER versus $E_b/N_0$ in a three-user DCSK system. Cubic map is used and $\alpha = 100$.

Bernoulli-shift map, cubic map and logistic map are used, respectively. It can be observed that using Bernoulli-shift map produces a higher BER compared with the other two cases. One possible reason is that the value of $\mathrm{var}[x_k^2]$ is larger for the Bernoulli-shift

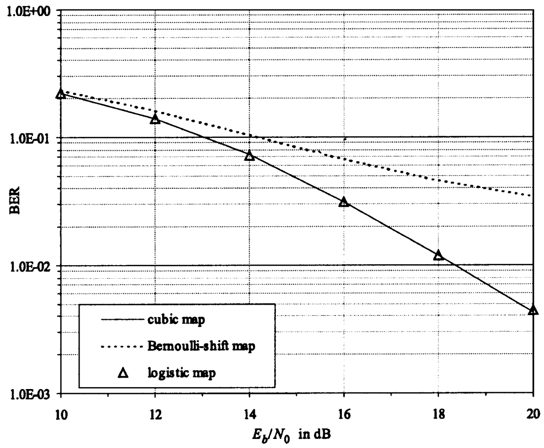


Fig. 7. Simulated BER versus $E_b/N_0$ in a three-user DCSK system. Bernoulli-shift map, cubic map and logistic map are used, respectively. $\alpha = 100$.

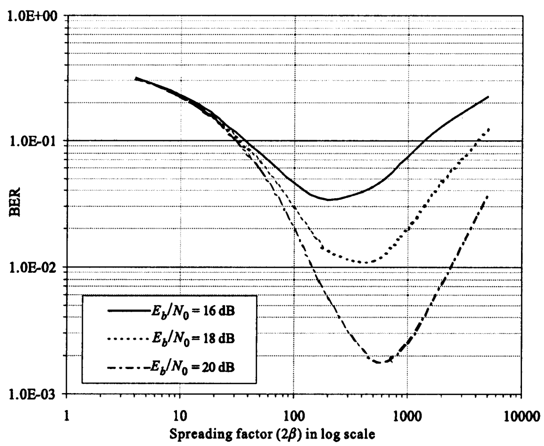


Fig. 8. Analytical BER versus spreading factor $(2\alpha)$ under different $E_b/N_0$ in a three-user DCSK system. Logistic map is used.

map. Finally, assuming the logistic map is used,we plot the analytical BER against the spreading factor $(2\alpha)$ under different $E_b/N_0$ for a three-user system. Fig. 8 shows that when the spreading factor increases initially, the BER improves. After the spreading factor has reached an optimum value, increasing the spreading factor further will deteriorate the BER.

## V. Conclusion

In this paper, two multiple-access techniques for use with CSK and DCSK under a noisy condition are analyzed thoroughly. An MAS technique and an analytical method are developed to derive the BERs. The techniques developed are applicable to other multiple-access schemes. Finally, the DCSK scheme is known to be suboptimal in the amplitude modulated version. An extension of the present numerical analysis to the FM-DCSK scheme and the multiple-access FM-DCSK scheme based on Walsh functions [16] would also be worth studying.

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