

# A Longer Look at the Asymmetric Dependence between Hedge Funds and the Equity Market

Byoung Uk Kang, Francis In, Gunky Kim, and Tong Suk Kim\*

## Abstract

This paper reexamines, at a range of investment horizons, the asymmetric dependence between hedge fund returns and market returns. Given the current availability of hedge fund data, the joint distribution of longer-horizon returns is extracted from the dynamics of monthly returns using the filtered historical simulation; we then apply the method based on copula theory to uncover the dependence structure therein. While the direction of asymmetry remains unchanged, the magnitude of asymmetry is attenuated considerably as the investment horizon increases. Similar horizon effects also occur on the tail dependence. Our findings suggest that nonlinearity in hedge fund exposure to market risk is more short term in nature, and that hedge funds provide higher benefits of diversification, the longer the horizon.

## I. Introduction

Returns on many hedge funds exhibit a nonlinear relationship with market returns (see, e.g., Fung and Hsieh (2001), Lo (2001), Mitchell and Pulvino (2001), Agarwal and Naik (2004), and Brown and Spitzer (2006)). This nonlinearity, or asymmetry, in dependence is often of a form where returns are more strongly correlated in bear markets than in flat and bull markets.<sup>1</sup> The implications of such

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\*Kang, afbkang@inet.polyu.edu.hk, School of Accounting and Finance, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong; In, francis.in@buseco.monash.edu.au, Department of Accounting and Finance, Monash University, Clayton, VIC 3168, Australia; G. Kim, gkim@uow.edu.au, School of Mathematics and Applied Statistics, University of Wollongong, Wollongong, NSW 2522, Australia; and T. Kim, tskim@business.kaist.ac.kr, KAIST Business School, KAIST, 87 Hoegiro, Dongdaemoon-gu, Seoul, 130-722, Korea. This paper is part of Kang's dissertation at KAIST that was written under the guidance of the other authors while he was a visiting doctoral student at Monash University. The authors acknowledge useful comments on earlier versions of this paper from Jose Blanchet, Louis Cheng, Timothy Chue, Ben Jacobsen, Samuel Kou, Yoonjung Lee, Peter Mackay, Mujtaba Mian, Wilson Tong, Yong Wang, Steven Wei, and seminar participants at Harvard University; the 2007 Joint Conference with Allied Korea Finance Associations in Dogo, Korea; the 2nd Conference on Asia-Pacific Financial Markets (CAFPM) in Seoul, Korea; the 15th Theories and Practices of Securities and Financial Markets (SFM) Conference in Kaohsiung, Taiwan; Massey University; and the Hong Kong Polytechnic University. The authors are also most grateful to Stephen Brown (the editor) and Andrew Patton (the referee) for particularly helpful suggestions. Any remaining errors are the authors' responsibility.

<sup>1</sup>Agarwal and Naik (2004) conjecture two possible sources of this nonlinearity. First, hedge funds may employ trading strategies that lose money during market downturns, but whose profits made

asymmetric dependence are important in portfolio allocation and risk management involving hedge funds. A hedge fund investor with some existing exposure to market risk (e.g., who fails to incorporate such asymmetry in her portfolio decision) may overstate the diversification benefits offered by hedge funds. The utility loss from the resulting suboptimal portfolio choice becomes more substantial as the magnitude of asymmetry increases (see, e.g., Ang and Chen (2002), Patton (2004), and Hong, Tu, and Zhou (2007)).

Despite the economic significance of its implications, our understanding of the asymmetric dependence between the market and hedge fund returns arises largely from previous empirical studies that are based on data reported at a monthly frequency. In this paper, we attempt to expand the literature by examining a wider range of horizons. The primary questions we address are: Is the asymmetric dependence between the market and hedge fund returns still present at horizons exceeding 1 month? How does lengthening the investment horizon alter, if at all, the magnitude of asymmetry in the dependence structure? In the context of hedge funds, a longer-horizon analysis is particularly important because hedge fund investors are often confronted with liquidity restrictions, such as a lockup provision or a redemption notice period, and this effectively forces investors to take a longer-term view; accordingly, learning about the longer-horizon dependence structure between the market and hedge fund returns is more relevant to investors' decision making.<sup>2</sup>

To date, very little has been said about the implications of investment horizon on cross-asset dependence, beyond linear correlation. The portfolio choice literature shows that predictability of asset returns can introduce a wedge between the correlation structures of short-horizon returns and those of long-horizon asset returns (see, e.g., Campbell and Viceira (2005)). Statistical theory suggests that even when returns are unpredictable, the correlation coefficient can still be a function of the investment horizon, depending on whether the returns are additive or multiplicative (see, e.g., Levy and Schwarz (1997), Levy, Guttman, and Tkatch (2001)). However, there is no such empirical or theoretical guidance on the type or degree of asymmetry to expect in longer-horizon dependence, given a particular type and degree of asymmetry in short-horizon dependence. To our knowledge, this paper is among the first to consider horizon effects on asymmetric dependence between asset returns.

A common problem in hedge fund research is that hedge fund return histories are generally short, rarely extending past 15 years. In this light, we do not

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during the market upturns are unrelated to the extent to which the market goes up. Examples include the risk arbitrage strategy investigated by Mitchell and Pulvino (2001). Second, regardless of trading strategy, managers could create (either directly or indirectly through dynamic trading) a payoff similar to that from writing a put option in order to improve their Sharpe ratio or to respond to their incentive contract.

<sup>2</sup>There has been recognition in the literature of the need for a better sense of investor returns accounting for lockup and notice periods. The first article to discuss this is Agarwal and Naik (2000), who argue that investors should have sufficient information about the performance of hedge funds over a long period before committing money to them. These authors find that persistence in the performance of hedge funds depends on return horizon, with longer-horizon returns showing less evidence of persistence. According to Getmansky, Lo, and Makarov (2004), a finding of persistence at a short horizon can be ascribed to the presence of illiquid securities in hedge fund portfolios.

directly analyze long-horizon returns. Rather the profile, by investment horizon, of dependence structure is derived from the dynamics of monthly returns. This approach is in the spirit of the recent long-term asset allocation literature, which extracts the moments of multiperiod asset returns from the assumed evolution of single-period returns (see, e.g., Campbell and Viceira (2005), Jurek and Viceira (2005), and Bansal and Kiku (2007)). Provided that the properties of monthly returns are well preserved, we can consistently estimate the dependence structure between returns over any desired horizon.

Our empirical analysis proceeds in 2 steps. First, we construct the joint distribution of holding period returns on the market and a given hedge fund, for a series of investment horizons: To span the various lockup and notice periods, the investment horizons we consider range from 1 quarter to 5 years. To accomplish this, we employ the filtered historical simulation (FHS) of Barone-Adesi, Bourgoin, and Giannopoulos (1998) and Barone-Adesi, Giannopoulos, and Vosper (1999). The FHS generates correlated pathways for market and hedge fund returns, allows serial correlation and time-varying volatility, and avoids assumptions about the joint conditional distribution of the market and hedge fund returns. Given that holding period returns are computed by time aggregating the simulated monthly return series, and that the simulated monthly returns under FHS preserve cross-asset dependence along with other aspects in the original data, the joint distributions of holding period returns will contain unbiased information about the longer-horizon dependence structure between market and hedge fund returns.

The next step is to uncover dependence structures embedded in the simulated joint distributions of holding period returns for the investment horizons under consideration. To this end, we use the method based on copula theory. A copula-based approach is natural in situations where the association between the variables is of primary interest, since the effect of the dependence structure can easily be separated from that of the marginals (see Sklar (1959)). In addition, copulas permit an examination of joint behavior at the tails of distribution, because measures of such tail behavior, known as *tail dependence*, can be directly expressed in terms of the copula associated with its joint distribution (see, e.g., Joe (1997)).

We use monthly Standard and Poor's (S&P) 500 and hedge fund index returns from January 1994 to May 2007 to simulate the joint distributions of holding period returns. We focus on a list of hedge fund investment strategies or styles characterized by strong asymmetric dependence on the market. Tail risk borne by these hedge fund strategies is of recent interest due to its close association with the concept of contagion. According to Boyson, Stahel, and Stulz (2007), there is a high probability of contagion among hedge funds, and thus diversification across strategies does not offer protection against extreme negative return on a specific strategy. Similarly, Brown and Spitzer (2006) find that forming a portfolio of hedge funds concentrates rather than dissipates tail risk exposure. While these authors note the difficulties in the cross-sectional diversification of tail risk, we ask in this paper whether such risk can nevertheless be diversified over time. In so doing, we can also address some of the suggestions of Poon, Rockinger, and Tawn (2004), who propose investigating the time-aggregating properties of extreme values and the effect of investment time horizon on tail dependence, among other issues.

Three main results emerge. First, we find that asymmetry in the dependence between market and hedge fund returns is not confined to a particular time horizon, but evident across all holding periods; hence, for a given horizon, the dependence for downside moves is always greater than for upside moves. Second, however, the magnitude of asymmetry is not invariant to investment horizon: The asymmetry is most severe at the quarterly horizons and decreases considerably in extent as the horizon lengthens to 5 years. Finally, a similar horizon effect occurs on the lower tail dependence (i.e., the probability of joint negative events), as it appears inversely related to the length of the investment horizon. In a series of robustness checks, we verify that our findings are not unduly driven by the choice of market index, by the weighting method selected to form hedge fund indexes, or by the choice of simulation methodology.

These results relate to and are consistent with those of Breyman, Dias, and Embrechts (2003) and Dias and Embrechts (2007) in that the dependence structure changes as a function of the time horizon. These authors investigate high-frequency exchange rates at 6 different time horizons and find that the dependence structure is best described by a  $t$ -copula with successively larger degrees of freedom as the time horizon increases. Unlike these authors, we focus on a data set characterized by asymmetric dependence and employ models that accommodate varying extents of asymmetry therein. Less related to our work is Ang and Chen (2002), who pioneered the development of a statistic for testing asymmetries in correlations. In their empirical work using U.S. stock portfolios, the authors observe no discernable pattern in the degree of correlation asymmetries across daily, weekly, and monthly frequencies. Conceptually, however, asymmetric correlations between 2 variables do not necessarily indicate the existence of asymmetry in their association, since asymmetric correlations can be attributable to purely marginal aspects, such as skewness. Our paper focuses exclusively on the intrinsic association between assets and asymmetries therein.

The remainder of this paper is organized as follows. Section II discusses our research design. Section III studies the dependence structure of the market and hedge fund returns over various holding periods. Section IV provides some robustness checks of our results. And Section V summarizes and interprets the results.

## II. Research Design

We begin by recalling one of the fundamental results in copula theory, from Sklar (1959). Sklar's Theorem states that any joint distribution can be represented in terms of the marginals and a dependence function, termed the copula.<sup>3</sup> While information concerning individual variables (e.g., mean, standard deviation, skewness, and kurtosis) is entirely determined by the marginal distributions, the dependence relationship between the variables is completely described by the copula. This allows us to separately treat each variable's marginal distribution and the dependence relation that couples the marginals into a joint distribution.

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<sup>3</sup>See, for example, Joe (1997) for such representation of the joint distribution.

Given this, our research procedure is summarized as follows: First, we construct the joint distribution of  $s$ -period returns on the market and a given hedge fund strategy, which we denote by  $\mathcal{H}^{(s)}(R_{m,t_0+s}, R_{h,t_0+s})$ , where  $R_{i,t_0+s}$  is the return on the market ( $i = m$ ) or the hedge fund ( $i = h$ ) bought at time  $t_0$  and held up to  $t_0 + s$ .<sup>4</sup> By Sklar's Theorem, we can write the joint distribution as

$$(1) \quad \mathcal{H}^{(s)}(R_{m,t_0+s}, R_{h,t_0+s}) = \mathcal{C}^{(s)}(\mathcal{F}^{(s)}(R_{m,t_0+s}), \mathcal{G}^{(s)}(R_{h,t_0+s})),$$

and accordingly, we can separate out  $\mathcal{C}^{(s)}$ , the intrinsic dependence between  $R_{m,t_0+s}$  and  $R_{h,t_0+s}$ , from the marginal distributions  $\mathcal{F}^{(s)}$  and  $\mathcal{G}^{(s)}$ . Our next step is to explore the profile of dependence structures  $\mathcal{C}^{(s)}$  across the various horizons. The specific investment horizons we consider are 1 quarter, 6 months, 1 year, 3 years, and 5 years; hence,  $s \in \{3, 6, 12, 36, 60\}$ . These investment horizons span almost the entire range of minimum holding periods that could be imposed on a typical hedge fund investor via a lockup provision and a redemption notice period.<sup>5</sup>

Note that the subscript on the holding period return incorporates both horizon and time dimensions. In this paper, we consider only the average market condition by deriving the *unconditional* distributions, so that we can isolate the impact of the horizon. However, by formulating conditional distributions, our procedure could also be used to examine the effect of time-varying market conditions on the dependence structure between  $R_{m,t_0+s}$  and  $R_{h,t_0+s}$ .

### A. Simulating a Joint Distribution

We now describe how we construct the (unconditional) joint distribution of  $s$ -period returns,  $\mathcal{H}^{(s)}$ , from the dynamics of single-period returns,  $r_{m,t}$  and  $r_{h,t}$ . A main advantage of the FHS algorithm for this purpose is also discussed along the way. Suppose that  $r_{m,t}$  and  $r_{h,t}$  evolve, respectively, as

$$(2) \quad r_{m,t} = \mu_{m,t} + \sigma_{m,t}\epsilon_{m,t} \quad \text{and}$$

$$(3) \quad r_{h,t} = \mu_{h,t} + \sigma_{h,t}\epsilon_{h,t},$$

where the innovations,  $\epsilon_{m,t}$  and  $\epsilon_{h,t}$ , are independent and identically distributed (i.i.d.) with zero mean and unit variance, and their cumulative distribution functions (CDFs) are denoted by  $\mathcal{F}$  and  $\mathcal{G}$ , respectively;  $\mu_{i,t}$  and  $\sigma_{i,t}$ , for  $i \in \{m, h\}$ , are measurable with respect to information about the return process available up to time  $t - 1$ . We assume that the joint distribution of  $\epsilon_{m,t}$  and  $\epsilon_{h,t}$  is

$$(4) \quad (\epsilon_{m,t}, \epsilon_{h,t}) \sim \mathcal{H} = \mathcal{C}(\mathcal{F}, \mathcal{G}),$$

<sup>4</sup>A single investment period corresponds to 1 month, since the data employed in our empirical work are sampled at a monthly frequency.

<sup>5</sup>According to the Lipper TASS hedge fund database, the lockup period ranges up to 7.5 years but mostly clusters around 1 year; the notice period ranges between 0 and 365 days and exhibits more variability. In effect, the lockup period could be longer than specified because some funds allow redemptions only at the end of the calendar year. For example, if the initial investment made in January 2007 is subject to a 1-year lockup, the earliest withdrawal could be made only at December 2008; hence, the effective lockup is 2 years. The redemption frequency, in a few cases, is as low as biennial or triennial, thereby making the effective lockup period even longer.

where  $\mathcal{C}$  is the copula associated with  $\mathcal{H}$ , the bivariate CDF of  $\epsilon_{m,t}$  and  $\epsilon_{h,t}$ . It is also assumed that the copula  $\mathcal{C}$  and the marginals  $\mathcal{F}$  and  $\mathcal{G}$  are constant over time.

Generally,  $\mathcal{H}^{(s)}$  is not known analytically, even for a known innovation distribution  $\mathcal{H}$ ; hence, we adopt a simulation approach. One commonly used approach is Monte Carlo simulation. This method generates innovations  $(\epsilon_{m,t}, \epsilon_{h,t})$  from assumed distributions with estimated parameters (i.e.,  $\hat{\mathcal{F}}$ ,  $\hat{\mathcal{G}}$ , and  $\hat{\mathcal{C}}$ ).<sup>6</sup> An obvious concern is that they may carry model misspecification and parameter estimation errors, which can be nonnegligible, especially given the short history of hedge fund returns.

The FHS algorithm makes minimal assumptions about underlying innovation distributions and draws the innovations from the empirical distribution of data. Below we summarize how we generate a path of  $r_{m,t}$  and  $r_{h,t}$  (with length equal to  $s$ ) using the FHS (see Barone-Adesi et al. (1998), (1999) for details):

- i) Specify a model for the dynamics of the conditional mean  $\mu_{i,t}$  and volatility  $\sigma_{i,t}$  in each of equations (2) and (3). Estimate equations (2) and (3) separately, while making no assumptions about  $\mathcal{F}$  and  $\mathcal{G}$  and using quasi-maximum likelihood estimation (QMLE). Recover estimates of past realizations of the standardized residuals,  $\hat{\epsilon}_{m,t}$  and  $\hat{\epsilon}_{h,t}$ .
- ii) Make  $s$  random draws (with replacement) of a past date and use each date's realization of the vector  $(\hat{\epsilon}_{m,t}, \hat{\epsilon}_{h,t})$  as a simulated innovation from  $\mathcal{H}$ ; denote the  $j$ th drawn vector of innovations as  $(\epsilon_{m,t_0+j}, \epsilon_{h,t_0+j})$ . Obtain  $(r_{m,t_0+j}, r_{h,t_0+j})$  by combining the estimated models (2) and (3) with simulated innovations  $(\epsilon_{m,t_0+j}, \epsilon_{h,t_0+j})$ ,  $j=1, 2, \dots, s$ .

Here, the initial values  $\mu_{i,t_0+1}$  and  $\sigma_{i,t_0+1}$ , for  $i \in \{m, h\}$ , are set to their unconditional levels to reflect average market conditions. Each simulated path produces a single pair of  $s$ -period returns on the market and the hedge fund, that is,  $(\prod_{j=1}^s (1 + r_{m,t_0+j}) - 1, \prod_{j=1}^s (1 + r_{h,t_0+j}) - 1)$ , and this can be envisaged as an i.i.d. draw from the density function corresponding to  $\mathcal{H}^{(s)}$ . We simulate 10,000 paths to create the simulated joint distribution of  $s$ -period returns.<sup>7</sup>

The core of FHS is the resampling of the historical standardized residuals,  $\hat{\epsilon}_{m,t}$  and  $\hat{\epsilon}_{h,t}$ . It can be visualized that under FHS, draws from  $\mathcal{H}$  are made by randomly drawing, with replacement, the vector of  $\hat{\epsilon}_{m,t}$  and  $\hat{\epsilon}_{h,t}$  from the set of past realizations. Therefore, neither modeling nor parameter estimation is required for  $\mathcal{F}$ ,  $\mathcal{G}$ , and  $\mathcal{C}$ ; the unknown true distributions are estimated nonparametrically by using the empirical distribution of  $(\hat{\epsilon}_{m,t}, \hat{\epsilon}_{h,t})$ . By resampling the vector of  $\hat{\epsilon}_{m,t}$  and  $\hat{\epsilon}_{h,t}$  rather than resampling separately, we ensure that, along with the

<sup>6</sup>The method first generates independent draws of  $(\mathcal{U}_t, \mathcal{V}_t)$  from the estimated copula,  $\hat{\mathcal{C}}$ , and then applies the inverse of estimated CDF for each marginal to  $\mathcal{U}_t$  and  $\mathcal{V}_t$  (i.e.,  $\epsilon_{m,t} = \hat{\mathcal{F}}^{-1}(\mathcal{U}_t)$ ,  $\epsilon_{h,t} = \hat{\mathcal{G}}^{-1}(\mathcal{V}_t)$ ) to obtain  $(\epsilon_{m,t}, \epsilon_{h,t})$ . See, for example, Cherubini, Luciano, and Vecchiato (2004) for details.

<sup>7</sup>Separately,  $\prod_{j=1}^s (1 + r_{i,t_0+j}) - 1$  can be viewed as an i.i.d. draw from the univariate density functions corresponding to  $\mathcal{F}^{(s)}$  ( $i = m$ ) or  $\mathcal{G}^{(s)}$  ( $i = h$ ). Thus, 10,000 paths also create the simulated marginal distribution of  $s$ -period returns on the market, and of  $s$ -period returns on the hedge fund.

statistical artifacts in marginal distributions, intrinsic association between the 2 marginals is preserved in simulated return pathways.<sup>8</sup> Note that we are looking at the longer-horizon properties of returns imputed from the single-period returns. Hence, preserving the characteristics of the original monthly returns is important in consistently estimating the joint distributions of returns over longer holding periods; the FHS that we use for simulation does exactly that.

## B. Identifying Dependence Structures

Sklar's Theorem indicates that the dependence structure of a joint distribution can be isolated by transforming the bivariate variable  $(X, Y)$  to uniformly distributed marginals  $(F(X), G(Y))$ , where  $F$  and  $G$  are the marginal CDFs of  $X$  and  $Y$ , respectively. Looking at  $(F(X), G(Y))$  instead of  $(X, Y)$  reveals the aspect of dependence exclusively. Accordingly, we extract the effect of the dependence structure,  $\mathcal{C}^{(s)}$ , by transforming the simulated data  $(R_{m,t_0+s}, R_{h,t_0+s})$  into  $(\mathcal{F}^{(s)}(R_{m,t_0+s}), \mathcal{G}^{(s)}(R_{h,t_0+s}))$ , for a given  $s \in \{3, 6, 12, 36, 60\}$ . For quantitative description of the nature of the dependence, we apply a broad class of parametric copula functions to the transformed data. As such, the type of dependence structure between market and hedge fund returns and the magnitude of asymmetry therein will be established for the various investment horizons.

Our estimation procedure of the parametric copulas is essentially the semi-parametric approach proposed by Genest, Ghoudi, and Rivest (1995): We do not specify functional forms for marginal distribution functions but use the simulated marginals,  $\mathcal{F}^{(s)}$  and  $\mathcal{G}^{(s)}$ , to map the data into the uniform space. Once the transformed data are plugged into a parametric copula function, the dependence parameters are estimated via the maximization of a pseudo log-likelihood function (see, e.g., Kim, Silvapulle, and Silvapulle (2007) for details of this approach and comparison of different methods). For statistical inference about the estimates of dependence parameters, we rely on the asymptotic distributions derived in Genest et al., which is valid for i.i.d. data. Recalling that our simulated data can be viewed as a set of i.i.d. observations drawn from  $\mathcal{H}^{(s)}$ , the asymptotic theory given in Genest et al. is directly applicable without time filtering.<sup>9</sup>

Besides a separate treatment of dependence and marginal aspects, copulas contain information about the joint behavior in the tails of distributions, namely tail dependence. Tail dependence measures the dependence between variables during extreme events and is defined as the probability of an extremely low (high) value of one variable, given that an extremely low (high) value of the other is

<sup>8</sup>This statement is true only to the extent that there is no time variation in correlation or, more generally, copula between  $\hat{\epsilon}_{m,t}$  and  $\hat{\epsilon}_{h,t}$ . We thank the referee for this important insight. In Section III.B, we address this issue by performing tests for constant correlation between  $\hat{\epsilon}_{m,t}$  and  $\hat{\epsilon}_{h,t}$ .

<sup>9</sup>For time-series data, many previous authors apply autoregressive (AR) or generalized autoregressive conditional heteroskedasticity (GARCH) filters before pursuing the semiparametric approach of Genest et al. (1995). This common practice in the literature is justified theoretically by the recent works of Chen and Fan (2006) and Kim, Silvapulle, and Silvapulle (2008).

observed, or vice versa. Clearly, as below, this definition of tail dependence can be equivalently expressed in terms of the copula

$$\begin{aligned}\lambda_L &\equiv \lim_{\varepsilon \rightarrow 0} \Pr[F(X) \leq \varepsilon | G(Y) \leq \varepsilon] = \lim_{\varepsilon \rightarrow 0} \Pr[G(Y) \leq \varepsilon | F(X) \leq \varepsilon] \\ &= \lim_{\varepsilon \rightarrow 0} \frac{C(\varepsilon, \varepsilon)}{\varepsilon},\end{aligned}$$

provided that the limit exists.<sup>10</sup> A bivariate copula  $C$  is said to have lower tail dependence if  $\lambda_L \in (0, 1]$ , and no lower tail dependence if  $\lambda_L = 0$ . In our application, the value of tail dependence implied from  $C^{(s)}$  is an indication of the likelihood that both the market and the fund simultaneously exhibit extreme returns over a holding horizon of  $s$  months. By estimating the tail dependence corresponding to  $C^{(s)}$  for each  $s \in \{3, 6, 12, 36, 60\}$ , we will be able to document the impact of investment time horizon on tail dependence.

We use 2 complementary methods to estimate tail dependence. First, we estimate tail dependence parametrically by imputing from the estimated dependence parameters given the assumed functional form of parametric copula (see, e.g., Breymann et al. (2003)). Second, we use the nonparametric estimation procedure, whereby the tail dependence is computed based directly on the extreme observations (see, e.g., Schmidt and Stadtmüller (2006)). Generally, the parametric estimates are reliable only if the underlying model is correctly specified. In this regard, the nonparametric estimates have the benefits of imposing no structure on the data. However, as pointed out by Pritsker (2006), implementing FHS with a short historical sample could increase variability and skewness, especially in the tails of the simulated distributions; the parametric estimates, which are based on the entire set of simulation outcomes, may be less subject to such a problem. In light of these 2 issues, we conduct both parametric and nonparametric estimations of tail dependence.

It is also important to note that the copula captures features of the joint distribution that are invariant under monotonic transformations of the marginal variables. Consequently, the results are unaffected by whether we use log returns (i.e., additive) or simple returns (i.e., multiplicative), or by whether we annualize returns or not.

### III. Empirical Analysis

Section III.A describes our data. Section III.B implements the FHS. Section III.C, the heart of this paper, examines various aspects of the dependence of hedge funds on the market index.

#### A. Data

We use data provided by the TASS database as our hedge fund sample. As of July 2007, the TASS database includes 7,761 individual hedge funds, a majority

<sup>10</sup>The upper tail dependence,  $\lambda_U$ , can be defined in a similar fashion (see, e.g., Joe (1997)).



of which report returns net of all fees on a monthly basis. For each fund, TASS provides an array of additional information, including investment strategy, assets under management (AUM), and redemption information. As a market index, we use the S&P 500 for our discussion of the main results and employ the Morgan Stanley Capital International (MSCI) World Index in Section IV to check the robustness of our findings.

In this paper, we concentrate on the following 5 hedge fund strategies: convertible arbitrage (CA), emerging market (EM), event driven (EVD), fund of funds (FOF), and long/short equity (LSE).<sup>11</sup> As of July 2007, these strategies encompass 71.6% of all hedge funds contained in the TASS database. We focus on these particular strategies because of their strong asymmetric dependence on the market (see, e.g., Mitchell and Pulvino (2001), Agarwal and Naik (2004), and Brown and Spitzer (2006)). As will be shown later, other common hedge fund strategies seem better described as having either symmetric dependence on or independence from the market and are thus excluded from further analysis. After all, our goal is to examine how the *asymmetric* dependence established at short (i.e., monthly) horizons alters as investment horizon lengthens; in this respect, it is natural as well as necessary that we choose these 5 styles for our analysis.

For tractability, we construct indexes of individual funds within each of the 5 style categories. To be included in the index, funds must report returns net of fees, and they must report returns on a monthly basis. Funds should also have more than \$5 million of time-averaged AUM. This is to reduce any bias that might be caused by very small funds. Funds that report returns denominated in currencies other than U.S. dollars or funds with no AUM information are also excluded. The remaining sample includes 4,527 funds, of which 160 are CA, 304 are EM, 449 are EVD, 1,017 are FOF, and 1,378 are LSE. We base our discussion of the main results on equal-weighted indexes of hedge funds, and in Section IV we check the robustness of our findings using value-weighted indexes.

Our sample period used to estimate the dynamics of single-period returns in equations (2) and (3) covers January 1994–May 2007. We choose January 1994 as the starting date because hedge fund data are thin prior to the 1990s. In results available from the authors, however, we show that our main results do not change greatly when an extended historical sample from January 1990 is used instead.

## B. GARCH Estimates

Implementation of the FHS begins with fitting a conditional volatility model,  $\mu_{i,t}$  and  $\sigma_{i,t}$ , for  $i \in \{m, h\}$ , in equations (2) and (3), to historical return data. Importantly, the selected model should remove serial correlation and volatility clustering from the return series and thus produce i.i.d. standardized residuals. Ensuring i.i.d. here is an important criterion in the model specification because the standardized residuals will form a set of innovations in the FHS (see Barone-Adesi et al. (1998), (1999)). For the dynamics of monthly market returns, we specify a GARCH(1,1) process, since returns are serially uncorrelated at this

<sup>11</sup> See <http://www.hedgeindex.com> for descriptions of each investment style.

frequency but show strong GARCH effects; hence,  $\mu_{m,t} = \mu_m$  and  $\sigma_{m,t}^2 = \omega_m + a_m \epsilon_{m,t-1}^2 + b_m \sigma_{m,t-1}^2$ . While there are many conditional volatility models in the literature, the GARCH(1,1) process is the most widely used to capture heteroskedasticity, due to its tractability.

It is now well recognized that hedge fund returns exhibit strong positive serial correlation (see, e.g., Asness, Krail, and Liew (2001), Getmansky et al. (2004)). Thus, for the dynamics of hedge fund returns, we specify a moving average (MA) model to filter the serial correlation. We apply different orders of MA terms to each hedge fund style and settle on a model that produces i.i.d. (standardized) residuals in the most parsimonious way. The selected orders are mostly 1 or 2 (i.e.,  $\mu_{h,t} = \mu_h + \theta_1 \epsilon_{h,t-1}$  or  $\mu_{h,t} = \mu_h + \theta_1 \epsilon_{h,t-1} + \theta_2 \epsilon_{h,t-2}$ ). Unlike serial correlation, heteroskedasticity is not present in all fund styles. For those that exhibit volatility dynamics, GARCH(1,1) models are specified (i.e.,  $\sigma_{h,t}^2 = \omega_h + a_h \epsilon_{h,t-1}^2 + b_h \sigma_{h,t-1}^2$ ). Specification for each strategy is indicated in Table 1.

TABLE 1  
Parameters for Filtered Historical Simulations

Table 1 reports the estimates of models for single-period (i.e., monthly) return dynamics using data from January 1994 through May 2007. The estimated model for the S&P 500 Index returns has the form  $r_{m,t} = \mu_m + \sigma_{m,t} \epsilon_{m,t}$ ,  $\sigma_{m,t}^2 = \omega_m + a_m \epsilon_{m,t-1}^2 + b_m \sigma_{m,t-1}^2$ , and the estimated models for the hedge fund returns, while different across the strategies, are nested within the following specification:  $r_{h,t} = \mu_h + \theta_1 \epsilon_{h,t-1} + \theta_2 \epsilon_{h,t-2} + \sigma_{h,t} \epsilon_{h,t}$ ,  $\sigma_{h,t}^2 = \omega_h + a_h \epsilon_{h,t-1}^2 + b_h \sigma_{h,t-1}^2$ . The precise specification for each strategy is indicated in the table. The abbreviations for different hedge fund strategies are convertible arbitrage (CA), emerging market (EM), event driven (EVD), fund of funds (FOF), and long/short equity (LSE). All parameters except for those of CA and EVD are estimated using the QMLE procedure, and parameters of CA and EVD are estimated by the ordinary least squares (OLS) method. The corresponding standard errors are provided in parentheses. LB(15), LB<sup>2</sup>(15), and LB<sup>x</sup>(15) are the Ljung-Box (1978) test statistics for up to 15th-order serial correlation in the (standardized) residuals, the squared (standardized) residuals, and the cross product with the market residual, respectively. The  $\chi^2$  value is the Engle and Sheppard (2001) statistic with 5 lags for the null of constant correlation between the market's and fund's residuals against an alternative of dynamic conditional correlation. The corresponding *p*-values are provided in square brackets.

Coefficient	S&P 500	CA	EM	EVD	FOF	LSE
$\mu_i$	1.040 (0.273)	0.795 (0.143)	1.517 (0.351)	1.027 (0.136)	0.629 (0.117)	1.130 (0.190)
$\theta_1$		0.529 (0.077)	0.298 (0.080)	0.406 (0.077)	0.302 (0.081)	0.265 (0.085)
$\theta_2$		0.244 (0.077)		0.176 (0.078)		
$\omega_i$	0.265 (0.340)		0.337 (0.358)		0.052 (0.059)	0.164 (0.158)
$a_i$	0.130 (0.040)		0.129 (0.146)		0.124 (0.050)	0.124 (0.056)
$b_i$	0.859 (0.052)		0.855 (0.105)		0.851 (0.071)	0.853 (0.051)
LB(15)	9.166 [0.869]	10.249 [0.673]	13.677 [0.474]	7.618 [0.868]	15.083 [0.373]	17.390 [0.236]
LB <sup>2</sup> (15)	6.025 [0.979]	3.624 [0.995]	22.800 [0.064]	2.445 [0.999]	13.416 [0.494]	15.945 [0.317]
LB <sup>x</sup> (15)		2.662 [0.999]	6.907 [0.960]	2.498 [0.999]	8.749 [0.890]	8.655 [0.895]
$\chi^2$ value	5.052 [0.537]	4.522 [0.606]	4.522 [0.606]	3.364 [0.762]	9.691 [0.138]	11.070 [0.086]

As mentioned earlier, we estimate the parameters of the specified models using QMLE. The QMLE method treats  $\epsilon_{i,t}$ ,  $i \in \{m, h\}$  as if it is normally

distributed, that is,  $\mathcal{F}, \mathcal{G} = \Phi$ , where  $\Phi$  is the standard univariate normal distribution function. Although this amounts to estimating a model using a distributional assumption that is not necessarily true, QMLE delivers a consistent and asymptotically normal estimator (Bollerslev and Wooldridge (1992)). Hence, the parameters and the conditional volatilities,  $\hat{\sigma}_{i,t}$ , obtained from the estimation recover estimates of standardized residuals,  $\hat{\epsilon}_i$ , for  $i \in \{m, h\}$ , at each past date. The coefficient estimates and their standard errors are reported in Table 1. Diagnostics confirm that residuals are without any dependency in the 1st or 2nd moments.

Table 1 also presents the results of 2 tests for the null of constant correlation between the residuals  $\hat{\epsilon}_{m,t}$  and  $\hat{\epsilon}_{h,t}$ : a Ljung-Box (LB) (1978) test on the cross products of the residuals, and the test of Engle and Sheppard (2001) against an alternative of dynamic conditional correlation. The results show no evidence of violation of the assumption underlying the FHS, that is, the assumption of constant correlation or, more generally, constant copula between the residuals.<sup>12</sup> In the general implementation of the FHS, however, this assumption should not be taken for granted and should be tested in each individual case to ensure the validity of subsequent analyses. Indeed, given the dynamic nature of hedge fund strategies, it is quite possible that the correlation constancy may be overturned in a future study when a longer time series is available.<sup>13</sup> One possible refinement of the FHS method in this regard would be to estimate a multivariate volatility model (e.g., Engle (2002)) to capture time-varying correlations. One could then resample the *jointly* standardized residuals, which may reasonably be assumed to have constant correlation over time.<sup>14</sup>

At this stage, it is instructive to take a look at dependence structure in historical standardized residuals,  $(\hat{\epsilon}_{m,t}, \hat{\epsilon}_{h,t})$ , although we do not attempt to model it for simulation purpose. In Figure 1, we plot  $\hat{\mathcal{F}}(\hat{\epsilon}_{m,t})$  against  $\hat{\mathcal{G}}(\hat{\epsilon}_{h,t})$ , where  $\hat{\mathcal{F}}$  and  $\hat{\mathcal{G}}$  are the empirical CDFs of the 2 residuals; this plot is more helpful than a plot of  $\hat{\epsilon}_{m,t}$  against  $\hat{\epsilon}_{h,t}$  because this transformation removes the influence of marginal aspects. For the purpose of comparison, the figure also presents analogous scatterplots for several other investment styles regularly featured in hedge fund research. Figure 1 reveals that for the 5 styles considered, the points tend to concentrate near the vertex  $(0, 0)$  in the unit square, implying greater dependence for joint negative events than for joint positive events, and also that such asymmetry seems less prevalent, if present at all, in other common hedge fund styles. These observations are consistent with the literature that has indicated the existence or nonexistence of asymmetry in a variety of contexts, such as beta asymmetries in up and down markets (Lo (2001)), nonlinear option-like payoffs (Mitchell and Pulvino (2001), Agarwal and Naik (2004)), and tail nonneutrality of hedge funds (Brown and Spitzer (2006)). Recall that  $(\hat{\epsilon}_{m,t}, \hat{\epsilon}_{h,t})$  defines the background driving process for the dynamics of single-period returns in our simulation. Thus, it is

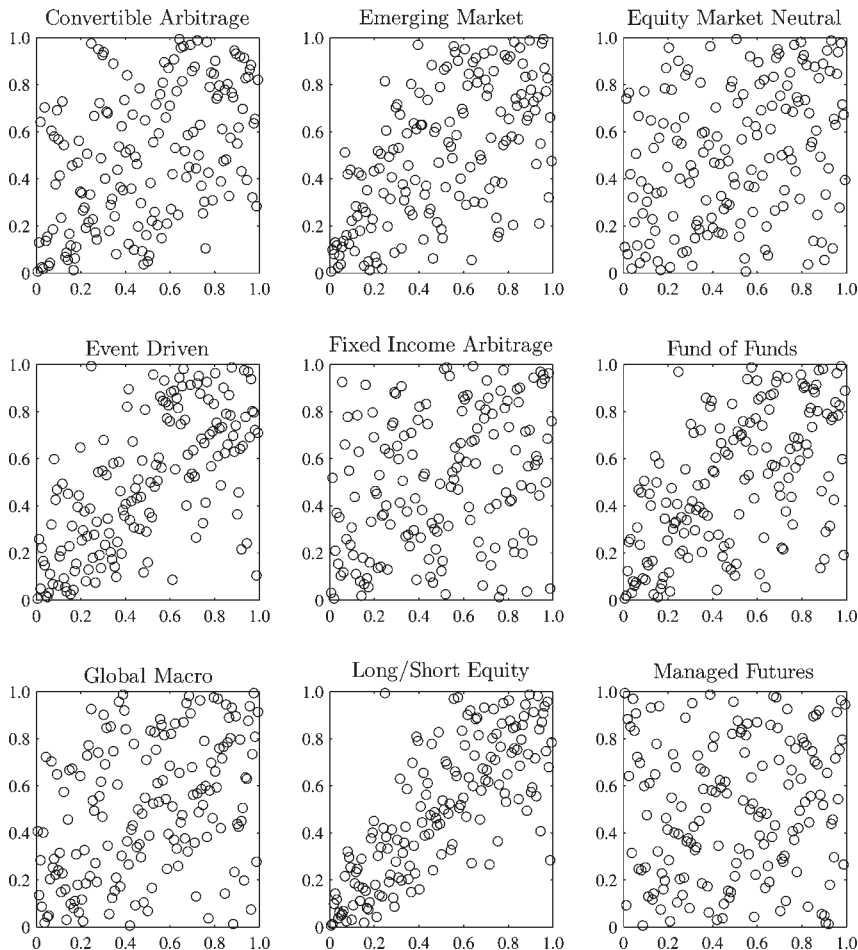
<sup>12</sup>We find that these results are robust to the choice of lag length.

<sup>13</sup>In addition, the assumption of constant correlation becomes less appropriate, the longer the sample (Pritsker (2006)).

<sup>14</sup>We thank the referee for this important suggestion on possible refinement of the FHS method under time-varying correlation.

FIGURE 1  
Scatter Plots of Transformed Residuals

Figure 1 shows the scatter plots of transformed residuals of market returns,  $\mathcal{F}(\hat{\epsilon}_{m,t})$ , versus transformed residuals of hedge fund returns,  $\mathcal{G}(\hat{\epsilon}_{h,t})$ . The transformed residuals of market returns are shown on the horizontal axis, and the transformed residuals of hedge fund returns are shown on the vertical axis.



expected that the dependence we observe here will exert the most direct influence on the dependence structure between 1-month holding period returns; but, how this dependence changes with the investment horizon remains to be seen.<sup>15</sup>

<sup>15</sup>In effect, the dependence structure between the simulated 1-month holding period returns is identical to that between the historical standardized residuals. Recall that in our simulation, the initial values,  $\mu_{i,t}$  and  $\sigma_{i,t}$ , for  $i \in \{m, h\}$  are set to predetermined levels. This means that first month returns (i.e., 1-month holding period returns) are monotonic transformations of historical standardized residuals. As noted in Section II.B, dependence is invariant under monotonic transformations of the marginal variables.

### C. Dependence Structure at Different Investment Horizons

We now present the main results of this paper: the estimation results for the dependence models. Before formally specifying alternative copula functions, we first provide a visual overview of  $\mathcal{C}^{(s)}$  across investment horizons. To do so, we use the measures of exceedance correlations presented in Longin and Solnik (2001), Ang and Chen (2002), and Hong et al. (2007). An exceedance correlation at a threshold level is defined as the correlation between 2 variables given that both variables exceed that threshold. We follow Patton (2004) and transform the data before subjecting them to the computation, such that

$$\rho_s(q) \equiv \begin{cases} \text{corr}[\mathcal{F}^{(s)}(R_{m,t_0+s}), \mathcal{G}^{(s)}(R_{h,t_0+s}) | \mathcal{F}^{(s)}(R_{m,t_0+s}) \leq q, \\ \quad \mathcal{G}^{(s)}(R_{h,t_0+s}) \leq q], & \text{if } q \leq 0.5, \\ \text{corr}[\mathcal{F}^{(s)}(R_{m,t_0+s}), \mathcal{G}^{(s)}(R_{h,t_0+s}) | \mathcal{F}^{(s)}(R_{m,t_0+s}) \geq q, \\ \quad \mathcal{G}^{(s)}(R_{h,t_0+s}) \geq q], & \text{if } q \geq 0.5. \end{cases}$$

By construction, the shape of this exceedance correlation plot, as a function of threshold quantile  $q$ , will directly depend on the underlying copula, having removed the influence of marginal distributions. Figure 2 shows the exceedance correlations for CA with the market at the 5 different time horizons.<sup>16</sup>

From Figure 2, we can already see some implications of the time horizon. First, we observe that the exceedance correlations for  $q \leq 0.5$  are always greater than the exceedance correlations for  $q \geq 0.5$ , indicating that the dependence between market and hedge fund returns maintains the same form of asymmetry across all horizons. Second, however, we find that the distance between the exceedance correlations for the  $q$ th and the  $(1 - q)$ th quantiles tends to decrease as we move to longer horizons, suggesting that the magnitude of the asymmetry in the dependence may be inversely related to the length of the investment horizon.

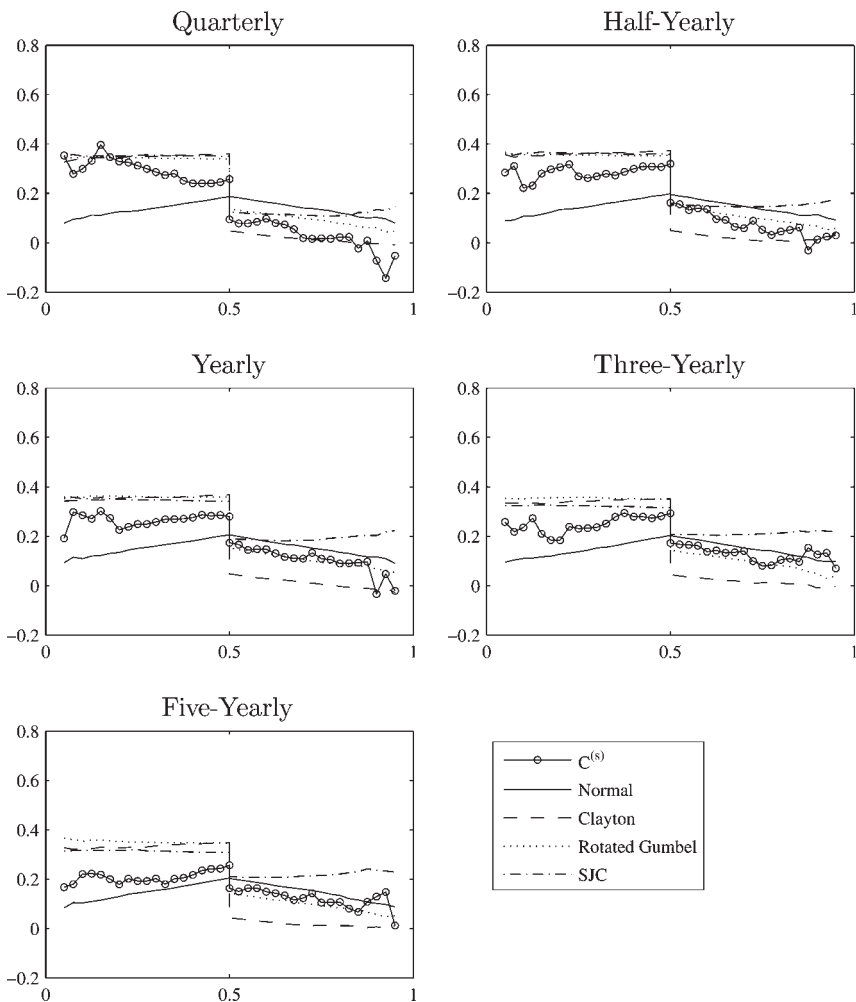
Along with the exceedance correlations implied from  $\mathcal{C}^{(s)}$ , Figure 2 also plots the exceedance correlations suggested under assumption of the following 4 alternative parametric copulas: the normal, rotated Gumbel, Clayton, and symmetrized Joe-Clayton (SJC) copulas. The normal (or Gaussian) copula is often taken as the benchmark copula in the literature, and the rotated (or survival) Gumbel, Clayton, and SJC copulas are among the few reported in the literature that are capable of matching the form of asymmetry indicated in Figure 2. As will be shown, none of these basic models, taken alone, adequately explains the data, but they provide useful guidance as to the class of parametric copula families we can build upon when formulating a more flexible model. The dependence parameters used for generating the plots are obtained by fitting each model to the transformed data. The functional forms of these copulas are given in the Appendix.

Clearly, the normal copula, which implies a symmetric dependence structure, fails to capture larger exceedance correlations for downside moves ( $q \leq 0.5$ ) than for upside moves ( $q \geq 0.5$ ). The failure of the normal copula model is more marked at shorter horizons, but considerably less so at longer horizons.

<sup>16</sup>Plots for other hedge fund styles are qualitatively similar and are omitted for the sake of brevity. However, they are available from the authors.

FIGURE 2  
Exceedance Correlations between Transformed Returns

Figure 2 plots the exceedance correlations between transformed holding period returns,  $\mathcal{F}^{(s)}(R_{m,t_0+s})$  and  $\mathcal{G}^{(s)}(R_{h,t_0+s})$ , at horizons of 3, 6, 12, 36, and 60 months. The horizontal axis shows the threshold quantile, and the vertical axis shows the correlation between convertible arbitrage and the market return given that both exceed that quantile.



The rotated Gumbel and Clayton copulas, which impose asymmetric dependence, do produce asymmetries between upside and downside exceedance correlations but tend to overstate their extent, especially at longer horizons. Finally, the SJC copula, which allows for but does not impose asymmetric dependence, also produces larger exceedance correlations for downside moves than for upside moves, but unlike the rotated Gumbel and Clayton copulas, its upside exceedance correlations do not decay as  $q$  goes to 1. This is due to the fact that the estimated dependence parameters of the SJC copula induce nonzero upper tail dependence.

Given that the exceedance correlations implied from  $\mathcal{C}^{(s)}$  taper off to 0 at a similar rate to those implied from the other copulas that impose upper tail independence (i.e.,  $\lambda_U = 0$ ), there seems no need for modeling upper tail dependence for our simulated returns.

### 1. Mixture Copulas

Our observations thus far compel us to be more flexible in specifying a model of the dependence structure between market and hedge fund returns. The above models, although capturing the direction of asymmetry, are far from satisfactory for our data in terms of the magnitude of asymmetry. That is, for a given model, the variation in dependence parameter(s), when fitted to different data sets, tells us virtually nothing about differences in the magnitude of asymmetry across the data sets (see, e.g., Figure 2).

With the aim of gaining flexibility, we follow Hu (2006), Hong et al. (2007), and Rodriguez (2007), among others, in specifying a *mixture of copulas*. By mixing (i.e., weighted averaging) carefully chosen component copulas, we can capture dependence structures that do not belong to any individual copula. Mixture copulas considered in the literature are often formed by mixing a symmetric copula (often, an elliptical model) and an asymmetric Archimedean copula.<sup>17</sup> In our application, we choose the rotated Gumbel or Clayton copulas for the asymmetric component, and the normal copula for the symmetric component in the mixture. The choice of the asymmetric component is straightforward given the observed form of asymmetry in our simulated data; the choice of the symmetric component is motivated by traditional approaches based on the Gaussian assumption. Hong et al. argue for the inclusion of the normal copula in the mixture on the basis that the normality assumption is widely used in both theoretical and empirical studies, hence it might be extreme to rule it out entirely. Hu notes that the estimated weight on the normal copula is informative about the role of the Gaussian dependence structure between financial markets.

Thus, the final 2 alternative models considered are a mixture of the normal copula and the rotated Gumbel copula (MIX1), specified as

$$C_{\text{MIX1}}(u, v; w, \rho, \delta) = w \cdot C_N(u, v; \rho) + (1 - w) \cdot C_{\text{RG}}(u, v; \delta),$$

and a mixture of the normal copula and the Clayton copula (MIX2), specified as

$$C_{\text{MIX2}}(u, v; w, \rho, \theta) = w \cdot C_N(u, v; \rho) + (1 - w) \cdot C_C(u, v; \theta),$$

where  $w \in [0, 1]$  is the weight on the normal copula, and  $\rho$ ,  $\delta$ , and  $\theta$  are dependence parameters of the normal (denoted by  $C_N$ ), rotated Gumbel (denoted by  $C_{\text{RG}}$ ), and Clayton (denoted by  $C_C$ ) copulas, respectively.

The shape of the exceedance correlation plot in Figure 2 lends support to these specifications, following the observations that the exceedance correlations

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<sup>17</sup>Some authors specify mixtures that include 2 asymmetric Archimedean copulas as their components, where one generates lower tail dependence and the other generates upper tail dependence. We opt not to consider these specifications, due to the seemingly absent upper tail dependence in our simulated data.

from  $\mathcal{C}^{(s)}$  wander between those implied from the normal copula and those from the Clayton or rotated Gumbel copulas, both of which the mixture copulas nest as special cases. While any positive  $w$  would correspondingly attenuate the overstated degree of asymmetry associated with the rotated Gumbel or Clayton copulas, the above specifications let the data determine  $w$ , which thereby summarizes the degree of asymmetry in the data. According to Hong et al. (2007),  $w$  is the key parameter of the mixture models that controls the degree of asymmetry in the dependence structure.

Figure 3 highlights that the flexibility gained by specifying mixture copulas justifies the extra complexity. Similar to Breymann et al. (2003), we plot, for each model and for each investment horizon, the deviation of the Bayesian information criterion (BIC) of the model from that of the MIX1 copula.<sup>18</sup> By inspection, the mixture models stand out as clearly preferable to all other alternative copulas; only a small difference is observed between the MIX1 and MIX2 copulas. Hence, we focus our discussion of the dependence parameter estimates on these 2 mixture models.

## 2. Estimation Results

Tables 2 and 3 present estimation results for the MIX1 and MIX2 copulas, respectively; tables detailing all the models are available from the authors. Throughout, results are estimated with great precision, due to the large size of our simulated data; hence, all estimates presented below are significantly different from 0 or are significantly different from that implying independence, unless otherwise stated.<sup>19</sup>

The message from the dependence parameter estimates is clear. The weight parameter,  $w$ , increases monotonically with the length of the investment horizon, confirming that dependence structures at longer horizons are more symmetric (or less asymmetric) than at shorter horizons; in all cases (even at the 5-year horizon),  $w$  stays significantly below unity, indicating that the asymmetry continues to exist in the longer-horizon dependence structure, although to a decreasing extent. Taking as an example the dependence between the market and the CA index (Panel A of Table 2), the MIX1 copula assigns about 21% of the weight to the normal copula at the quarterly horizon. The weight tends to increase as we move to longer horizons, and by the 5-year horizon, about 67% of the weight is assigned to the normal copula. Similarly, in the case of MIX2 copula (Panel A of Table 3), about 39% of the weight is assigned to the normal copula at the quarterly horizon, and it gradually increases to 73% by the 5-year horizon. In any case, no discernable pattern obtains for the other dependence parameters. To ensure that our evidence is sound, we also apply the  $J_\rho$  statistic of Hong et al. (2007) to our (transformed) data and draw very similar implications (not reported): We find

<sup>18</sup>Note that the BIC puts a heavier penalty on the number of model parameters than the Akaike information criterion employed in Breymann et al. (2003).

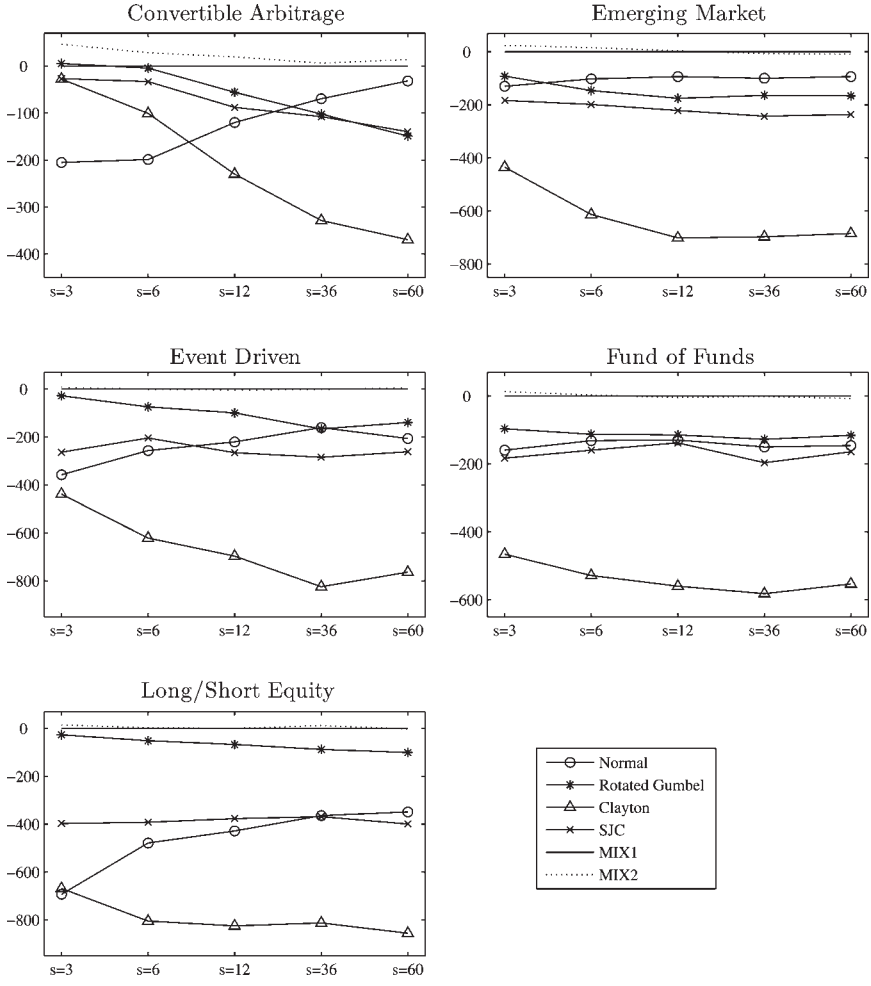
<sup>19</sup>We could estimate with even greater precision by simply simulating more draws. These results, however, should be interpreted keeping in mind that the standard errors reported here condition on *simulated* distribution functions. Interpreting such distributions themselves as approximations of the *true* distributions would mean increasing the standard errors nontrivially. We thank the referee for bringing this to our attention.



FIGURE 3

Comparison of the Bayesian Information Criterion Values across Alternative Copula Models

Figure 3 compares the Bayesian information criterion (BIC) values of alternative copula models across different holding periods. The quantity plotted on the vertical axis is the BIC of the MIX1 copula minus the BIC of the model (i.e.,  $BIC(MIX1) - BIC(model)$ ).



that, across all horizons and strategies, the null hypothesis of symmetric correlation is strongly rejected, which is consistent with the significant weight on the asymmetric component. More importantly, we also find that the obtained  $J_\rho$  statistic tends to decline as the investment horizon increases.

Notice as an aside that the estimated value of the weight parameter assigned by the MIX2 copula is always greater than that in the MIX1 copula. This is perhaps due to the fact that the asymmetry predicted from the Clayton copula is more severe than in the rotated Gumbel copula, even when they are fitted to the same data (see, e.g., Figure 2); consequently, the MIX2 copula, which employs the Clayton copula as its asymmetric component, puts more weight on the normal

TABLE 2  
Estimation of the MIX1 Copula Model

The first 6 columns of Table 2 report the estimates of the parameters in the MIX1 copula using the semiparametric approach proposed by Genest et al. (1995). The MIX1 copula is specified as  $C_{MIX1}(u, v; w, \rho, \delta) = w \cdot C_N(u, v; \rho) + (1 - w) \cdot C_{RG}(u, v; \delta)$ , where  $w \in [0, 1]$  is the weight on the normal copula, and  $\rho$  and  $\delta$  are dependence parameters of the normal (denoted by  $C_N$ ) and rotated Gumbel (denoted by  $C_{RG}$ ) copulas, respectively. The 7th column of the table reports the resultant lower tail dependence estimates, which is related to the estimated dependence parameters by  $\lambda_L(\hat{\vartheta}) = (2 - 2^{1/\delta})(1 - w)$  under the specification of MIX1 copula. Given that the semiparametric estimates are asymptotically normally distributed, and that  $\lambda_L$  is a suitably smooth function of the estimates, the standard errors are approximated by the  $\delta$  method and are given in the 8th column of the table. The last 2 columns of the table report the 1%-quantile dependence estimates,  $\hat{\lambda}_{1\%}$ , and the corresponding standard errors. The 1%-quantile dependence is calculated as  $\hat{\lambda}_{1\%} = C^{(s)}(0.01, 0.01)/0.01$ , and the standard errors are computed by jackknife procedure. Est. and Std. stand for estimate and standard error, respectively.

	<i>w</i>		$\rho$		$\delta$		$\lambda_L(\hat{\vartheta})$		$\hat{\lambda}_{1\%}$	
	Est.	Std.	Est.	Std.	Est.	Std.	Est.	Std.	Est.	Std.
<i>Panel A. Convertible Arbitrage</i>										
<i>s</i> = 3	0.207	0.083	0.473	0.134	1.410	0.051	0.290	0.027	0.440	0.075
<i>s</i> = 6	0.252	0.072	0.501	0.099	1.441	0.052	0.286	0.026	0.400	0.063
<i>s</i> = 12	0.429	0.059	0.533	0.045	1.427	0.051	0.214	0.025	0.290	0.054
<i>s</i> = 36	0.550	0.056	0.528	0.032	1.378	0.053	0.156	0.025	0.220	0.047
<i>s</i> = 60	0.674	0.060	0.497	0.032	1.415	0.095	0.120	0.025	0.230	0.048
<i>Panel B. Emerging Market</i>										
<i>s</i> = 3	0.486	0.059	0.628	0.034	1.670	0.075	0.250	0.028	0.490	0.071
<i>s</i> = 6	0.569	0.053	0.651	0.026	1.686	0.084	0.212	0.027	0.390	0.062
<i>s</i> = 12	0.606	0.051	0.652	0.021	1.657	0.076	0.189	0.029	0.340	0.058
<i>s</i> = 36	0.585	0.050	0.639	0.020	1.666	0.068	0.201	0.028	0.320	0.082
<i>s</i> = 60	0.593	0.050	0.639	0.020	1.610	0.063	0.188	0.028	0.280	0.053
<i>Panel C. Event Driven</i>										
<i>s</i> = 3	0.276	0.049	0.704	0.035	1.783	0.044	0.380	0.030	0.500	0.071
<i>s</i> = 6	0.393	0.050	0.719	0.024	1.780	0.051	0.318	0.031	0.430	0.065
<i>s</i> = 12	0.429	0.047	0.694	0.024	1.843	0.058	0.310	0.028	0.410	0.064
<i>s</i> = 36	0.545	0.047	0.716	0.016	1.746	0.059	0.233	0.028	0.410	0.077
<i>s</i> = 60	0.499	0.048	0.707	0.018	1.781	0.055	0.262	0.029	0.420	0.065
<i>Panel D. Fund of Funds</i>										
<i>s</i> = 3	0.460	0.050	0.661	0.025	1.561	0.048	0.238	0.028	0.380	0.062
<i>s</i> = 6	0.502	0.053	0.637	0.026	1.586	0.058	0.225	0.029	0.330	0.057
<i>s</i> = 12	0.505	0.053	0.630	0.027	1.587	0.060	0.224	0.029	0.340	0.058
<i>s</i> = 36	0.519	0.049	0.650	0.021	1.565	0.048	0.213	0.027	0.270	0.052
<i>s</i> = 60	0.503	0.050	0.623	0.024	1.553	0.050	0.217	0.028	0.330	0.057
<i>Panel E. Long/Short Equity</i>										
<i>s</i> = 3	0.234	0.044	0.851	0.017	2.083	0.041	0.463	0.031	0.540	0.068
<i>s</i> = 6	0.294	0.045	0.815	0.019	2.115	0.050	0.432	0.031	0.450	0.067
<i>s</i> = 12	0.323	0.042	0.804	0.019	2.080	0.049	0.409	0.028	0.460	0.068
<i>s</i> = 36	0.377	0.044	0.781	0.018	1.992	0.050	0.364	0.029	0.450	0.067
<i>s</i> = 60	0.401	0.044	0.772	0.016	1.961	0.047	0.345	0.029	0.440	0.066

copula than the MIX1 copula to generate a similar degree of asymmetry. It is also interesting to find that the estimated value of correlation,  $\rho$ , in the mixtures is greater than that in the normal copula. This finding is consistent with Hong et al. (2007), who note that controlling asymmetry in the sample increases the correlation of the rest of the sample.

Turning to tail dependence, we first look at tail dependence estimates imputed from the estimated dependence parameters,  $\lambda(\hat{\vartheta})$ . The 4th column of Tables 2 and 3 presents results for lower tail dependence estimates,  $\lambda_L(\hat{\vartheta})$ ; results for upper tail counterparts are omitted, since both mixture copulas imply no upper tail dependence. From the tables, we see a clear horizon effect similar to that observed with the degree of asymmetry in the overall dependence. Specifically, we

TABLE 3  
 Estimation of the MIX2 Copula Model

The first 6 columns of Table 3 report the estimates of the parameters in the MIX2 copula using the semiparametric approach proposed by Genest et al. (1995). The MIX2 copula is specified as  $C_{\text{MIX2}}(u, v; w, \rho, \theta) = w \cdot C_N(u, v; \rho) + (1 - w) \cdot C_C(u, v; \theta)$ , where  $w \in [0, 1]$  is the weight on the normal copula, and  $\rho$  and  $\delta$  are dependence parameters of the normal (denoted by  $C_N$ ) and Clayton (denoted by  $C_C$ ) copulas, respectively. The 7th column of the table reports the resultant lower tail dependence estimates, which are related to the estimated dependence parameters by  $\lambda_L(\hat{\vartheta}) = 2^{-1/\theta}(1 - w)$  under the specification of MIX2 copula. Given that the semiparametric estimates are asymptotically normally distributed, and that  $\lambda_L$  is a suitably smooth function of the estimates, the standard errors are approximated by the delta method and are given in the 8th column of the table. Est. and Std. stand for estimate and standard error, respectively.

	$w$		$\rho$		$\theta$		$\lambda_L(\hat{\vartheta})$	
	Est.	Std.	Est.	Std.	Est.	Std.	Est.	Std.
<i>Panel A. Convertible Arbitrage</i>								
$s = 3$	0.389	0.058	0.484	0.044	0.782	0.071	0.252	0.021
$s = 6$	0.438	0.052	0.519	0.037	0.805	0.074	0.238	0.021
$s = 12$	0.556	0.045	0.544	0.026	0.747	0.075	0.176	0.020
$s = 36$	0.642	0.044	0.540	0.022	0.624	0.076	0.118	0.019
$s = 60$	0.729	0.047	0.507	0.022	0.727	0.130	0.105	0.019
<i>Panel B. Emerging Market</i>								
$s = 3$	0.648	0.040	0.637	0.017	1.197	0.115	0.197	0.020
$s = 6$	0.713	0.036	0.655	0.015	1.209	0.138	0.162	0.019
$s = 12$	0.747	0.033	0.654	0.013	1.135	0.126	0.137	0.019
$s = 36$	0.750	0.032	0.639	0.012	1.195	0.126	0.140	0.019
$s = 60$	0.748	0.032	0.637	0.012	1.064	0.108	0.131	0.019
<i>Panel C. Event Driven</i>								
$s = 3$	0.556	0.030	0.699	0.013	1.407	0.079	0.271	0.019
$s = 6$	0.628	0.031	0.720	0.012	1.291	0.086	0.217	0.020
$s = 12$	0.666	0.029	0.696	0.011	1.536	0.105	0.213	0.019
$s = 36$	0.721	0.029	0.715	0.010	1.228	0.100	0.159	0.019
$s = 60$	0.692	0.029	0.709	0.010	1.316	0.094	0.182	0.019
<i>Panel D. Fund of Funds</i>								
$s = 3$	0.616	0.035	0.664	0.016	0.922	0.073	0.181	0.019
$s = 6$	0.662	0.036	0.641	0.015	0.991	0.092	0.168	0.020
$s = 12$	0.675	0.036	0.634	0.015	0.987	0.097	0.161	0.020
$s = 36$	0.677	0.032	0.649	0.013	0.936	0.078	0.154	0.019
$s = 60$	0.670	0.033	0.626	0.014	0.920	0.080	0.155	0.019
<i>Panel E. Long/Short Equity</i>								
$s = 3$	0.539	0.024	0.823	0.007	1.823	0.070	0.315	0.018
$s = 6$	0.602	0.025	0.795	0.008	2.026	0.097	0.282	0.018
$s = 12$	0.606	0.025	0.794	0.008	1.897	0.087	0.274	0.018
$s = 36$	0.638	0.025	0.769	0.008	1.795	0.097	0.246	0.018
$s = 60$	0.649	0.025	0.760	0.007	1.747	0.082	0.236	0.018

find that, across all considered hedge fund styles,  $\lambda_L(\hat{\vartheta})$  exhibits a monotonically decreasing pattern as investment horizon increases. For example, the  $\lambda_L$  of the market and the CA index implied from the MIX1 copula is 29% at the quarterly horizon, but only 12% at the 5-year horizon. The corresponding numbers for the MIX2 copula are 25% and 11% at horizons of 1 quarter and 5 years, respectively. Recalling that  $\lambda_L$  estimates the probability that both the market and the hedge fund simultaneously experience an extremely low return, our findings mean that such risk of a joint crash can be reduced by more than 50%. However, this is not surprising given the fact that  $\lambda_L$  is related to the dependence parameters by  $\lambda_L(\vartheta) = (2 - 2^{1/\delta})(1 - w)$  under the MIX1 copula, and by  $\lambda_L(\vartheta) = 2^{-1/\theta}(1 - w)$  under the MIX2 copula; the greater weight on the normal copula corresponds to lower value of  $\lambda_L$ . Hence, this finding is largely attributable to the behavior of the weight parameter,  $w$ , within the parametrization of the mixture models.

To supplement our evidence, we also compute nonparametric estimates of tail dependence based on several predetermined threshold levels.<sup>20</sup> The threshold levels we consider are the 1%–5% quantiles of distributions  $\mathcal{F}^{(s)}$  and  $\mathcal{G}^{(s)}$ , which correspond to 100–500 observations in the left tail of each distribution. Having been based on a finite threshold, the nonparametric estimates will generally be different from the parametric estimates that correspond to the asymptotic case where the threshold goes to 0%. In our application, for example, we find that nonparametric estimates are greater than their parametric counterparts, and in many cases the difference becomes larger as the cutoff quantile moves from 1% to 5%. Nonetheless, we find the cross-horizon pattern in nonparametric estimates to be similar, regardless of the level of threshold used, and more importantly, it is consistent with that in the parametric estimates,  $\lambda_L(\hat{\vartheta})$ . Here, for brevity, we report results corresponding to a threshold of a 1% quantile in the last column of Table 2. Overall, our evidence suggests that the estimation method of tail dependence may affect the specific values of the estimates but it does not affect the horizon pattern of tail dependence.

We also compute the nonparametric estimates for upper tail dependence, using 100–500 observations in the right tail of each distribution (not reported). Similar to Longin and Solnik (2001), we compare, for a given threshold value, nonparametric estimates with those suggested under the assumption of the normal copula. We find the nonparametric estimates to be always lower than the theoretical values derived from the normal copula, confirming no upper tail dependence across all horizons and strategies.

### 3. Goodness-of-Fit Test

To evaluate the copula models, we follow Junker and May (2005) in formulating a bivariate  $\chi^2$  test.<sup>21</sup> We divide each axis into  $r = 9, 11, \text{ or } 13$  equidistant intervals, so we test over  $r^2 = 81, 121, \text{ or } 169$  cells. The choice is due to Moore (1986), who suggests that a reasonable number of equiprobable cells falls between  $2L^{2/5}$  and double that value. Notice that, as pointed out by Klugman and Parsa (1999), the standard asymptotic theory of the test is not valid, since we do not know exact functional forms for marginal distributions, but use the simulated marginals  $\mathcal{F}^{(s)}$  and  $\mathcal{G}^{(s)}$  to map the data into the uniform space. In this light, we use the modified  $\chi^2$  test suggested by Dobrić and Schmid (2005), which is designed to account for use of the empirical marginal distributions. The critical value of the modified test is determined as the  $(1 - \alpha)$ th quantile of the  $\chi^2$

<sup>20</sup>We also estimate the optimal threshold level, for each horizon, via a simple plateau-finding algorithm, but differences in the resulting levels of optimal threshold do not help direct comparison across horizons. See Schmidt and Stadtmüller (2006) for asymptotic properties of the nonparametric estimator.

<sup>21</sup>This procedure, based on Rosenblatt's (1952) transformation, proves convenient because within the (auxiliary) null hypothesis of the test, the expected number of realizations in each cell is directly proportional to its area; hence, the test space  $[0, 1]^2$  can be easily separated into rectangles of equal area without having to merge cells with small expected frequency as in Genest and Rivest (1993) and others.

distribution with degree of freedom of  $(r - 1)(r - 1) - m$ , where  $m$  is the number of dependence parameters.<sup>22</sup>

In Table 4, we present  $p$ -values from the bivariate  $\chi^2$  test for the null hypothesis that the copula model is correctly specified.<sup>23</sup> We report only the results for the mixture copulas and for the case of  $r = 11$ , for brevity; tables with details for all models are available from the authors. Overall, the performance of the mixture copulas seems impressive given the substantial volume of data. In many cases, the mixture copulas pass the  $\chi^2$  test with  $p$ -values of more than 10%. By comparison, other alternative copulas are rejected soundly by the data, with  $p$ -values of virtually 0%, irrespective of horizon and fund style. For some cases, however, the situation is not as satisfactory. For example, the mixture models fail all 3  $\chi^2$  tests at the quarterly horizon, regardless of fund style. For LSE, the mixture models are rejected at several other horizons as well. Nevertheless, the results here should be interpreted keeping in mind that, with 10,000 observations, it is not surprising to see fairly intricate models rejected. The graphical displays similar to Figure 2 indicate that the fits are actually quite good; to highlight this, we provide in Figure 4 the exceedance correlation plots for the quarterly horizon, where the mixture models exhibit the worst fit.

#### IV. Robustness Checks

This section performs robustness checks of the main results. First, we consider an alternative market index, the MSCI World Index. We next consider using value-weighted hedge fund indexes. Finally, we conduct the analysis based on an alternative bootstrapping methodology. For the sake of parsimony, we only present the robustness checks for EVD and LSE funds; the results for other styles are very similar.<sup>24</sup>

Panel A of Table 5 presents results when returns on the MSCI World Index are used as market returns. As in the other panels of the table, the first 3 columns for each style report dependence parameter estimates for the MIX1 copula, the 4th column presents lower tail dependence estimates implied from the estimated dependence parameters, and the last column reports 1%-quantile dependence estimates. By inspection, we find no substantial difference in the horizon pattern of the estimates of interest from those obtained using the S&P 500: As the horizon increases, the weight parameter,  $w$ , continues to rise and the lower tail dependence,  $\lambda_L(\hat{\vartheta})$  and  $\hat{\lambda}_{1\%}$ , keeps on falling.

In the hedge fund industry, fewer than 25% of hedge funds manage more than 75% of the industry's capital (Fung and Hsieh (2004)). Given that an equal-weighting scheme may bias the index toward small-cap funds, we recompute all

<sup>22</sup>In Dobrić and Schmid (2007), the property of related goodness-of-fit test, proposed in Breyermann et al. (2003), is also investigated for the case where marginal distributions are estimated by their empirical counterparts.

<sup>23</sup>The results reported here are subject to the same note of caution given in footnote 19.

<sup>24</sup>The choice of funds is motivated by their status. EVD funds are those, in the first place, responsible for making the asymmetric nature of hedge fund returns part of our wisdom since Mitchell and Pulvino (2001) and Agarwal and Naik (2004). The LSE style represents the single largest strategy, according to TASS.

TABLE 4  
Goodness-of-Fit Tests

Table 4 reports the results of the bivariate  $\chi^2$  test proposed by Junker and May (2005). Panel A presents the results for the test that  $C^{(s)}$  is the realization of the MIX1 copula, while Panel B presents the results for the test that  $C^{(s)}$  is the realization of the MIX2 copula. Each axis of the test space  $[0, 1]^2$  is divided into  $r = 11$  equidistant intervals. Following Dobrić and Schmid (2005), the corresponding  $p$ -values, given in parentheses in percentage form, are adjusted for the use of the empirical marginal distributions. The abbreviations for different hedge fund strategies are convertible arbitrage (CA), emerging market (EM), event driven (EVD), fund of funds (FOF), and long/short equity (LSE).

Strategy	$s = 3$	$s = 6$	$s = 12$	$s = 36$	$s = 60$
<i>Panel A. MIX1 Copula</i>					
CA	181.18 (0.00)	125.38 (2.78)	118.21 (7.06)	107.47 (21.96)	93.92 (56.97)
EM	201.29 (0.00)	187.38 (0.00)	116.11 (9.04)	107.88 (21.15)	109.74 (17.75)
EVD	192.53 (0.00)	107.59 (21.72)	95.78 (51.60)	173.51 (0.00)	132.88 (0.91)
FOF	199.72 (0.00)	128.96 (1.66)	98.37 (44.21)	110.76 (16.06)	116.62 (8.53)
LSE	199.31 (0.00)	165.94 (0.00)	149.43 (0.05)	173.99 (0.00)	192.82 (0.00)
<i>Panel B. MIX2 Copula</i>					
CA	139.51 (0.31)	116.16 (8.99)	112.04 (14.10)	106.84 (23.23)	87.75 (73.84)
EM	174.58 (0.00)	177.14 (0.00)	106.21 (24.54)	111.46 (14.96)	109.55 (18.08)
EVD	178.83 (0.00)	96.46 (49.64)	91.25 (64.53)	155.46 (0.02)	128.60 (1.76)
FOF	185.27 (0.00)	123.83 (3.44)	88.50 (71.94)	117.80 (7.42)	116.93 (8.22)
LSE	188.93 (0.00)	166.78 (0.00)	141.45 (0.22)	166.32 (0.00)	191.73 (0.00)

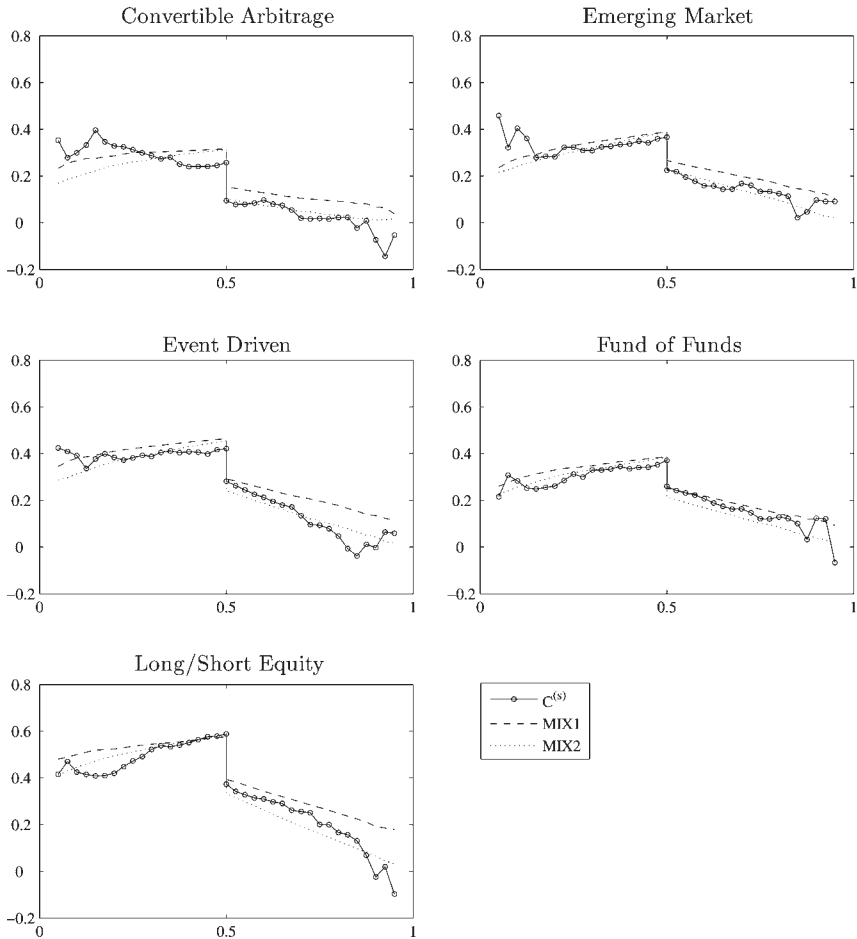
the results based on value-weighted hedge fund indexes using the S&P 500 as the market index. However, using AUM as weights in a hedge fund index has its own problems, including the frequent occurrence of discontinuities in the historical series of AUM (see Fung and Hsieh (2004) for other issues). Similar to Aragon (2007), we calculate value-weighted returns for a given month  $t$  based on funds' returns in month  $t$  and funds' asset size available at the nearest past month within the 12 months preceding month  $t$ . We drop 19,428 (8.56%) of the observations due to missing AUM data. The results are presented in Panel B of Table 5 and are similar to those obtained using equal-weighted hedge fund indexes.

The results thus far have relied on the FHS method to generate the samples of pairs of returns. This method, as described in Section II.A, is based on parametric mean and variance models with nonparametric innovation distribution. An alternative approach would be to use a purely nonparametric simulation method, such as a block bootstrap; the "block" part maintains (asymptotically) any time-series dependence in the data, without the need for specifying parametric models. In Panel C of Table 5, we present the results obtained using the stationary bootstrap of Politis and Romano (1994).<sup>25</sup> The results are overall robust to the choice

<sup>25</sup>We thank the referee for suggesting this additional robustness check. In implementing the stationary bootstrap, the average block length is determined by using the algorithm of Politis and White (2004). Specifically, for the market and the fund, we apply this algorithm to the return, the squared

FIGURE 4  
The Implied Exceedance Correlations from the Mixture Copulas

Figure 4 plots the exceedance correlations between transformed holding period returns,  $\mathcal{F}^{(s)}(R_{m,t_0+s})$  and  $\mathcal{G}^{(s)}(R_{h,t_0+s})$ , for the quarterly horizon, where the mixture models exhibit the worst fit. The horizontal axis shows the threshold quantile, and the vertical axis shows the correlation between a given hedge fund style and the market return given that both exceed that quantile.



of simulation method, but it is observed that the cross-horizon pattern in the quantile dependence estimates becomes less clear as the cutoff quantile moves from 5% to 1%. This is not very surprising because this *unfiltered* historical simulation reduces the range of simulation outcomes,<sup>26</sup> and is thereby less effective than the

return, and the cross product of the returns. Then we use the largest of these lengths as the average block length.

<sup>26</sup>Applying the FHS over an  $s$ -month horizon using our data set of 161 monthly returns, the number of possible pathways is  $161^s$ ; applying a block bootstrap with a fixed block length  $l$ , it reduces to  $161^{\lceil s/l \rceil}$ , where  $\lceil s/l \rceil$  denotes the nearest integer larger than or equal to  $s/l$ . A similar point can be made for the stationary bootstrap, where the block lengths are *random*.

TABLE 5  
Results from Robustness Checks

Table 5 reports the estimates of the parameters in the MIX1 copula, the resultant lower tail dependence estimates, and the 1%-quantile dependence estimates, using variations on the original data set. Panel A presents results when returns on the MSCI World Index are used for the market returns instead of returns on the S&P 500 Index. Panel B considers an alternative choice of index weight, the asset under management. Similar to Aragon (2007), value-weighted returns for a given month  $t$  are calculated based on funds' returns in month  $t$  and funds' asset size available at the nearest past month within the 12 months preceding the month  $t$ . Panel C conducts the analysis based on the stationary bootstrap of Politis and Romano (1994). The nonparametric estimates of tail dependence in this panel are computed using 5% as a cutoff quantile.

	Event Driven					Long/Short Equity				
	$w$	$\rho$	$\delta$	$\lambda_L(\hat{\vartheta})$	$\hat{\lambda}_{1\%}$	$w$	$\rho$	$\delta$	$\lambda_L(\hat{\vartheta})$	$\hat{\lambda}_{1\%}$
<i>Panel A. MSCI World</i>										
$s = 3$	0.498	0.689	1.834	0.271	0.510	0.513	0.796	2.166	0.303	0.510
$s = 6$	0.518	0.694	1.836	0.261	0.460	0.560	0.785	2.212	0.278	0.460
$s = 12$	0.548	0.698	1.899	0.253	0.440	0.616	0.785	2.182	0.241	0.430
$s = 36$	0.583	0.698	1.816	0.223	0.400	0.582	0.786	1.970	0.242	0.430
$s = 60$	0.658	0.686	1.793	0.180	0.370	0.632	0.794	1.805	0.196	0.370
<i>Panel B. Value-Weighted Hedge Fund Indexes</i>										
$s = 3$	0.293	0.655	1.731	0.359	0.470	0.266	0.854	1.929	0.417	0.540
$s = 6$	0.363	0.652	1.744	0.326	0.450	0.333	0.831	1.843	0.362	0.500
$s = 12$	0.437	0.663	1.752	0.290	0.400	0.362	0.785	1.855	0.349	0.470
$s = 36$	0.504	0.695	1.681	0.243	0.340	0.290	0.788	1.850	0.387	0.460
$s = 60$	0.433	0.697	1.703	0.282	0.430	0.369	0.776	1.755	0.325	0.350
<i>Panel C. Unfiltered Historical Simulation</i>										
$s = 3$	0.379	0.771	1.622	0.290	0.560	0.291	0.850	2.014	0.418	0.642
$s = 6$	0.454	0.730	1.708	0.273	0.438	0.302	0.836	2.014	0.411	0.518
$s = 12$	0.629	0.652	1.954	0.213	0.384	0.208	0.830	2.018	0.467	0.536
$s = 36$	0.752	0.638	2.088	0.150	0.370	0.381	0.747	1.961	0.357	0.530
$s = 60$	0.880	0.679	1.582	0.054	0.350	0.576	0.727	1.892	0.237	0.490

FHS in generating tail events beyond the historical record. Nonetheless, using less extreme thresholds (e.g., the 5% quantile) whereby such a problem is less acute, we obtain a similar horizon pattern of the estimates as in the benchmark case; the 5%-quantile dependence estimates are shown instead in this panel.

## V. Conclusion

Perhaps due to the paucity of hedge fund data, studies to date have been largely restricted to short-horizon returns. This is despite the fact that, given lockups and other restrictions on liquidity, the most relevant horizon for an individual's investment decision can be substantially longer. This paper expands our understanding of hedge funds' exposure to market risk by investigating the dependence structure of market and hedge fund returns over various holding periods. By examining a wider range of horizons, we uncover the impact of investment horizon on several aspects of the funds' dependence structure. First, we find that the asymmetry in their dependence is not confined to a particular time horizon but is evident at all holding periods that span various lockup and notice periods. Second, unlike the form of asymmetry, the magnitude of asymmetry is not invariant to investment horizon and decreases considerably in extent as holding period lengthens. Finally, we find that a similar horizon effect occurs on the lower tail dependence, as it appears inversely related to the length of the investment horizon; the upper tail counterpart remains independent across all horizons.



There are currently enough data to analyze monthly or quarterly returns, but insufficient information to afford 1-year or longer holding periods as the basic unit of time. Perhaps it is commonplace for investors who make decisions that will lock up their investments for several quarters to come to rely on short-horizon returns for inference. Our results suggest that these investors should be aware that the downside exposure of their hedge fund investment to the market is not as large as it appears; if any nonlinear or tail risk exposure is found, then its magnitude may be thought of as an upper bound for the possible magnitude using longer, more relevant horizons. Indeed, at the horizon over which their investment is made, hedge funds are able to provide much higher benefits of diversification than at shorter horizons.

Of course, when more data become available, a converse situation might arise where an investor who plans to hold a position in a fund for  $s$  months arbitrarily chooses to use longer-horizon returns. In such cases, the investor faces 2 biases that may seriously underestimate the downside risk associated with her investment. For example, in testing for the null hypothesis of zero tail dependence, which has been the subject of recent papers by Brown and Spitzer (2006), Boyson et al. (2007), and Patton (2009), the measured tail dependence is closer to 0 than the  $s$ -month estimates, and the number of observations employed in the test decreases. These 2 factors decrease the power of the test and may mislead the investor into inferring “tail neutrality.”

## Appendix

This appendix provides the functional forms of the dependence functions used in this paper, and those of related copulas. Also provided are the parameter space(s) and the implied tail dependence coefficients (see, e.g., Joe (1997) for further details on these copulas, and see Patton (2006) for details on the symmetrized Joe-Clayton copula).

### Normal copula

$$C_N(u, v; \rho) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v)),$$

$$\rho \in (-1, 1),$$

$$\lambda_L = 0, \lambda_U = 0.$$

### Gumbel copula

$$C_G(u, v; \delta) = \exp\{-((-\log u)^\delta + (-\log v)^\delta)^{1/\delta}\},$$

$$\delta \in [1, \infty),$$

$$\lambda_L = 0, \lambda_U = 2 - 2^{1/\delta}.$$

### Clayton copula

$$C_C(u, v; \theta) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta},$$

$$\theta \in [-1, \infty) \setminus \{0\},$$

$$\lambda_L = 2^{-1/\theta}, \lambda_U = 0.$$

### Rotated Gumbel copula

$$C_{RG}(u, v; \delta) = u + v - 1 + C_G(1 - u, 1 - v; \delta),$$

$$\delta \in [1, \infty),$$

$$\lambda_L = 2 - 2^{1/\delta}, \lambda_U = 0.$$

### Joe-Clayton copula

$$C_{JC}(u, v; \lambda_U, \lambda_L) = 1 - (1 - \{[1 - (1 - u)^\kappa]^{-\gamma} + [1 - (1 - v)^\kappa]^{-\gamma} - 1\}^{-1/\gamma})^{1/\kappa},$$

where  $\kappa = 1/\log_2(2 - \lambda_U)$ ,  $\gamma = -1/\log_2(\lambda_L)$ ,

$$\lambda_L \in (0, 1), \lambda_U \in (0, 1),$$

$$\lambda_L = 2^{-1/\gamma}, \lambda_U = 2 - 2^{1/\kappa}.$$

**Symmetrized Joe-Clayton copula**

$$C_{\text{SJC}}(u, v; \lambda_U, \lambda_L) = 0.5(C_{\text{JC}}(u, v; \lambda_U, \lambda_L) + C_{\text{JC}}(1 - u, 1 - v; \lambda_L, \lambda_U) + u + v - 1),$$

$$\lambda_L \in (0, 1), \lambda_U \in (0, 1),$$

$$\lambda_L = 2^{-1/\gamma}, \lambda_U = 2 - 2^{1/\kappa}.$$

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