A Learning–based Variables Assignment Weighting Scheme for Heuristic and Exact Searching

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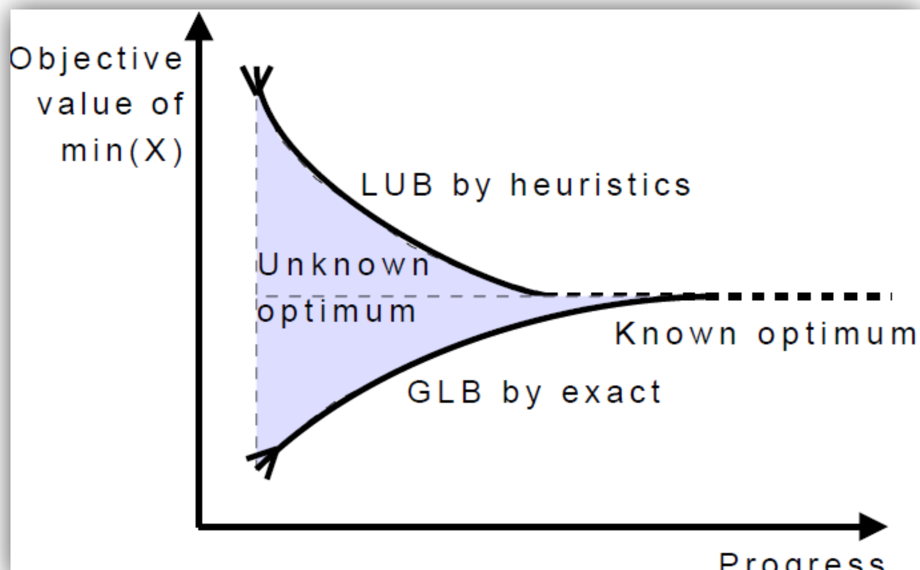
Outlines

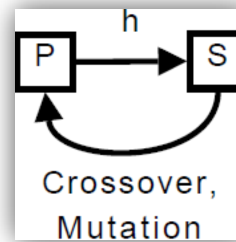
Opportunity and background

- Many combinatorial optimizations are NP-hard
  - "…no good algorithms…" (Edmonds, 1967)
  - The larger, the much more difficult to solve ☹

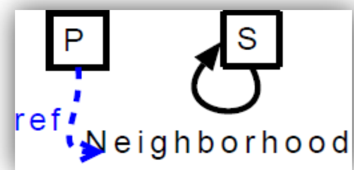


A typical problem solving progress

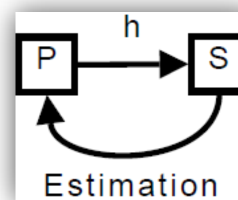
- Different metaheuristics have been proposed to improve searching (h), e.g.,
  - GA



  - LS

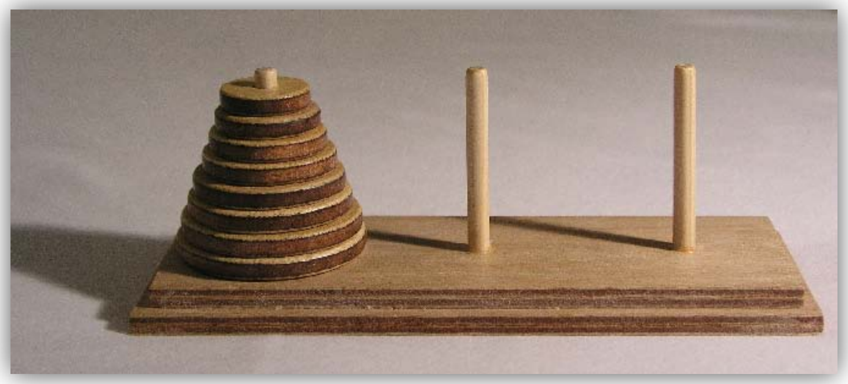


  - EDAs



An inspiring game

- The game of *Tower of Hanoi* consists of:
  - Three rods,
  - A number of disks of different sizes.
- The puzzle starts with the disks in a neat stack in ascending order of size on one rod.
- The objective is to move the stack to another rod, obeying:
  - No disk on top of a smaller one
  - One disk at a time.
- To unveil the solving rules, play with 2 or 3 disks at first.
  - ***Learn from a small sample***



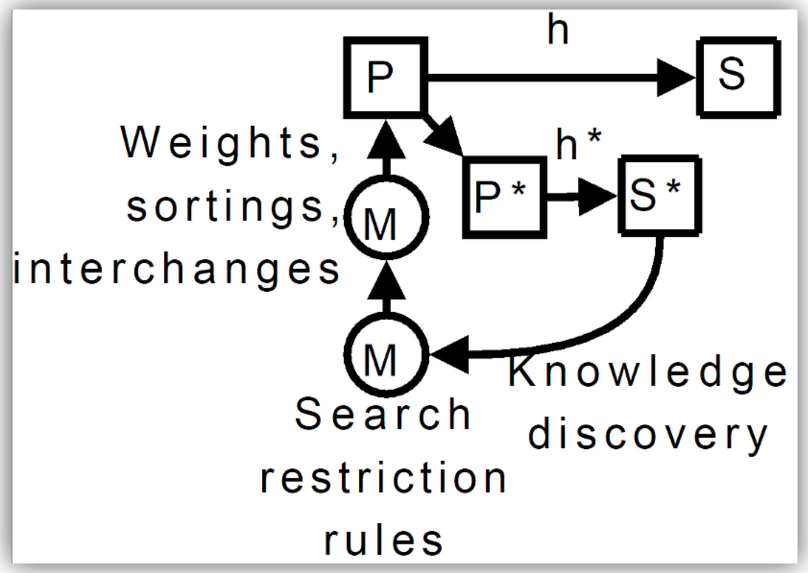
A model of Tower of Hanoi (8 disks, Photo brought from Wikipedia)

Objective and assumptions

- The objective is to improve searching through learning-based revisions of assignments of variables
- Basic assumptions
  - A recognizable problem
  - Similar decision rules for each variable
- Notes
  - The smaller the problem is, the much easier (NP-hardness ☺)
  - The 1st assumption makes learning possible
  - The 2nd assumption further enables learning from a part of the problem (variables), it implicitly enables learning from near-optimal solutions
  - Large-scale problems are preferred

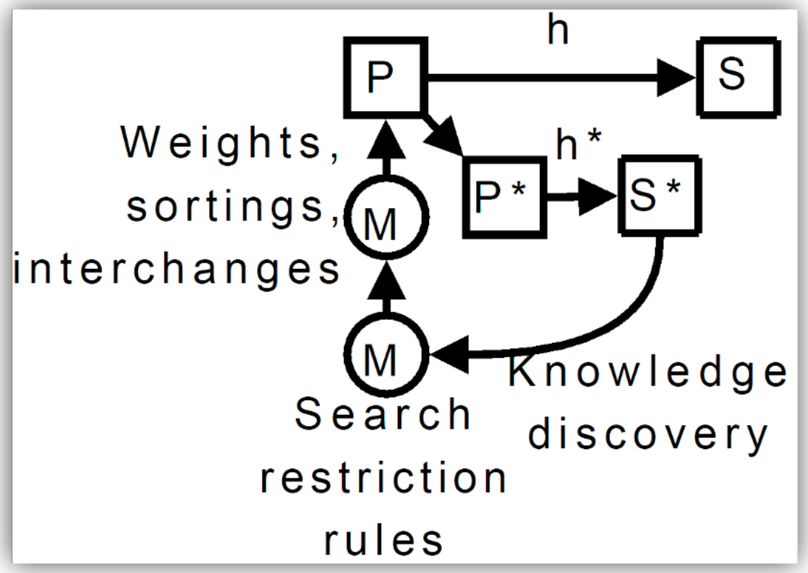
The proposed method

- The phases of the proposed method are:
  - 1. Start with a problem “P”
  - **2**. Find a *small* “representative” part “P*”
  - 3. Obtain a good solution “S*” quickly
  - **4**. Obtain rules about assignments from “S*” as complete as possible
  - 5. Interpret the rules to weights, sorting, or interchanges of possible assignments of the variables
  - 6. Reform the assignment process of heuristic (sometimes exact) searching “h”



The proposed method

- Notes
  - Size(P*) << size(P)
  - h* ≠ h (not necessarily same, nor necessarily heuristic)
  - The indirect way of using the learning results
    - Rules with confidences from 100% down to 1% are potentially useful.
  - Interpretations for different heuristics:
    - Weights for value assignments
    - Sorting for tests of local search
    - Interchanges for tests of binaries
    - …

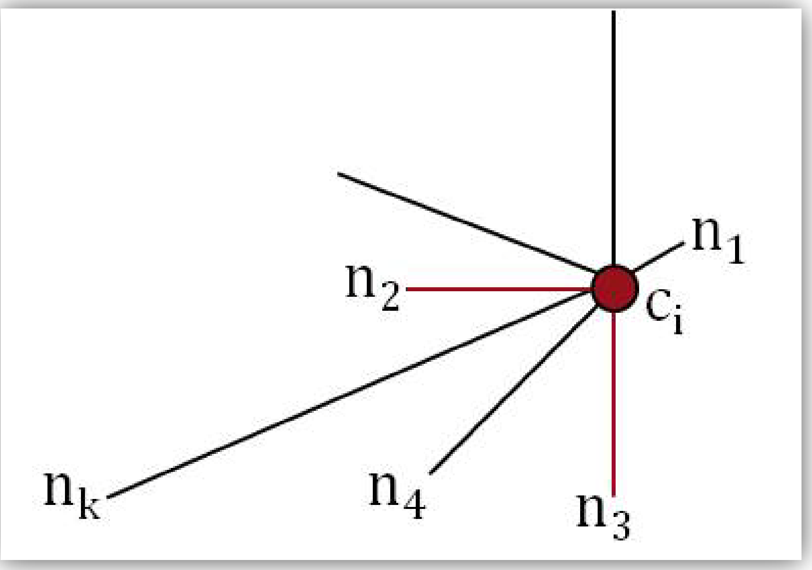


Traveling salesman as an example

- The Euclidean traveling salesman problem (TSP): finding a shortest tour that visits all given spatial points (cities).
  - Hamilton circle: two edges for each city
  - Most of very long edges are not possible to appear in the optimal tour(s)
- How does the method work?
  - Indentify a weight for each edge candidate of each city
  - Reorder and reform the possible
- How to indentify the weights?
  - Learn from a part of the given problem, with a set of attributes for the edge candidates

Traveling salesman: attributes

- The attributes of an edge (c_i, n_j) for a city c_i
  - G1 Global nearest
  - R1, R2, R3 Length indices comparing to (c_i, n_1), (c_i, n_2), (c_i, n_3)
  - S1, S2 Whether $d(c_i, n_1) \le d(c_i, n_2)/2$, $d(c_i, n_2) \le d(c_i, n_3)/2$
  - P1, P2, P3 R1-R3 of n_j
  - Q1, Q2 S1, S2 of n_j
  - Ag, Ah Minimal / maximal angular gap around c_i
  - An Number of directions around c_i
  - Opt Whether appears in the training sample or not



Traveling salesman: sample data

- Learning samples

G1	R1	R2	R3	S1	S2	P1	P2	P3	Q1	Q2	Ag	Ah	An	Opt
			...					...						...
0	3	1	1	1	0	4	3	2	0	0	3	10	7	0
0	9	3	3	1	0	6	6	2	0	1	3	10	7	1
0	9	3	3	1	0	10	4	2	1	1	3	10	7	1
			...					...						...

- Sample rules ("Opt=1" only)

Id	Rule	Support	Confidence
1	R1=3, S1=1, Q1=1 => Opt=1	0.013	1.000
2	P1=3, S1=1, Q1=1 => Opt=1	0.013	1.000
3	R1=3, S1=1, Q2=0 => Opt=1	0.012	1.000
...	...	...	...
30	G1=1 => Opt=1	0.022	0.913
...	...	...	...
983	R3=8 => Opt=1	0.048	0.010

XUE et al: A Learning-based Searching Reform Scheme (EURO XXIV, Lisbon, 2010)

Traveling salesman: revising the assignments

- Weights of edge candidates
  - Highest confidence of the rule that implies the edge should be in optimal tour (Opt=1)
  - Range [0, 1]
- Possible usage:
  - Direct value assignments (dispatching rules),
  - Grouping for a rank-based constructive heuristic,
  - Sorting for tests of searching, e.g., by Distance × (1-weight) (WD)
  - Interchanges for tests of binaries. The weights descending
- For those candidate sets not determine by Euclidean distance, a pseudo-distance could be defined.
  - E.g., a pseudo-distance = ln(α-value+1) for the α-nearness

Traveling salesman: test 1 (local search)

- Inputs
  - 32 large Euclidean TSPs from industry, geography and random generation, grouped, ranging from 3,000 to 1,000,000 cities.
- Objective algorithm
  - 5,2-Opt (100 runs) initialized by Greedy, on 5-sized candidate sets
- Parameters (Class Association Rules, CARs)
  - P* = 3,000 cities with a closest density (and same aspect ratio)
  - Min confidence of learning = 0.01
  - Min support of learning = 0.001
  - Learn from 50-sized (if applicable) candidate sets, find the best 5
- Optional parameters
  - Length control of rules: |antecedent| < 6 (learns much faster without much loss of rules)

Traveling salesman: test 1

- Groups of instances to test

Category	VLSI(BK)	E(BK)	TSPLIB(Optimum)
3k	lsn3119(9114*) lta3140(9517*) fdp3256(10008*)	E3k.0(40634081*)E3k.1(40315287*) E3k.2(40303394*)E3k.3(40589659*) E3k.4(40757209)	pr2392(378032) pcb3038(137694) fnl4461(182566)
10k	dga9698(27724) xmc10150(28387)	E10k.0(71865826)E10k.1(72031630) E10k.2(71822483)	pla7397(23260728) brd14051(469385)
31k	pbh30440(88328) xib32892(96757)	E31k.0(71865826) E31k.1(72031630)	pla33810(66048945)
100k	sra104815(251433)	E100k.0(225787421) E100k.1(225659006)	pla85900(142382641)
316k	ara238025(578775) lra498378(2168067)	E316k.0(401307462)	-
1M	lrb744710(1612132)	E1M.0(713189834)	-

* Also proved optimal

Traveling salesman: results of test 1

- Average quality (% excess BK) comparison (G+5-Opt)

		G+5-Opt @ NN			G+5-Opt @ Quadrant			G+5-Opt @ α-nearness		
		Avg	Avg/WD	Imp(%)	Avg	Avg/WD	Imp(%)	Avg	Avg/WD	Imp(%)
VLSI	3k	3.889	2.663	**31.5**	0.695	0.649	6.7	0.361	**0.327**	**9.3**
	10k	4.236	3.300	**22.1**	0.863	0.693	**19.7**	0.526	**0.503**	**4.5**
	31k	4.169	2.913	**30.1**	0.814	0.642	**21.2**	0.454	**0.437**	**3.7**
	100k	6.657	6.467	2.9	0.842	0.752	**10.7**	0.339	**0.328**	**3.2**
	316k	9.959	7.950	**20.2**	1.183	0.917	**22.5**	-	-	-
	1M	4.682	4.385	6.3	0.857	0.762	**11.1**	-	-	-
E	3k	0.703	0.487	**30.7**	0.346	0.338	2.3	0.156	**0.156**	0.3
	10k	0.862	0.490	**43.1**	0.375	0.370	1.4	0.179	**0.178**	0.2
	31k	1.262	0.659	**47.8**	0.527	0.526	0.2	0.343	**0.341**	0.6
	100k	1.851	0.646	**65.1**	0.438	0.434	0.9	0.252	**0.250**	0.8
	316k	1.660	0.679	**59.1**	0.430	0.422	1.9	-	-	-
	1M	1.176	0.911	**22.5**	0.381	0.379	0.5	-	-	-
TSPLIB	3k	0.456	0.358	**21.4**	0.340	0.321	5.4	0.143	**0.134**	**6.5**
	10k	2.878	2.234	**22.4**	0.427	0.395	7.6	**0.253**	0.278	-10.1
	31k	2.297	1.677	**27.0**	0.913	**0.517**	**43.4**	0.560	0.617	-10.2
	100k	2.065	1.476	**28.5**	0.761	**0.445**	**41.5**	0.932	0.978	-4.9

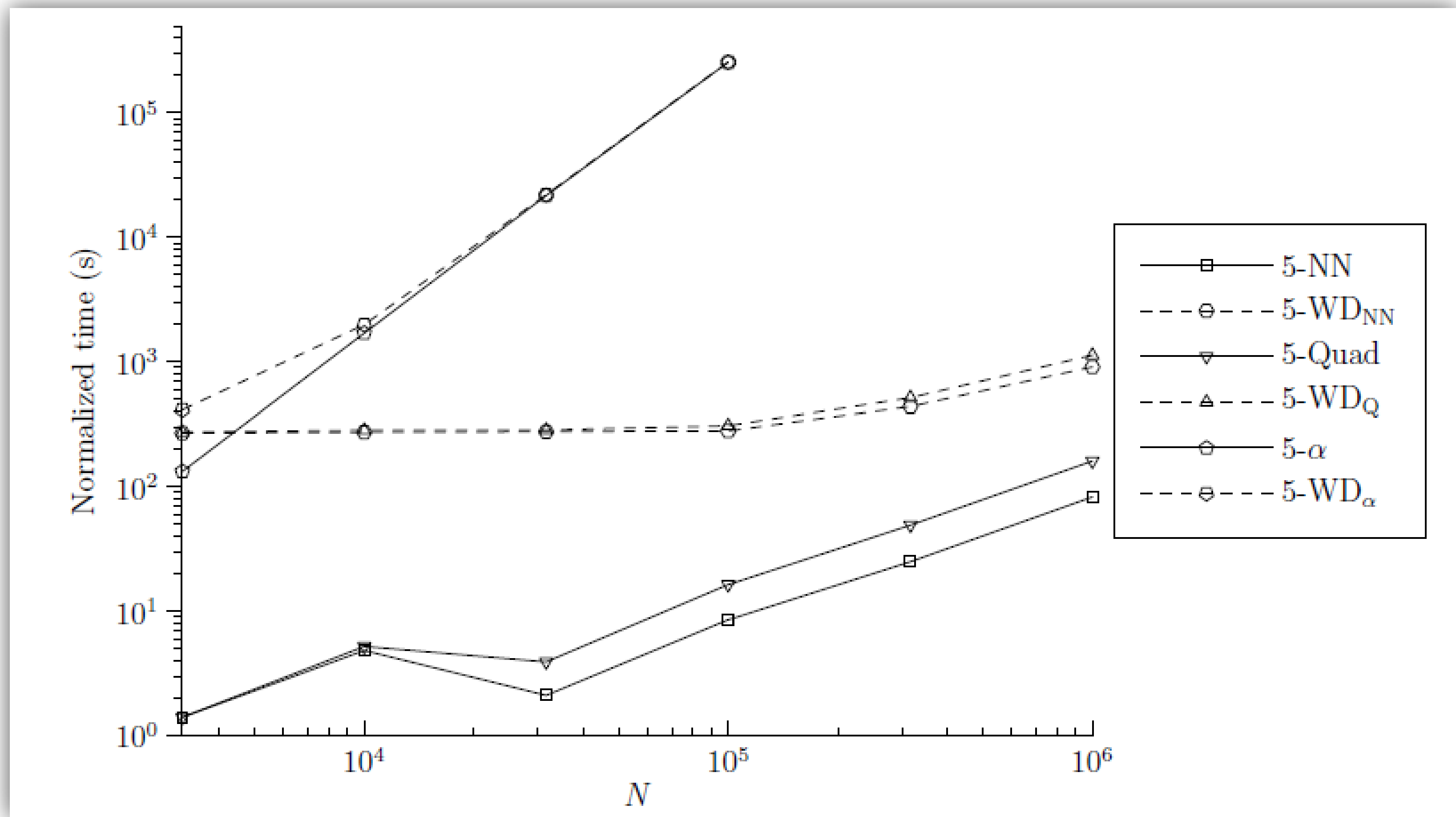
Traveling salesman: results of test 1

❖ Average quality (% excess BK) comparison (G+2-Opt)

		G+2-Opt @ NN			G+2-Opt @ Quadrant			G+2-Opt @ α-nearness		
		Avg	Avg/WD	Imp/%	Avg	Avg/WD	Imp(%)	Avg	Avg/WD	Imp(%)
VLSI	3k	5.196	4.234	**18.5**	2.177	2.019	7.3	1.376	1.625	**-18.1**
	10k	5.949	4.943	**16.9**	2.716	2.144	**21.1**	2.101	1.956	6.9
	31k	5.660	4.221	**25.4**	2.421	2.162	**10.7**	1.675	2.052	**-22.5**
	100k	8.101	7.945	1.9	2.472	2.344	5.2	1.244	2.006	**-61.3**
	316k	11.503	4.942	**57.0**	3.004	2.746	8.6	-	-	-
	1M	6.125	5.710	6.8	2.505	2.380	5.0	-	-	-
E	3k	2.250	1.616	**28.2**	1.412	1.645	**-16.5**	0.791	0.996	**-25.8**
	10k	1.849	1.575	**14.8**	1.439	1.495	-3.8	0.756	1.113	**-47.3**
	31k	2.007	1.648	**17.9**	1.604	1.678	-4.6	0.881	1.314	**-49.1**
	100k	2.320	1.611	**30.6**	1.553	1.550	0.2	0.791	1.237	**-56.5**
	316k	2.764	2.370	**14.3**	2.154	2.179	-1.2	-	-	-
	1M	2.235	1.884	**15.7**	1.452	1.460	-0.6	-	-	-
TSPLIB	3k	1.735	1.470	**15.3**	1.554	1.433	7.8	0.793	0.751	5.3
	10k	3.832	3.371	**12.0**	1.817	2.040	**-12.3**	1.177	1.485	**-26.1**
	31k	3.176	2.610	**17.8**	2.469	2.232	9.6	1.694	1.914	**-13.0**
	100k	3.017	2.591	**14.1**	2.211	2.022	8.5	1.589	1.978	**-24.5**

Traveling salesman: results of test 1

- Set up time costs (Normalized, dash = weighted distance)



XUE et al: A Learning-based Searching Reform Scheme (EURO XXIV, Lisbon, 2010)

Traveling salesman: results of test 1

- Average time cost comparison (Normalized, G+5-Opt)

		G+5-Opt @ NN			G+5-Opt @ Quadrant			G+5-Opt @ α-nearness		
		Avg	Avg/WD	Imp/%	Avg	Avg/WD	Imp(%)	Avg	Avg/WD	Imp(%)
VLSI	3k	2.20	2.27	**-3.1**	1.93	1.48	23.0	2.32	2.21	**4.6**
	10k	8.09	7.84	**3.2**	8.72	6.64	**23.9**	10.32	9.69	**6.1**
	31k	33.20	31.79	**4.3**	37.66	29.40	**21.9**	42.88	46.18	**-7.7**
	100k	88.57	86.00	2.9	147.62	133.05	**9.9**	158.95	169.70	**-6.8**
	316k	479.84	421.65	**12.1**	675.39	649.52	**3.8**	-	-	-
	1M	1123.66	949.97	15.5	1665.11	1500.81	**9.9**	-	-	-
E	3k	2.60	2.47	**5.1**	1.68	1.87	-11.6	1.98	2.08	-5.3
	10k	9.86	10.28	**-4.2**	7.94	7.56	4.8	8.91	8.56	3.9
	31k	41.81	45.40	**-8.6**	37.18	36.69	1.3	47.20	43.73	7.4
	100k	140.30	156.68	**-11.7**	141.62	139.84	1.3	161.85	167.15	-3.3
	316k	503.66	568.11	**-12.8**	601.57	596.53	0.8	-	-	-
	1M	2141.66	2432.96	**-13.6**	2986.15	3033.61	-1.6	-	-	-
TSPLIB	3k	2.71	2.68	**1.3**	2.18	1.84	15.6	1.88	4.07	**-116.7**
	10k	12.88	13.57	**-5.3**	12.60	11.17	11.3	13.40	13.57	-1.3
	31k	97.50	97.02	**0.5**	81.40	65.16	**19.9**	84.44	97.27	-15.2
	100k	149.99	159.61	**-6.4**	174.31	166.20	**4.7**	134.96	150.73	-11.7

Traveling salesman: results of test 1

- Average time cost comparison (Normalized, G+2-Opt)

		G+2-Opt @ NN			G+2-Opt @ Quadrant			G+2-Opt @ α-nearness		
		Avg	Avg/WD	Imp/%	Avg	Avg/WD	Imp(%)	Avg	Avg/WD	Imp(%)
VLSI	3k	0.30	0.30	0.0	0.28	0.30	-7.1	0.24	0.30	**-25.0**
	10k	1.42	1.36	4.1	1.16	1.33	**-15.0**	1.10	1.27	**-15.8**
	31k	9.21	7.33	**20.4**	6.97	7.37	**-5.7**	6.21	4.27	**31.3**
	100k	32.23	31.02	3.8	26.58	26.58	0.0	22.74	28.06	**-23.4**
	316k	170.32	156.59	8.1	137.81	148.35	-7.6	-	-	-
	1M	460.39	341.91	**25.7**	275.10	297.95	-8.3	-	-	-
E	3k	0.48	0.57	**-20.0**	0.53	0.72	**-36.4**	0.31	0.35	**-11.5**
	10k	2.53	2.72	-7.6	2.70	2.91	-7.9	1.72	2.08	**-21.3**
	31k	12.20	12.38	-1.5	11.73	13.90	**-18.5**	8.34	10.25	**-22.9**
	100k	48.62	54.44	**-12.0**	50.53	51.11	-1.1	33.61	48.62	**-44.6**
	316k	205.92	212.52	-3.2	222.06	232.22	-4.6	-	-	-
	1M	914.88	932.81	-2.0	1175.30	1141.33	2.9	-	-	-
TSPLIB	3k	0.36	0.40	**-11.1**	0.36	0.40	**-11.1**	0.24	0.30	**-25.0**
	10k	1.56	1.76	**-13.0**	1.62	1.94	**-19.6**	1.22	1.59	**-31.0**
	31k	7.00	5.85	**16.5**	4.91	5.20	-5.9	4.41	5.34	**-21.3**
	100k	15.27	15.01	1.8	13.39	16.15	**-20.6**	13.79	16.55	**-20.0**

Traveling salesman: test 2 (branch-and-bound)

- Inputs
  - 10 problems (30 cities), 5 are Euclidean random and 5 are subproblems of the first 5 instances of the VLSI data set:

Category	Problem (source)				
Random	E30.0	E30.1	E30.2	E30.3	E30.4 (generator)
VLSI	xqf30 (xqf131)	xqg30 (xqg237)	pma30 (pma343)	pka30 (pka379)	bcl30 (bcl380)

- Objective algorithm
  - A branch-and-bound, LB by minimum spanning 1 tree.
- Parameters (Class Association Rules, CARs)
  - P* = half (15) problems with a closest density
  - Min confidence of learning = 0.01
  - Min support of learning = 0.05
  - Learn from all candidate sets

Traveling salesman: results of test 2

	Problem	Optimum	Expanded branches			Δ time (%)
			BnB	BnB-WD	Δ (%)	
Random	E30.0	4620393	957	287	**-70.01**	**-61.46**
	E30.1	4539405	1370	1135	**-17.15**	**-31.84**
	E30.2	4778537	327	141	**-56.88**	**-37.50**
	E30.3	4779040	835	1189*	42.40*	80.92
	E30.4	4739803	610	976	60.00	84.87
VLSI	xqf30	128	258	214	**-17.05**	-5.19
	xqg30	158	379	143	**-62.27**	**-36.90**
	pma30	195	866	645	**-25.52**	**-15.33**
	pka30	184	563	547	-2.84	-8.35
	bcl30	149	46	45	-2.17	-4.23

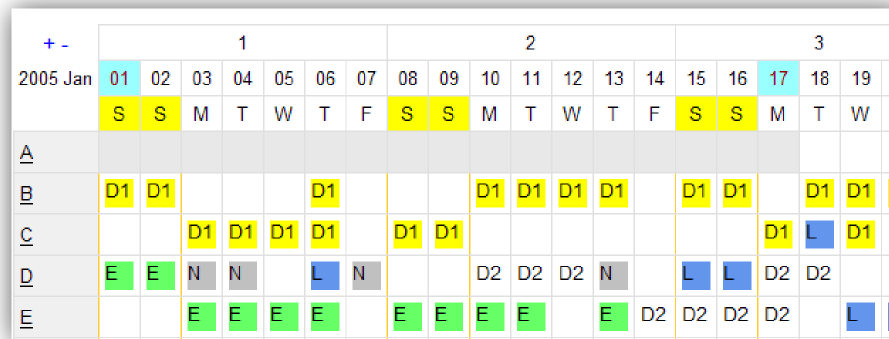
**: when support threshold = 0.2; it was 61201 (!) when 0.05.*

Traveling salesman: results interpretation

- ☺
  - Depending on the search depth, local search **can** be significantly benefited on different candidate sets (NN, Quadrant, α-nearness) over different families (especially industrial) of problems
  - It seems that BnB **can** be significantly benefited, but risks might be there especially when problem is small.
  - The additional time cost is pretty low in very large problems
- ☹
  - Less effective in random than industrial ETSP
  - Less effective for the α-nearness than the NN and the Quadrant candidate sets

Staff rostering as another example

❖ Staff rostering
  - Determine shifts for demands
  - Construct work timetables*



+ -	1							2							3					
2005 Jan	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	2
	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T
A																				
B	D1	D1				D1				D1	D1	D1	D1		D1	D1		D1	D1	D
C			D1	D1	D1	D1		D1	D1								D1	L	D1	
D	E	E	N	N		L	N			D2	D2	D2	N		L	L	D2	D2		
E			E	E	E	E		E	E	E	E		E	D2	D2	D2	D2		L	L

❖ Attributes
  - ID, CN   Employee ID, Contract ID (group)
  - S1, S2   Shift on yesterday, on the day before yesterday
  - SQ   Length of current consecutive working days
  - DW   Day of week
  - St, Ed   Level ($\log_2$) of days from the beginning, to the end
  - LD   Absolute difference of the current employee's workload against the average workload (till yesterday, rounded to integer).
  - JB   Shift to determine

Staff rostering: tests

- Inputs
  - Problems (>10 staff, >20 days, fixed number of shifts) from http://www.cs.nott.ac.uk/~tec/NRP/
  - A set of enlarged problems (no day/shift on/off constraints, enlarged to same employees, 3 months)
- Objective algorithm
  - 4-Hybrid VDS (10 runs) initialized by Greedy
- Parameters (CARs)
  - P* = half scheduling period, or those before
  - Min confidence of learning = 0.01
  - Min support of learning = 0.05*

*: Less training examples (~1,000) than in TSP (~100,000)

Staff rostering: results

❖ Comparisons on two groups of problems

Problem	BK	4-HVDS avg	4-HVDS stddev	4-HVDS time(s)	4-HVDS /Weighted avg	4-HVDS /Weighted stddev	4-HVDS /Weighted time(s)	Δ time (%)
BCV-2.46.1(46x28)	1572*	**1576**	8.7	631.8	1582	10.8	616.2	-2.47
BCV-3.46.1(46x26)	3280^	3314	7.4	1590	**3307**	11.7	1808	13.7
BCV-3.46.2(46x26)	894*^	**896.1**	1.8	1148	898	1.6	1014	-11.7
BCV-6.13.1(13x30)	768	884.9	101.9	211.1	**833.5**	82.1	204.6	-3.07
BCV-A.12.1(12x31)	1294^	2217	493.5	1678	**1983**	403.2	2003	19.4
BCV-A.12.2(12x31)	1953^	**2440**	188.8	2819	2486	298.5	2160	-23.4
ORTEC01(16x31)	270*^	2254	915.5	29.4	**2128**	1731	26.2	-10.9
QMC-1(19x28)	13*	**31.3**	3	61.6	34.7	2.9	50.1	-18.7
SINTEF(24x21)	0*	9	1.9	12.6	**8.8**	2.3	13.5	6.92
Valouxis-1(16x28)	20*	**422**	7.9	6.2	476	98.3	4.6	-26
* Also proved optimal; ^ found by the Hybrid VDS								
EBCV-4.13.1 (13x3m)	-	155.8	28.6	352.3	**153.9**	98.8	413.6	17.4
EBCV-5.4.1 (4x3m)	-	525.9	132.3	0.8	**462.7**	0.5	1.5	89.6
EGPost-B (8x3m)	-	3223	1939	68	**2599**	1411	63.2	-7.1
EMillar-2Shift-DATA1(8x3m)	-	3650	97.2	8.5	**3640**	51.6	6.9	-18.2
EMillar-2Shift-DATA1.1(8x3m)	-	3640	51.6	1.6	**3620**	42.2	2.7	68.3
EValouxis-1 (16x3m)	-	1656	252.8	109.3	**1632**	161.2	143.8	31.5

XUE et al: A Learning-based Searching Reform Scheme (EURO XXIV, Lisbon, 2010)

Staff rostering: results interpretation

- ☺
  - Fits large-scale problems better
  - According to *limited* evidences, the Hybrid VDS can be benefited in quality, if certain criteria (such as “large-enough”) are met
- ☹
  - Although the additional time costs by machine learning are low, the iteration time increases by some percent
  - Preliminary tests only. There might be some other reasons for the quality change (i.e., possibly no improvements by the learning in fact)…

Discussion

- Some characteristics:
  - The parameters of learning (including non-CARs) are easy to determine: set to (feasibly) minimal values
  - The design of decision attributes is the key to a successful application: decentralized, able to borrow the attributes from human heuristics
- Beyond the cases, more challenges await
  - Heuristics/ CO problems incompatible (not homogeneous)?
  - Problems with *many arbitrary global* constraints (e.g., SAT)
  - Constraint satisfaction methods (e.g., revising backtracks like those in BnB?)
  - An encapsulated general purpose (or a list of purposes) optimization program module

Conclusion and future works

- We present an efficient metaheuristic-like approach
  - Small-problem-oriented learning (thus fast)
  - Enhance problem solving with the rules learnt
  - Transparent to the embedded heuristic
- We find the results of tests encouraging.
- We hope it unveils a direction to take the power of machine learning in large-scale optimization.
- Possible future works
  - An general guide of designing the attributes
  - Special plan guide for special industrial practice
  - Challenges listed on last page

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