A study of coupled flexural-longitudinal wave motion in a periodic dual-beam structure with transverse connection

Yi Yun and Cheuk Ming Mak[a)]
Department of Building Services Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China

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A theoretical study of the multi-coupling flexural and longitudinal waves that propagate in a periodic dual-beam-type waveguide with structural connection branches is conducted. The analytical equations of the transfer matrix method are derived for the wave transmission with consideration given to the fully flexural and longitudinal motions that are tri-coupled at each connection. Based on this transfer matrix method, numerical calculation is performed to investigate the characteristic wave-types that propagate in a semi-infinite periodic structure. The complex wave-coupling phenomena in the periodically connected dual-beam waveguide are then analyzed numerically. Remarkably, it is found that three symmetric and three antisymmetric types of characteristic coupled waves propagate in a periodic structure. The numerical results show that the energy contribution of the coupled waves with respect to the source excitation depends on the forbidden band of the wave-types and on the energy ratios and combination of wave-types. This study promotes the fundamental understanding and prediction of coupled acoustic waves in multi-layered frame structures. The long-term significance is that it may lead to a more effective control method for structure-borne sound transmission in a multi-layered coupling structure.
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I. INTRODUCTION

A number of building structures, bridges, container ship structures, and steel-reinforced concrete constructions are built from an assembly of a number of same or similar structure elements, of which the frames are typically coupled in an identical manner to form a so-called "spatially periodic structure." The excited vibration and transmission of the mechanical waves in these structures—from side to side in a bridge or layer to layer in a building—often give rise to structure-borne noise problems in the connected spaces and can sometimes even be harmful to the stability of the entire structure. When they propagate through frame structures that contain many connection branches, these structure-borne sound waves are coupled and reflected by each connector. The reflected and transmitted waves are then coupled and reflected again by other connectors. This process is physically repeated and sets off infinite multi-interactions between the coupling connections and propagating waves in a periodic structure, which forms the dispersion bands of structure-borne sound waves.

Early on, the dispersion bands of waves in periodic waveguides were studied for the electro-magnetic waves in solids,[1] thus promoting our basic understanding of the properties of conductors, semi-conductors, and the like. Since the late 1980s, the optical wave bands in media with periodical modulation have been extensively studied, and these studies have led to a number of practical applications, including the advanced design of photonic crystals[2] and waveguide devices.[3] All of these studies have brought researchers deeper insight into the dispersion properties of periodic structures and have helped them to develop methods for the theoretical calculation of wave propagation. In acoustics, the classical problem of plane sound wave transmission in one-dimensional periodic media can be tackled in an exact manner via the transfer matrix method.[4] The theoretical computations of band structures have also been well-documented for sound waves in periodic acoustic structures by Kushwaha and Mod.[5] Enhanced wave transmission was modeled in rib-reinforced floors about 50 years ago by using a beam that was periodically loaded with eccentric attachments because of wave coupling.[6] Four different methods of calculating the structure-borne sound propagation in beams with many non-resonant discontinuities were demonstrated by Heckl,[7] and three of these methods took the coupling between longitudinal and flexural waves into account. The fundamental and central ideas in the area of periodic system characterization was introduced by Mead.[8] In this context, a quadratic and well-posed spectral problem was studied to determine the wave propagation constants of a periodic system. This work was extended by Mead,[9] which proposed a second order matrix equation leading to the propagation constants of a periodic system. Several years ago, a mathematical model for the coupling of waves that propagate in a periodically supported Timoshenko beam was presented by Heckl.[10] Furthermore, the propagation characteristics of coupled longitudinal and flexural waves in beam-type transmission paths with asymmetric loads in the form of resonant columns were theoretically analyzed[11] and experimentally examined[12] by Friss and Ohlrich. However, little understanding of the fundamental physical propagation characteristics of the coupling acoustic

[a)]Author to whom correspondence should be addressed. Electronic mail: becmmak@polyu.edu.hk

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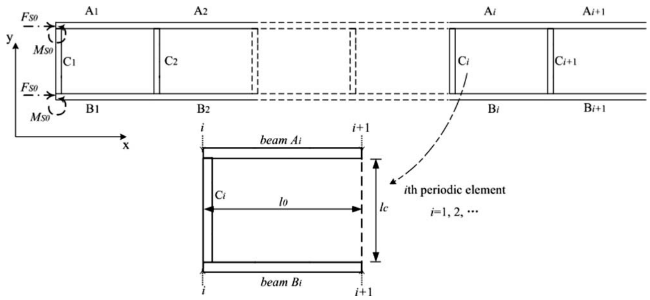

FIG. 1. Scheme of the semi-infinite periodic dual-beam structure and the excitations.

waves in multi-layered structures has been gained from these studies. It is because they are commonly concerned with models of a single-channel waveguide that comprises the independent beam-type components or uncoupled wave transmission path in the structure.

Therefore, the analytical study reported in this paper investigated the characteristics of the multi-coupling flexural and longitudinal waves that propagate in a periodic dual-beam-type waveguide with structural connection branches. The propagation of waves in a semi-two-dimensional system was adopted because the coupling interaction between the waves at the connections and transmission paths through a three-dimensional structure is so complicated that the theoretical predictions based on the series of approximations can be very different from the actual experimental observations. The transfer matrix method is developed by using the concept based on the propagation constants[8,9] of the waves in a periodic structure so that it avoids the problems from inversion of ill-conditioned matrices and the cumulative errors. The developed method is therefore explicit and appropriate for the calculation of the coupled waves in the periodic beam structure.

II. THEORETICAL MODEL AND ANALYTICAL FUNDAMENTALS

A. Simple model of a periodic dual-beam structure with a transverse connection

This paper examines the band structure of flexural-longitudinal wave propagation in a dual-beam coupling structure that is periodically connected with transverse branches. A simplified model is shown in Fig. 1. The structure-borne sound consists of the flexural waves and longitudinal waves that propagate in two horizontal beams—A and B are coupled at each connection with a vertical branch C_i. The beams and branches discussed theoretically are even, straight, isotropic, and homogeneous, and the following physical parameters are assumed. $\rho_{(1,2,3)}$=the density of beams A and B and branch C_i, $B_{(\mathrm{a,b,c})0}$=the bending stiffness of beams A and B and branch C_i, $E_{(\mathrm{A,B,C})}$=Young's modulus of beams A and B and branch C_i, $k_{(\mathrm{A,B,C})f}$=the flexural wave numbers of beams A and B and branch C_i, and $k_{(\mathrm{A,B,C})l}$ =the longitudinal wave numbers corresponding to the acoustic speeds of the longitudinal wave $c_{(1,2,3)}$ of beams A and B and branch C_i. The characteristics of the wave-types and the energy transmission of the coupled flexural-longitudinal waves in a semi-infinite periodic dual-beam structure are calculated for analysis in a case study.

B. Wave transfer matrix and the propagation constants of the characteristic wave-types

In the analytical model of this research, the coupled wave components of the complex velocity (horizontal, vertical, and rotational) and force (horizontal, vertical, and moment) vectors are used for describing the coupled wave motions and response in the dual-beam structure. For mathematical derivation, all of the analytical equations in this paper are based on the harmonic wave of separate frequency ω_n with time dependence suppressed. Normally the longitudinal and flexural waves in a beam can be expressed in the form of the independent wave velocity components. In addition, the longitudinal-flexural waves can be described by the beam velocities and the corresponding forces caused by the wave motions. At the connections of every periodic element, the vector consisting of velocity and force components can be related to the vector of the flexural-longitudinal waves that propagate through the beams in a matrix form as follows:

$$\begin{bmatrix} V_n \\ F_n \end{bmatrix} = [S_{VF}][v_w], \tag{1}$$

$$[v_w] = \begin{bmatrix} v_{wA} \\ v_{wB} \end{bmatrix} = [S_{VF}]^{-1} \begin{bmatrix} V_n \\ F_n \end{bmatrix}, \tag{2}$$

where the velocity vectors of the flexural and longitudinal wave components are expressed as

$$[v_{wA}] = [v_{Af}^{+}\ v_{Afj}^{+}\ v_{Af}^{-}\ v_{Afj}^{-}\ v_{Al}^{+}\ v_{Al}^{-}]^T,$$

$$[v_{wB}] = [v_{Bf}^{+}\ v_{Bfj}^{+}\ v_{Bf}^{-}\ v_{Bfj}^{-}\ v_{Bl}^{+}\ v_{Bl}^{-}]^T,$$

of which "+" donates the wave components propagating in the positive x-direction, "−" donates the components going in the negative x-direction, f, fj, and l donate the propagating flexural, nearfield flexural, and longitudinal wave components, respectively. The vectors of the velocities and forces of the two beam channels, indicated by the subscripts a and b, are expressed as

$$[V_n] = [V_{ya}\ V_{yb}\ \omega_a\ \omega_b\ V_{xa}\ V_{xb}]^T,$$

$$[F_n] = [F_{ya}\ F_{yb}\ M_a\ M_b\ F_{xa}\ F_{xb}]^T,$$

where V_x, V_y, and ω are the x-degree, y-degree, and rotational velocities in the beam, and F_x, F_y, and M are the x-degree force, y-degree force, and moment acting on a beam. To describe the relationship between the independent flexural-longitudinal waves and the velocities-forces in two beams, the waves to motions-actions transfer matrix $[S_{VF}]$ takes on the matrix form:

$$[S_{VF}] = [S_{V1}\ S_{V2}\ S_{V3}\ V_{F1}\ S_{F2}\ S_{F3}]^T,$$

$$S_{Vi}=\begin{bmatrix} D_{Vai} & \mathbf{O}_{6\times 1} \\ \mathbf{O}_{6\times 1} & D_{Vbi} \end{bmatrix}, \quad S_{Fi}=\begin{bmatrix} D_{Fai} & \mathbf{O}_{6\times 1} \\ \mathbf{O}_{6\times 1} & D_{Fbi} \end{bmatrix}, \qquad (3)$$

in which the matrix are derived by

$$D_{V(a,b)1}=\begin{bmatrix} 1\\1\\1\\1\\0\\0 \end{bmatrix}, \quad D_{V(a,b)3}=\begin{bmatrix} 0\\0\\0\\0\\1\\1 \end{bmatrix},$$

$$D_{Va2}=\begin{bmatrix} -jk_{Af}\\ -k_{Af}\\ jk_{Af}\\ k_{Af}\\ 0\\ 0 \end{bmatrix}, \quad D_{Vb2}=\begin{bmatrix} -jk_{Bf}\\ -k_{Bf}\\ jk_{Bf}\\ k_{Bf}\\ 0\\ 0 \end{bmatrix},$$

$$D_{Fa1}=\begin{bmatrix} jR_{AF}\\ -R_{AF}\\ -jR_{AF}\\ R_{AF}\\ 0\\ 0 \end{bmatrix},$$

and

$$D_{Fb1}=\begin{bmatrix} jR_{BF}\\ -R_{BF}\\ -jR_{BF}\\ R_{BF}\\ 0\\ 0 \end{bmatrix}, \quad D_{Fa2}=\begin{bmatrix} R_{AM}\\ -R_{AM}\\ R_{AM}\\ -R_{AM}\\ 0\\ 0 \end{bmatrix},$$

$$D_{Fb2}=\begin{bmatrix} R_{BM}\\ -R_{BM}\\ R_{BM}\\ -R_{BM}\\ 0\\ 0 \end{bmatrix}, \quad D_{Fa3}=\begin{bmatrix} 0\\0\\0\\0\\ R_{Al}\\ -R_{Al} \end{bmatrix}, \quad D_{Fb3}=\begin{bmatrix} 0\\0\\0\\0\\ R_{Bl}\\ -R_{Bl} \end{bmatrix},$$

$$R_{AM}=\frac{B_{a0}k_{Af}^2}{j\omega_n}, \quad R_{BM}=\frac{B_{b0}k_{Bf}^2}{j\omega_n}, \quad R_{AF}=\frac{B_{a0}k_{Af}^3}{j\omega_n},$$

$$R_{BF}=\frac{B_{b0}k_{Bf}^2}{j\omega_n}, \quad R_{Al}=\rho_1 c_1, \quad R_{Bl}=\rho_2 c_2.$$

It is clear that the y-degree velocities and forces, rotational velocities, and moments in the beams A and B resulted from the flexural wave motions, while the x-degree velocities and forces in two beams resulted from the longitudinal wave motions. It should be noted that the wave-coupling effect in a periodic dual-beam structure is caused by the vertical connections. By introducing the dynamic continuity conditions at the interfaces that are vertically connected with the branch beams, the relationship between the velocities and the forces of the coupled flexural and longitudinal waves at the connections of dual-beam structure can be characterized as a 12×12 coupling transfer matrix that can be expressed as

$$\begin{bmatrix} V_n \\ F_n \end{bmatrix}_i^+ = [W_C]\times\begin{bmatrix} V_n \\ F_n \end{bmatrix}_i^+, \qquad (4)$$

where

$$\begin{bmatrix} V_n \\ F_n \end{bmatrix}_i^- \quad \text{and} \quad \begin{bmatrix} V_n \\ F_n \end{bmatrix}_i^+$$

denote the velocity and force vectors of the beam on the left and right side of the connection points on beam A-B with C_i. Based on the dynamic equilibrium on two sides of a branch C_i, the wave-coupling matrix $[W_C]$ is given by

$$[W_C]=\begin{bmatrix} I_6 & O_6 \\ -Z_{Cw}\times T_{Cv} & I_6 \end{bmatrix}, \qquad (5)$$

in which the transfer elements are given by

$$Z_{Cw}=\begin{bmatrix} Z_{Cl} & O_{2\times 4} \\ O_{4\times 2} & Z_{Cf} \end{bmatrix},$$

$$Z_{Cl}=\begin{bmatrix} R_{cl} & -R_{cl} \\ -R_{cl}\phi_{Cl}^{-j} & R_{cl}\phi_{Cl}^{j} \end{bmatrix},$$

$$Z_{Cf}=\begin{bmatrix} \Omega_{Mc} & -\Omega_{Mc} & \Omega_{Mc} & -\Omega_{Mc} \\ -\phi_{Cf}^{-j}\Omega_{Mc} & \phi_{Cf}^{-1}\Omega_{Mc} & -\phi_{Cf}^{j}\Omega_{Mc} & \phi_{Cf}\Omega_{Mc} \\ j\Omega_{Fc} & -\Omega_{Fc} & -j\Omega_{Fc} & \Omega_{Fc} \\ -j\phi_{Cf}^{-j}\Omega_{Fc} & \phi_{Cf}^{-1}\Omega_{Fc} & j\phi_{Cf}^{j}\Omega_{Fc} & -\phi_{Cf}\Omega_{Fc} \end{bmatrix},$$

$$T_{Cf}=\begin{bmatrix} 1 & 1 & 1 & 1 \\ \phi_{Cf}^{-j} & \phi_{Cf}^{-1} & \phi_{Cf}^{j} & \phi_{Cf} \\ -jk_{Cf} & -k_{Cf} & jk_{Cf} & k_{Cf} \\ -jk_{Cf}\phi_{Cf}^{-j} & -k_{Cf}\phi_{Cf}^{-1} & jk_{Cf}\phi_{Cf}^{j} & k_{Cf}\phi_{Cf} \end{bmatrix}^{-1},$$

$$T_{Cv}=\begin{bmatrix} T_{Cl} & O_{2\times 4} \\ O_{4\times 2} & T_{Cf} \end{bmatrix}, \quad T_{Cl}=\begin{bmatrix} 1 & 1 \\ \phi_{Cl}^{-j} & \phi_{Cl}^{j} \end{bmatrix}^{-1},$$

$$R_{cl}=\rho_3 c_3, \quad \phi_{Cl}=e^{k_{Cl}h}, \quad \phi_{Cf}=e^{k_{cf}h},$$

$$\Omega_{Mc}=\frac{B_{0C}k_{Cf}^2}{j\omega_n}, \quad \Omega_{Fc}=\frac{B_{0C}k_{Cf}^2}{j\omega_n}.$$

The new flexural and longitudinal waves that are excited in the branch will result in the velocities and forces acting on the two connection sides of the branch with the beams A and B, so that the flexural and longitudinal waves will be coupled there. Moreover, the transfer matrix of the longitudinal and flexural waves propagating in the continuous beam period (whose length $=d$) is given by

$$[v_w]_{i+1}^- = [P_{wv}][v_w]_i^+.$$

$$[P_{wv}] = \begin{pmatrix} P_{Af} & O & O & O \\ O & P_{Al} & O & O \\ O & O & P_{Bf} & O \\ O & O & O & P_{Bl} \end{pmatrix},$$

$$P_{Af} = \begin{pmatrix} e^{-jk_{Af}d} & 0 & 0 & 0 \\ 0 & e^{-k_{Af}d} & 0 & 0 \\ 0 & 0 & e^{jk_{Af}d} & 0 \\ 0 & 0 & 0 & e^{k_{Af}d} \end{pmatrix},$$

$$P_{Bf} = \begin{pmatrix} e^{-jk_{Bf}d} & 0 & 0 & 0 \\ 0 & e^{-k_{Bf}d} & 0 & 0 \\ 0 & 0 & e^{jk_{Bf}d} & 0 \\ 0 & 0 & 0 & e^{k_{Bf}d} \end{pmatrix}, \quad (6)$$

$$P_{Al} = \begin{pmatrix} e^{-jk_{Al}d} & 0 \\ 0 & e^{jk_{Al}d} \end{pmatrix}, \quad P_{Bl} = \begin{pmatrix} e^{-jk_{Bl}d} & 0 \\ 0 & e^{jk_{Bl}d} \end{pmatrix}.$$

It is generally understood that the flexural and longitudal waves can propagate independently through the dual-beam part between the connnection branches without coupling. Therefore, the transfer relationship of the waves in the continuous beam part can be described by using the diagonal matrices. On the whole, the coupled wave transmission in the periodic structure can be expressed as

$$\begin{bmatrix} V_n \\ F_n \end{bmatrix}_{i+1}^{-} = [U_e] \times \begin{bmatrix} V_n \\ F_n \end{bmatrix}_{i}^{-}, \quad (7)$$

where the entire periodic transfer matrix is given

$$[U_e] = [S_{VF}][P_{wv}][S_{VF}]^{-1}[W_C].$$

According to the Bloch wave theory,[13] for a linear acoustic system, when the acoustic waves are propagating through a semi-infinite one-dimensional periodic structure, the wave motions can be described as the characteristic wave-types of Bloch waves. Then the relationship between velocity vector $[V_n]_i$ and force vector $[F_n]_i$ at the two periodic connection points nearby satisfies the form

$$[V_n]_{(i)}^{-} = \sum_{j=1}^{N} v_{j,(i)}[\xi_{jn}], \quad [F_n]_{(i)}^{-} = \sum_{j=1}^{N} f_{j,(i)}[\zeta_{jn}], \quad (8)$$

$$\begin{bmatrix} v \\ f \end{bmatrix}_{(i+1)} = e^{\mu} \begin{bmatrix} v \\ f \end{bmatrix}_{(i)}. \quad (9)$$

This represents a problem on the eigenvalue vector for the transfer matrix $[U_e]$, where $\mu_j = \pm(\mu_{jR} + j \cdot \mu_{jI})$ are the pair of jth eigen values—the frequency-dependent complex propagation constants for the corresponding pair of the N characteristic wave-types ($N=6$ for this periodic structure). Correspondingly the characteristic wave-types are formulated by the eigenvectors $[\xi_{jn}\zeta_{jn}]^T$, which take on the form $[\xi_{jn}] = [X_{Vxa}^j, X_{Vxb}^j, X_{\omega a}^j, X_{\omega b}^j, X_{Vya}^j, X_{Vyb}^j]^T$ and $[\zeta_{jn}] = [X_{Fxa}^j, X_{Fxb}^j, X_{Ma}^j, X_{Mb}^j, X_{Fya}^j, X_{Fyb}^j]^T$. As the "attenuation constant" of the coupled wave type, the real part μ_{jR} expresses the exponential decay rate for the jth characteristic wave-type that propagates through a periodic beam element, whereas the imaginary part μ_{jI} is defined as the "phase constant," of which the cosine value describes the phase transfer of the jth characteristic wave-type that propagates through each element. If the propagation constants of the positive-going wave-types are defined as $\mu_j = \mu_{jR} + \cdot \mu_{jI} (0 \le |\mu_{jI}| < 2\pi)$, then, correspondingly, the real and imaginary parts of the propagation constants ought to be negative. For the frequency-dependent wave propagation in a periodic structure, the frequency domain is classified into pass bands, i.e., frequencies at which the coupled waves travel through the periodic structure with little loss, and forbidden bands, i.e., frequencies at which the coupled waves propagating in the periodic structure are evanescent. In an ideal case as the damping factor is negligible, a pair (positive and negative-going) of characteristic wave-types yield up to a pair of pure imaginary propagation constants at any frequency within the pass bands, which indicates that the coupled waves propagating through the periodic structure will not decay. On the other side, the real parts of propagation constants are nonzero at the frequencies within the forbidden bands, which indicates that the coupled waves will be attenuated as propagating through the periodic structure. The zone of larger attenuation constants, i.e., $|\mu_{jR}|$ means that the corresponding wave-type is in the stronger forbidden band of the periodic structure. In the semi-infinite structure, only the positive-going propagation constant $\mu_j = -|\mu_{jR}| - j|\mu_{jI}|$ is the reasonable solution, because neither $+|\mu_{jI}|$ nor $+|\mu_{jR}|$ for a negative-going wave-type is physically possible for the phase retardation and energy decay in propagation.

III. ANALYSIS AND DISCUSSION

A. Settings and parameters used in computation

The numerical analysis and choice of the physical parameters for a semi-infinite periodic structure were designed to investigate and reveal the coupling effects of wave propagation. All of the computations and matrix manipulations were conducted using MATLAB. Aluminum was chosen as the beam material, of which Young's modulus is $E_0 = 6.9 \times 10^{10}$ N/m^2 with loss factor $\eta = 0.002$ and density $\rho_0 = 2700$ kg/m^3. The two equal beams are semi-infinite along the x-direction and periodically connected by the transverse beams with a same rectangular cross-section. The thickness and width of the beam cross-section are $h_0 = 11$ mm and $d_0 = 50$ mm, respectively, the periodic element length is $l_0 = 550$ mm and the length of transverse connection beam is $l_c = 500$ mm. The results in the frequency domain computed for the analysis and the discussion herein of the characteristic coupled waves are normalized by using the non-dimensional frequency parameter Ω_n,[11] which is given by

$$\Omega_n = (k_f l_0)^2 = \omega_n (12\rho_0/E_0)^{1/2}(l_0/h_0), \quad (10)$$

where k_f is the real wave number for the free flexural waves that are propagating in beams A and B.

B. Propagation constants and the nature of the wave-types

Basically, there are six characteristic wave-types for the coupled waves that propagate in the semi-infinite periodic waveguide, which all contain both positive-going and negative-going longitudinal and flexural wave motions in the beams because of the multi-coupling at the beam connections. These characteristic wave-types can be divided into two groups—symmetric and antisymmetric types—based on the different phase relationships of the wave motions between beams A and B. Herein the symmetric wave-types are named because the phase differences of the y-degree velocities between beam A and beam B are π and the phase differences of the x-degree velocities between two beams are 0. They are like the mirror images from the symmetry axis of the dual-beam structure in x-direction. For the motions of antisymmetric wave-types, the phase differences of the y-degree velocities between beam A and beam B are 0 and the phase differences of the x-degree velocities between two beams are π. They are like the inverted images from the symmetry axis of the dual-beam structure in x-direction. A further step to describe the propagation characteristics of the wave-types is to use the dispersion of the propagation constants. The computed results for the attenuation constants μ_R and $\cos(\mu_I)$ for the characteristic wave-types in the periodic beam structure are plotted in Figs. 2(a)–2(c). It can be seen from Fig. 2(a) that the symmetric flexural-longitudinal wave motion is governed by two types: α-I and α-II. It is found that more attenuation zones belong to wave-type α-I, and they fall off slowly and have broad forbidden bands. Those that belong to wave-type α-II, which, for the most part, belong to the frequency region below $\Omega_n=330$, have pass bands, but two strong forbidden bands from nearly $\Omega_n=25-47$ and 133–177, where the attenuation constants of α-II fall off rapidly and have sharp peaks at around two significant symmetric resonant modes of the connecting beam branch. It should be noted that the values of $\cos(\mu_I)$ are equal to 1 or -1 in most regions of the forbidden bands. It can be seen from Fig. 2(a) that the two curves of $\cos(\mu_I)$ for the two wave-types overlap at certain normalized frequencies where the attenuation constant is non-zero. This implies that the propagation constants almost become complex conjugates with the non-zero attenuation constant. Similarly, it can be seen from Fig. 2(b) that the antisymmetric flexural-longitudinal wave propagation is governed by two wave-types: β-I and β-II. It is found that more attenuation zones that correspond to the forbidden bands of type β-I fall off slowly and have broad bands. Those of type β-II in most regions below $\Omega_n=310$ have pass bands, but two significant forbidden bands from nearly 62–72 and 219–271, where the attenuation constants of β-II fall off rapidly and have two sharp peaks at around two strong antisymmetric resonant modes of the connecting beam branch.

Strong wave coupling occurs in the forbidden band gaps of the coupled longitudinal-flexural waves, as they are strongly attenuated through the periodic structure. In Fig. 2(c), the attenuation constants and $\cos(\mu_I)$ of the predominantly near-field wave-types are plotted as symmetric and antisymmetric types. For the predominantly near-field waves, the attenuation constants are obviously larger than those for the other wave-types, and all of the $\cos(\mu_I)$ values are almost equal to 1 or -1, which indicates that the energy of predominantly near-field waves decays dramatically as the waves propagate. As these two wave-types are in the strong forbid-

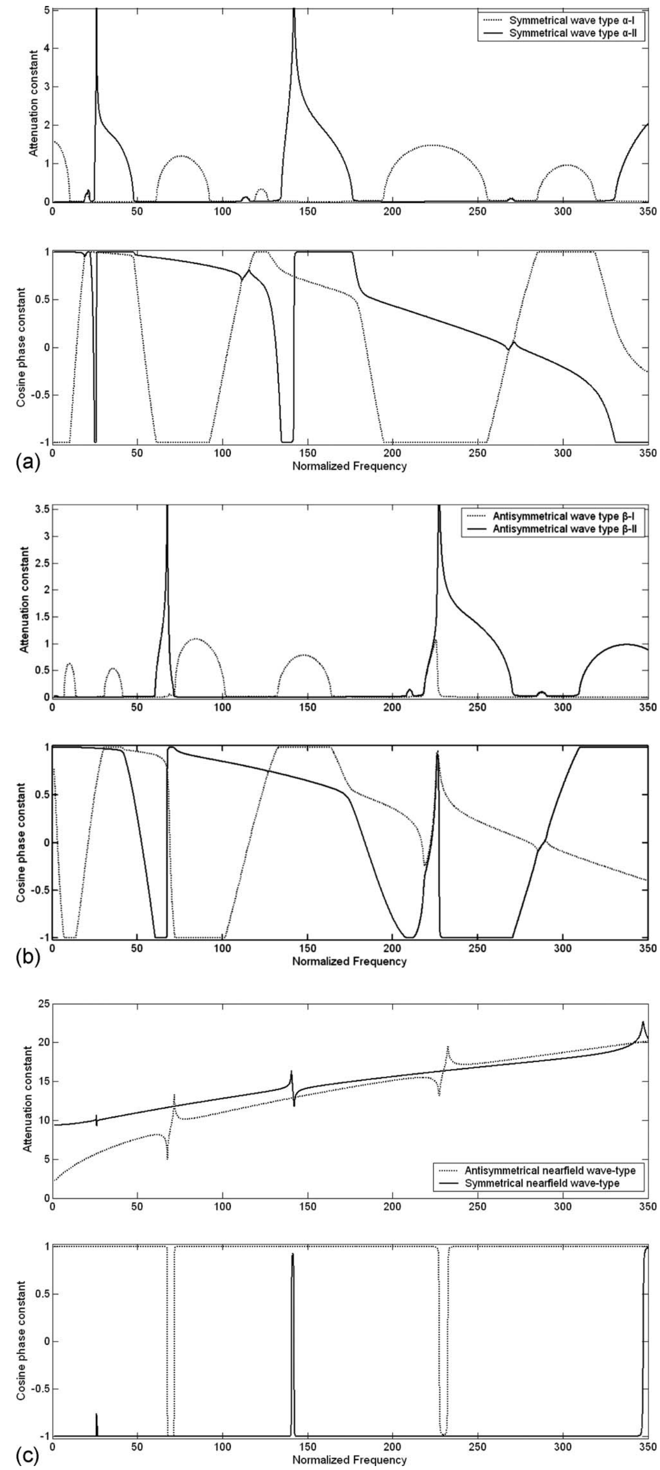

FIG. 2. Propagation constants of characteristic wave-types. (a) μ_R and $\cos(\mu_I)$ of the symmetric flexural-longitudinal wave-types: α-I and α-II. (b) μ_R and $\cos(\mu_I)$ of the antisymmetric flexural-longitudinal wave-types: β-I and β-II. (c) μ_R and $\cos(\mu_I)$ of the symmetric and antisymmetric predominantly near-field wave-types.

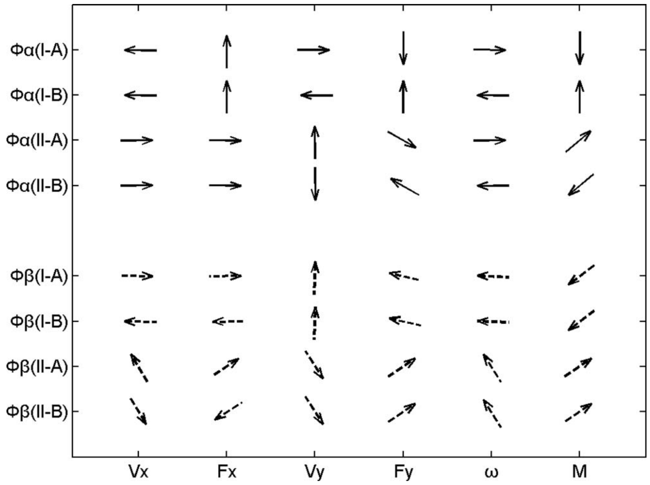

FIG. 3. Phase relationship of characteristic flexural-longitudinal wave vectors in normalized frequency $\Omega_n = 150$.

den band regions, they can be ignored in the consideration of structure-borne sound transmission through a periodic structure.

Figure 3 shows the indicative results of the phase behavior of force-velocity vectors and further illustrates the phase relationship of the motions and actions between two beams. The phase vectors of the coupled flexural-longitudinal waves at the connection between beams A and B are chosen in the normalized frequency $\Omega_n = 150$. Remarkably, it can be observed that for symmetric wave-types α-I and II, the phases of y-degree velocity V_y and rotational velocity ω of beam A are the reverse of those of beam B, and the phase of longitudinal velocity V_x of beam A is the same as that of beam B. In contrast, for antisymmetric wave-types β-I and II, it is found that the phases of y-degree velocity V_y and rotational velocity ω of beam A are the same as those of beam B, whereas the phase of longitudinal velocity V_x of beam A is the reverse of that of beam B. As the frequency is chosen from the pass bands of wave-types α-I and β-II, the phase vectors of their forces and moments point in different directions than their velocities and rotational velocities, which, in total, results in the positive energy flow constantly propagating along the periodic beams. However, the frequency is in the forbidden bands of wave-types α-I and β-II, and almost all of the phase vectors of their forces and moments are perpendicular to the phase vectors of their velocity fields, which indicates that the energy flow cannot continuously propagate through the periodic structure because of energy losses.

C. Excited waves in a semi-infinite periodic structure

In this section, the effect of wave coupling on the response of an ideal semi-infinite periodic structure to two synchronous point excitations is investigated via simulation using the analytical transfer matrix method. Two types of harmonic source excitations that synchronously act on the left side of the semi-infinite beams A and B (along the x-axis) are considered. They are defined as the standardized synchronous longitudinal (x-degree) forces of amplitude $F_{S0} = E_0 S_0$ and the synchronous moments of amplitude M_{S0}

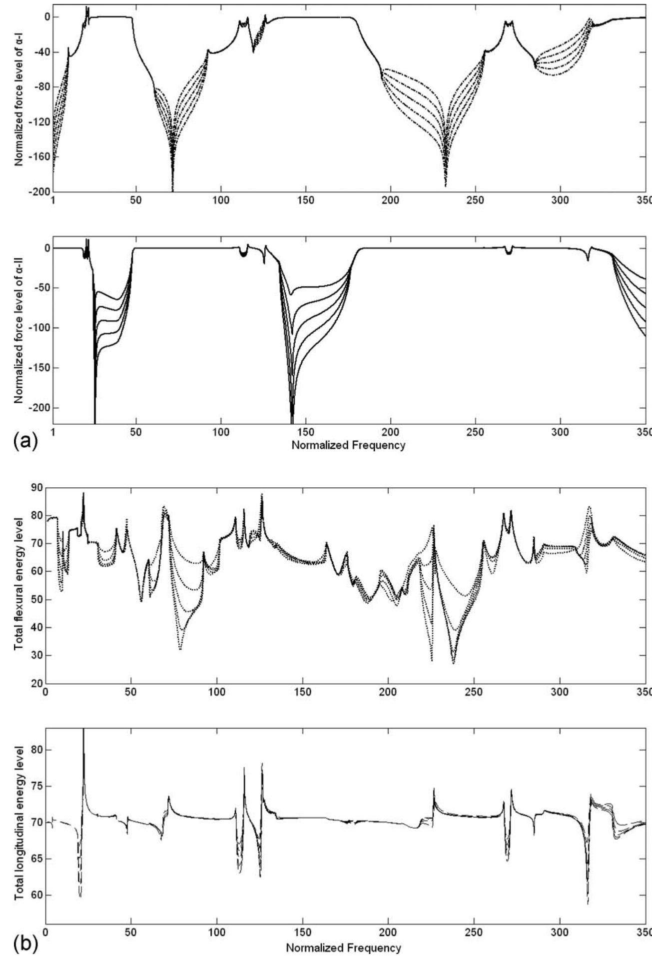

FIG. 4. Amplitude and energy transmission of coupled waves in response to the excitation of synchronous longitudinal forces. (a) The normalized force levels of the symmetrical wave-types α-I and α-II that propagate through the first to fifth beam elements. (b) The total longitudinal and flexural energy levels of the five beam elements for synchronous longitudinal force excitation.

$= (E_0 I_0)/l_0$, where I_0 is the second moment of area of the beam elements and S_0 is its cross-sectional area. The normalized force/moment levels of the propagating wave-types through the first to fifth beam elements are plotted in Figs. 4(a) and 5(a) in the conditions of being excited by the longitudinal forces and moments, which are defined as $\hat{f}_{in}^{j} = 20\log(|\mathbf{f}_{i,j}|/|F_{\mathrm{excit}}|)$, where $\mathbf{f}_{i,j}$ corresponds to the normalized eigenvector ζ_{jn} satisfying the condition that $X_{FxA}^{j} = 1$ is being excited by the synchronous longitudinal forces, and $X_{MA}^{j} = 1$ is the excitation of the synchronous moments. In addition, the variations in the total flexural and longitudinal energy levels of every beam element are plotted in Figs. 4(b) and 5(b) in the forms given by

$$LE_{\mathrm{long}} = 10\log\left[\rho_0 S_0(|v_l^+|^2 + |v_l^-|^2)\int_{l_0}\cos^2 k_l x \cdot dx\right], \quad (11)$$

$$LE_{\mathrm{flex}} = 10\log \rho_0 S_0\left[\frac{1}{2}(|v_f^+|^2 + |v_f^-|^2)l_0 + |v_f^+ v_f^-|\int_{l_0}\cos(2k_f x)\cdot dx\right], \quad (12)$$

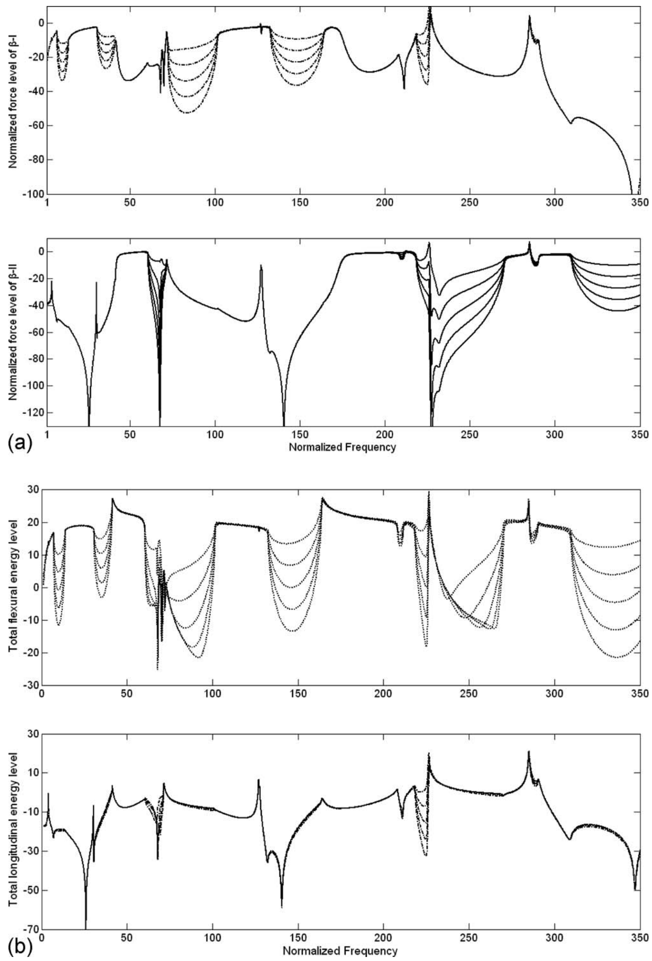

FIG. 5. Amplitude and energy transmission of coupled waves in response to the excitation of synchronous moments. (a) The normalized force levels of symmetrical wave-types β-I and β-II that propagate through the first to fifth beam elements. (b) The total longitudinal and flexural energy levels of the five beam elements for synchronous moment excitation.

Herein the energy unit –J is suppressed because of the use of standardized excitations.

For a periodic structure that is being excited by the synchronous longitudinal forces F_{S0}, the normalized force levels of the symmetrical wave-types α-I and α-II that propagate through the first to fifth beam elements are plotted and shown in Fig. 4(a). The near-field wave-types are neglected, as they decay significantly after propagating through a few elements, and the antisymmetric waves are omitted because they cannot be excited in this case. Notably, it can be seen from Fig. 4(a) that wave-type α-I is excited at a low level and is attenuated significantly within the frequency regions of about $\Omega_n=0-11$, 65–90, 195–256, and $\Omega_n=285-320$ (which belong to the main forbidden bands of α-I) as it propagates through the structure, whereas wave-type α-II is excited at a high level (i.e., the excited forces are near to the source excitation forces) and propagates through the structure without significant attenuation at those frequency regions. Similarly, wave-type α-II is excited at a low level and is attenuated significantly within the frequency regions of about $\Omega_n=25-48$, 136–180, and $\Omega_n=330$ and above (which belong to the strong forbidden bands of α-II) as it propagates through the structure, whereas wave-type α-I is excited at a high level and propagates through the structure without significant attenuation at those frequency regions. Figure 4(b) shows the total longitudinal and flexural energy levels of the five beam elements. It should be noted from this figure that the total longitudinal energy is excited at a considerable level (near to 73), and the waves propagate through the structure without significant energy loss at most frequency regions, except for certain narrow zones that belong to the small forbidden bands of wave-type α-I or α-II. Two prominent gaps in the curves of the total flexural energy level at the frequency regions that correspond to the strong forbidden bands of wave-type α-I can be observed, as the total flexural energy level is mainly due to the coupling effect of the structure. On the other hand, the total longitudinal energy level holds relatively steady at most frequencies, as the total longitudinal energy level is mainly due to the direct effect of the longitudinal source exciting forces. In fact, these two prominent gaps indicate that wave-type α-I contributes most of the energy to the total flexural energy level, compared with wave-type α-II, at those frequency regions. This means that the energy contribution of coupled waves with respect to source excitation depends not only on the forbidden band of the wave-types but also on the energy ratios and combination of wave-types.

For a structure that is being excited by synchronous longitudinal forces M_{S0}, the normalized force levels of symmetrical wave-types β-I and β-II that propagate through the first to fifth beam elements are plotted separately in Fig. 5(a). The near-field wave-types and the antisymmetric waves are again neglected. Notably, it can be seen from Fig. 5(a) that the excited wave type β-I is excited at a low level and is attenuated significantly within the frequency regions of about $\Omega_f=5-10$, 30–40, 72–99, 132–164, and from 220 to 229, which belong to the main forbidden bands of β-I as it propagates through the structure. Besides, the wave-type β-II is excited at a high level where the excited moments are near the source excitation moments and the wave-type β-II propagates through the structure without significant attenuation at those frequency regions. Similarly, wave-type β-II is excited significantly at a low level and is attenuated strongly within the frequency regions of about $\Omega_f=11-21$, 218–270, and $\Omega_f=311$ and above (which belong to the strong forbidden bands of β-II) as it propagates through the structure, whereas wave-type β-I is excited at a high level and propagates through the periodic structure without significant energy loss at those frequency regions. Figure 5(b) shows the total longitudinal and flexural energy levels of the five beam elements for synchronous moment excitation. A comparison of the shapes of the curves of the total flexural energy level in Fig. 5(b) with those in Fig. 5(a) shows that the propagating flexural energy at frequencies approximately lower than 225 is mainly due to the transmission of wave-type β-I, whereas the propagating flexural energy at frequencies approximately higher than 225 is mainly due to the transmission of wave-type β-II. Figure 5 again illustrates that the energy contribution of coupled waves with respect to source excitation depends on the forbidden band of the wave-types and on the energy ratios and combination of wave-types.

IV. CONCLUSION

A new model based on a multi-coupling wave transfer matrix has been developed to study the phenomena of the coupled flexural-longitudinal waves that propagate in tri-coupled dual-channel periodic beam-type waveguides. A lightly damped semi-infinite structure that consists of two equally thin semi-infinite beams connected with resonant branches has been numerically analyzed. The connection branches are the beams perpendicularly connected at regular intervals. This type of waveguide can simulate a one- to two-dimensional model of a column-beam frame for modern steel-concrete buildings or bridges simply. The computed results of the complex propagation constants that govern the transmission of wave-types in periodic structures have clearly revealed the characteristics of pass and forbidden bands and the wave-coupling phenomena. It is found that there are six characteristic coupled wave-types that propagate through such a structure, and these can be divided into symmetric and antisymmetric groups of flexural-longitudinal and predominantly near-field characteristic wave-types. Their properties under different excitations are quantified from the computed transmission of the normalized amplitudes of the coupled wave-types together with the maximum flexural and longitudinal energies along the wave-carrying components. It has been revealed that the structure-borne sound energy from the synchronous longitudinal excitations at two beams mainly propagate through the periodic structure in the form of one or two types of symmetric characteristic coupled flexural-longitudinal waves. In contrast, the structure-borne sound energy from the synchronous rotational sources that excite dual-channel beams mainly propagate along the periodic structure in the form of one or two types of antisymmetric characteristic coupled flexural-longitudinal waves. These results demonstrate that the energy contribution of coupled waves with respect to source excitation depends on the forbidden band of the wave-types and on the energy ratios and combination of wave-types.

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