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Lightning Surge Propagation on a Single Conductor in Free Space

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Abstract—**This paper introduces an analysis of lightning surge propagation on a conductor without any returning current path. The conductor can be either a free-space or grounded conductor as long as the reflected surge from the ground has not arrived. Unlike a TEM transmission line, this conductor is characterized with time/position-variant surge impedance as surge current attenuates during its propagation. In this paper a simplified formula was derived. Using the unique parameter – attenuation coefficient of current, an iterative method was developed to evaluate actual propagation characteristics. This method was verified numerically, and is much more efficient in calculation and easier in implementation. It is found that the surge impedance is affected by the waveform of an impulse current source, but is independent of the slope of a ramp current source. It increases quickly if the source current has a short rising time or failing time. The simplified formula generates over-estimated results, but the difference decreases with increasing distance to the source. The proposed method can be used to address surge voltage on the tower upon the arrival of a reflected surge from the ground.**

Keywords-lightning; surge; impedance; propagation; vertical conductor

I. INTRODUCTION

Surge propagation on vertical structures above the ground is a long standing issue in lightning surge analysis. This is particularly important in predicting lightning overvoltage on electrical power systems as well as providing reference information for surge protection design. In the past decades, a number of experimental and theoretical studies have been carried out to investigate surge impedance [1-15] and the corresponding current attenuation phenomenon [16-19]. These studies provided useful information for researches and engineers in understanding surge propagation behaviors along vertical structures and designing effective lightning protection.

The investigations into surge impedance have been carried out for a long time. In 1934, Jordan in [1] proposed the first theoretical equation based on the Neumann inductance equation. After that, several surge impedance equations were proposed using the field theory [2-4]. These equations all assume that the vertical conductor is a cylinder or an equivalent cylinder, and the surge propagation speed is assumed to be the same as the velocity of light. In these equations, surge impedance is expressed as a function of conductor height and radius, and is a time-invariant constant. The authors in [5] took the influence of a return stroke into consideration and developed corresponding equations of surge impedance. The surge impedance is not only affected by the height and radius, but also affected by the route of the current lead wire and the position of the current source. In these studies, the current attenuation in surge propagation was not taken into account. The author in [6] analyzed surge current in a dipole antenna fed with a unit step voltage. This configuration is similar to those discussed in [5]. Again current attenuation on the line was not taken into account. Numerical methods are also available to study lightning surges on the vertical structures. These includes Method of Moment [7-8], partial element equivalent circuit (PEEC) [9-11,21], finite-difference finite-time method [17]. Except the PEEC method, other methods are generally applicable to the surge current analysis only.

Though it is known that further analysis on surge behavior on vertical conductors needs time-variant surge impedance expressions [10-12], the question of "how to define surge impedance" obstructs the understanding of this problem. Authors in [10] quoted some definitions of surge impedance in the time domain as below;

- Transient surge impedance defined by voltage $v(t)$ and current $i(t)$ at the top of a conductor:

$$z(t)=\frac{v(t)}{i(t)} \tag{1}$$

- Surge impedance defined by voltage $v(t)$ and current of a single value:

$$z(t)=\frac{v(t)}{\max[i(t)]} \tag{2}$$

where the current is of either a step wave or a ramp wave.

Furthermore, the voltage in the equations above can be defined differently, such as:

- Line integral of electric field from the ground to the top of a conductor.
- Potential difference between the top and bottom of a conductor.
- Measured value from the top of a conductor to a point far enough on the ground surface.

In this paper, definition of surge impedance given in (1) is adopted. Voltage is defined as the potential with reference to

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a point at infinity. Quasi-static definition of voltage (line integral of electric field) is not adequate in addressing surge propagation as the inductive effect is not taken into account. A comparison with quasi-static voltage was addressed in [12].

An important issue associated with the surge impedance is the attenuation of surge current along the conductor. This phenomenon was observed in both experimental and numerical results of the surge propagation on perfect vertical conductors, especially near the source region [17-20]. A "scatter theory" was proposed by authors in [17] to explain the current attenuation. The surge associated with an electromagnetic wave in a non-zero thickness vertical conductor does not transmit in TEM mode. The attenuation can be attributed to the "scattered waves" generated during its propagation. Authors in [18] provided experimental results for this analysis. Though the surge current on the vertical conductor encounters great attenuation and distortion, the propagating voltage waveform keeps unchanged on the condition that the electromagnetic wave propagates [19].

This paper presents a further investigation into lightning surge propagation along a single conductor without any returning current path. This conductor can be either a free-space conductor or a grounded conductor as long as the reflected surge from the ground has not arrived. Two issues are addressed, that is, time- and position-dependence of both current attenuation and surge impedance. Surge impedance of a single conductor without considering current attenuation is discussed first. The influence of an upward lead wire representing the return stroke current path is taken into consideration. Attenuation of surge current on the conductor is then addressed, as well as the impact on surge impedance. An efficient iterative algorithm is presented for the evaluation of both surge impedance and current attenuation on a free-space conductor. The proposed method is validated with the simulation results obtained from the PEEC method. Finally a discussion of surge impedance against source current waveforms is provided.

II. Surge Impedance without Considering Current Attenuation

When a surge current source is placed in series between two conductor segments, surges will be generated and propagate upwards and downwards. Fig. 1(a) shows a simplified configuration for surge propagation on a vertical line above the ground. These surges have the same current values at Point X due to the current continuity. This configuration is similar to that of a vertical grounded structure struck by lightning. The return stroke can be viewed as a lumped current source placed between the grounded conductor and the lead wire when the current and voltage on the grounded object are of concern. The return stroke current propagates upwards along the lead wire and the discharge current propagates downwards to the ground.

Note that the ground has no effect on the surge propagation if the reflected surge from the ground has not arrived. Without loss of generality, a single conductor with neither the ground nor a returning current path is then analyzed in this paper. As shown in Fig. 1(b) this conductor is separated into two segments at Point X: upper and lower segments carrying surges propagating in opposite directions.

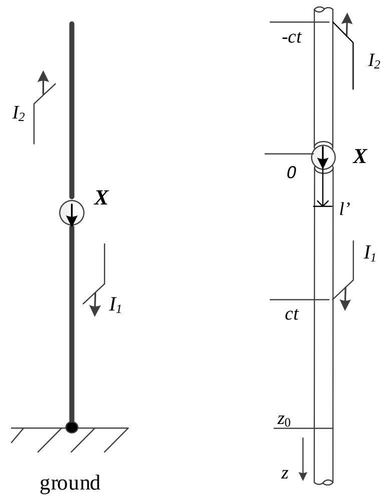
A. *Surge voltage on a single conductor*

According to Maxwell equations, electric scalar potential ϕ at point z on a conductor (Fig. 1(b)) can be expressed by magnetic vector potential A_z and electric field E_z in the z direction, as follows;

$$\begin{aligned} \frac{\partial \phi(z,t)}{\partial z} &= -E_z(z,t) - \frac{\partial A_z(z,t)}{\partial t} \\ A_z(z,t) &= \int \frac{\mu_0 I(l', t - R/c)}{4\pi R} dl' \end{aligned} \tag{3}$$

where $I(l', t - R/c)$ is a retarded surge current on the conductor, and is equal to zero when $t < R/c$. Both c and R are respectively the velocity of an electromagnetic wave in free space and the distance between the source-current point and observation point. On a perfect conductor electric field E_z in (3) is identically zero. As the resistance on a lossy conductor can be added into the surge impedance as a separate item [9], the lossy wire is not discussed in this paper.

Throughout this paper, potential $\phi(z,t)$ in (3) is selected to be the voltage for addressing surge propagation on the conductor. This voltage is determined by conductor geometry, observation position and time, and is not affected by the path selected for the evaluation.



(a) (b)
Fig. 1 Configuration of a vertical line over the ground
(a) Vertical conductor over the ground
(b) Single line in free space

Assume that the surge current does not attenuate during its propagation. Voltage $\phi(z,t)$ at position z and time t on a conductor of radius r is given by

$$\phi(z,t) = \frac{\mu_0}{4\pi} \cdot \int_z^{ct} \frac{\partial}{\partial t} \int_{\frac{l-ct}{2}}^{\frac{l+ct}{2}} \frac{I(0, t - \frac{|l'|}{c} - \frac{|l-l'|}{c})}{\sqrt{(l-l')^2 + r^2}} dl' dl \tag{4}$$

where $I(0,t)$ is the source current at Point X. The derivation of (4) is given in Appendix A.

B. *Surge impedance of a single conductor*

Assume that the source current has a ramp waveform expressed by Kt. Voltage in (4) can be then evaluated analytically. Note that a return stroke current could be approximated by a series of ramp currents with different time delay. By using the definition of transient surge impedance given in (1) with the voltage given in (4), surge impedance at point z is obtained,

$$Z(z,t) = \frac{\mu_0 c}{2\pi}\left[\ln\frac{\sqrt{ct-z}}{r} - 1 + \frac{ct}{ct-z}\ln 2ct - \frac{ct+z}{ct-z}\ln\sqrt{ct+z} + \frac{r}{ct-z}\right] \quad (5)$$

for $t \geq z/c$. Note that slope K of a ramp current is constant, and has been cancelled out in (5). Surge impedance of a single conductor is then independent of the ramp slope. It is determined by time t and positon z as well as conductor radius r. At Point X ($z = 0$), surge impedance of the vertical conductor reduces to

$$Z(0,t) = 60\left(\ln\frac{2ct}{r} - 1\right) \quad (6)$$

Fig. 2 shows time-domain curves of surge impedance at different positions on a conductor with the radius of 5mm. For comparison, the time delay of each curve in the figure is removed. It is noted that that surge impedance increases with time, but the changing rate becomes small as time goes on. The surge impedance increases as well when the distance to the source point is increased. However, the change of surge impedance will be small if the observation point is away from the source.

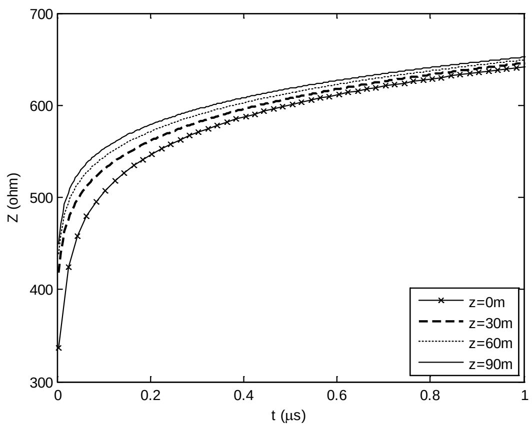


Fig.2 Surge impedance on a single conductor with the radius of 5mm

Analytic formulas of surge impedance have been derived in the literature by using different approaches. These formulas are normally used to estimate the surge at the top of a grounded conductor just before a reflected surge from the ground reaches the top of the conductor. Table 1 shows some of the surge impedance formulas together with the proposed formula of (6). It is noted that these expressions are very similar. Only the coefficients or the constants of the formulas are different. The difference might be caused by; (a) the upward lead wire is not taken into account, or (b) a step function is adopted for the current source. Note that the revised Jordan's expression [16] is the same as the proposed formula when $t = 2H/c$. However, the revised Jordan's expression was derived with the following assumptions:

- The effect of the conductor image is considered;
- The effect of the upward lead wire is not considered.

Table 1 Analytic formulas of surge impedance for a vertical conductor with height *H* and radius *r* above the ground

Revised Jordan's formula [16]	$Z = 60\ln(4H/r) - 60$
Wagner's formula [3]	$Z = 60\ln(2\sqrt{2}H/r)$
Sargent's formula [4]	$Z = 60\ln(\sqrt{2}H/r) - 60$
CIGRE formula [22]	$Z = 60\ln[\cot[0.5tan^{-1}(r/H)]]$
Proposed formula (6)	$Z = 60\ln(4H/r) - 60$

III. CHARACTERISTICS OF SURGES WITH CURRENT ATTENUATION BEING CONSIDERED

Surge propagation on a free-space single conductor excited by a ramp current source is investigated numerically with the PEEC method. The cylindrical conductor has the radius of 5mm, and the length of 150m in both upper and lower segments, as shown in Fig. 1(b). It is connected to the perfect ground at the lower end, and to a ramp current source of Kt ($K = 5kA/\mu s$) at Point X on the conductor. In the PEEC method, wires are divided into short elements which are represented by coupled lumped-parameter circuit components. In this model the conductor is equally divided into 2500 elements. Surge voltage and current in the coupled network are analyzed using a circuit approach. Both surge current and voltage are calculated in the frequency domain. The time domain solution is obtained using an inverse Fourier Transform technique (*ifft*).

A. *Surge current on the conductor*

Fig. 3 shows time-domain curves of the surge current at different positions along the conductor. In the figure the time delay of individual current curves is removed. As the reflected surge is not an issue of concern in this paper, all the curves are padded with zeros when a reflected surge from the ground or a discontinuity arrives. This is for easy identification of curves at different locations. It is found in the figure that the surge current attenuates or the slope of its waveform decreases with increasing distance to the observation point. The change, however, becomes less at a farther position.

Attenuation coefficient of surge current $p(z,t)$ is introduced in this paper for surge evaluation. It is defined by a ratio of the current I with propagation attenuation to the current I' without propagation attenuation, as follows:

$$p(z,t) = \frac{I(z,t)}{I'(z,t)} = \frac{I(z,t)}{I_0(t-z/c)} \quad (7)$$

for $t \geq z/c$. Attenuation coefficient of the surge current is affected by conductor radius as well. Fig. 4 shows time-domain curves of attenuation coefficient at different positions for two conductor radii. The time delay of each curve is removed. It is found that current attenuation is significant in the early time period, and becomes small when time goes on. Current attenuation is significant as well as near the source point. It is also found that attenuation coefficient approaches one when the radius becomes small.

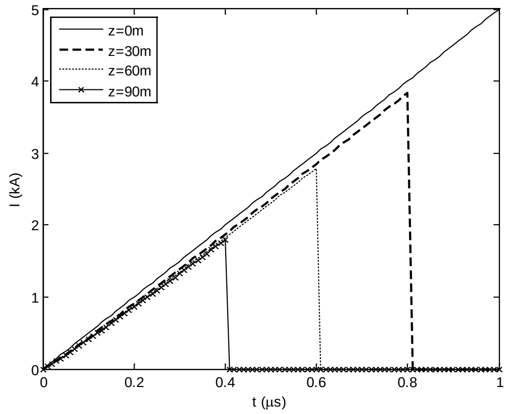


Fig. 3 Current curves on the lower segment with time delay being removed

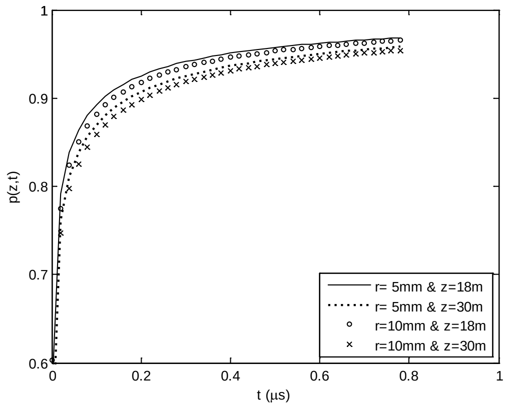


Fig. 4 Attenuation coefficient $p(z,t)$ of surge current at *z*=18m and 30m with conductor radius *r*=5mm and 10mm

B. *Surge voltage on the conductor*

Fig. 5 shows the curves of the surge voltage at different positions along the conductor. The time delay of individual voltage curves is removed again, and each curve is padded with zeros when a reflected surge comes back from the ground. It is found that the voltage curves at different positions on the conductor match very well. This indicates that there is no voltage attenuation during its propagation. This phenomenon can be explained theoretically using the wave propagation equation. The detail is given in Appendix B.

As the surge voltage does not attenuate during its propagation along the conductor, an equation can be established between current attenuation and surge impedance. Note that at any point on the conductor voltage can be expressed by

$$\phi(z,t) = I(z,t) \cdot Z(z,t), \quad (8)$$

and voltage ϕ remains the same everywhere on the conductor. The relationship of surge currents $I_0(t)$ and I(z,t) at Point X and position *z* is given by

$$p(z,t) = \frac{I(z,t)}{I_0(t-z/c)} = \frac{Z(0,t-z/c)}{Z(z,t)}. \quad (9)$$

for $t \geq z/c$.

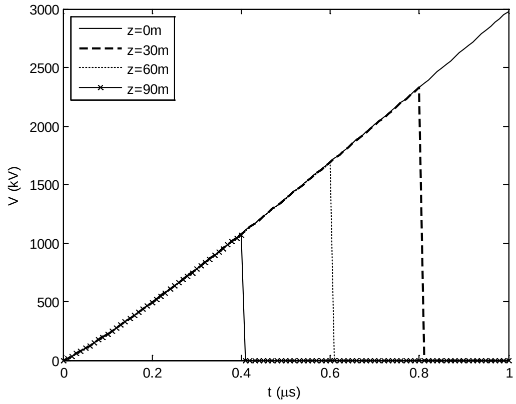


Fig. 5 Voltage curves on the lower segment with time delay being removed.

IV. ALGORITHM FOR SURGE IMPEDANCE AND CURRENT EVALUATION

An iterative method is presented in this section for the evaluation of surge propagation on a free-space single conductor. This procedure does not request a detailed knowledge of circuit modeling. With the information of conductor geometry and current source, surge current, voltage and impedance can be obtained directly. Two particular requirements imposed in Section II are relaxed, that is, (1) the source current waveform can be arbitrary and (2) the surge current attenuates during its propagation.

Assume source current $I_0(t)$ at Point X is already given, and attenuation coefficient $p(z,t)$ are known. Let discrete values $z_i = i\Delta z$, and $t_j = j\Delta t$ (position index i and time index $j = 0, \ldots, N$) and $\Delta z = c\Delta t$. With the results in Appendix A, discrete potential $A_{i,j} = A_z(z_i, t_j)$ is expressed by

$$A_{i,j} = \frac{\mu_0}{4\pi} \int_{\frac{z_i - ct_j}{2}}^{\frac{z_i + ct_j}{2}} \frac{I(l', t - |z_i - l'|/c)}{\sqrt{(z_i - l')^2 + r_1^2}} dl' \quad (10)$$

where $ct_j > z_i$. r_1 is the radius of the conductor. To evaluate (10), the interval of l' is divided into three sub-segments, that is, $[0.5(z_i - ct_j)$, 0], $[0, z_i]$ and $[z_i, 0.5(z_i + ct_j)]$. By replacing $dl' = -dl'$ in the first interval of $[0.5(z_i - ct_j)$, 0], vector potential $A_{i,j}$ turns to

$$A_{i,j} = \frac{\mu_0}{4\pi}\left[\int_0^{\frac{ct_j - z_i}{2}} \frac{I(l', t-(z_i + l')/c)}{\sqrt{(z_i + l')^2 + r_1^2}} dl' + \int_0^{z_i} \frac{I(l', t-(z_i - l')/c)}{\sqrt{(z_i - l')^2 + r_1^2}} dl' + \int_{z_i}^{\frac{z_i + ct_j}{2}} \frac{I(l', t-(l' - z_i)/c)}{\sqrt{(z_i - l')^2 + r_1^2}} dl' \right] \quad (11)$$

It is noted from (7) that $I(l', t)$ can be expressed with source current I_0 and attenuation coefficient $p(l', t)$. Three surge currents in (11) are then expressed by

$$\begin{aligned}
I(l', t_j - \frac{z_i + l'}{c}) &= I_0(t_j - \frac{z_i - 2l'}{c}) \times p(l', t_j - \frac{z_i + l'}{c}) \\
&\approx \sum_{k=0}^{(j-i)/2} I_{0,j-i+2k} \times p_{k,j-i-k} \times U_k(l') \\
I(l', t_j - \frac{z_i - l'}{c}) &= I_0(t_j - \frac{z_i}{c}) \times p(l', t_j - \frac{z_i - l'}{c}) \\
&\approx \sum_{k=0}^{i} I_{0,j-i} \times p_{k,j-i+k} \times U_k(l') \\
I(l', t_j - \frac{l' - z_i}{c}) &= I_0(t_j - \frac{2l' - z_i}{c}) \times p(l', t_j - \frac{l' - z_i}{c}) \\
&\approx \sum_{k=0}^{(j+i)/2} I_{0,j+i-2k} \times p_{k,j+i-k} \times U_k(l')
\end{aligned} \tag{12}$$

where $I_{0,j}$ is the discrete source current at $t = t_j$, and $p_{i,j}$ is the discrete attenuation coefficient at $z = z_i$ and $t = t_j$. In (12) pulse function $U_k(l')$ is defined by

$$U_k(l') = \begin{cases} 1 & k\Delta z \le l' < (k+1)\Delta z \\ 0 & otherwise \end{cases} \tag{13}$$

Substituting (12) in (11) yields

$$\begin{aligned}
A_{i,j} = \frac{\mu_0}{4\pi}\Bigg[&\sum_{k=0}^{(j-i)/2} I_{0,j-i+2k} \times p_{k,j-i-k} \times f_{1,i}(k) + \\
&\sum_{k=0}^{i} I_{0,j-i} \times p_{k,j-i-k} \times f_{2,i}(k) + \\
&\sum_{k=i}^{(j+i)/2} I_{0,j+i-2k} \times p_{k,j+i-k} \times f_{2,i}(k) \Bigg]
\end{aligned} \tag{14}$$

where function $f_{1,i}(k)$ is defined by

$$\begin{aligned}
f_{1,i}(k) &= \frac{\mu_0}{4\pi}\int_{k\Delta z}^{(k+1)\Delta z} \frac{dl'}{\sqrt{(i\Delta z + \Delta z/2 + l')^2 + r^2}} \\
&= \frac{\mu_0}{4\pi} \ln \frac{k + 1.5 + i + \sqrt{(k + 1.5 + i)^2 + (r/\Delta z)^2}}{k + 0.5 + i + \sqrt{(k + 0.5 + i)^2 + (r/\Delta z)^2}}
\end{aligned} \tag{15a}$$

and $f_{2,i}(k)$ is defined by

$$f_{2,i}(k) = \frac{\mu_0}{4\pi}\int_{k\Delta z}^{(k+1)\Delta z} \frac{dl'}{\sqrt{(i\Delta z + \Delta z/2 - l')^2 + r^2}}, \tag{15b}$$

and reduces to

$$f_{2,i}(k) = \begin{cases} \frac{\mu_0}{4\pi} \ln \frac{k + 0.5 - i + \sqrt{(k + 0.5 - i)^2 + (r/\Delta z)^2}}{k - 0.5 - i + \sqrt{(k - 0.5 - i)^2 + (r/\Delta z)^2}} & i \ne k \\ \frac{\mu_0}{2\pi} \ln \frac{0.5 + \sqrt{0.5^2 + (r/\Delta z)^2}}{r/\Delta z} & i = k \end{cases}$$

Note in Appendix B that vector potential propagates downwards without any attenuation, that is, $A_{i,j} = A_{i-k,j-k}$ $(k = 1, \ldots)$. If any difference of vector potential at different positions along the conductor is observed, attenuation coefficient $p_{i,j}$ will be adjusted. The vector potential will be updated again.

In the proposed algorithm, the following error of vector potential ($ERR^{(m)}$) in the m-th iteration is checked

$$ERR^{(m)} = \sum_{i=0}^{N}\sum_{j=0}^{N} \frac{A_{i,j}^{(m)} - A_{i+1j+1}^{(m)}}{A_{i,j}^{(m)}} \tag{16}$$

If $ERR^{(m)}$ is greater than a pre-set value, attenuation coefficient $p_{i+1,j+1}^{(m+1)}$ will be updated with

$$p_{i+1,j+1}^{(m+1)} = \Delta p_{i,j}^{(m+1)} + p_{i,j}^{(m+1)} \tag{17}$$

In (17) difference of attenuation coefficient $\Delta p_{i,j}^{(m+1)}$ is determined by

$$\Delta p_{i,j}^{(m+1)} = \Delta p_{i,j}^{(m)} \times e^{\beta \times \frac{A_{i,j}^{(m)} - A_{i+1,j+1}^{(m)}}{A_{i,j}^{(m)}}} \tag{18}$$

where β is a damping coefficient. A minimum value of the damping coefficient could be set for fast convergence of the iterative method. Prior to the calculation with (17), a boundary condition has to be set, that is, $p_{0,j}^{(m+1)} = 1$ for any j. This means that the current at Point X does not have any attenuation. After $p_{i,j}^{(m+1)}$is obtained, voltage $\phi_{i,j}^{(m+1)}$ can be calculated using (B6). The flow chart of this iterative procedure is given in Fig. 6.

V. SIMULATION RESULTS AND COMPARISONS

Propagation characteristics of a surge on a free-space single conductor can be analyzed by applying the iterative method proposed in Section IV. This method has been coded on the MATLAB platform. With the source current and conductor radius, this MATLAB program returns attenuation coefficient of the surge current, as well as surge voltage, current and impedance at any time and any position along the conductor.

A. *Comparison with PEEC results*

A numerical comparison with the PEEC results has been made. The configuration of a conductor system for comparison is illustrated in Fig. 1(b). Both conductor and source parameters and are given in Section III, as well as PEEC simulation parameters. In the simulation with the proposed iterative method, both distance and time steps Δz and Δt were selected to 1.5m and 5ns, respectively. It is noted that simulation time in a PC with i7-4790 CPU at 3.6GHz and 16GB RAM is 27.5 sec. with the iterative method, and 3hr 15min. with the PEEC method.

Fig. 7 shows the surge current at z=30m on the conductor calculated with (1) the iterative method and (2) the PEEC method. It is noted that these two curves match very well. The average error is less than 1%. The source current at Point X is also presented in the figure for reference. Fig. 8 shows surge impedance at $z = 0m$ and $z = 30m$ calculated with these two methods. When the current attenuation is taken into consideration, the calculated impedance matches with the

PEEC result. The average difference between the calculated surge impedance and simulated one is 0.2% for the time greater than 0.2μs.

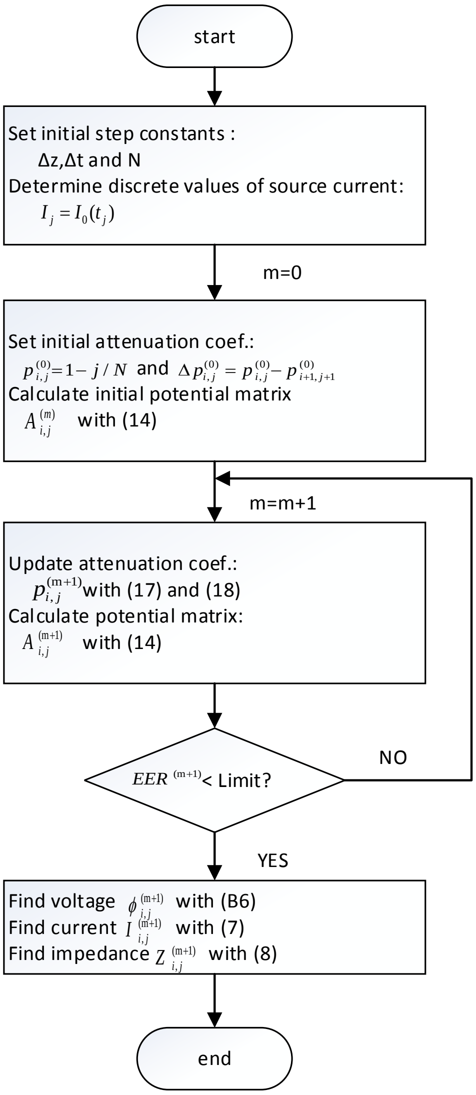


Fig. 6 Iterative method for surge evaluation

A relatively large difference of these curves is observed in the early time period in which the current is relatively small. This difference arises from using the frequency-domain PEEC method. Because both *fft* and *ifft* techniques are used to obtain the time-domain result, a small non-zero current is observed in the PEEC result even before the surge arrives. This leads to a relatively large error.

For comparison, surge impedance calculated with the simplified formula (5) is presented in the figure as well. It is noted that there is a large difference between the results using the simplified formula and the iterative method. The surge impedance is generally over estimated if the theoretical formula is used, as the current attenuation is not taken into consideration.

B. *Surge impedance with arbitrary current waveforms*

It is noted in Section II that simplified surge impedance given in (6) is not affected by the slope of a ramp current source. It is then interesting to know whether the surge impedance is affected by source current waveform when the current attenuation is taken into account. Simulation of surge impedance with different current waveforms has been carried out using the proposed iterative method. Three groups of representative waveforms were selected for evaluation, that is, (1) ramp waveform, (2) impulse waveform for the 2nd stroke lightning return stroke current, and (3) impulse waveform with a slow decaying or rising rate.

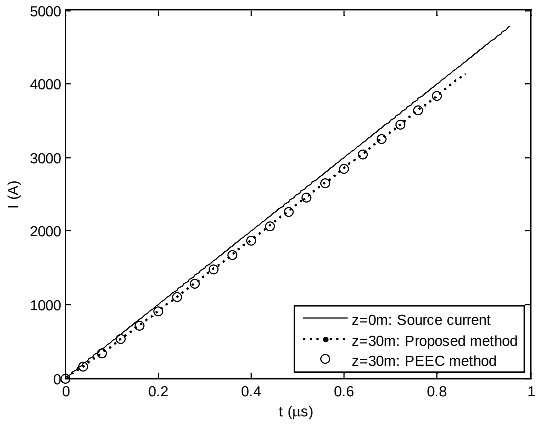


Fig. 7 Surge current on the vertical conductor under a ramp current source

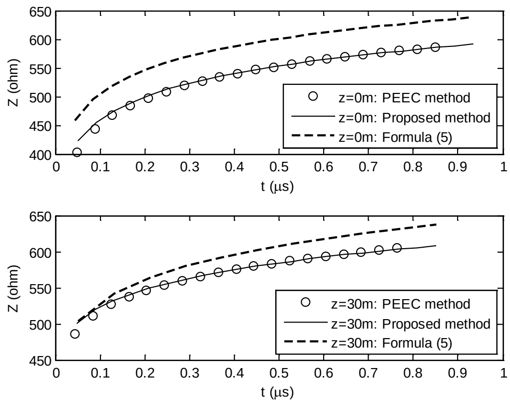


Fig. 8 Surge impedance on the vertical conductor under a ramp current source

Fig. 9 shows all the waveforms used for a surge current injected at Point X. The front time of an impulse current varies from 0.25μs or 0.38μs, and the time to half-peak varies from 1.7μs to 100μs approximately. The impulse current has a fixed magnitude of 10kA. The ramp current has a slope of either 10kA/μs or 20kA/μs. Fig. 10 shows the curves of surge impedance under five different current waveforms. It is noted that the slope of a ramp current does not affect the surge impedance. This is the same as that observed from the simplified formula of (6). The surge impedance under an impulse current is, however, significantly different from that under a ramp current. The surge impedance generally varies with the front time and the time to half-peak. It increases quickly if the front time is short, and remains the same if there is no difference in the tail. The surge impedance generally continues to increase no matter what waveform the tail has. The surge impedance will increases quickly if the time to half-peak is short. Therefore, the surge impedance under a ramp current generally has a lower impedance than that under an impulse current.

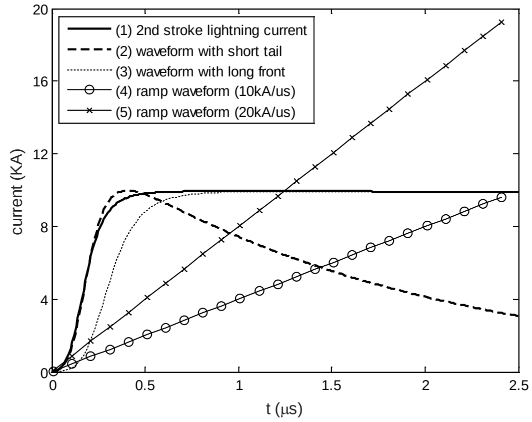


Fig. 9 Waveforms of a surge current propagating on the conductor

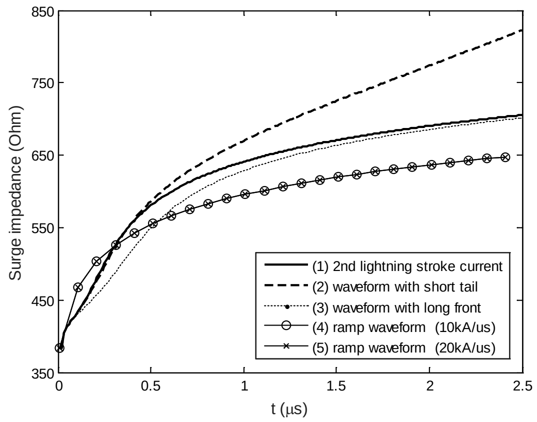


Fig. 10 Surge impedance with different waveforms of a surge current

Although surge impedance is not affected by the slope of a ramp function, it is indeed changed by the shape of the source current waveform, as seen in Fig. 10. This is because surge impedance of a standalone line is time-variant. Assume that the source current is made of two ramp functions $R_1(t)$ and $R_2(t)$ applied at $t = 0$ and $t = t_1$ respectively. Surge impedances at point z associated with these two current components are then expressed by $Z(z,t)$ and $Z(z,t-t_1)$. The resultant impedance of the current source $Z_t(z,t)$ is given by

$$Z_t(z,t) = \frac{Z(z,t)R_1(t)p(z,t) + Z(z,t-t_1)R_2(t)p(z,t-t_1)}{R_1(t)p(z,t) + R_2(t)p(z,t-t_1)} \quad (19)$$

where $p(z,t)$ is the attenuation coefficient of surge current at position z. Generally $Z_t(z,t) \neq Z(z,t)$ as surge impedance varied with time. Impedance $Z_t(z,t)$ will be equal to $Z(z,t)$ only if $Z(z,t)$ does not vary with time, that is, $Z(z,t) = Z(z,t-t_1)$.

VI. Conclusion

This paper addressed the propagation of a lightning surge along a single conductor without any returning current path. This conductor can be either a free-space conductor or a grounded conductor as long as the reflected surge from the ground has not arrived. Similar to a traditional transmission line, surge propagation can be characterized using surge impedance. This surge impedance, however, varies with time and position. A simplified formula of surge impedance was derived with the assumption of ramp current excitation and no current attenuation. By using the attenuation coefficient of current, a simple iterative method was then presented for the evaluation of surge propagation without these constraints. The iterative method was validated numerically using the PEEC method. It is much more efficient in calculation and easier in implementation.

Unlike a traditional transmission line, a surge current attenuates during its propagation even on a lossless line while the surge voltage does not attenuate. The corresponding surge impedance varies with conductor radius, and is affected by surge current waveform. It is found that the surge impedance increases continuously with increasing time and distance from the source point, as long as the current source does not change its polarity. The surge impedance increases quickly if the surge current has a short rising time or failing time. Generally speaking, the surge impedance under a ramp current source is lower than that under an impulse current source. What is more, it does not vary with the slope of the ramp current. The surge impedance calculated with the simplified formula is normally higher than the actual value, as the attenuation of a current surge in its propagation is not taken into account. The proposed method can be used to address surge voltage on the top of a grounded tower or voltage between two points on the tower upon the arrival of a reflected surge from the ground. It could be extended to solve wave propagation problems for the lines associated with a discontinuity or the ground.

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APPENDIX A

Applying (3) for the conductor given in Fig. 1(b) yields voltage $\phi(z,t)$ at position *z* and time *t*, that is,

$$\phi(z,t)-\phi(z_0,t)=\int_z^{z_0}\frac{\partial}{\partial t}A_z(l,t)dl \qquad \text{(A1)}$$

where reference position z_0 is selected in such a way that the surge has not arrived at that point yet, that is, $z_0 > ct$. Therefore, voltage $\phi(z_0,t)$ is identically zero. Alternatively, z_0 can be located at infinity to which a virtual wire is extended from the conductor.

Note that two current surges propagate upwards and downwards on the conductor, respectively, as shown in Fig. 1(c). Assume that at time *t* the two surges arrive at *ct* on the lower segment of the conductor, and –*ct* on the upward segment. Voltage $\phi(z,t)$ in (A1) can be then expressed by

$$\phi(z,t)=\frac{\mu_0}{4\pi}\int_z^{ct}\left(\frac{\partial}{\partial t}\int_{-ct}^{ct}\frac{I(l',t-\frac{|l-l'|}{c})}{\sqrt{(l-l')^2+r^2}}dl'\right)dl \qquad \text{(A2)}$$

where l and l' are respectively the variables of integration along the conductor for scalar potential and vector potential. Voltage $\phi(z,t)$ is identically zero if $z > ct$ (travel distance of the surge at time *t*).

Considering there is no current attenuation on the conductor, the retarded current is expressed by

$$I(l',t-\frac{|l-l'|}{c})=I(0,t-\frac{|l'|}{c}-\frac{|l-l'|}{c}) \qquad \text{(A3)}$$

It is noted that the retarded time of $t-|l'|/c-|l-l'|/c$ must be greater than zero at any point and at any moment. The following inequalities yield

$$\begin{aligned} &ct > l-2l' \quad \text{for } l'<0 \\ &ct \ge l+2l' \quad \text{for } l'\ge 0 \end{aligned} \qquad \text{(A4)}$$

The upper and lower limits of variable l' can be worked out from (A4). Then voltage $\phi(z,t)$ in (A2) reduces to

$$\phi(z,t)=\frac{\mu_0}{4\pi}\cdot\int_z^{ct}\frac{\partial}{\partial t}\int_{\frac{l-ct}{2}}^{\frac{l+ct}{2}}\frac{I(0,t-\frac{|l'|}{c}-\frac{|l-l'|}{c})}{\sqrt{(l-l')^2+r^2}}dl'dl \qquad \text{(A5)}$$

Appendix B

In free space, with the Lorentz condition both vector potential $\boldsymbol{A}$ and scalar potential (voltage) ϕ satisfy wave equations, as follows:

$$\begin{aligned} \nabla^2\mathbf{A}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} &= -\mu J \\ \nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} &= -\frac{\rho}{\varepsilon} \end{aligned} \qquad \text{(B1)}$$

where both J and ρ is the current density and the charge density in the medium. Assume that the conductor is made of perfectly conducting material. The current and charge are situated on the conductor surface if a surge propagates along the conductor shown in Fig. 1(b). Consequently, both electric field strength $\boldsymbol{E}$ and magnetic flux density *B* are identically equal to zero within the conductor.

Note that vector potential $\boldsymbol{A}$ reduces to A_z (z direction). Both vector potential A_z and voltage ϕ within the conductor remain constant on its cross section, and satisfy the following 1D wave equations

$$\frac{\partial^2 A_z}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 A_z}{\partial t^2} = 0$$
$$\frac{\partial^2 \phi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = 0 \tag{B2}$$

A solution of (B2) for the downward surge is given by

$$A_z(z,t) = F(z-ct)$$
$$\phi(z,t) = G(z-ct) \tag{B3}$$

where $F(\cdot)$ and $G(\cdot)$ are wave functions for the vector potential and voltage, respectively.

Assume that vector potential $A_{i,j} = A_z(z_i, t_j)$, where $z_i = i\Delta z$, $t_j = j\Delta t$ ($i, j = 0,1, \ldots N$ and $\Delta z = c\Delta t$). A discrete equation can be obtained from (B3), as follows:

$$A_{i,j} = A_{i-k,j-k} = F(i\Delta z - j\Delta z) \tag{B4}$$

where k is an arbitrary integer ($k \leq i$ and $k \leq j$). This is the discrete propagation equation of vector potential without any attenuation.

Similarly, a discrete wave equation without any attenuation for voltage $\phi_{i,j} = \phi(z_i, t_i)$ can be derived, as follows:

$$\phi_{i,j} = \phi_{i-k,j-k} \tag{B5}$$

Voltage $\phi_{i,j}$ can be determined directly with (A1), and is expressed by

$$\begin{aligned} \phi_{i,j} &= \int_{z_i}^{ct_j} \frac{\partial A_z}{\partial t} dl \\ &\approx c \cdot \sum_{k=i}^{j} (A_{k,j} - A_{k,j-1}) \\ &\approx c \cdot A_{i,j} \end{aligned} \tag{B6}$$

Note $A_{i,j} = 0$ for $i \geq j$, as the surge has just arrived at $z = z_i$ when $t = t_j$.