Flow noise from spoilers in ducts

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(Received 17 April 2008; revised 28 March 2009; accepted 10 April 2009)

Measurements of flow noise produced by strip spoilers in the air duct of a ventilation system and radiated from an open exhaust termination unit into a reverberation chamber have been made. The results agree with the previous work of Nelson and Morfey [J. Sound Vib. 79, 263–289 (1981)].

Prediction of flow noise produced by multiple spoilers requires the values of the ratio of the mean drag forces that act on the spoilers, the phase relationship between the fluctuating drag forces that act on the spoilers, and the coherence function of the noise sources. The latter is empirically derived from the measured results, where the predicted results agree well with the experimental results within 3 dB at most frequencies except for very high frequencies. © 2009 Acoustical Society of America. [DOI: 10.1121/1.3127129]

PACS number(s): 43.50.Nm, 43.50.Ed, 43.50.Cb, 43.28.Ra [JWP] Pages: 3756–3765

I. INTRODUCTION

Flow noise is produced by in-duct elements such as dampers, sensors, bends, transition pieces, duct corners, branch points, or even splitter attenuators. At long distances from the fans in an air duct, flow noise produced by in-duct elements can be very serious. Our objective is to enable the prediction of the level and spectral content of the flow noise that is produced by the multiple elements in the air duct of a ventilation system.

The current design guides that are usually adopted, such as the ASHRAE handbook and the CIBSE guide, provide design methods for the prediction of flow noise only from a single, isolated in-duct element in an air ductwork system. Wilson and Iqbal observed that these methods seriously underestimate the levels of flow noise in practical systems.

Measured data that have been reported in previous studies can only be used with confidence on systems that incorporate the same configurations and carry the same airflows. Attempts based on simplified theories that have been made using the limited data and equations are not applicable to systems with very different configurations.

Gordon conducted a series of experiments and produced a free-field scaling law radiation model relating the sound power radiated from an element to the geometrical and flow parameters involved to collapse his measured sound power data into a “generalized spectrum.” However, the experimental results he obtained were at low Strouhal numbers and high Mach numbers that are different from the air duct flow in a ventilation system (at high Strouhal numbers and low Mach numbers). Heller and Widnall clarified the acoustical significance of the duct that encloses the noise source.

A method for predicting flow noise was produced and verified experimentally (at high Strouhal numbers and low Mach numbers) by Nelson and Morfey. Oldham and Ukpoho then rewrote the Nelson–Morfey equations. They determined the appropriate values of the open area ratio and of the characteristic dimension to be applied to a flow spoiler in a circular or a square duct. They focused on the sound field that is due to a single duct element such as a strip spoiler. Flow noise can be produced by the interaction of a moving fluid with a single duct element or a combination of duct elements. Ukpoho and Oldham experimented with two sound sources (in-duct spoilers) to find that the flow noise increases when two duct elements are considered and that this increase is frequency-dependent. The latter conclusion shows that it is intuitively the same as the two sources in the active control of the sound in an air duct. Owing to the flow interaction between the duct elements, the random (partially coherent) field case is likely to be the general case in ventilation systems. Mak and Yang applied a model of partially coherent sources to formulate the sound power level due to duct spoilers. This model considers the acoustic interaction of two in-duct spoilers based on the earlier work of Nelson and Morfey. Mak later modified the Mak–Yang equations to determine the sound power radiated by the interaction of more complicated spoilers in circular ducts. He assumed a coherence function and compared the measured

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values of Ukpoho and Oldham\textsuperscript{18} with the predicted values of the modified Mak–Yang equations and found that these predicted values agreed well with the general trend of the measured values. Mak and Oldham\textsuperscript{23,24} approach was later used to modify the Mak–Yang equations to produce a turbulence-based prediction technique for flow noise.\textsuperscript{25} Recent work by Mak and Au\textsuperscript{26} has confirmed the usefulness of the approach. Mak\textsuperscript{27} then extended Mak and Yang’s prediction method to the prediction of multiple flow noise sources.

Airflow, acoustic, and force measurement data are collapsed into normalized spectra with the aid of these derived predictive equations. This is a prediction technique for the flow-generated noise produced by multiple spoilers in the air duct of a ventilation system.

II. THEORY AND PREDICTIVE EQUATIONS OF NOISE PRODUCED BY IN-DUCT FLOW SPOILER(S)

Mak adopted the concept of partially coherent sound fields to formulate the sound powers that are produced by the interaction of multiple spoilers at frequencies below and above the cut-on frequency of the lowest transverse duct mode. An example of three flow spoilers in an infinite air duct is shown in Fig. 1.

Two equations were obtained to determine the sound power that is generated by the interaction of multiple spoilers, one of which corresponds to frequencies, $f_c$, below the cut-on frequency, $f_0$, and one of which corresponds to frequencies above it. For $N (N>2)$ elements, the following are derived. For $f_c<f_0$,

$$\Pi_N = K^2(S) \times \Gamma_1 \times I_1.$$  \hspace{1cm} (1)

For $f_c>f_0$,

$$\Pi_N = K^2(S) \times \Gamma_2 \times I_2.$$  \hspace{1cm} (2)

The power term $\Gamma_1$ from the first spoiler below the cut-on frequency is

$$\Gamma_1 = \{\rho_0 A[\sigma^2(1-\sigma)]^2 C_p^2 U_0^4/16c_0^2\}.$$ 

The power term $\Gamma_2$ from the first spoiler above the cut-one frequency is

$$\Gamma_2 = \{\rho_0 \pi A^2(S)^2[\sigma^2(1-\sigma)]^2 \times \left[ C_p^2 U_0^4/24c_0^2 \right]^2 \times \left[ 1 + (3\pi c_0/4\omega_c)(a + b)/A \right] \}.$$ 

The interaction term $I_1$ below the cut-on frequency is

$$I_1 = \left\{ \sum_{i=1}^{N} \xi^2_i + 2 \sum_{i=1}^{N-1} \sum_{j=1}^{N-2} \left[ \sqrt{\gamma_{i(i+1)}} \times \cos(\omega_k d_{i(i+1)}/c_0) \cos\phi_{i(i+1)}(\omega_c)\xi_{i+1} \right] ight. 
+ \gamma_{i(i+1)} \times \cos(\omega_k d_{i(i-1)(i+j)}/c_0) \right\} \xi_{i+1}.$$ 

The interaction term $I_2$ above the cut-on frequency is

$$I_2 = \left\{ \sum_{i=1}^{N} \xi^2_i + 2 \sum_{i=1}^{N-1} \sum_{j=1}^{N-2} \left[ \sqrt{\gamma_{i(i+1)}} \Omega_{i(i+1)} \times \cos(\phi_{i(i+1)}(\omega_c)\xi_{i+1} \right] ight. 
+ \gamma_{i(i+1)} \Omega_{i(i+1)} \cos(\phi_{i(i-1)(i+j)}(\omega_c)\xi_{i+1} \xi_{i+1}) \right\}.$$ 

where $N > 2$, $N$, $i$, and $j$ are integers and $j=1,2,\ldots,(N-2)$.

In the above equation, a value is ignored if any one of its subscripts is zero or greater than $N$. $Q_{i(i+1)}$ is given by

$$Q_{i(i+1)} = \Omega_{i(i+1)} \Psi_{i(i+1)} \times \left[ k^2 ab/6\pi + k(a + b)/8 \right],$$

and $e=kd_{i(i+1)}$. $Q_{(i-1)(i+j)}$ is given by $Q_{(i-1)(i+j)} = \Omega_{(i-1)(i+j)} \Psi_{(i-1)(i+j)}$, where

$$\Omega_{(i-1)(i+j)} = \frac{k^2 ab}{6\pi} \left[ \sin e + \frac{2 \cos e}{e^2} - \frac{2 \sin e}{e^3} \right] + \frac{k(a + b)}{8} \left[ J_0(e) - J_1(e) \right],$$

$$\Psi_{(i-1)(i+j)} = \frac{k^2 ab}{6\pi} + \frac{k(a + b)}{8}.$$
\(P_N\) is the infinite-duct values of the radiated sound power; \(f_0\) is the cut-on frequency of the first transverse duct mode (i.e., the least non-zero value of the cut-on frequency is defined by \(f_0=(c_0/2\pi)(m\pi/a)^2+(n\pi/b)^2\), where \(m,n=0,1,\ldots\)); \(c_0\) is the ambient speed of sound; \(\rho_0\) is the ambient air density; \(\gamma_{ij}\) is the coherence function of the \(i\)th spoiler and the \(j\)th spoiler; \(\omega_i\) is the center frequency of the measurement band; \(\phi_{ij}(\omega_i)\) is the phase of the cross-power spectral density of the source volume of the \(i\)th sound source and the \(j\)th sound source; \(\zeta_i\) is a constant ratio of the mean drag forces acting on the \(i\)th spoiler and the first spoiler; \(\Delta P_s\) is the static pressure drop across a spoiler (Pa); and \(C_D\) is the drag coefficient determined by

\[
C_D = \frac{\Delta P_s}{\frac{1}{2} \rho_0 U_c^2 \sigma^2 (1 - \sigma)}.
\]

Comparing the above expressions with those obtained by Nelson and Morfey\(^6\) for the sound power generated by an in-duct spoiler, the interaction factor \(\beta_N\) can be defined as follows:

\[
\beta_N = \begin{cases} 
I_1, & f_c < f_0, \\
I_2, & f_c > f_0.
\end{cases}
\]

Furthermore, if the sound power that is due to an in-duct spoiler is denoted as \(\Pi_N\), a simple relationship between \(\Pi_N\), the sound power that is due to multiple \((N)\) spoilers and that due to a single spoiler, is then obtained as follows:

\[
\Pi_N = \Pi_S \times \beta_N,
\]

where \(\Pi_S\) can be obtained by using the prediction method provided by Nelson and Morfey,\(^6\) and \(\beta_N\) can be determined by experiments.

We define \(\zeta_i\) as the constant ratio of the mean drag forces acting on the \(i\)th spoiler and the first spoiler. The first spoiler is closest to the inlet of the air flow:

\[
\zeta_i = \frac{\bar{F}_{z_i}}{\bar{F}_{z1}},
\]

where \(\bar{F}_{z_i}\) is the mean drag force acting on the \(i\)th spoiler counted from the inlet of the air flow, and \(\bar{F}_{z1}\) is the mean drag force acting on the first spoiler. The mean drag force acting on the \(i\)th spoiler can be expressed as \(\bar{F}_{z_i}=A \Delta P_s\).

Han et al.\(^8\) and Han and Mak\(^9\) suggested that the phase of the cross-power spectral density of the source volume of the \(i\)th sound source and the \(j\)th sound source, \(\phi_{ij}(\omega_c)\), can be given by

\[
\phi_{ij}(\omega_c) = \delta_{ij} - k M \bar{d}_{ij},
\]

where \(\delta_{ij}\) is the difference between the phases of the total fluctuating drag forces acting on the \(i\)th spoiler and the \(j\)th spoiler: \(\delta_{ij} = \theta_i(\omega_c) - \theta_j(\omega_c)\), where \(\theta_i(\omega_c)\) and \(\theta_j(\omega_c)\) are the phases of the fluctuating drag force acting on the \(i\)th spoiler, respectively, \(k\) is the wave number, \(M=U/c_0\), and \(\bar{d}_{ij} = d_{ij}/(1-M^2)\).

The coherence functions, \(\gamma_{ij}\), of the noise sources are obtained as follows.

III. EXPERIMENTAL PROCEDURE

A. Design of the experimental rig

The test rig used for the determination of the sound power generated by flat plate spoiler(s) and their interactions are shown in Fig. 2. Air flow was provided by a centrifugal fan driven by a variable speed motor. The fan was vibration-isolated by springs and was enclosed in a lined acoustic enclosure of \(1.22 \times 1.22 \times 1.22\) m\(^3\). Fan noise was attenuated on both the upstream and downstream sides by silencers and acoustically lined elbows. The 0.1 m\(^2\) test duct was made of steel and was enclosed by 25-mm-thick absorptive lining to reduce breakout noise from the duct. This arrangement yielded a quiet, fully developed air flow at the first test piece counted from the inlet of air flow, which was situated approximately 1.75 m from the duct entrance section. The total length of the duct was 5.45 m. The duct was passed into a 70 m\(^3\) reverberation chamber with an outlet cone of 0.16 \(\times 0.16\) m\(^2\) and a length of 0.3 m for acoustic measurements. The reverberation chamber was provided with lined outlet ducts that allowed air to escape without allowing noise from outside to penetrate. The entire system was located in another 200 m\(^3\) reverberation chamber with a closed door so that the level of sound that was measured in the system was always well above the background noise level.

B. The spoilers used in the experiment

The spoilers used in the experiment, as shown in Fig. 3, were selected to test the validity of Mak’s\(^27\) prediction method for multiple flow spoilers. The spoiler plates were made from 1-mm-thick steel plate. These plates were fixed by springs and force transducers between the flanges of two adjoining sections of the test duct. The gap was sealed with compressed foam rubber. The spoilers provided a rigid obstruction to the flow in the duct and were not able to vibrate significantly in the air stream. Six different spoiler geometries were tested, each with at least four duct flow velocities. Three of the spoilers were vertical strips of plate placed centrally in the air stream. Their height was that of the test duct,
and their widths were 0.025, 0.05, and 0.075 m, respectively. The other three geometries consisted of plates that protruded symmetrically from both sides of the duct, leaving a central vertical strip of the duct open. Again, the plates had the same height as the duct, and the widths of the two side plates were equal. The total widths in the three configurations were 0.025, 0.05, and 0.075 m, respectively. Eleven tests were conducted, as shown in Table I. In Tests 1–4, only a single spoiler was inserted into position p1. In Tests 5–9, two spoilers were inserted at two different positions p1 and p2. In Tests 10 and 11, three spoilers were inserted at three different positions p1, p2, and p3. The three spoiler positions are shown in Fig. 2.

### C. Airflow measurements

The velocity profile in the empty test duct was measured. A pitot tube was used to sample the dynamic pressure at specified points in the duct cross-section at the position shown in Fig. 4. Plots of the duct velocity profile at several test velocities at the 0.1 m side are shown in Fig. 5. The slight asymmetry of the velocity profiles that were measured is due to the non-symmetric inlet conditions and is not regarded as serious in the experiments. This procedure was used to calculate the mean duct velocity over the 5.4 m length at five duct velocities. The mean velocity calculated for each of the five velocities was found to have a linear relationship with the velocity measured at the “calibrated” position at the center of the duct (as shown in Fig. 6). Value of the mean duct velocity was determined by using this single calibrated position of the pitot tube. The pitot tube was removed from the duct when undertaking acoustic measurements.

The static pressure drop across the various spoilers that were tested was measured using two piezometric rings located at the positions p1, p2, and p3 shown in Fig. 2. Each ring consisted of four static pressure tappings, one in each

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**TABLE I.** Eleven spoiler configurations tested. See Fig. 2. (p1 = position at which the first test spoiler was inserted, p2 = position at which the second test spoiler was inserted, and p3 = position at which the third test spoiler was inserted.)

<table>
<thead>
<tr>
<th>No.</th>
<th>Spoiler(s) used in the experiments</th>
<th>Mean flow velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>r=0.025m at p1</td>
<td>10,15,20,25,30</td>
</tr>
<tr>
<td>2</td>
<td>r=0.05m at p1</td>
<td>10,14,18,22,26</td>
</tr>
<tr>
<td>3</td>
<td>r=0.025m at p1</td>
<td>15,20,25,30,35</td>
</tr>
<tr>
<td>4</td>
<td>r=0.05m at p1</td>
<td>10,15,20,25,30</td>
</tr>
<tr>
<td>5</td>
<td>r=0.025m at p1,r=0.025m at p2</td>
<td>10,15,20,25,30</td>
</tr>
<tr>
<td>6</td>
<td>r=0.025m at p1,r=0.05m at p2</td>
<td>10,14,18,22,26</td>
</tr>
<tr>
<td>7</td>
<td>r=0.05m at p1,r=0.05m at p2</td>
<td>10,13,16,19,22</td>
</tr>
<tr>
<td>8</td>
<td>r=0.075m at p1,r=0.075m at p2</td>
<td>10,11,12,13,14</td>
</tr>
<tr>
<td>9</td>
<td>r=0.025m at p1,r=0.05m at p2</td>
<td>10,14,18,22,26</td>
</tr>
<tr>
<td>10</td>
<td>r=0.025m at p1,r=0.025m at p2,r=0.025m at p3</td>
<td>10,13,16,19,22</td>
</tr>
<tr>
<td>11</td>
<td>r=0.05m at p1,r=0.05m at p2,r=0.05m at p3</td>
<td>10,12,14,16,18</td>
</tr>
</tbody>
</table>

---

FIG. 3. Cross-section of the duct with two main types and various sizes of the flat plate strip spoilers used (shaded area). (a) Centrally placed strip spoilers; r=0.025, 0.05, 0.075 m. (b) The geometries consisted of trip plates protruding symmetrically from both sides of the duct, leaving a central vertical strip of the duct open; r=0.025, 0.05, 0.075 m.

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FIG. 4. Positions in the duct cross-section at which a pitot tube was used to sample the dynamic pressure.
duct face. The downstream ring was far enough away (five times the duct dimensions) from the test spoiler to ensure that full static pressure recovery could take place in the wake of the flow obstructions under test.

D. Acoustic measurements

The temperature and relative air humidity in the reverberation chamber were measured with a sling psychrometer. During the measurements, the temperature ranged from 26.5 to 27.8 °C, and the relative humidity ranged from 53% to 59%. The reverberation time (RT) was measured by the impulse response method of the mean-length sequence (MLS), as shown in Fig. 7. This MLS signal was generated internally by DIRAC software installed on a notebook computer and fed to the omni-directional source B&K-type 4241, a dodecahedron loudspeaker that was placed at the corner of the chamber. A B&K-type sound level meter, which was connected to the computer through an external sound card, was located at 1.20 m above the floor at the predetermined positions. The door of the chamber was closed, and there were no personnel inside the chamber during the measurement process. Six different microphone positions were used for the frequencies between 200 and 10 kHz. All of the measured positions were located at least 1 m away from the chamber walls. The distance between the two microphones was larger than the half a wavelength of sound under consideration. The geometric mean of the measured RTs and the sound pressure levels (SPLs) were based on the measured RT and the measured SPLs at the six different randomly selected positions, respectively. About 50 sampled averaged RTs and averaged SPLs were obtained so that the mean of the averaged RTs and the averaged SPLs and the standard deviation, $S_D$, of the variation of the averaged RTs and averaged SPLs could be calculated, respectively. Assuming that the measured value of $S_D$ is the true standard deviation of a normal distribution of averaged RTs and averaged SPLs, it is found that the 95% confidence limits for the mean of the distribution estimated from $n_s$ samples ($n_s = 50$) are then given by $\pm 1.96S_D/\sqrt{n_s}$. The error is $\pm 1.96S_D/\sqrt{50}$, and the geometrical mean of the RTs and the SPLs should be around $1\sim3$ s and $\pm 1.5$ dB, respectively. The geometrical mean of the measured RT is shown in Fig. 8. The 95% confidence limits for the mean SPL are shown in Fig. 9. The results are better than those measured by Nelson and Morfey, as can be seen by the smaller errors at all frequencies (see Fig. 9). A sufficiently diffuse field was established in the middle of the frequency range so that more reliable results could be obtained. At high frequencies, the accuracy decreases again as the absorption in the room increases and the direct field of the duct exit encroaches on the reverberant field. The sound power level

\[ y = 1.0375x + 0.3412 \]

\[ R^2 = 0.9997 \]
that radiated from the duct exit was then calculated from

\[ SWL = SPL + 10 \log_{10} V_R - 10 \log_{10} RT - 14, \]

where \( V_R \) is the room volume (70 m\(^3\)), \( RT \) is the geometrical mean of the reverberation time(s), and SPL is the space-averaged sound pressure level in the chamber (dB).

The background noise due to the flow in the empty duct was monitored at five test velocities. The ambient background noise level was also measured after each test. The measured noise levels were kept at least 10 dB above the ambient level, and the background flow noise level at the given test velocity in the duct.

E. Force measurements

The arrangement of springs and force transducers is shown in Fig. 10. The plates were fixed by springs and force transducers between the flanges of the two adjoining sections of the duct, and the gap sealed with a compressed foam rubber seal. For the centrally placed strip spoiler, two transducers mounted on the duct supporting flanges were required to measure the total fluctuating drag force. For the two side strip spoilers, four transducers on the duct supporting flanges were required to measure the total fluctuating drag force. The springs used to provide support for the plates were selected to have stiffness sufficient to place the mechanical resonant frequency well below 200 Hz, the lowest 1/3 octave band measuring frequency of the reverberation chamber of 70 m\(^3\), and also below 50 Hz, which is the frequency of the alternating current supply in Hong Kong. The frequency response of the force transducers was flat over the measuring frequency range. Their sensitivity was 11 mV/N. The force transducers were connected to NI LABVIEW equipment via a synchronized multi-channel signal acquisition of type NI cRio-9233 through pre-amplifiers, as indicated in Fig. 10. The magnitude and phases of the total fluctuating forces acting on the spoilers at various duct flow velocities were measured in real time and converted into a frequency domain by the NI equipment.

IV. ANALYSIS AND DISCUSSION OF THE RESULTS

A. Normalization of the experimental results for a single flow spoiler

The results measured for the sound power that radiated from the end of the duct for a single spoiler can be normalized according to Nelson and Morfey.\(^{16}\) Their equations, and those of Mak\(^{27}\) and Mak and Yang,\(^{20,21}\) were developed for an infinite duct. The duct used, however, was of finite length with a considerable length-absorbent lining upstream of the test duct. It has been assumed that the spoiler is transparent with a considerable length-absorbent lining upstream of the duct, and also below 50 Hz, which is the frequency of the alternating current supply in Hong Kong. The frequency response of the force transducers was flat over the measuring frequency range. Their sensitivity was 11 mV/N. The force transducers were connected to NI LABVIEW equipment via a synchronized multi-channel signal acquisition of type NI cRio-9233 through pre-amplifiers, as indicated in Fig. 10. The magnitude and phases of the total fluctuating forces acting on the spoilers at various duct flow velocities were measured in real time and converted into a frequency domain by the NI equipment.

The result for one of the single spoilers tested, normalized\(^{16}\) with the measured values of \( C_D \) used and the sound power levels is shown in Fig. 11, which denotes 120 +20 \( \log_{10} K(S) \) and relates to the infinite-duct values of the radiated sound power \( P_s \) at each test velocity. For most of the spoilers tested, the calculated values of \( C_D \) varied by around 0.5%–5% over the range of the test velocities used.

The drag coefficient, \( C_D \), used in the normalization of the results for the single spoilers was evaluated from the measurements of the static pressure drop across the spoilers based on Eq. (3). The values of \( C_D \) for each spoiler were found by averaging the values calculated from the measurements of \( \Delta P_s \) and \( U_c \), at each test velocity. For most of the spoilers tested, the calculated values of \( C_D \) varied by around 0.5%–5% over the range of the test velocities used.

The result for one of the single spoilers tested, normalized\(^{16}\) with the measured values of \( C_D \) used and the sound power levels is shown in Fig. 11, which denotes 120 +20 \( \log_{10} K(S) \) and relates to the infinite-duct values of the radiated sound power \( P_s \) in Eqs. (1) and (2). In view of the vastly different forms of the two equations for frequencies above and below the cut-on frequency of the first transverse duct mode, the collapse of the experimental data for all of the single spoilers tested is good in view of the widely differing drag coefficients associated with the various spoilers. Nelson and Morfey\(^{16}\) reported that there was an error in their original normalized spectrum and that their trend lines need to be displaced vertically downwards by 6 dB. The scatter associated with the experimental points of Fig. 11 is comparable with the range of corrected trend lines in their normal-
ized spectrum. This suggests that the results reported here agree well with their corrected trend lines; Fig. 12 shows a comparison of the values of the normalized results for all of the single spoilers tested and a trend line based on simple linear relationships over the range of measurements. Scattering was observed at low Strouhal numbers (or low frequencies), which was also seen in the work of Oldham and Ukpo.17

B. Determination of the coherence function of noise sources

The assumptions used in the determination of the coherence function, $\gamma_{ij}^2$, are as follows:

1. The values of the coherence function should be between 0 and 1, i.e., $0 \leq \gamma_{ij}^2 \leq 1$. If $\gamma_{ij}^2$ takes the value of 1 at certain frequencies, then this means that the $i$th and $j$th sound sources are fully coherent at those frequencies. If $\gamma_{ij}^2$ is equal to zero at certain frequencies, then this means that the $i$th and $j$th sound sources are incoherent at those frequencies.

2. The coherence function should be dependent on the magnitude and phases of the fluctuating drag forces acting on each spoiler at various frequencies.

3. The coherence function is inversely related to the distance between the $i$th and $j$th spoilers. When one flow spoiler is far away from another, the coherence function between the flow noise sources should be small when the other parameters are fixed.

4. The coherence function should be dependent on the mean duct flow air velocity.

The experimental data for the two flow spoilers in Test 5 were used to obtain $\sqrt{\gamma_{ij}^2}$ in the predictive equations for multiple spoilers. The relationship between $\sqrt{\gamma_{ij}^2}$ and the other parameters, such as the phases and magnitude of the fluctuating drag forces, $F_i$, acting on the spoilers, the distance between the two noise sources, $d_{ij}$, and the mean flow air velocity, $U$, were analyzed by the Statistical Package for the Social Sciences (SPSS). The square of $\sqrt{\gamma_{ij}^2}$ will give the coherence function $\gamma_{ij}^2$ and the value of $\sqrt{\gamma_{ij}^2}$ will therefore also be between 0 and 1.

It was found from the SPSS analysis of the data that there was an approximately linear relationship between $\sqrt{\gamma_{ij}^2}$ and $\log(|F_i(\omega)/F_j(\omega)\cos(\theta_i(\omega)-\theta_j(\omega))|)$ with a correlation coefficient of 0.84 between them. It was therefore assumed that $\sqrt{\gamma_{ij}^2}$ may be a logarithmic function that contains the factor of $|F_i(\omega)/F_j(\omega)\cos(\theta_i(\omega)-\theta_j(\omega))|$. $F_i(\omega)$ and $F_j(\omega)$ are the magnitudes of the fluctuating drag force acting on the $i$th and $j$th spoilers, respectively. To ensure that the value of the coherence function was between 0 and 1, the following preliminary formula was obtained:

$$\gamma_{ij}^2 = \log_{10}\left(1 + \frac{15}{Ud_{ij}}\frac{F_i(\omega)}{F_j(\omega)}^2 \cos^2(\theta_i(\omega) - \theta_j(\omega))\right),$$

where $F_i(\omega)$ and $\theta_i(\omega)$ are the magnitude and phases, respectively, of the fluctuating drag force acting on the $i$th spoiler.

Together with the values of the other parameters, such as the phases of the cross-power spectral density of the source volumes and the ratio of the mean drag forces, the interaction factor $\beta_N$ in the predictive equations can be obtained.

C. Comparison between predicted and measured results

Mak’s predictive equations for multiple flow spoilers were used to predict the sound power levels produced by two or three flow spoilers in Tests 5–11. The predicted and measured results of the sound wave levels in the seven tests at a particular flow velocity are shown in Figs. 13–19. The error between the predicted and measured results was $\pm 0–3$ dB at most frequencies at a particular mean flow velocity. The deviation between the predicted and measured results at certain frequencies at or above 4 kHz at a particular flow may have been due to high frequency vibration modes of the flow spoiler or the duct system.

These predictive equations developed are useful for predicting the level and spectral contents of the flow-generated noise from multiple in-duct flow spoilers at the design stage. This prediction provides a normalized spectrum, using a table of parameters derived from experimental model measurements.
FIG. 13. Comparison of the measured values and predicted values of the SPL in Test 5 at $U=20$ m/s.

FIG. 14. Comparison of the measured values and predicted values of the SPL in Test 6 at $U=22$ m/s.

FIG. 15. Comparison of the measured values and predicted values of the SPL in Test 7 at $U=10$ m/s.

FIG. 16. Comparison of the measured values and predicted values of the SPL in Test 8 at $U=11$ m/s.

FIG. 17. Comparison of the measured values and predicted values of the SPL in Test 9 at $U=18$ m/s.

FIG. 18. Comparison of the measured values and predicted values of the SPL in Test 10 at $U=19$ m/s.
V. CONCLUSION

The collapse of the data found with single flow spoilers is similar to that observed by Nelson and Morfey and Oldham and Ukpocho. Together with the normalized spectrum and the coherence function, Mak’s predictive equations for multiple flow spoilers, one can now predict the sound power levels produced by two or three flow spoilers at most frequencies at various mean duct flow velocities.

It is suggested that the line trends reported in this study, together with the coherence function of the noise sources, the phase relationship between the fluctuating spoiler drag forces, and the ratio of the mean drag forces provide the basis of a generalized predictive technique. Further work is required to extend the work to the interactions of practical duct discontinuities such as bends and transition pieces.

ACKNOWLEDGMENTS

The work described in this paper was fully supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. PolyU 5229/05E).

NOMENCLATURE

\[ a, b = \text{duct cross-section dimensions (m)} \]
\[ A = \text{area of the duct cross-section determined by } A = (a \times b) \text{ (m}^2) \]
\[ A_s = \text{face area of flat plate spoiler (m}^2) \]
\[ A_c = \text{area of the duct constriction determined by } A_c = (a \times b - r \times b) \text{ (m}^2) \]
\[ c_0 = \text{ambient speed of sound (m/s)} \]
\[ C_D = \text{drag coefficient} \]
\[ d_{ij} = \text{distance between the } i\text{th spoiler and the } j\text{th spoiler (m)} \]
\[ f_c = \text{center frequency of measurement band (Hz)} \]
\[ f_0 = \text{cut-on frequency of the first transverse duct mode (Hz) defined by } f_0 = (c_0/2\pi)\sqrt{(m\pi/a)^2 + (n\pi/b)^2}, \text{ where } m, n = 0, ..., 1, ... \]
\[ F_i(\omega) = \text{magnitude of the fluctuating drag force acting on the } i\text{th spoiler (N)} \]
\[ \bar{F}_i = \text{mean drag force acting on the } i\text{th spoiler counted from the inlet of the air flow (N)} \]
\[ \bar{F}_{ci} = \text{mean drag force acting on the first spoiler (N)} \]
\[ i, j = \text{integers} \]
\[ I_1, I_2 = \text{interaction terms below and above the cut-on frequency} \]
\[ J_0, J_1 = \text{zero- and first-order Bessel’s functions} \]
\[ k = \text{wave number} \]
\[ K(S) = \text{ratio of fluctuating to steady-state drag forces on the spoilers} \]
\[ K^2(S) = \text{square of the ratio } K(S) \]
\[ m, n = \text{integers} \]
\[ M = \text{Mach number given by } M = U/c_0 \]
\[ N = \text{number of in-duct spoilers or in-duct elements} \]
\[ P_S = \text{static pressure (Pa)} \]
\[ \Delta P_S = \text{static pressure drop across a spoiler (Pa)} \]
\[ q = \text{volume flow rate (m}^3\text{/s)} \]
\[ r = \text{characteristic dimension of the in-duct element (m)} \]
\[ RT = \text{reverberation time (s)} \]
\[ \text{RT} = \text{geometrical mean of reverberation time (s)} \]
\[ S = \text{Strohal number determined by } S = f_r/U_c \]
\[ S_D = \text{standard deviation} \]
\[ \text{SPL} = \text{sound pressure level (dB)} \]
\[ \text{SWL} = \text{sound power level radiated from the duct exit (dB)} \]
\[ U = \text{mean duct flow velocity (m/s)} \]
\[ U_c = \text{flow velocity in the constriction (m/s) determined by } U_c = q/A_c = UA/A_c \]
\[ V_R = \text{room volume of the reverberation chamber (m}^3) \]
\[ \omega = \text{center radiant frequency of the measurement band (rad/s)} \]
\[ W = \text{sound power (W)} \]
\[ \Pi = \text{infinite-duct values of the radiated sound power due to multiple (N) spoilers (W)} \]
\[ c_0 = \text{ambient air density (kg/m}^3) \]
\[ \sigma = \text{open area ratio determined by } \sigma = A_c/A \]
\[ \gamma_{ij}^2 = \text{coherence function of the } i\text{th spoiler and the } j\text{th spoiler} \]
\[ \phi_{ij}(\omega) = \text{phase of the cross-power spectral density of the source volume of the } i\text{th sound source and the } j\text{th sound source} \]
\[ \zeta_i = \text{constant ratio of the mean drag forces acting on the } i\text{th spoiler and the first spoiler} \]
\[ \beta_N = \text{interaction factor} \]

FIG. 19. Comparison of the measured values and predicted values of the SPL in Test 11 at U=10 m/s.

NOMENCLATURE
\[ \phi_i(\omega) = \text{phase of the cross-power spectral density of} \]
\[ \delta_{ij} = \text{difference between the phases of the total} \]
\[ \text{fluctuating drag forces acting on the } i \text{th} \]
\[ \text{spoiler and the } j \text{th} \text{ spoiler determined by} \]
\[ \delta_{ij} = \theta_i(\omega) - \theta_j(\omega), \text{ where } \theta_i(\omega), \theta_j(\omega) \text{ phases} \]
\[ \text{of the fluctuating drag force acting on the } i \text{th} \]
\[ \text{spoiler and the } j \text{th} \text{ spoiler, respectively.} \]


2. CIBSE (The Chartered Institution of Building Services Engineers) guide B5 Noise and Vibration Control for HVAC, 7–9 (The Chartered Institution of Building Services Engineers London, May 2005).


