CHENG, Qi; WANG, Shengwei; YAN, Chengchu. Sequential Monte Carlo simulation for robust optimal design of cooling water system with quantified uncertainty and reliability. Energy, 2017, 118: 489-501.

https://doi.org/10.1016/j.energy.2016.10.051 This is the Pre-Published Version.

Sequential Monte Carlo Simulation for Robust Optimal Design of Cooling Water System with Quantified Uncertainty and Reliability

Qi Cheng, Shengwei Wang*, Chengchu Yan

Department of Building Services Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong

Abstract: Conventional design of cooling water systems mainly focused on the individual components of cooling water system, not the system as a whole. In this paper, a robust optimal design based on sequential Monte Carlo simulation is proposed to optimize the design of cooling water system. Monte Carlo simulation is used to obtain the cooling load distribution of required accuracy, power consumption and unmet cooling load. Convergence assessment is conducted to terminate the sampling process of Monte Carlo simulation. Under different penalty ratios and repair rates, this proposed design minimizes the annual total cost of cooling water system. A case study of a building in Hong Kong is conducted to demonstrate the design process and test the robust optimal design method. The results show that the minimum total cost could be achieved under various possible cooling load conditions considering the uncertainties of design inputs and reliability of system components.

Keywords: Robust optimal design, uncertainty-based design, sequential Monte Carlo simulation, cooling water system, reliability

---

* Corresponding author: Shengwei Wang, Email: beswwang@polyu.edu.hk, Phone: (852) 27665858, Fax: (852) 27746146

1. Introduction

A typical centralized heating, ventilation and air conditioning (HVAC) system is comprised of cooling water loop, chilled water loop and indoor air loop [1]. A cooling water loop consists of condensers of chillers, pumps, cooling towers and fans [2]. The condensers of chillers transfer the indoor cooling load and the heat generated by the compressors of chillers into the cooling water. The cooling water pumps circulate cooling water from the chiller condensers to the cooling towers. The heat load is finally rejected to the ambient through heat transfer and evaporation by cooling towers.

The sizing and selection of cooling water systems is one of the most important aspects in determining the energy performance of the HVAC systems [3-7]. According to ASHRAE Handbook [8], the thermal capacity of a cooling tower might be determined by following parameters, i.e. return and supply cooling water temperatures, inlet air wet-bulb temperature and design cooling water flow rate [9-13]. Design cooling water flow rate depends on the total heat rejection of condensers under given working conditions. The total heat rejection contains the design cooling capacity and heat of compression [11]. Due to the inevitable uncertainty of weather data, indoor occupants and internal heat gain, designers tend to select a design cooling capacity much larger than the peak duty (e.g., multiply a safety factor) in order that the plant can fulfil the cooling demand under any uncertain conditions for safety [14,15]. At the same time, additional cooling tower capacity is added in case that the ambient temperature is off-design or heat rejection varies from the design condition [9]. This may result in significant oversizing of design cooling capacity and cooling tower capacity and thus a large amount of energy wastes. In selecting the pumps for a cooling water system, considerations are mainly given to the static pressure and the system friction loss [8]. The pump inlet must have an adequate net positive suction pressure [16]. In addition, continuous contact with air introduces oxygen into the water and concentrates minerals that can cause scale and corrosion on a continuing basis [8]. Fouling factors and an increased pressure caused by aging of the piping must be taken into account in the design of cooling water pump [17]. However, research on cooling water systems has focused on the individual components of cooling systems, not the system as a whole [18]. Picón-Núnez et al. [19] proposed a methodology for designing coolers in the context of both process needs and cooling water system behavior. It considers the design of cooling systems in the context of piping costs, exchanger costs, pumping costs and its hydraulic and thermal performance. In addition, little attention has been placed to the interactions among cooling towers, cooling water pumps and condensers of chillers [20], even though changes to operating conditions of cooling water systems frequently happen.

Conventional optimal design of building energy systems is typically based on the annual cooling load under the predefined conditions, which is commonly subject to a deterministic model-based simulation [15, 21]. However, many researchers had taken the impacts of uncertainties and reliability into account when calculating cooling loads and evaluating the performance of building energy systems [22, 23, 24]. Eisenhower et al. [25] conducted an uncertainty study in the intermediate processes by performing decomposition, aiming to find the most important subsystem in modelling. Sun et al. [15] proposed a design method to size building energy systems considering uncertainties in weather conditions, building envelope and operation. Cheng et al. [26] proposed a probabilistic approach for uncertainty-based optimal design to size the chiller plant considering uncertainties of input parameters, which ensures that the chiller plant can operate at a high efficiency and the minimum annual total cost could be achieved under various possible cooling load conditions. Myrefelt [27] used actual data collected from buildings of seven large real estate operators to analyze the reliability of the HVAC systems. Gang et al. [28, 29] proposed a robust optimal design of cooling systems considering uncertainties of inputs and system reliability, which could obtain the optimal cooling systems with low cost and high robustness and provide a promising means for designers to make their best design decisions. Au-Yong et al. [30] investigated the maintenance characteristics of HVAC systems that affect the satisfaction of occupants, subsequently established a relationship between the characteristics and the satisfaction of occupants through questionnaire surveys and interviews and finally develop a regression model for prediction purpose. Peruzzi et al. [31] emphasized the importance of the reliability parameters considering financial (reduction of energy and maintenances costs), environmental and resources managing (both concerning the energy and staff) profits.

In reliability evaluation studies of substations, there are two main approaches applied: the analytical approach (a steady approach) and the Monte Carlo simulation approach. In HVAC fields, the analytical approach, such as Markov processes, is usually utilized for reliability modeling of aging equipment. Gang et al. [28, 29] proposed the Markov method to quantify the system reliability on the design of cooling systems. Cheng et al. [32] proposed a robust optimal design based on minimized life-cycle cost to optimize the design of chilled water pump systems while concerning the uncertainties of design inputs and models as well as the component reliability in operation. The advantages of the analytical approach include high accuracy and relatively fast computation speed. Its disadvantages are the inability to provide more detailed reliability information. For example, only the average steady probability distribution of states can be provided. Moreover, in some situations, transitions between some states do not have Markovian characteristics and therefore cannot be modeled by standard Markov processes [33, 34]. Compared to analytical methods, the Monte Carlo simulation is a powerful tool that can handle more conditions related to reliability evaluation (i.e., impact of severe weather, load variation) of systems [35, 36]. Moreover, the Monte Carlo simulation approach is capable of providing more comprehensive results.

In order to achieve the minimum total life-cycle cost under various possible cooling load conditions considering the uncertainties of design inputs and reliability of the components in operation, a sequential Monte Carlo simulation-based robust optimal design method is proposed in this paper. In order to achieve the minimum total cost, trials of simulations on different cooling water flows and different number/size of cooling water pump and cooling tower are conducted to obtain the optimum cooling water system. A series of so-called uncertainty “scenarios” generated by Monte Carlo simulation, is used to obtain the average cooling load distribution of required accuracy and average "unmet cooling load". Several indices are developed for the convergence assessment of average cooling load and average "unmet cooling load". Average cooling load is used to evaluate the operation cost of the cooling water system. "Unmet cooling load" is used to evaluate the availability risk cost of the cooling water system.

2. Optimization objective of the robust optimal design method

Figure1 shows the schematic of a cooling water loop. Identical constant-speed pumps are used to circulate the cooling water through the entire system and the pumps are assumed to work at the rated power. Identical cooling towers are used to reject the heat load to the ambient. Variable speed fans are used in the cooling towers.

In the cooling water loop, the energy balance is shown in Equation (1) and (2).

$$CL + Q_{compression} = \sum_{i=1}^{n} m_{fluid} \cdot c_{fluid} \cdot \left(T_{fluid,in} - T_{fluid,out}\right) \tag{1}$$

$$Q_{compression} = CL / COP_{chiller} \tag{2}$$

where, CL is cooling load, $Q_{compression}$ is heat of compression, m_{fluid} is the cooling water flow rate, c_{fluid} is the specific heat of water, $T_{fluid,in}$ is the return cooling water temperature, $T_{fluid,out}$ is the supply cooling water temperature.

The total heat load rejected by cooling towers is determined by building cooling load and the COP of chillers. Under a given range (i.e. the temperature difference between the supply cooling water temperature and return cooling water temperature) and approach (i.e. the temperature difference between supply cooling water temperature and wet-bulb temperature of inlet air), the design cooling flow rate is determined by the total heat load.

The objective of the proposed method is to ensure that the cooling water system operates at high efficiency over the entire cooling season and achieve the minimum total cost considering uncertainties of inputs and reliability of system components in operation. The total cost (TC_n) consists of annualized capital cost (CC_n), annual operation cost (OC_n) and annual availability risk cost (RC_n). Annualized capital cost includes the expense in purchasing/installing the pumps and cooling towers and associated components (equipment cost) and the spaces for accommodating them (space cost), which is determined by the number and size of pumps and cooling towers. Annual operational cost is the cost of electricity consumed by the pumps and fans in cooling towers in operation, which is mainly associated to the annual cooling load distribution and the energy efficiency of pumps and fans. Availability risk cost is the "expense" or service sacrifice penalty, which should be considered when the cooling demands cannot be fulfilled. In the cooling water loop, the overall total cost contains the total cost of cooling towers and the total cost of cooling water pumps, as shown in Equation (3). Figure 2 illustrates the conceptual relationship between the costs and system total capacity [32]. It is well-known that a larger system capacity means higher system reliability. The capital cost and operation cost increase as the system capacity increases. On the other hand, the availability risk cost decreases as the system total capacity increases. The total life-cycle cost is comprised of the capital cost, operation cost and availability risk cost, as shown in Equation (4) and (5). According to Figure 2, there should be a comprised system capacity to achieve the minimum total life-cycle cost, at which a comprised level of reliability is achieved [37].

$$TC_{n,all} = TC_{n,\mathrm{cot}} + TC_{n,cwp} \tag{3}$$

$$TC_{n,\mathrm{cot}} = CC_{n,\mathrm{cot}} + OC_{n,\mathrm{cot}} + RC_{n,\mathrm{cot}} \tag{4}$$

$$TC_{n,cwp} = CC_{n,cwp} + OC_{n,cwp} + RC_{n,cwp} \tag{5}$$

3. Computing procedure of the robust optimal design method

3.1 Procedure outline

Figure 3 shows the overall procedure of the proposed robust optimal design. It mainly addresses the determination of design cooling water flow, the pump head of cooling water pumps and number/size of cooling towers and cooling water pumps. Considering that the cooling towers and cooling water pumps are only manufactured in certain discrete sizes, trials on different design cooling water flow rates and different numbers/sizes of cooling towers and cooling water pumps are conducted to select the optimal cooling water system.

Searching range of design cooling water flow rate is assumed to be 1 to 2 times of the minimum design cooling water flow. The minimum design cooling water flow is equivalent to the required cooling water flow based on the design cooling capacity and the rated COP of chillers concerned. Under a given design cooling water flow rate, the pump head is determined by the hydraulic resistance distribution involving uncertainties. Then, the operation cost, unmet cooling load and capital cost are obtained under different numbers/sizes of cooling tower and cooling water pump. Under this given design cooling water flow rate, the optimal option of cooling water system is selected based on the minimized total costs of the cooling water pumps and cooling towers. Simulation trials of cooling water pumps start from two pumps (the minimum of two is assumed concerning the basic requirement for reliability and maintenance) until the total cost of pumps begins to increase. For the same reason, simulation trials of cooling towers also start from two cooling towers until the total cost of cooling towers in the life-cycle begins to increase. Eventually, among the options corresponding to various design cooling water flow rates, the option which has the minimum total cost is selected as the optimum design for application.

Equation (6) formulates the optimization problem for selecting the total design cooling water flow rates and numbers/sizes of cooling towers and cooling water pumps. Where, TC is the total cost, M is the design cooling water flow, m_{cot} is the individual capacity of cooling water, n_{cot} is the number of cooling towers, m_{cwp} is the individual capacity of cooling water pump, n_{cwp} is the number of cooling water pumps, M_{min} is the minimum design cooling water flow.

$$\text{find} \qquad M, m_{\text{cot}}, n_{\text{cot}}, m_{cwp}, n_{cwp}$$

$$\begin{aligned} &\text{minimize} && TC_{n,all}(M_i, m_{\mathrm{cot}}, n_{\mathrm{cot}}, m_{cwp}, n_{cwp}) \\ &\text{constraint} && M_i \ge M_{\min}, M_i \le 2M_{\min} \\ & && M_i = m_{\mathrm{cot}} \cdot n_{\mathrm{cot}} = m_{cwp} \cdot n_{cwp} \\ & && n_{\mathrm{cot}} \ge 2, n_{cwp} \ge 2 \end{aligned} \tag{6}$$

3.2 Sequential Monte Carlo simulation

Figure 4 shows the simulation procedure for obtaining the cooling load distribution, average operation cost and average “unmet cooling load”. Cooling load distribution is generated by the TRNSYS building energy model based on the uncertainties of design inputs. Average operation cost and average “unmet cooling load” are determined by the cooling load conditions, heat of compression and available cooling capacity. Unmet cooling load is the load difference when the available cooling capacity is less than the actual cooling load conditions. Based on the uncertainties of inputs (quantified on Matlab), cooling load conditions, generated by Monte Carlo simulations, are calculated by TRNSTS test platform. Heat of compression is calculated by the chiller model based on the uncertainties of inputs and cooling load conditions. Available cooling capacity is determined by the uncertainties of health states of components in the system, which can be calculated by the component reliability model. When some cooling towers fail, the capacity of cooling towers might not be able to meet the capacity of chiller plant. Heat of compression is determined by the COP of chiller plant. The COP is affected by the condensing temperature, which depends on the cooling water temperature. Convergence assessment is conducted to verify the cooling load distribution, average operation cost and average “unmet cooling load”. If not, more Monte Carlo sampling times of simulation are conducted until these three values converge.

3.3 Implementation flowchart of the proposed design method

Quantification of pump head of cooling water pump

The simplified model structure for the pressure-flow balance of the cooling water loop is presented earlier in Figure 1. The overall pressure drop of the system can be mathematically described as in Equation (7), which consists of five parts of pressure drops, including (1) the pressure drop on the condensers of chillers, (2) the pressure drop on the fittings around pumps (including the pressure drop on the headers that direct the flow into/from each pump and the pressure drop on the valves), (3) the pressure drops on main supply and return pipelines, (4) the pressure drop measured from the operating water level in the cold water basin to the spray system (i.e. nozzle) and, (5) the pressure drop of nozzle required to effect proper distribution of the water to the fill.

$$\Delta p = \frac{S_{con}}{N_{chiller}} \cdot m_w^2 + \frac{S_{cwp}}{N_{cwp}} \cdot m_w^2 + S_{pipe} \cdot k_c \cdot m_w^2 + H_{\text{cot}} + S_{noz} \cdot m_w^2 \tag{7}$$

Where, Δp is the pressure drop of the entire cooling water loop, S_{con} is the coefficient of chiller condenser, $N_{chiller}$ is the number of chillers, S_{cwp} is the coefficient of cooling water pumps, N_{cwp} is the number of pumps, S_{pipe} is the coefficient of pipelines, k_c is the aging factor of pipes, H_{cot} is the height from the operating water level in the cold water basin to the spray system, S_{noz} is the coefficient of nozzles, m_w is the cooling water flow rate.

The pump pressure head is then determined by the hydraulic resistance coefficients and aging factor of the pipelines as well as the fluctuation of the cooling water flow. For a given design cooling load, the cooling water flow is influenced by the fluctuation (i.e. uncertainty) of the difference between return and supply cooling water temperatures. The cooling water flow usually fluctuates around the design cooling water flow considering the uncertainty of its heat transfer performance. The cooling water flow is assumed to be subject to normal distribution. Uniform distribution is used to describe the uncertainties of the hydraulic resistances of components. In addition, an artificial aging factor is adopted to account for the decrease in pipe diameter as the system ages. According to Equation (7), the distribution of pressure head can be generated and the design pressure head is assumed to be 99.6 percentile of the distribution.

Quantification of cooling load conditions

Monte Carlo simulation is employed to obtain a representative and reasonable cooling load distribution considering uncertainties. The calculation process can be illustrated by Equation (8). With the inputs $x_1, x_2, \ldots, x_n$ (e.g., the outdoor temperature, ventilation rate), the output y (the cooling load) can be obtained. The detailed process to obtain the output cooling load is shown in reference [26]. In this study, the uncertainties of the design inputs are computed by Matlab. Three types of distributions (including normal distribution, tri-angular distribution and uniform distribution) are used to describe the uncertainties of inputs [26]. Table 1 shows an example of the settings of uncertainties of the inputs. Combining the output uncertainties from Matlab, the TRNSYS building model is used to obtain the cooling load conditions.

$$Y = \left[y_1, y_2, \ldots, y_{8760}\right] = f\left(x_1, x_2, \ldots, x_n\right) \tag{8}$$

Model of heat of compression

Heat of compression is the amount of heat added to refrigerant during the compression process, which depends on the actual cooling load and the operating COP of chillers COP_{op}. Usually, the COP_{op} of chiller varies depending on the part load ratio (PLR). It is well understood that the larger the PLR, the higher COP once the impact of other operating parameters (e.g. condensing and evaporating temperatures) are separated [38, 39], as shown in Equation (9):

$$COP_{op} = \frac{273.15 + T_{eva}}{T_{con} - T_{eva}} \times \left(C_0 + C_1 \cdot PLR + C_2 \cdot PLR^2 + C_3 \cdot PLR^3\right) \tag{9}$$

where, T_{eva} and T_{con} are evaporating and condensing temperature (°C), respectively; C_0-C_3 are the correlation coefficients that can be identified from chiller catalogues or field measurement data. T_{eva} and T_{con} are calculated based on the part load ratio, temperature difference, refrigerant flow rate and cooling load. The outlet water temperature of the evaporator ($T_{eva,out}$) is set to be 7℃ in simulation tests, and the inlet water temperature of condenser ($T_{con,in}$) is assumed to have a difference of 5 K with the wet-bulb temperature of the cooling tower inlet air ($T_{wb,in}$) as shown in Equation (10).

$$T_{con,in} = T_{wb,in} + 5 \tag{10}$$

Model of available cooling load

Figure 5 is a reliability and maintainability history chart of a three-state machine. The state "Operate" indicates that the equipment currently resides in a working state (i.e. State 1). The lengths of this state are the holding times of being in working state. The holding time is random and determined by analysis of historical reliability and maintainability data. In practice, the mean time to failure (*MTTF*, $1/\lambda$) is often used to represent this holding time, as shown in Equation (11) [34]. The states "Maintenance" and "Failure" (i.e. State 0) indicate that the equipment currently resides in an inoperative (i.e. failure or maintenance) state. The lengths of these states are the holding times of being in this state. In practice, the mean time to repair (*MTTR*, $1/\mu$) is often used to represent this holding time, as shown in Equation (12) [34]. Given a reliability and maintainability history chart, the reliability indices, such as availability, $p_{availability}$ (percentage of time staying in a working state) and unavailability, $p_{unavailability}$ (percentage of time staying in a failure and maintenance state) can be calculated from the reliability and maintainability history chart above by Equation (13) and (14). Where, $t_{operate}$, t_{main} and t_{fail} are the total operation time, the total maintenance time, and the total failure time respectively in an entire period. λ is failure rate, μ is repair rate.

$$MTTF = \frac{1}{\lambda} = \sum t_{operate} \tag{11}$$

$$MTTR = \frac{1}{\mu} = \sum t_{main} + \sum t_{fail} \tag{12}$$

$$p_{availability} = \frac{MTTF}{MTTF + MTTR} \tag{13}$$

$$p_{unavailability} = \frac{MTTR}{MTTF + MTTR} \tag{14}$$

With the assumption that each component is independent and has no relationship with the other components, the probability of cooling water pump and cooling tower are assumed to be subject to the binary distribution $B(1, p_{availability})$, as shown in Equation (15) and (16). The total available cooling load of cooling water pumps and cooling towers are calculated by Equation (17) and (18). The unmet cooling load of cooling water pumps and cooling towers are calculated by Equation (19) and (20). Where,

$p_{availablility,cot}$ and $p_{availability,cwp}$ are the availabilities of cooling towers and cooling water pumps. $f_{cot}(i)$ and $f_{cwp}(i)$ are the states of cooling tower and cooling water pump. $CL_{available,cwp}$ and $CL_{available,cot}$ are the available cooling load of cooling water pumps and cooling towers, $CL_{ind,cwp}$ and $CL_{ind,cot}$ are the nominal capacity of cooling water pumps and cooling towers. $CL_{unmet,cot}$ and $CL_{unmet,cwp}$ are the unmet cooling loads of cooling towers and cooling water pumps. CL_{actual} is the annual cooling load.

$$f_{\cot}(i) = B(1,\ p_{availability,\cot}) \tag{15}$$

$$f_{cwp}(i) = B(1,\ p_{availability,cwp}) \tag{16}$$

$$CL_{available,cwp} = CL_{ind,cwp} \cdot \sum_{i=1}^{n_{cwp}} f_{cwp}(i) \tag{17}$$

$$CL_{available,\cot} = CL_{ind,\cot} \cdot \sum_{i=1}^{n_{\cot}} f_{\cot}(i) \tag{18}$$

$$CL_{unmet,\cot} = Max\,(CL_{actual} - CL_{available,\cot}, 0) \tag{19}$$

$$CL_{unmet,cwp} = Max\,(CL_{actual} - CL_{available,cwp}, 0) \tag{20}$$

<u>*Model of cooling water loop*</u>

Pump model

Cooling water pumps are constant speed pumps and they are assumed to work at their rated powers. Their electricity consumptions depend on the pressure drop (Δp_{cwp}), the cooling water flow rate (m_w) and pump efficiency (η_{cwp}) as shown by Equation (21) [40]. In this study, the pressure drop of the cooling water loop is equivalent to the pressure head of cooling water pump. The pump efficiency depends on the pump capacity.

$$OC_{pu} = \frac{m_w \Delta p_{cwp}}{102\eta_{pu}} \tag{21}$$

Cooling tower model

The electricity consumption of cooling tower fans is calculated based on the design efficiency of fan and load ratio of fan (Equation (22)). The fans of cooling towers are equipped with variable speed drives. The cooling tower model (TYPE510) in TRNSYS [41] is used in this study. The air at tower outlet is assumed to be saturated air. The load ratio of fan (γ_{air}) can be then calculated by Equation (23) and (24). Where, P_{fan} and $P_{fan,rated}$ are the power consumption of fans and the rated power consumption of fans. γ_{air} is the load ratio. a_0-a_3 are the correlation coefficients provided by the manufacturer. h_{sat} is the enthalpy of saturated air, h_{air} is the enthalpy of air.

$$P_{fan} = P_{fan,rated} \cdot \left(a_0 + a_1 \cdot \gamma_{air} + a_2 \cdot \gamma_{air}^2 + a_3 \cdot \gamma_{air}^3\right) \tag{22}$$

$$h_{sat} = h_{air}\left(T_{air,in}\right) + \frac{c_{fluid} \cdot m_{fluid} \cdot \left(T_{fluid,in} - T_{fluid,out}\right)}{m_{air} \cdot \left(1 - \exp\left[-\lambda_{design} \cdot \gamma_{air}^{-0.4}\right]\right)} \tag{23}$$

$$\lambda_{design} = \ln\left[\frac{h_{sat}\left(T_{fluid,out,design}\right) - h_{air}\left(T_{air,in,design}\right)}{h_{sat}\left(T_{fluid,out,design}\right) - h_{air}\left(T_{air,out,design}\right)}\right] \tag{24}$$

Convergence of Monte Carlo simulation

As mentioned above, the cooling load distribution, the operation cost and available cooling capacity are generated by a sequential Monte Carlo simulation. For the purpose of checking the convergence and terminating the sampling process, there are several different types of stop criteria in literature, such as coefficient of variance, maximum number of iterations and convergence band (i.e. also called as threshold). Among these criteria, the threshold is used to evaluate the uncertainty and reliability in this study.

As mentioned above, the convergence assessment needs to be conducted on the average cooling load distribution, average operation cost and average unmet cooling load. Two convergence criteria are applied as follows:

- The deviation of the cooling load distribution profile should be within its threshold B_w over a number of simulation trials (i.e. over convergence band length B_L).
- The deviations of the dimensionless operation cost and unmet cooling load should be within their threshold B_w over the same convergence band length B_L.

The profile deviation $f(n+i,n)$ is defined as the difference between average cooling load distribution profiles over ($n+i$) number of simulations and n number of simulations respectively, as shown in Equation (25) and Figure 6 [26].

$$f(n+i,n)=\frac{\sum_{1}^{k}\left[\left|p_{n+i}(j)-p_{n}(j)\right|\cdot\Delta CL_{j}\right]}{\sum_{1}^{k}\left[p_{n}(j)\cdot\Delta CL_{j}\right]} \tag{25}$$

where, $p_n(j)$ is the probability at the load CL_j over n trials of simulations, $p_{n+i}(j)$ is the probability at the load CL_j over $n+i$ trials of simulations. $\triangle CL_j$ is the cooling load interval and k is the total number of intervals.

The deviation $\Delta y(n+i,n)$ is defined as the difference between the average value of ($n+i$) number of simulations and the average value of n number of simulations respectively, as shown in Equation (26).This deviation is used to evaluate the convergence of the dimensionless operation cost and unmet cooling load.

$$\Delta y(n+i,n)=\frac{\left|y_{n+i}-y_{n}\right|}{y_{n}} \tag{26}$$

where, y_n is the average value over n trials of simulations, $y_{n+\mathrm{i}}$ is the average value over ($n+i$) trials of simulations.

4. Case study and evaluation of the proposed design optimization method

A case study on the cooling water system design for a building in Hong Kong is conducted to test and evaluate the proposed robust optimal design method. The performance of the system designed using the proposed robust optimal design method is compared with that using conventional design method, conventional optimal design method and uncertainty-based design method. The benefits and drawbacks of using sequential Monte Carlo simulation for unmet cooling load calculation are also investigated by comparing it with Markov method.

4.1 Outline of implementation

The main steps of design method implementation are summarized as follows:

- At first step, the minimum design cooling water flow rate and searching range of design cooling water flow rate are determined;
- Under each given design cooling water flow rate, the pressure head of cooling water pumps is determined by the hydraulic resistance distribution involving uncertainties;
- Then, the trials of different number of cooling towers and cooling water pumps on each given design cooling water flow rate are conducted;
- A sequential Monte Carlo simulation is used to obtain the cooling load distribution profile, operation cost and unmet cooling load;
- Markov method is used as a reference to validate the results of sequential Monte Carlo simulation on the "unmet cooling load";
- The optimal option under each design cooling water flow rate is selected based on the minimized total cost;
- Among these options, the option which has the minimum total cost is selected as the best option used in the building energy system.

4.2 Outline of Markov method for reliability assessment

In this study, Markov method is used as an alternative means to quantify the component reliability and available cooling load of towers as a reference method to evaluate the use of sequential Monte Carlo simulation method for the same purposes. Figure 7 shows the Markov process for equipment with normal and failure states. From normal state to failure state, the failure rate λ is used to represent the probability from one state to another. From failure state to normal state, the repair rate μ is used to represent the probability from one state to another. According to the equations of Markov process [32], the probability distribution of steady state can be calculated. Based on the cooling load distribution and the probability distribution of system state, the unmet cooling load and operation cost can be obtained. It is assumed that each component of cooling water system has two states only: normal (0) and failure (1). Figure 8 shows the states of a n-parallel system and possible transitions. State 0 symbolizes that all the components are available to operate and state n symbolizes that all the components are fail to work. The transition probability is determined by a state transition density matrix A (Equation (27)), which only involves the repair rate and failure rate [32].

$$A=\begin{bmatrix} (1-n\lambda) & n\lambda & 0 & 0 & 0\ \ldots & 0 & 0 & 0 \\ \mu & (1-\mu-(n-1)\lambda) & (n-1)\lambda & 0 & 0\ \ldots & 0 & 0 & 0 \\ 0 & \mu & (1-\mu-(n-2)\lambda) & (n-2)\lambda & 0\ \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & & & \\ 0 & 0 & 0 & 0 & 0\ldots & \mu & (1\text{-}\lambda\text{-}\mu) & \lambda \\ 0 & 0 & 0 & 0 & 0\ldots & 0 & \mu & (1-\mu) \end{bmatrix} \tag{27}$$

4.3 Determination of cooling load distribution and head of cooling water pumps

The design cooling capacity of the building is 5100kW. Three identical chillers (1700kW), the rated COPs of which are 6.4, are employed. According to Equation (1) and (2), the minimum design cooling water flow rate is 285L/s. The searching range of design cooling water flow rate is assumed to be 285~420L/s and the interval of the trials is 15L/s. The failure rates of pump and cooling tower are 0.0001/hour and 0.00001/hour respectively, which means that the total working time of cooling water pumps and cooling towers are 10,000 hours and 100,000 hours during an entire period (see Figure 5). The repair rates of both pumps and cooling towers are 0.002/hour, which means that totally 500 hours are needed to repair or maintain each of the pumps and cooling towers during the same period. Therefore, the availabilities of pumps and cooling towers are 0.9524 and 0.995 respectively.

For example, when the design cooling water flow rate is assumed to be 330L/s, the annual cooling load profile and annual unmet cooling load of cooling towers under five years can be obtained as shown in Figure 9. The annual cooling load profile is different over the five years and the annual unmet cooling load varies greatly under different years. Therefore, sufficient sampling is required to obtain the accurate cooling load distribution, operation cost and unmet cooling load.

The pressure head of cooling water pumps is determined by the cooling water flow rate, hydraulic resistance coefficient and aging factor of the pipelines. Table 2 shows the settings of pressure drops of components and aging factor, which are selected referring to the literature [40]. According to Equation (7), the distribution of pump pressure head can be generated as shown in Figure 10. The design pump pressure head is assumed to be 25.5m, which is equivalent to 99.6% of the distribution of the hydraulic resistance.

Then, the trials on different numbers/sizes of cooling towers and cooling water pumps are conducted based on the minimized total cost respectively. The cooling load distribution depends on the uncertainties of inputs and it is independent from the design cooling water flow rate. After conducting 780 times of Monte Carlo simulations [26], a cooling load distribution of sufficient accuracy is obtained based on the convergence assessment, as shown in Figure 11. The reference case is the cooling load distribution in the typical year without considering uncertainties.

4.4 Validation of sequential Monte Carlo simulation

As mentioned above, Markov method, which is described in detail in a textbook [32], is used to validate the simulation procedure and the use of sequential Monte Carlo simulation. Using Markov method, the steady probability distribution of system components can be obtained by solving the linear algebraic equations. According to the steady probability distribution of the system components and cooling load conditions, the steady unmet cooling load can be obtained. The steady unmet cooling load using Markov method is used to validate the converged unmet cooling load using sequential Monte Carlo simulation.

Figure 12 shows the average unmet cooling loads when using 3, 5 and 7 cooling towers using sequential Monte Carlo simulation and Markov method respectively. It is obvious that the average unmet cooling load is getting smaller when the number of cooling tower increases. Figure 12 (a) shows the average unmet cooling load when using 3 cooling towers under different simulation trials. The average unmet cooling load varies greatly when the simulation trial is less than 250. From 250 sampling times to 530 sampling times, the average unmet load cooling increases gradually. When the sampling times are over 530, the average unmet cooling load fluctuates around the value 7330kWh, which can be considered as the converged average unmet cooling load. Therefore, about 530 sampling times (years) are needed to obtain the accurate average unmet cooling load. The unmet cooling load using Markov method is equal to about 7314kWh, which is almost the same as that using sequential Monte Carlo simulation. Figure 12 (b) shows the average unmet cooling load when using 5 cooling towers under different simulation trials. About 530 sampling times (years) are needed to obtain the accurate average unmet cooling load and the average unmet cooling load fluctuates around the converged value 814kWh. The unmet cooling load using Markov method is also almost the same as the converged average unmet cooling load using the sequential Monte Carlo simulation. Figure 12 (c) shows the average unmet cooling load when using 7 cooling towers under different simulation trials. About 480 sampling times (years) are needed to obtain the accurate average unmet cooling load and the average unmet cooling load fluctuates around the converged value 100kWh. Therefore, using Markov method can obtain accurate unmet cooling load and consume less computation time compared to sequential Monte Carlo simulation. However, Monte Carlo simulation approach is capable of providing more comprehensive information, such as the detailed changes of unmet cooling load, compared with using Markov method.

Table 3 shows the converged average unmet cooling load under different options of cooling towers. The converged average unmet cooling load decreases rapidly when the number of cooling towers increases. When the number of cooling towers is large, further increase of the number will not result in obvious change of the average unmet cooling load any more.

The electricity price used in this study is 1 HKD/kWh, which is within the range of the typical rate in Hong Kong. Figure 13 shows the average operation costs when using 3, 5 and 7 cooling towers. It is obvious that the average operation cost is larger when more cooling towers are used. Figure 13 (a), (b) and (c) show the average operation costs when using 3, 5 and 7 cooling towers under different numbers of simulation trials respectively. It can be seen that the average operation costs have no obvious change under different simulation trials. The converged average operation costs when using 3, 5 and 7 cooling towers are 6.2×10^5, 6.62×10^5 and 6.88×10^5 respectively. Table 3 shows the converged average operation costs using different design options of cooling towers. The converged average operation cost increases when the number of cooling towers increases.

4.5 Optimal configuration of cooling water system

Annualized capital cost contains the equipment cost and space cost. The life cycle of the cooling water system is assumed to be 10 years. Equipment cost of cooling tower (110L/s) is 110,000HKD, referring to the data from a manufacturer. The cooling tower cost of other sizes are estimated using Equation (28) [42, 43].

$$EC = EC_0 \cdot \left(C / C_0\right)^{\alpha} \tag{28}$$

where, EC_0 is the equipment cost of the reference cooling tower with the capacity C_0. EC is equipment cost of cooling tower with the capacity C. α is the coefficient, which set to be 0.15 in this study [26]. The space cost of cooling tower is assumed to be 5000HKD/unit/year.

Table 4 shows the capital costs, annual availability risk costs and total costs of different numbers of cooling towers under three penalty ratios (i.e., 1, 10 and 100 HKD/kWh). It is obvious that the annualized capital cost becomes larger when the number of cooling tower increases. It can be seen that, when the number of cooling towers is small, the annual availability risk cost decreases rapidly when the number of cooling tower increases. It can also be observed that the total cost decreases when the number of cooling tower increases in certain range and increases when the number of cooling tower increases further. Since the availability risk cost is high when the number of cooling towers is small and the capital cost and operation cost is high when the number of cooling towers is large, there is a comprised number/size of cooling tower which has the minimum total cost. In this study, the penalty ratio is assumed to be 10HKD/kWh. Among options assessed, the option, 83L/s×4 cooling towers, has the minimum total cost 1.035×10^6HKD, which can be considered as the best option under the design cooling water flow, 330L/s. If the penalty ratio is 1HKD/kWh, the best cooling tower option under the design cooling water flow is 165L/s×2. If the penalty ratio is 100HKD/kWh, the best cooling tower option under the design cooling water flow is 55L/s×6. The designers can select the best option based on their specific concern on the predefined penalty ratio.

Table 5 shows the capital costs, annual availability risk costs and total costs of different numbers of pumps under three penalty ratios (i.e., 1, 10 and 100 HKD/kWh). It is also obvious that the annualized capital cost becomes larger when the number of pumps increases. It can be seen that the annual availability risk cost of pumps is larger than that of cooling towers because of the larger failure rate of pumps. It can also be observed that the total cost decreases when the number of cooling tower increases. Among these options, the option 41L/s×8 pumps has the minimum total cost 1.119×10^{6}HKD, which can be considered as the best option in principle under the design cooling water flow of 330L/s. In practice, the number of cooling water pumps should be integer times of the number of chillers for the convenient capacity control when the cooling water pumps are constant speed pumps. Therefore, the option 41L/s×6 pumps may be considered as the best design option under the design cooling water flow 330L/s. It means that using more cooling water pumps may be a better option considering the uncertainties and reliability. If the penalty ratio is 1HKD/kWh, the best option under the design cooling water flow is 110L/s×3 pumps. If the penalty ratio is 100HKD/kWh, the best option under the design cooling water flow is 55L/s×6 pumps. The designers can select the best option based on their specific concern on the penalty ratio. Therefore, the best option of the cooling water system consists of 83L/s×4 cooling towers and 55L/s×6 pumps under the design cooling water flow rate 330L/s.

After conducting the trials on other design cooling water flow rates within the range between 285 L/s and 420 L/s, the minimum total costs are computed corresponding to each design flow rate respectively as shown in Table 6. When the design cooling water flow rate increases from 285L/s to 375L/s, the total cost of cooling tower increases because the operation cost and capital cost increase obviously and the availability risk cost decreases slightly. For cooling water pumps, since the decrease of availability risk cost is larger than the increase of operation cost and capital cost, the total cost of pumps decreases. Therefore, the total cost of the cooling water system decreases. When the design cooling water flow rate is over 375 L/s (i.e. 375 L/s to 420L/s), the total costs of both the cooling towers and pumps increase, which result in the increase of total cost of the cooling water system. Among the options assessed, the option with 120L/s×3 cooling towers and 60L/s×6 pumps has the minimum total cost (i.e. 2.166×10^6 HKD) compared with other options. This option selected therefore has better robustness to uncertainties and system reliability.

Table 7 shows the best design options under the other repair rates. The required design cooling water flow rate decreases when the repair rate increases (i.e. the availabilities of cooling tower and pump increase). The users can choose the preferred repair rate based on their specific level or efficiency of handling the problems such as maintenance and failure.

4.6 System performance using different design methods

Table 8 shows the results of uncertainty-based design, conventional design and robust optimal design. It can be seen that the unmet cooling load of uncertainty-based design is much larger than that of conventional design and robust optimal design. Compared with the total costs of conventional design (2.801×10^6HKD) and uncertainty-based optimal design (4.65×10^6HKD), the total cost under robust optimal design (2.166×10^6HKD) is reduced by about 22.7% and 53.4% respectively when the penalty ratio is 10 HKD/kW. To achieve the minimum annual total cost, the option with 120L/s×3 cooling towers and 60L/s×6 cooling water pumps can be selected as the optimum design option. This option has the minimum total cost and it also has good robustness considering the uncertainties of design inputs and reliability of system components.

5. Conclusion

This paper presented a robust optimal design method which is based on a sequential Monte Carlo simulation to achieve the minimum annual total cost of cooling water system considering both uncertainties of design inputs and reliability of system components in operation. It is realized by optimizing the design cooling water flow rate, the number/size of cooling towers and the number of cooling water pumps. The design method is tested and evaluated by conducting a case study. Based on the results, conclusions can be made as follows:

- Annual average cooling load and annual unmet cooling load varies largely when considering uncertainties. Sufficient sampling times are required to obtain the accurate cooling load distribution, operation cost and unmet cooling load. Sequential Monte Carlo simulation can be effectively used to obtain the accurate cooling load distribution, operation cost and unmet cooling load by quantifying the uncertainties of design inputs and the reliability of system components.

- Using Markov method can obtain accurate unmet cooling load and consume less computation time compared to sequential Monte Carlo simulation. However, Monte Carlo simulation approach is capable of providing more comprehensive information than Markov methods such as the detailed changes of unmet cooling load.

- The penalty ratio and repair rate can affect the determination of design cooling water flow rate and thus the selected best option. The optimal design cooling water flow rate is larger at the higher penalty ratio. The results also show that the design cooling water flow reduces when the repair rate increases.

- The design option of cooling water systems can be selected by achieving the minimum total cost when considering uncertainties and system reliability. The selected cooling water system has the good robustness towards the uncertainties of design inputs and system reliability. The results of the case study show that the total cost of optimized system can be reduced significantly (totally 22.7%) compared with the conventional design.

It is worth noticing that the optimization output may be slightly different from the best one in principle as not all options/combinations are tested due to the interval selected in the tests and limitations on the available sizes of cooling towers and pumps in practice.

Acknowledgements

The research presented in this paper is financially supported a general research grant (5267/13E) of the Research Grant Council of Hong Kong SAR and a grant of the MTR Corporation Limited.

Reference

1. L. Lu, W. Cai, Y.C. Soh, L. Xie, S. Li. HVAC system optimization—condenser water loop. Energy Conversion and Management 2004, 45(4): 613-630.
2. ASHRAE. ASHRAE handbook—fundamentals. American Society of Heating, Refrigerating and Air-Conditioning Engineers 2009.
3. G.Y. Jin, W.J. Cai, L. Lu, E.L. Lee, A. Chiang. A simplified modeling of mechanical cooling tower for control and optimization of HVAC systems. Energy conversion and management 2007, 48(2): 355-365.
4. M.A. Bernier. Thermal performance of cooling towers. ASHRAE journal 1995, 37(4).
5. H. Crowther, J. Furlong. Optimizing chillers & towers. ASHRAE Journal 2004, 46(7): 34.
6. S.W. Wang, C.C. Yan, F. Xiao. Quantitative energy performance assessment methods for existing buildings. Energy and Buildings 2012, 55: 873-888.
7. C.C. Yan, S.W. Wang, F. Xiao. A simplified energy performance assessment method for existing buildings based on energy bill disaggregation. Energy and

Buildings 2012; 55: 563-574.

8. ASHRAE Handbook. HVAC systems and equipment, Atlanta, GA, 2012.
9. Stanford III H W. HVAC water chillers and cooling towers: fundamentals, application, and operation. CRC Press, 2011.
10. Energy Efficiency Guide for Industry in Asia. United Nations Environment Program, 2006.
11. Cooling tower fundamentals. Marley Cooling Tower Company, 1983.
12. N. Milosavljevic, P. Heikkilä. A comprehensive approach to cooling tower design. Applied Thermal Engineering 2001, 21(9): 899-915.
13. A.K.M. Mohiuddin, K. Kant. Knowledge base for the systematic design of wet cooling towers. Part I: Selection and tower characteristics. International Journal of Refrigeration 1996, 19(1): 43-51.
14. F. Domínguez-Muñoz, J.M. Cejudo-López, A. Carrillo-Andrés. Uncertainty in peak cooling load calculations. Energy and Buildings 2010; 42(7): 1010-1018.
15. Y. Sun, L. Gu, C.F.J. Wu, G. Augenbroe. Exploring HVAC system sizing under uncertainty. Energy and Buildings 2014; 81: 243-252.
16. S.K. Mansfield. Oversized pumps--a look at practical field experiences. ASHRAE Transactions 2001; 107: 573.
17. R.C.E. Ahlgren. Why did I buy such an oversized pump?. ASHRAE Transactions 2001; 107: 566.
18. J.K. Kim, R. Smith. Cooling water system design. Chemical Engineering Science 2001, 56(12): 3641-3658.
19. M. Picón-Núnez, G.T. Polley, L. Canizalez-Dávalos, J. M. Medina-Flores. Short cut performance method for the design of flexible cooling systems. Energy 2011, 36(8): 4646-4653.
20. M.H. Panjeshahi, A. Ataei, M. Gharaie, R. Parand. Optimum design of cooling water systems for energy and water conservation. Chemical Engineering Research and Design 2009, 87(2): 200-209.
21. Ashouri A, Petrini F, Bornatico R, Ben MJ. Sensitivity analysis for robust design of building energy systems. Energy 2014, 76: 264-275.

22. D.H.W. Li, S.L. Wong, J.C. Lam. Climatic effects on cooling load determination in subtropical regions. Energy Conversion and Management 2003; 44(11):1831-1843
23. Y.Y. ıldız, Z.D. Arsan. Identification of the building parameters that influence heating and cooling energy loads for apartment buildings in hot-humid climates. Energy 2011; 36(7):542 4287-4296.
24. W.J. Gang, G. Augenbroe, S.W. Wang, C. Fan, F. Xiao. An uncertainty-based design optimization method for district cooling systems. Energy 2016, 102: 516-527.
25. B. Eisenhower, Z. O'Neill, V.A. Fonoberov, I. Mezić. Uncertainty and sensitivity decomposition of building energy models. Journal of Building Performance Simulation 2011; 5(3):171-184.
26. Q. Cheng, S.W. Wang, C.C. Yan, F. Xiao. Probabilistic approach for uncertainty-based optimal design of chiller plants in buildings. Applied Energy 2015.
27. S. Myrefelt. The reliability and availability of heating, ventilation and air conditioning systems. Energy and Buildings 2004; 36(10): 1035-1048.
28. W.J. Gang, S.W. Wang, F. Xiao, D.C. Gao. Robust optimal design of building cooling systems considering cooling load uncertainty and equipment reliability. Applied Energy 2015; 159: 265-275.
29. W.J. Gang, S.W. Wang, C.C. Yan, F. Xiao. Robust optimal design of building cooling systems concerning uncertainties using mini-max regret theory. Science and Technology for the Built Environment 2015, 21(6): 789-799.
30. C.P. Au-Yong, A.S. Ali, F. Ahmad. Improving occupants' satisfaction with effective maintenance management of HVAC system in office buildings. Automation in Construction 2014; 43: 31-37.
31. L. Peruzzi, F. Salata, A. De Lieto Vollaro, R. De Lieto Vollaro. The reliability of technological systems with high energy efficiency in residential buildings. Energy and Buildings 2014; 68: 19-24.
32. Q. Cheng, S.W. Wang, C.C. Yan. Robust optimal design of chilled water systems in buildings with quantified uncertainty and reliability for minimized life-cycle

cost. Energy and Buildings 2016, 126: 159-169.

33. X. Bai, S. Asgarpoor. Fuzzy-based approaches to substation reliability evaluation. Electric Power Systems Research 2004, 69(2): 197-204.
34. H. Ge, S. Asgarpoor. Parallel Monte Carlo simulation for reliability and cost evaluation of equipment and systems. Electric Power Systems Research 2011, 81(2): 347-356.
35. R.N. Allan. Reliability evaluation of power systems. Springer Science & Business Media, 2013.
36. R. Billinton, H. Chen, R. Ghajar. A sequential simulation technique for adequacy evaluation of generating systems including wind energy. Energy Conversion, IEEE Transactions on, 1996, 11(4): 728-734.
37. R. Billinton, W. Zhang. Cost related reliability evaluation of bulk power systems. International journal of electrical power & energy systems 2001; 23(2): 99-112.
38. T. Hong, L. Yang, D. Hill, W. Feng. Data and analytics to inform energy retrofit of high performance buildings. Applied Energy 2014; 126: 90-106.
39. C.C. Yan, S.W. Wang, F. Xiao, D.C. Gao. A multi-level energy performance diagnosis method for energy information poor buildings. Energy 2015, 83: 189-203.
40. S.W. Wang, J. Burnett. Online adaptive control for optimizing variable-speed pumps of indirect water-cooled chilling systems. Applied Thermal Engineering 2001; 21 (11): 1083–1103.
41. TRNSYS. TRNSYS 16 documentation; 2004. <http://sel.me.wisc.edu/trnsys>.
42. M. Taal, I. Bulatov, J. Klemeš, P. StehlíK. Cost estimation and energy price forecasts for economic evaluation of retrofit projects. Applied Thermal Engineering 2003; 23(14): 1819-1835.
43. K.M. Guthrie. Capital cost estimating. Chemical Engineering 1969; 3:114–142.

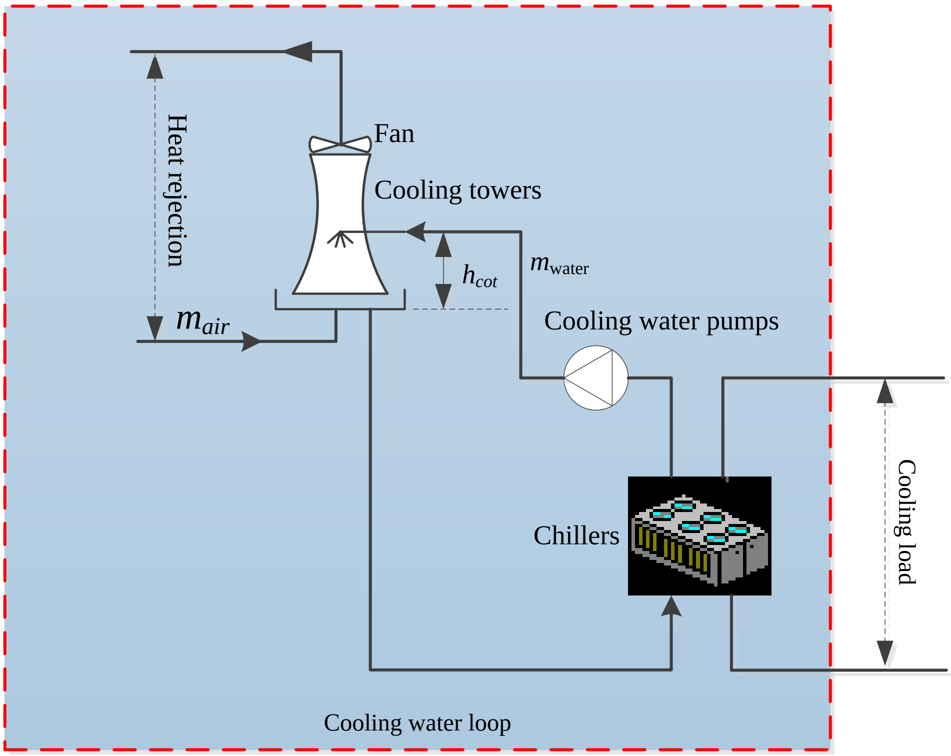


Fig.1 Schematic of a cooling water loop

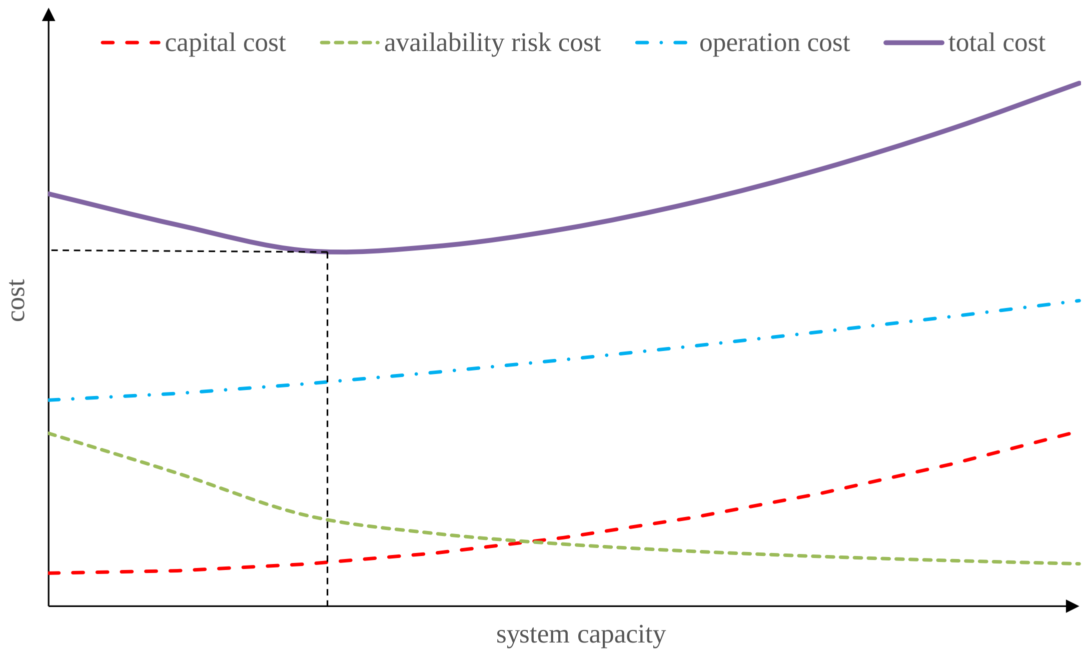


Fig.2 Total cost *vs* system capacity

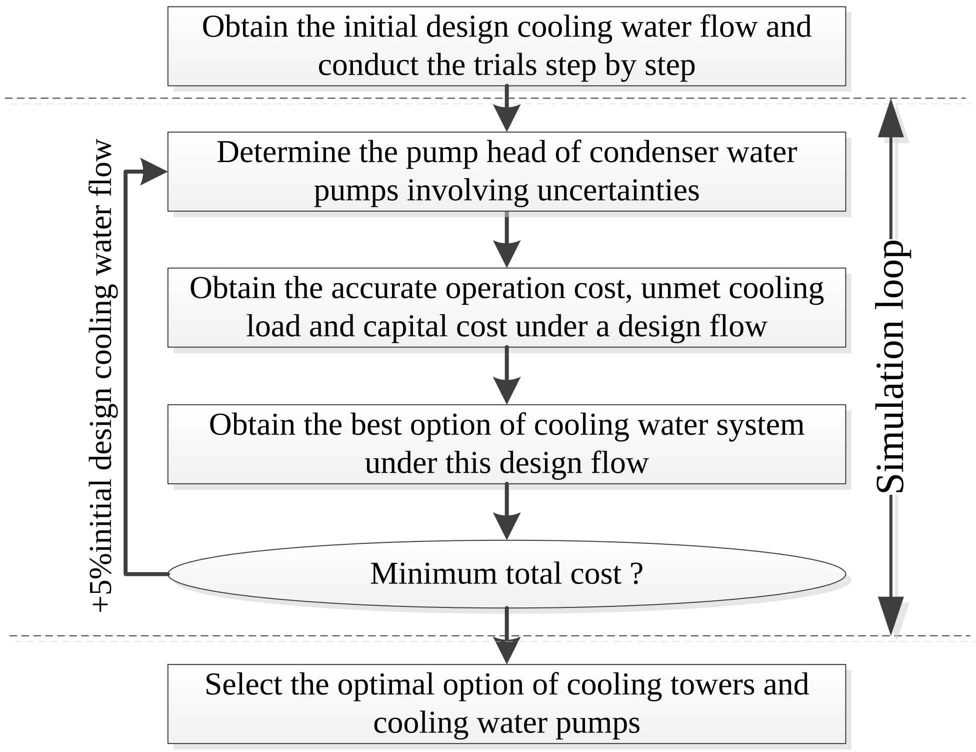


Fig.3 Design optimization procedure of cooling water system

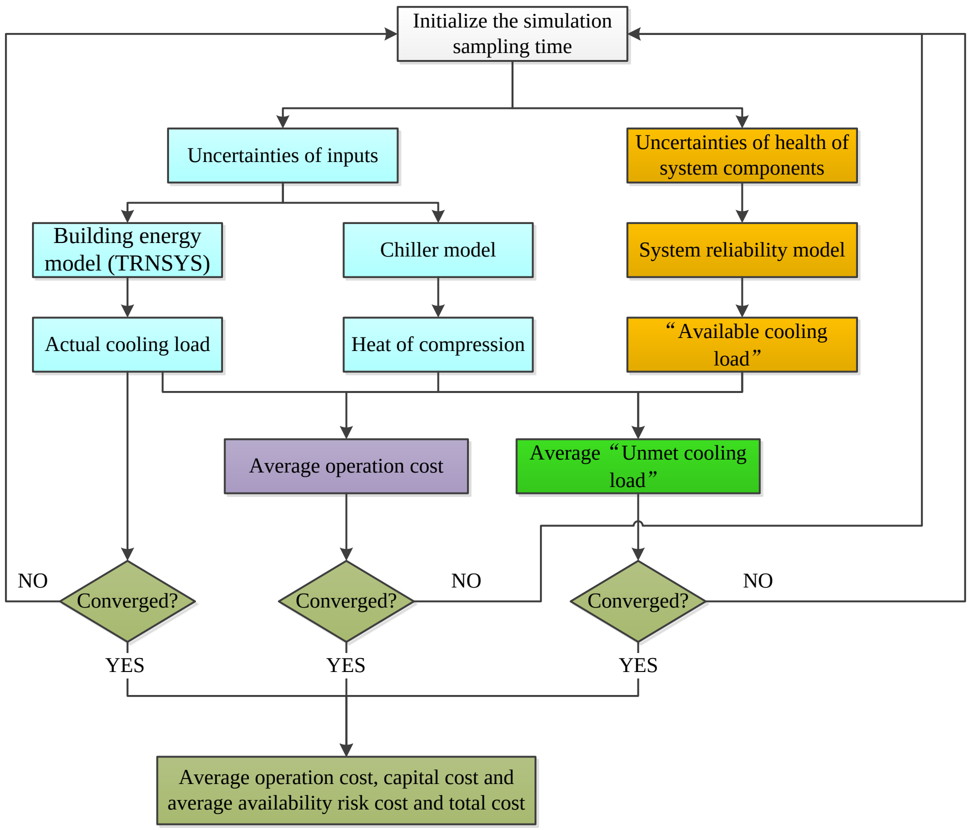


Fig.4 Simulation procedure for obtaining the accurate cooling load distribution, average operation cost and average "unmet cooling load"

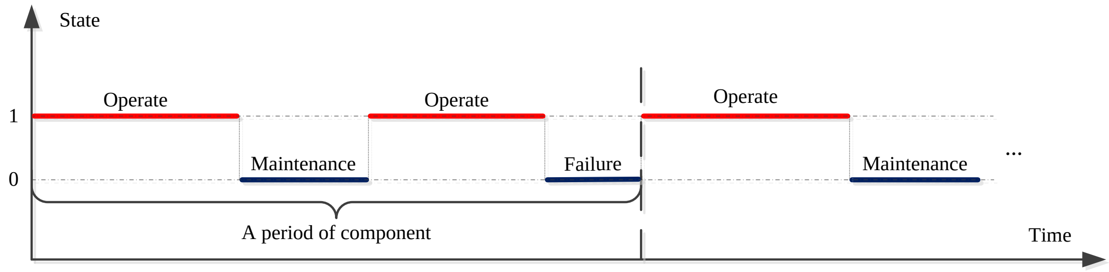


Fig.5 Health states of a component in the life cycle

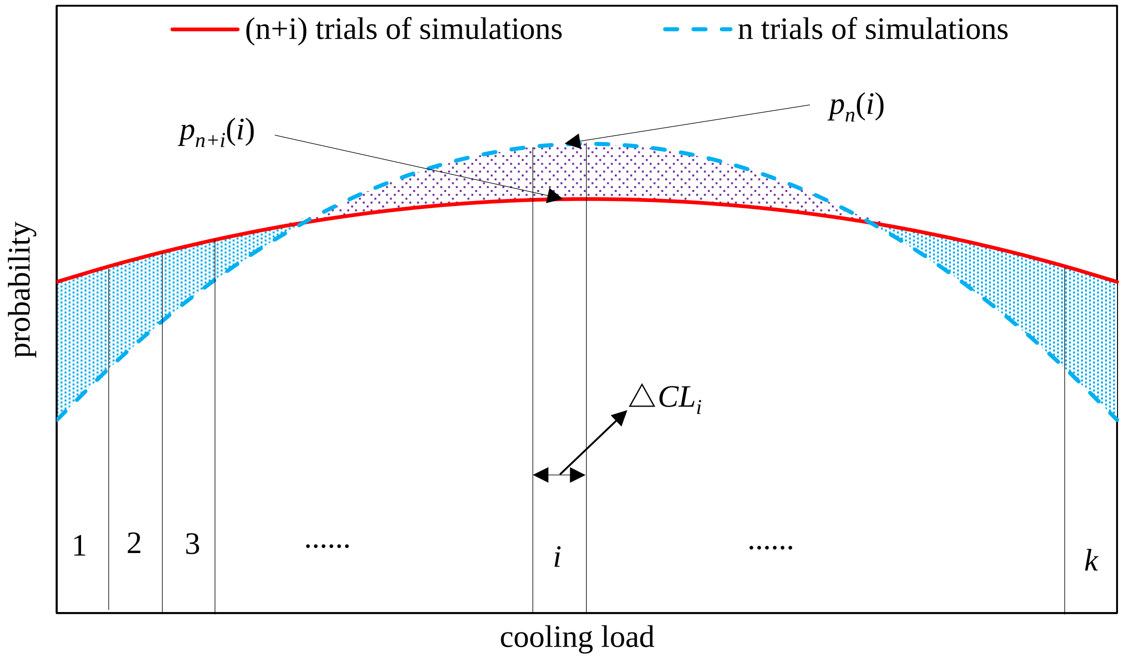


Fig.6 Difference between cooling load distributions over simulation of two different simulation numbers

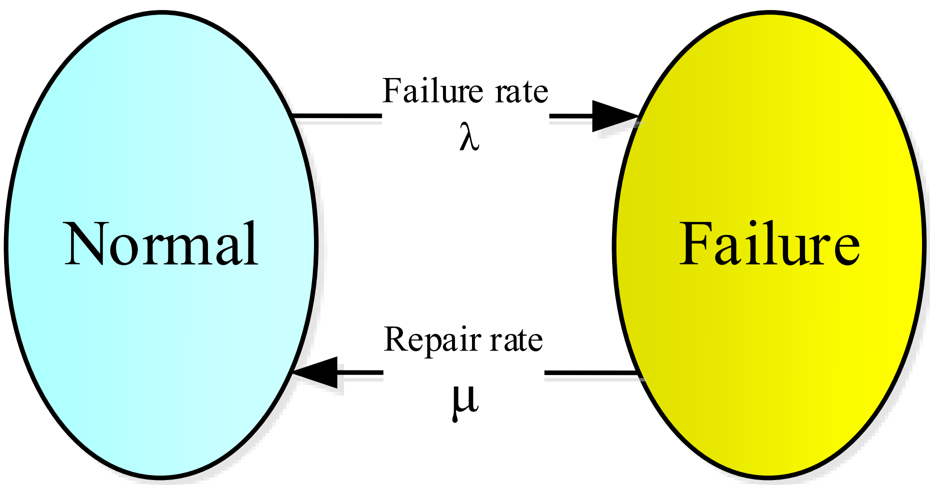


Fig.7 Two-state Markov process

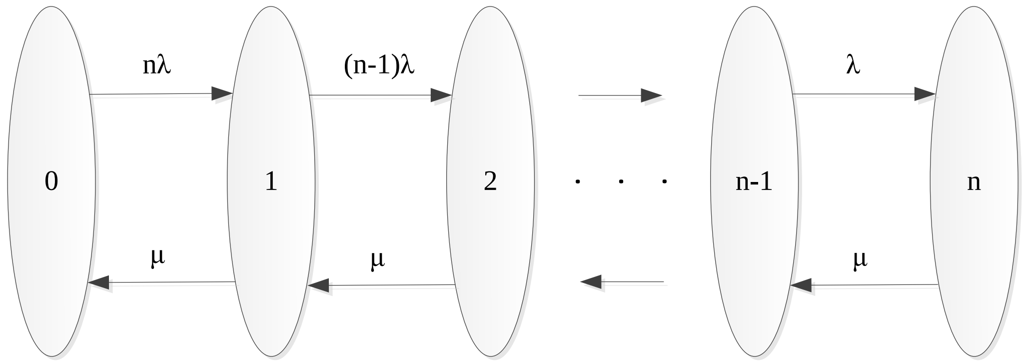


Fig.8 States of a *n*-parallel system and possible transitions

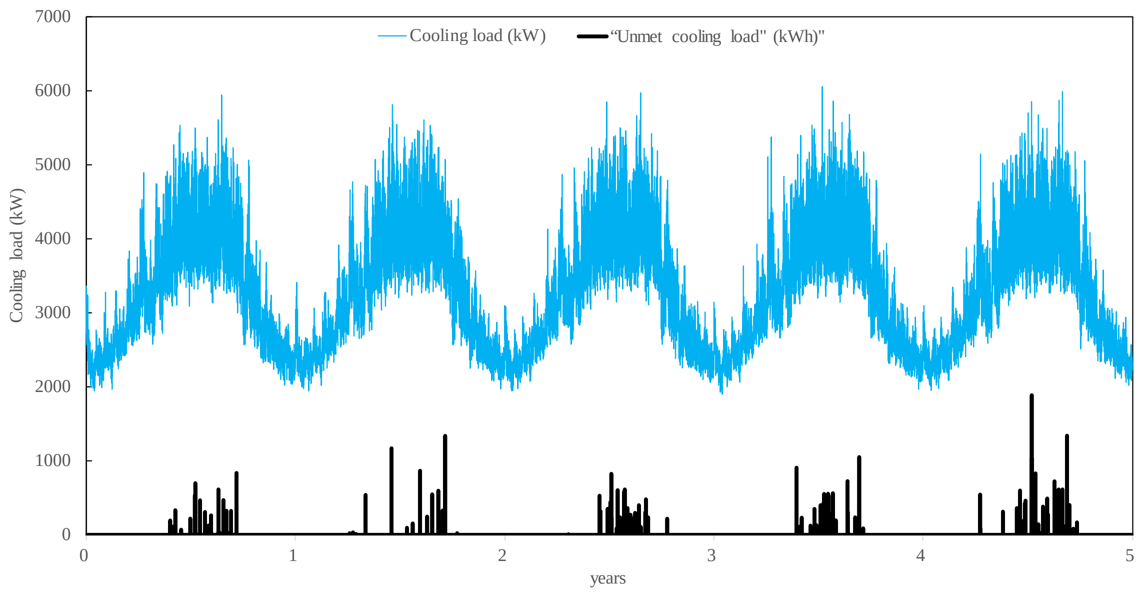


Fig.9 Annual cooling load and unmet cooling load of cooling towers

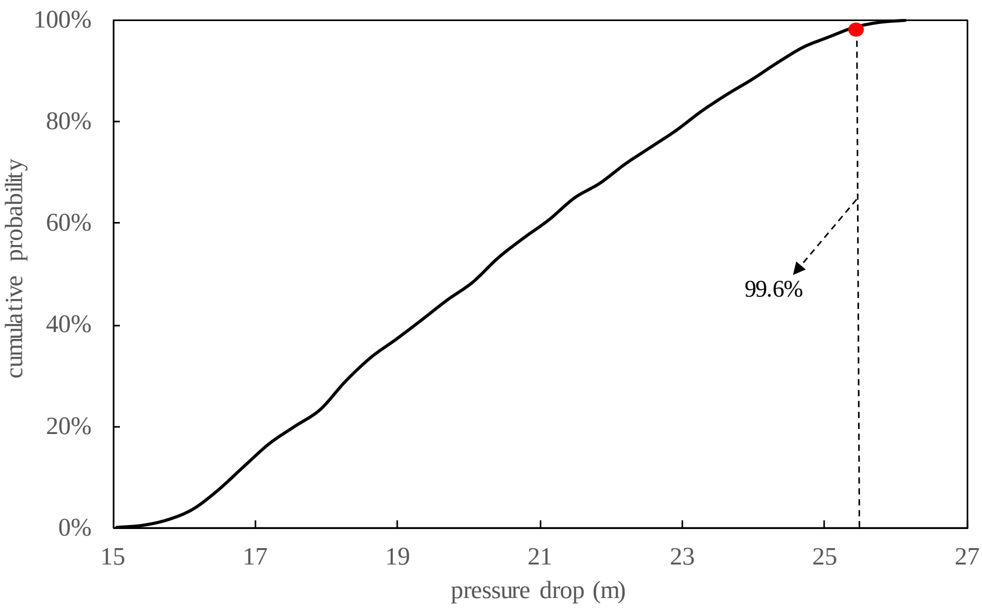


Fig.10 Accumulative probability distribution of the overall pressure drop

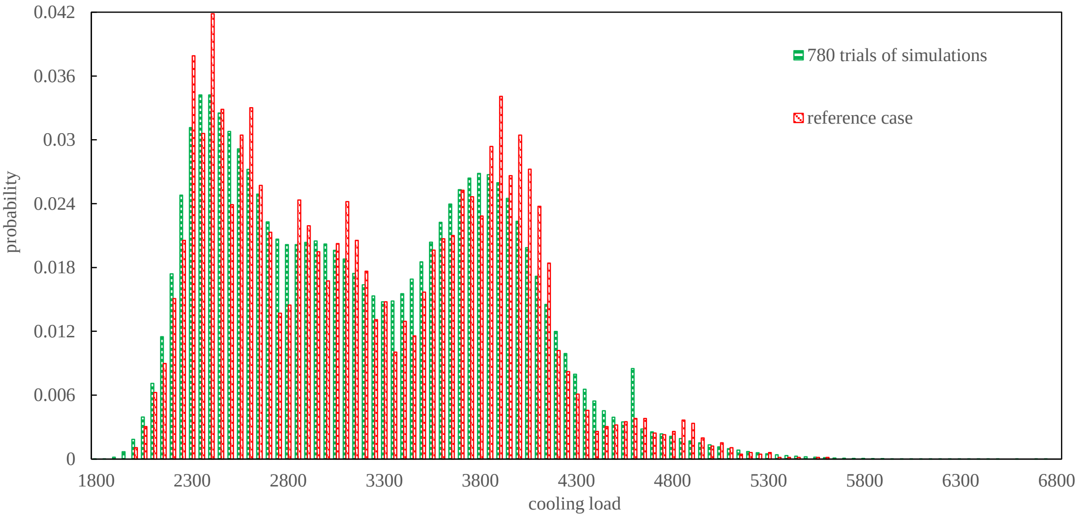
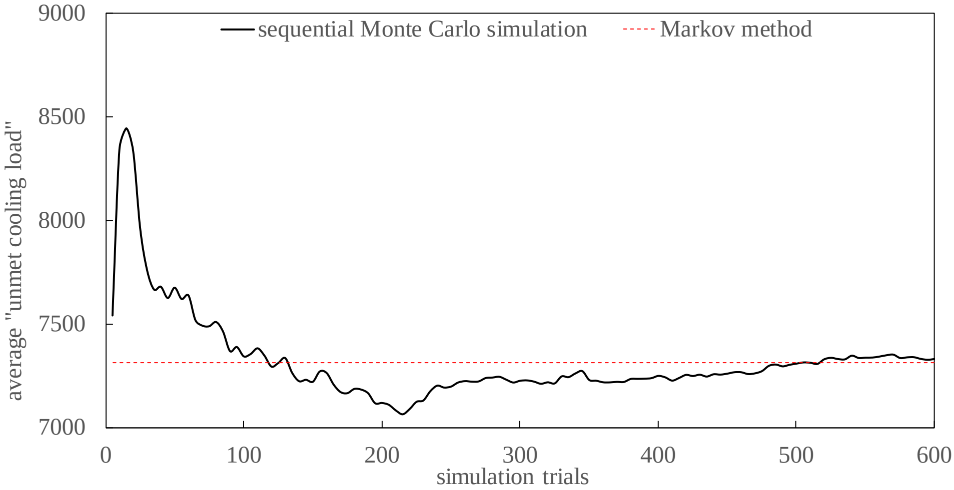
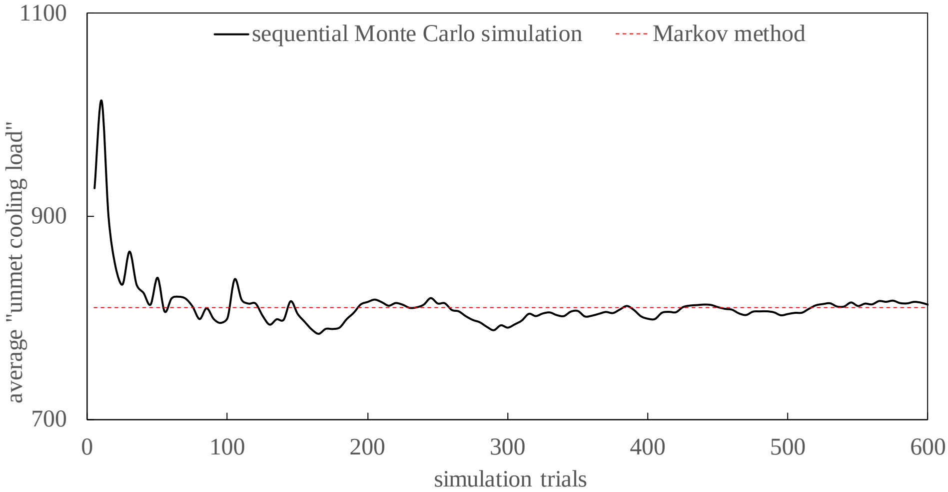


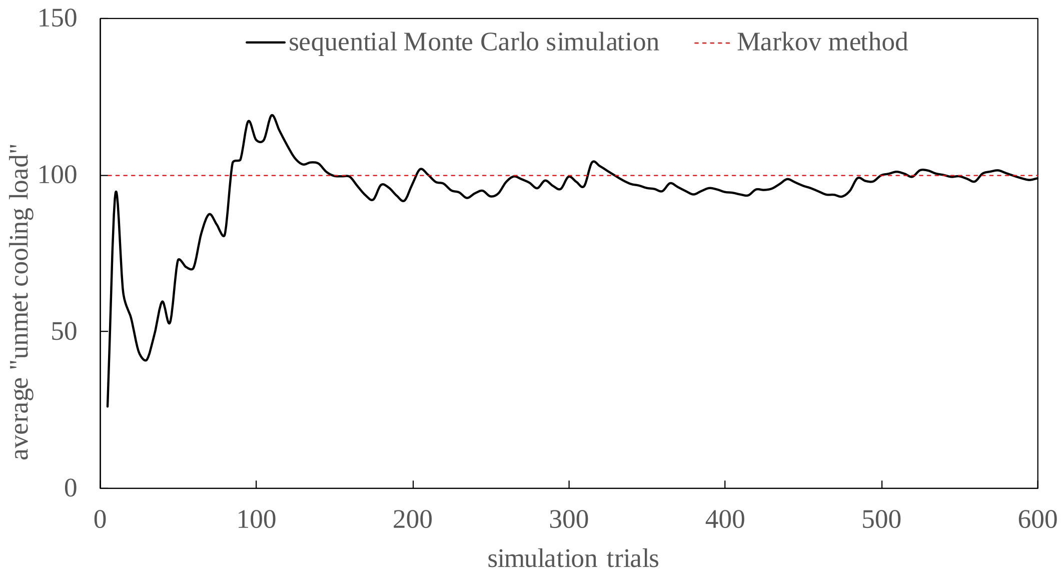
Fig.11 Distribution of annual cooling load



(a) 3 cooling towers

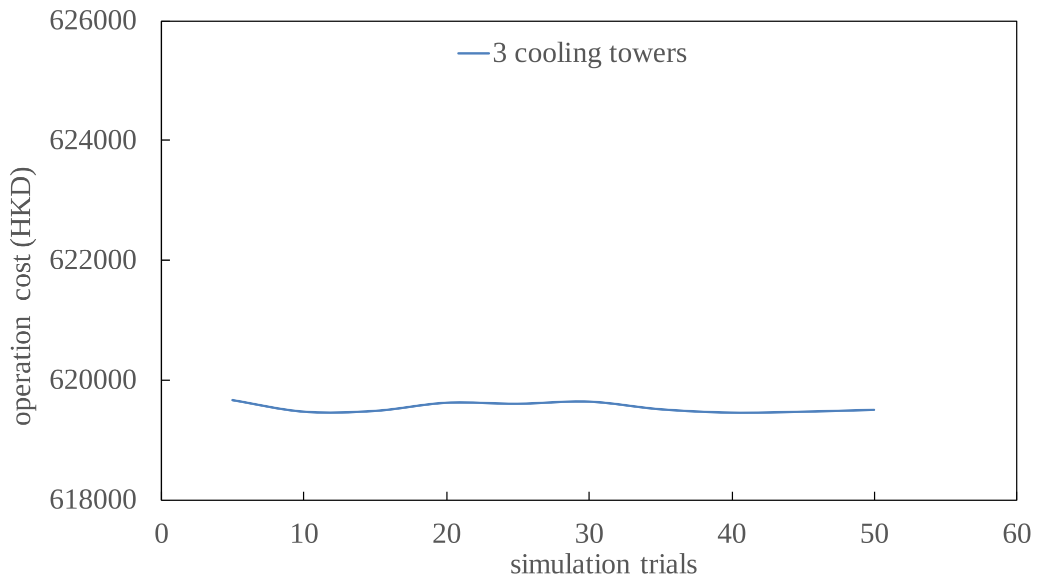


(b) 5 cooling towers

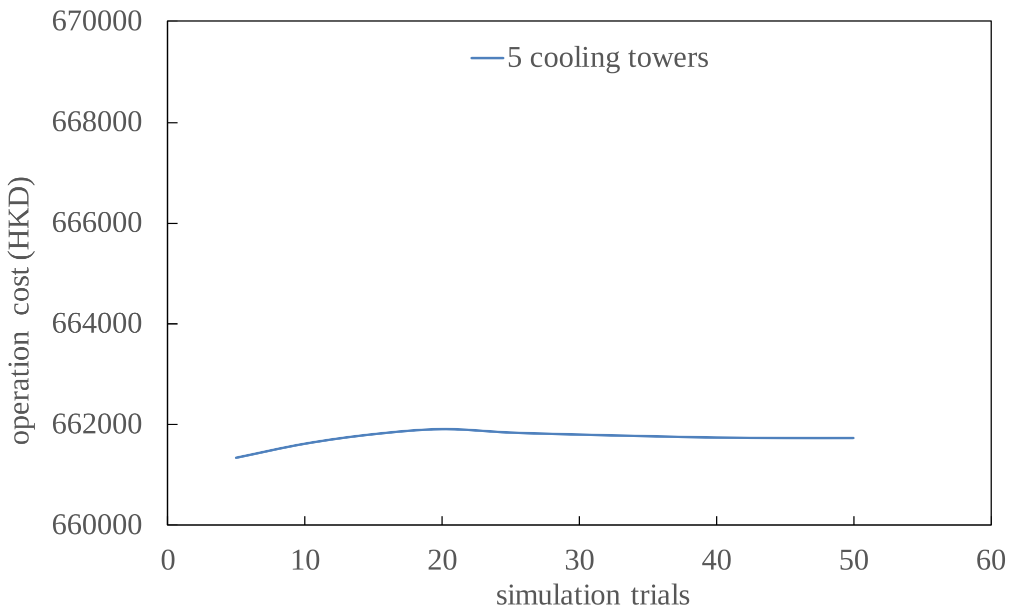


(c) 7 cooling towers

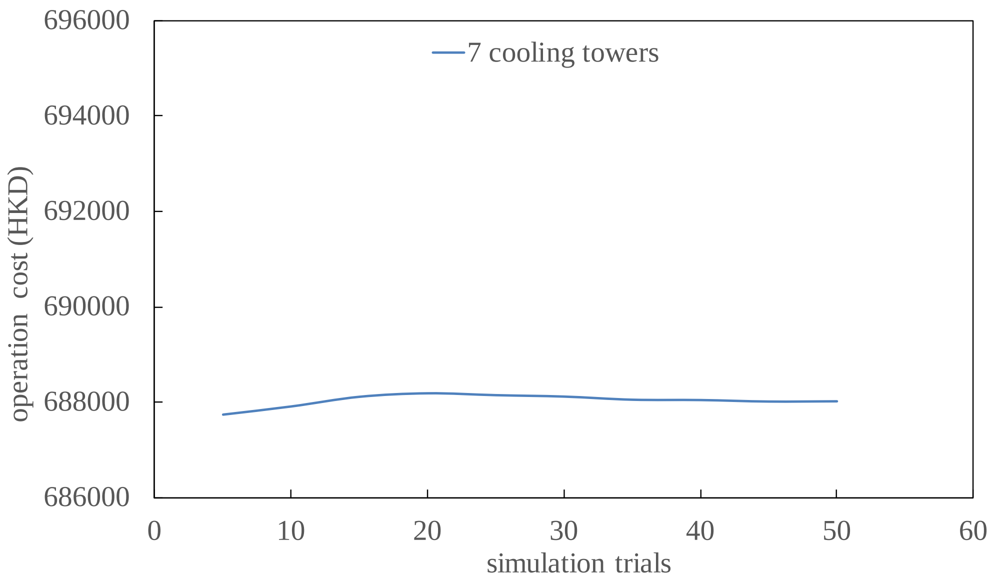
Fig.12 Average unmet cooling load vs number of simulation trials when using different tower numbers



(a)



(b)



(c)

Fig.13 Average operation cost under different simulation trials

Table 1 Distributions of stochastic inputs

Parameters	Distributions
Outdoor temperature (℃)	$N(0,1)$
Relative Humidity (%)	$N(0,1.35)$
Number of Occupants	$T(0.3,1.2,0.9)$
Infiltration rate (m^3/s)	$U(2.7, 3.3)$

Equipment rejection load (kW)	U(376, 464)
Remarks: N(μ, σ) - normal distribution with mean value μ and standard deviation σ; U(a, b) - uniform distribution between a and b; T (a, b, c) - triangular distribution with lower limit a, upper limit b and mode c.	

Table 2 Settings of hydraulic parameters

Parameters	Pressure drop of fittings (m)	Uncertainty
Condenser of Chiller	5.8	U (0.9,1.1)
Pump	7.2	U (0.9,1.1)
Pipe	5.7	U (0.9,1.1)
Nozzle	4.4	U (0.9,1.1)
Height	2.6	-
Cooling water flow	-	1+N(0,0.05)
Aging factor of pipes	15%	-

Table 3 Converged average unmet cooling load and average operation cost of different cooling tower options

Options (Size (L/s) ×number)	165×2	110×3	83×4	66×5	55×6	47×7	41×8
Average unmet cooling load (kWh)	26504	7341	1961	814	325	100	60
Unmet cooling load/total cooling load (%)	0.12	0.035	0.009	0.004	0.002	0	0
Operation cost (10^3 HKD)	570	620	643	661	676	688	698

Table 4 Annual availability risk cost (10^3HKD) and total cost (10^3HKD) of different cooling tower design options

Penalty ratio	1	10	100

(HKD/kWh)									
Option (size (L/s)×number)	*CC*	*RC*	*TC*	*CC*	*RC*	*TC*	*CC*	*RC*	*TC*
165×2	313	26.5	910	313	265	1148	313	2650	3534
110×3	345	7.3	972	345	73.3	1038	345	733	1698
83×4	371	2	1017	***371***	***19.6***	***1035***	371	196	1211
66×5	393	0.81	1055	393	8.1	1063	393	81	1136
55×6	412	0.3	1090	412	3.3	1093	412	33	1122
47×7	430	0	1118	430	1	1119	430	10	1128
41×8	446	0	1144	446	1	1145	446	6	1150
Remarks: CC- capital cost, *RC*- availability risk cost, *TC*- total cost									

Table 5 Annual availability risk cost (10^3HKD) and total cost (10^3HKD) of different pump design options

Penalty ratio (HKD/kW)	1			10			100		
Option (size (L/s)×number)	*CC*	*RC*	*TC*	*CC*	*RC*	*TC*	*CC*	*RC*	*TC*
165×2	70	325	1129	70	3250	4054	70	32500	33298
110×3	91	114	978	91	1140	2003	91	11400	12249
83×4	111	55.3	962	111	553	1460	111	5530	6441
66×5	129	35.4	977	129	354	1296	129	3540	4480
55×6	146	20.1	994	***146***	***201***	***1175***	146	2010	2982
47×7	162	12.4	1016	162	124	1127	162	1240	2243
41×8	178	8.9	1039	178	89	1119	178	890	1923
Remarks: CC- capital cost, *RC*- availability risk cost, *TC*- total cost									

Table 6 Best design options of cooling water system under different design cooling water flow rates (penalty ratio:10HKD/kW)

Design cooling water flow	300	330	345	***360***	375	390

(L/s)							
Cooling towers	Best options (size (L/s) ×number)	60×5	83×4	115×3	***120×3***	125×3	130×3
	Total cost (10^3HKD)	1,026	1,035	1,067	***1,076***	1,098	1,130
Cooling water pumps	Best options (size (L/s) ×number)	50×6	55×6	58×6	***60×6***	63×6	65×6
	Total cost (10^6HKD)	1.422	1.175	1.107	***1.090***	1.087	1.113
Total cost (10^6HKD)		2.448	2.210	2.178	***2.166***	2.185	2.243

Table 7 Best design options under different repair rates

Repair rate		0.001	0.002	0.003	0.004	0.005
Design cooling water flow (L/s)		375	360	345	315	315
Cooling towers	Best options (size (L/s) ×number)	125×3	120×3	115×3	79×4	105×3
	Total cost (10^3HKD)	1,072	1,076	1,044	1,009	1,001
Cooling water pumps	Best options (size (L/s) ×number)	63×6	60×6	58×6	53×6	53×6
	Total cost (10^3HKD)	1,284	1,090	1,020	999	975

Table 8 Best options using different design methods (penalty ratio:10HKD/kW)

	Conventional design	Uncertainty-based design	Robust optimal design
Design cooling water flow (L/s)	345	285	360

Cooling towers	Best options (size (L/s) ×number)	115×3	95×3	120×3
	Total cost (10^6HKD)	1.067	1.122	1.076
Cooling water pumps	Best options (size (L/s) ×number)	115×4 (one standby)	95×4 (one standby)	60×6
	Total cost (10^6HKD)	1.734	3.528	1.090
Total cost (10^6HKD)		2.801	4.650	2.166

Highlights

- A robust optimal design method is developed for cooling water systems.
- Sequential Monte Carlo simulation is used to quantify the impact of reliability.
- It is capable of providing more comprehensive information than Markov method.
- The penalty ratio and repair rate can significantly affect the design option.
- Total system cost is reduced greatly using proposed method.