Transient Wave-Blockage Interaction and Extended Blockage Detection in Elastic Water Pipelines

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Abstract:

Extended partial blockages are common in pressurized water pipelines and can result in the wastage of energy, the reduction in system carrying capacity and the increased potential for contamination. This paper investigates the transient wave-blockage interaction and its application to extended blockage detection in pipelines, where blockage-induced changes to the system resonant frequencies are observed. The frequency shifting is first inspected and explained in this study through wave perturbation analysis, providing a theoretical confirmation for the result that unlike discrete blockages, extended blockages cause resonant frequency shifts in the system. Furthermore, an analytical expression is derived for the relationship between the blockage properties and the resonant frequency shifts and is used to detect the blockages in this study. The obtained results are validated through both numerical applications and laboratory experiments, where the accuracy and efficiency of the developed method for extended blockage detection are tested.

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1. Introduction

Pressurized conduits transporting fluids such as freshwater, seawater, storm-water, wastewater, oil, and blood often experience partial blockages during their lifetime. The blockages begin in the form of a small increase in the wall roughness that grows with time from physical or chemical processes and can eventually block a sizeable portion of the pipe cross sectional area (Stephens, 2008). These blockages result in the wastage of energy, a reduction in the pipe carrying capacity and the increased potential for contamination. In addition, the severely throttled flows from blockages cause flow redistribution in the pipe network and can result in the overpressure of pipes and the development of leaks. It is therefore of paramount importance to detect blockages so that they are dealt with in a timely manner.

Transient-based methods, where a transient signal is injected into the conduit and the response measured at specified locations, is a promising approach for detecting defects in pipes and have been used in the detection of discrete blockages, leaks, and assessment of pipe wall condition (Liggett and Chen, 1994, Brunone, 1999, Brunone and Ferrante, 2001, Vitkovsky et al., 2000, Wang et al., 2002, Wang et al., 2005, Ferrante and Brunone 2003, Covas et al., 2004, Mohapatra et al., 2006, Sattar et al., 2008, Lee et al., 2004, Lee et al., 2006, Lee et al., 2008, Stephens, 2008, Duan et al., 2011a, Duan et al., 2011b, Duan et al., 2012, Duan et al., 2013, Mohapatra and Chaudhry, 2011, Meniconi et al., 2009, Meniconi et al., 2011 and Meniconi et al., 2013). The tenet of this approach is that a measured pressure wave signal in a conduit is modified by, and thus contains information on, the conduit properties.

Stephens et al. (2005), Brunone et al. (2008) and Duan et al. (2012) proposed that blockages in pipes are divided into two categories—discrete and extended blockages—according to its relative length to the total pipeline length. In the context of discrete blockages, Contractor (1965) shows that a discrete partial blockage causes a partial reflection of a waterhammer wave where the amplitude of the reflected wave provides information on the severity of the constriction and the arrival time of the reflected wave provides the location of the blockage. The findings in Contractor (1965) have been confirmed and used for blockage detection by Wang et al. (2005) and Meniconi et al. (2009, 2011 and 2012). Wang et al. (2005) showed that a discrete blockage in a pipe system introduces a frequency dependent damping to the transient signal and developed a technique for locating and sizing discrete blockages based on this damping. Mohapatra et al. (2006), Sattar et al. (2008) and Lee et al. (2008 and 2013) found the effect of the blockage in time translates to a pattern being imposed onto the amplitudes of the resonant responses from the system and this pattern can be used to detect and locate the discrete blockages in the frequency domain.

Field tests by Stephens et al. (2005) and laboratory experiments by Meniconi et al. (2012) found that extended blockages have significantly different impacts on the system responses compared to discrete blockages and discrete blockage detection techniques are not applicable for extended blockages. Stephens et al. (2005) shows that severe wall deterioration is often associated with a reduction in the pipe flow area as well as wavespeed, with nearly 40% reduction in both parameters observed in the field. Similarly, extended changes in pipe wall thickness and material was found to produce changes in the wavespeed in the laboratory studies of Hachem and Schleiss (2011, 2012a and 2012b) and Tuck et al. (2012). Duan et al. (2012) and Tuck et al. (2012) show that extended blockage changes the amplitude as well as the position of resonant responses from the system. An analytical expression for the blockage-induced changes in the system resonant frequencies was derived in Duan et al. (2012) and was used for detecting extended blockages in pipelines. To determine the properties of the blockage, an optimization process coupled with a Genetic Algorithm (GA) was used to fit the observed resonant frequencies with the theoretical expression. This approach was verified using numerical as well as experimental results in Duan et al. (2012, and 2013) and Meniconi et al. (2013). It was found from these studies that the solution process is time consuming and its efficiency decreases significantly with the number of blockages in the system. A simplified form to the original analytical equations was developed in Duan et al. (2013) and the computational efficiency was increased by sacrificing the accuracy of the solution.

This paper further investigates the effect of extended blockages on the system frequency response and proposes an improvement to the frequency domain method for detecting extended blockage in pipes. The frequency shifts due to wave-blockage interaction is inspected using wave perturbation analysis and the expression for the resonant frequencies shifts proposed in Duan et al. (2012) is simplified using a first order approximation and the result is validated numerically and experimentally.

2. System Frequency Response-Based Extended Blockage Detection

The analytical expression for the frequency response of extended partial blockage pipeline system in Duan et al., (2012) is derived using the transfer matrix method, where the one-dimensional (1-D) waterhammer equations are linearized in the frequency domain (Chaudhry, 1987). For the blockage-free pipeline in Fig. 1(a), Chaudhry (1987) defines the system resonant frequencies as the solutions to the following equation,

$$\cos(\lambda_0 \omega_{rf0}) = 0, \quad (1)$$

where ω_{rf0} = resonant frequency of the blockage-free pipe system, $\lambda_0 = \frac{L_0}{a_0}\sqrt{1 - \mathrm{i}\frac{gA_0}{\omega_{rf0}} R_f}$ = wave propagation coefficient, A_0 = cross-sectional area of uniform (blockage-free) pipe, D_0 = pipe diameter, L_0 = pipe length, a_0 = acoustic wavespeed of uniform pipe, g = gravitational acceleration, $\mathrm{i} = \sqrt{-1}$, and $R_f = R_{fS} + R_{fU}$ = friction damping factor, with R_{fS} and R_{fU} representing the steady and unsteady friction components respectively and the subscript "0" represents the uniform system.

V ťkovsk ý et al. (2003) derived the frequency domain expressions for unsteady friction damping for laminar and turbulent flows based on Zielke (1968) and Vardy and Brown (1996). For turbulent flows, $R_{fS} = \frac{fQ}{gDA^2}$ and $R_{fU} = \frac{2\mathrm{i}\omega_{rf}}{gA}\left(\frac{1}{C^*} + \frac{\mathrm{i}\omega_{rf} D^2}{4\nu}\right)^{-1/2}$, where $C^* = \frac{7.41}{R_e^{\kappa}}$ is shear decay coefficient with $\kappa = \log_{10}\left(\frac{14.3}{R_e^{0.05}}\right)$, R_e = Reynolds number and ν = kinematic viscosity.

If neglecting the friction effect (i.e., R=0), the frequency corresponding to the k^{th} resonant peak in a blockage-free (uniform) pipeline system in Fig. 1(a) can be obtained as, $\omega_{rf0}(m) = (2m-1)\omega_{th0}$ where m is a real positive integer and $\omega_{th0} = 2\pi\frac{a_0}{4L_0}$ is the theoretical or fundamental frequency of the uniform (blockage-free) pipeline system. The result shows that the spacing between the resonant peaks in a blockage-free system is a constant and the resonant peaks are spaced equally along the frequency axis.

The resonant frequencies for the system with an extended blockage in Fig. 1(b), are determined by Duan et al. (2012 and 2013) as,

$$\begin{aligned}&(Y_1+Y_2)(Y_2+Y_3)\cos[(\lambda_1+\lambda_2+\lambda_3)\omega_{rfb}]+(Y_1-Y_2)(-Y_2-Y_3)\cos[(\lambda_1-\lambda_2-\lambda_3)\omega_{rfb}]\\&-(Y_1+Y_2)(Y_2-Y_3)\cos[(\lambda_1+\lambda_2-\lambda_3)\omega_{rfb}]-(Y_1-Y_2)(-Y_2+Y_3)\cos[(\lambda_1-\lambda_2+\lambda_3)\omega_{rfb}]=0\end{aligned}, \qquad (2)$$

where $Y=-\frac{a}{gA}\sqrt{1-\mathrm{i}\frac{gAR_f}{\omega_{rfb}}}$ = characteristic impedance of pipeline; ω_{rfb} = resonant frequencies of the blocked pipe case; and subscripts 1, 2, 3 denote pipe sections numbered from upstream to downstream. For convenience, the frequency associated with the k^{th} resonant peak, $\omega_{rf}(k)$ is written as ω_{rf} .

The result of Eq. (2) shows that the resonant frequencies of the system with an extended blockage are different from the uniform pipe case of Eq. (1). These differences are functions of the characteristic impedance Y and the wave propagation constant λ of each pipe section, and are related to the length, location and severity of the extended blockages. The system frequency responses for both blockage-free and blocked systems are obtained using the procedure in Lee et al. *(*2004 and 2008) and the results are plotted in Fig. 2. The figure shows clear differences in the resonant peak frequencies as well as the magnitudes of the resonant peak responses between the two cases.

Duan et al. (2012 and 2013) obtained the properties of the extended blockages by inversely calibrating the resonant frequencies calculated in Eq. (2) to data obtained from the numerical and experimental tests. Despite its success for detecting extended blockages, the inverse calibration procedure demonstrated in Duan et al. (2012 and 2013) using Eq. (2) is time consuming and the efficiency is greatly reduced with an increasing number of blockages. Moreover, as shown in Eq. (2), the expression for the resonant frequencies in a system with an extended blockage is complicated and the effect of the blockage length, location and severity on the resonant frequencies cannot be clearly seen from the expression. Further analysis on the result of Eq. (2) is required to address these issues.

It is necessary to point out that the determination of the resonant frequencies does not require the system to be forced into a state of resonance and any signal with a wide range of frequencies can give the resonant frequencies through analysis in the frequency domain. Details on this frequency domain analysis approach can be found a recent state-of-the-art paper by Lee et al. (2013).

3. Understanding the Properties of Extended Blockage Induced Frequency Shifting

While the blockage-induced shifts in the system resonant frequencies in Fig. 2 have been shown in experimental data and numerical simulations (Stephens et al., 2005, Brunone et al., 2008, Duan et al., 2012 and Duan et al., 2013), the physical understanding for this change remains unclear. This effect is first investigated using wave perturbation analysis followed by the quantitative derivation of the expression for the frequency shifting. The 1-D wave equation for a conduit with varying pipe cross-sectional area is given in Duan et al. (2011c) as,

$$A\frac{\partial^2 P}{\partial t^2} = a^2 \frac{\partial}{\partial x}\left(A\frac{\partial P}{\partial x}\right), \tag{3}$$

where P = pressure response in the time domain; x = longitudinal coordinate along the pipeline with x = 0 as the centre of the pipe system as well as the extended blockage; and t = time coordinate. Note that a frictionless pipe with a constant wave speed is considered in this analysis to highlight the interaction between the transient wave and blockage. Eq. (3) is first re-written as,

$$\underbrace{\frac{\partial^2 P}{\partial t^2} - a^2\frac{\partial^2 P}{\partial x^2}}_{\text{lhs}} = \underbrace{a^2\frac{\partial \ln A}{\partial x}\frac{\partial P}{\partial x}}_{\text{rhs}}. \tag{4}$$

where the difference between the blockage-free (i.e. rhs = 0) and extended blockage situations (i.e. rhs ≠ 0) is clearly shown as a function of the spatial variation of the flow cross-sectional area,

$\frac{\partial \ln A}{\partial x}$. Furthermore, by considering an incident pressure wave with a certain frequency (ω) impinging on the blockage from the boundary end, $P = \hat{P}(x,\omega)e^{-iwt}$, where $\hat{P}(x,\omega)$ is the amplitude of the propagating wave in the pipeline, Eqs. (3) and (4) can now be simplified as,

$$\frac{\partial}{\partial x}\left(A\frac{\partial \hat{P}}{\partial x}\right) + \frac{\omega^2}{a^2}A\hat{P} = 0, \tag{5}$$

$$\underbrace{\frac{\partial^2 \hat{P}}{\partial x^2} + \frac{\omega^2}{a^2}\hat{P}}_{\text{LHS}} = \underbrace{-\frac{\omega^2}{a^2}\frac{\partial \ln A}{\partial x}\frac{\partial \hat{P}}{\partial x}}_{\text{RHS}}. \tag{6}$$

Thus, Eq. (5) is the well-known *Sturm-Liouville* type equation (Zettl, 2005) and can usually be analyzed by the perturbation method (Mei et al., 2005).

To investigate the effect of an extended blockage on the wave propagation, the pipeline in Fig. 1(b) is used and reflections from the either ends of the pipeline are ignored to highlight the effect the blockage on the wave propagation (i.e. reflection-free end boundaries). A similar derivation has been carried out by Mei et al. (2005) for open channel flows. Eq. (5) is first solved by wave perturbation analysis and for each section of the pipeline, Eq. (5) is satisfied as,

$$\frac{\partial}{\partial x}\left(A_j\frac{\partial \hat{P}_j}{\partial x}\right) + k_j^2 A_j \hat{P}_j = 0, \tag{7}$$

where $k_j = \frac{\omega_j}{a_j}$ is wave number, and j = 1, 2, 3 identifies the pipe sections shown in Fig. 1(b). Under this condition, the pressure head responses for the three pipe sections from an incident wave with amplitude P_0 and frequency ω_0 originating from $+\infty$ are obtained as follows (note that x = 0 corresponds to the middle of the extended blockage in Fig. 1b):

$$\begin{aligned} \hat{P}_1 &= I_1 \mathrm{e}^{\mathrm{i}k_1(x+0.5l_2)} + R_1 \mathrm{e}^{-\mathrm{i}k_1(x+0.5l_2)}, \quad x < -0.5l_2 \\ \hat{P}_2 &= I_2 \mathrm{e}^{\mathrm{i}k_2 x} + R_2 e^{-\mathrm{i}k_2 x}, \quad -0.5l_2 < x < 0.5l_2 \\ \hat{P}_3 &= I_3 \mathrm{e}^{\mathrm{i}k_3(x-0.5l_2)} + R_3 \mathrm{e}^{-\mathrm{i}k_3(x-0.5l_2)}, \quad x > 0.5l_2 \end{aligned} , \tag{8}$$

where l_2 here refers to the length of blockage section in Fig. 1(b); I and R are amplitudes of incident and reflected waves. Therefore under the conditions of reflection-free end boundaries, $I_3 = P_0$ since the incident wave in this pipe section is the known wave originating from the downstream boundary at $x = +\infty$. In addition, since the reflection from the upstream boundary is ignored, $R_1 = 0$. This produces four remaining unknowns (I_1, I_2, R_2, and R_3) in the solutions given by Eq. (8). These unknowns are evaluated from the enforcement of mass and momentum conservation at the pipe junctions (i.e., $x = 0.5l_2$ and $x = -0.5l_2$) and leads to:

$$\begin{aligned} \hat{P}_1 &= \hat{P}_2 && \text{at} \quad x = -0.5l_2 \\ A_1 \frac{\partial \hat{P}_1}{\partial x} &= A_2 \frac{\partial \hat{P}_2}{\partial x} && \text{at} \quad x = -0.5l_2 \\ \hat{P}_2 &= \hat{P}_3 && \text{at} \quad x = 0.5l_2 \\ A_2 \frac{\partial \hat{P}_2}{\partial x} &= A_3 \frac{\partial \hat{P}_3}{\partial x} && \text{at} \quad x = 0.5l_2 \end{aligned} . \tag{9}$$

Combining Eq. (8) and Eq. (9) gives,

$$R_3 = \left[\frac{(1+s_{12})(1-s_{23})\mathrm{e}^{2\mathrm{i}k_2 l_2} + (1-s_{12})(1+s_{23})}{(1-s_{12})(1-s_{23}) + (1+s_{12})(1+s_{23})\mathrm{e}^{2\mathrm{i}k_2 l_2}}\right] P_0 , \tag{10}$$

where $s_{12} = \frac{k_1 A_1}{k_2 A_2}$ and $s_{23} = \frac{k_2 A_2}{k_3 A_3}$. If a single blockage is considered (i.e., $A_1 = A_3$ and $a_1 = a_3$), which implies $k_1 = k_3$ and $s_{12} = s_{23}^{-1}$. As a result, Eq. (10) becomes:

$$R_3 = -\frac{\left(1 - \mathrm{e}^{-2\mathrm{i}k_2 l_2}\right)}{\left(1 - \xi_s^2 \mathrm{e}^{-2\mathrm{i}k_2 l_2}\right)} \xi_s P_0 , \text{ with } \quad \xi_s = \frac{1 - s_{12}}{1 + s_{12}} . \tag{11a}$$

where ξ_s is a measure of the radial constriction imposed by the blockage. It is instructive to consider a blockage with small radial constriction (i.e., $|\xi_s| << 1$) which simplifies Eq. (11a) to:

$$R_3 \approx -\left(1 - \mathrm{e}^{-2\mathrm{i}k_2 l_2}\right)\xi_s P_0 . \tag{11b}$$

Substituting Eq. (11b) into Eq. (8), the pressure head at downstream pipe section (i.e., for $x > 0.5 l_2$) is:

$$\hat{P}_3(x) = \underbrace{P_0 \mathrm{e}^{\mathrm{i}k_3(x-0.5l_2)} + P_0 \mathrm{e}^{-\mathrm{i}k_3(x-0.5l_2)}}_{(a)} + \underbrace{\left[\xi_s P_0 \mathrm{e}^{-2\mathrm{i}k_2 l_2} \mathrm{e}^{-\mathrm{i}k_3(x-0.5l_2)} - \left(1+\xi_s\right) P_0 \mathrm{e}^{-\mathrm{i}k_3(x-0.5l_2)}\right]}_{(b)} , \tag{12}$$

in which the term (a) represents the wave propagation in a blockage-free pipeline while term (b) represents the effect of the extended blockage on the wave field. In particular, the presence of the $\mathrm{e}^{-2\mathrm{i}k_2 l_2}$ in term (b) clearly shows that the blockage induces a frequency shift, while the presence of ξ_s shows that the blockage induces a change in wave amplitude. In addition, since $\mathrm{e}^{-2\mathrm{i}k_2 l_2} \to 1$ as l_2 tends to zero (or more precisely as $k_2 l_2$ tends to zero), the expression provides a theoretical proof that a discrete blockage (blockage with a negligible length) causes a change in the wave amplitude but no phase shift. This result is consistent with Wang et al. (2005), Mohapatra et al., (2006), Lee et al. (2008 and 2013) and Sattar et al., (2008), where a blockage of a sufficiently short length can be approximated as a lumped local loss (discrete blockage), which changes the amplitude of the system resonant responses but not their frequencies. The quantity of this frequency shifting by the extended blockage is further analyzed in the next section.

It is insightful to investigate Eq. (11) further. Since, $k_2 l_2 = \frac{\omega_0}{a_2} l_2$ and, $\omega_0 = \frac{2\pi}{L_0} a_0$, where L_0 and a_0 are the wave length and speed of the incident wave, then $k_2 l_2 = 2\pi \frac{a_0}{a_2} \frac{l_2}{L_0}$. The blockage induced variations of wave amplitude and phase angle versus $\frac{\omega_0}{\omega_2} = \frac{a_0}{a_2} \frac{l_2}{L_0}$ are investigated for the small blockage case of $|\xi_s| << 1$, and the results are plotted in Fig. 3. The results demonstrate that the reflected wave amplitude and frequency varies periodically with $\frac{\omega_0}{\omega_2} = \frac{a_0}{a_2}\frac{l_2}{L_0}$. As a result, it is clear that the blockage selectively reflects some waves more than others and highlights that the detection methods that focus on using wave generation mechanisms of limited frequency content are only effective for blockages with certain lengths. In fact, according to Eq. (11), maximum reflection occurs if

$$2k_2 l_2 = (2m-1)\pi \Rightarrow \frac{a_0}{a_2}\frac{l_2}{L_0} = \frac{\omega_0}{\omega_b} = \frac{(2m-1)}{4}, \tag{13}$$

where *m* is an integer. This condition is referred to in the gravity waves literature as resonance condition (e.g., Mei et al., 2005).

4. Analytical Derivation of Extended Blockage Induced Frequency Shifting

The numerical and experimental studies in Duan et al. (2012 and 2013) have shown that both steady and unsteady friction do not shift the system resonant frequencies and the assumption of $R = 0$ is therefore justified in the following derivation to highlight the effect of the extended blockage. The full effect of friction is taken into account for the comparisons with numerical and experimental data later in the paper. For the case of a single extended blockage in Fig. 1(b),

$$Y_1 = Y_3 = Y_0 = -\frac{a_0}{gA_0},\; Y_2 = -\frac{a_2}{gA_2},\; \lambda_1 = \frac{l_1}{a_0},\; \lambda_2 = \frac{l_2}{a_2}\;\; \lambda_2 = \frac{l_3}{a_0}, \text{ and } \lambda_0 = \frac{L_0}{a_0}. \tag{14}$$

By defining the changes in the characteristic impedance and wave propagation coefficient imposed by the extended blockage section as $\delta Y = Y_2 - Y_0$ and $\delta\lambda = (\lambda_1 + \lambda_2 + \lambda_3) - \lambda_0$, the following results can be obtained,

$$\varepsilon_Y = \frac{\delta Y}{Y_0} = \frac{Y_2 - Y_0}{Y_0}, \text{ and } \varepsilon_\lambda = \frac{\delta\lambda}{\lambda_0} = \frac{(\lambda_1 + \lambda_2 + \lambda_3) - \lambda_0}{\lambda_0}, \tag{15}$$

where ε_Y and ε_λ = the changes of the characteristic impedance and wave propagation coefficient relative to the values of original blockage-free case respectively. Furthermore, by defining,

$$\varepsilon_A = \frac{\delta A}{A_0} = \frac{A_0 - A_2}{A_0}, \ \varepsilon_L = \frac{l_2}{L_0}, \text{ and } \varepsilon_a = \frac{\delta a}{a_0} = \frac{a_0 - a_2}{a_0}, \tag{16}$$

gives,

$$\varepsilon_Y = \frac{\varepsilon_A - \varepsilon_a}{1 - \varepsilon_A}, \text{ and } \varepsilon_\lambda = \varepsilon_L \frac{\varepsilon_a}{1 - \varepsilon_a}, \tag{17}$$

where ε_A, ε_L, and ε_a represent the pipe area, pipe length and pipe celerity changes imposed by the extended. Previous studies such as Stephens et al. (2005) and Stephens (2008) have found that areas of severe wall deterioration are often associated with a reduction in the pipe flow area as well as wavespeed, with nearly 40% reduction in both parameters observed in the field. The results of Eq. (15) through to Eq. (17) demonstrate that the changes in the system parameters imposed by the extended blockage (ε_A, ε_a, and ε_L) result in changes in the pipe characteristic impedance and wave propagation coefficients (ε_Y and ε_λ). That is, a severe blockage that imposes a significant change in cross-sectional area (ε_A), wavespeed (ε_a) or has a long blockage length (ε_L) produces large values of ε_Y and/or ε_λ.

By defining $\omega_{rfb} = \omega_{rf0} + \Delta\omega_{rf}$ (or $\Delta\omega_{rf} = \omega_{rfb} - \omega_{rf0}$) and by combining Eq. (14) through Eq. (17) where $\Delta\omega_{rf}$ is the size of the shift in resonant frequency between the uniform and blocked cases for a particular resonant peak, the result of Eq. (2) can be rewritten in terms of ε_A, ε_a, ε_L and $\Delta\omega_{rf}$. Using a first order approximation and after trigonometric transformations and rearrangements, the resonant frequency shift normalized by the theoretical frequency of the uniform pipeline can be expressed as,

$$\Delta\omega_{rf} = \frac{C_u}{C_d}, \tag{18}$$

where C_u and C_d are coefficients relating to the extended blockage parameters defined as Eq. (A4) in the appendix of the paper.

It is necessary to note that under most practical situations the blockage causes a reduction in the pipe celerity and ε_a is normally much smaller than 1 with $\varepsilon_a \sim 0.4$ observed in the literature for extreme cases (e.g., Hachem and Schleiss, 2012a, Hachem and Schleiss, 2012b, Stephens et al., 2005 and Stephens, 2008). Based on ε_a <<1, Eq. (18) can be further simplified with regard to ε_A and ε_L as,

$$\frac{\Delta\omega_{rf}}{\omega_{th0}} \approx \frac{2}{\pi}\frac{\varepsilon_A}{2-\varepsilon_A}\left[\sin\left(2\lambda_1\omega_{rf0}\right) - \sin\left(2\lambda_3\omega_{rf0}\right) - \frac{\varepsilon_A}{2-\varepsilon_A}\sin\left(2\varepsilon_L\lambda_0\omega_{rf0}\right)\right], \tag{19a}$$

Or in terms of the resonant peak number (m) as,

$$\begin{aligned}\frac{\Delta\omega_{rf}}{\omega_{th0}} &\approx \frac{2}{\pi}\frac{\varepsilon_A}{2-\varepsilon_A}\left[\sin[(4m-2)\lambda_1\omega_{th0}] - \sin[(4m-2)\lambda_3\omega_{th0}] - \frac{\varepsilon_A}{2-\varepsilon_A}\sin[(4m-2)\varepsilon_L\lambda_0\omega_{th0}]\right] \\ &= (-1)^{k+1}\frac{4}{\pi}\frac{\varepsilon_A}{2-\varepsilon_A}\sin[(2m-1)\varepsilon_L\lambda_0\omega_{th0}]\left(\sin[(2m-1)(\lambda_1-\lambda_3)\omega_{th0}] - \frac{\varepsilon_A}{2-\varepsilon_A}\cos[(2m-1)\varepsilon_L\lambda_0\omega_{th0}]\right)\end{aligned}.$$

(19b)

The detailed derivations for Eq. (19), Eq. (19a) and Eq. (19b) are shown in the appendix of the paper. The simplified result of Eq. (19) shows that the resonant frequencies are shifted in a periodic pattern and that the pattern is dependent on the size, length and location of the extended blockage (ε_A, ε_L and λ_3). This result is consistent with the findings of Duan et al. (2012 and 2013). Furthermore, when the longitudinal length of the blockage is very small (i.e., $l_2 \sim 0$) such that ε_L tends to zero, the magnitude of the frequency shift also approaches zero according to Eq.

(19b) and is a further confirmation for the behavior of discrete blockages previously shown using Eq. (12). Moreover, once its accuracy is confirmed, Eq. (19) provides an efficient alternative to the complicated Eq. (2) for determining the blockage properties through an inverse calibration process. In the following sections numerical and laboratory experiments are conducted and used for the verification of Eq. (19).

5. Numerical Validation and Results Analysis

The single pipeline systems in Figs. 1 (a) and (b) are used for validating the results of Eq. (19a) and Eq. (19b). The reservoir head at the upstream end of the system is fixed at 50m and the valve at downstream end is initially fully open. The reservoir and boundary valve are connected by a single pipeline of 1000m length and 0.5m diameter. The steady state flowrate in the pipeline is $0.1m^3/s$ and the transient signal is caused by the sudden and full closure of the end boundary valve. The pressure head trace is measured at the upstream face of the valve. A total of 6 blockage cases (labeled as cases no. *N*1 through to *N*6) with a wide range of ε_A and ε_L values are tested numerically and the parameter settings are shown in Table 1.

To initially highlight the impact of different blockage constriction severities on the shift pattern of Eq. (19), the wavespeeds for all sections (original and constricted) are fixed at 1000m/s (i.e., ε_a =0 but $\varepsilon_L \neq 0$). That is, the effect of ε_A is first inspected in this section while the effect of ε_a will be tested later in the paper. The numerical tests are conducted by a 1-D method of characteristics (MOC) model (Wylie et al., 1993 and Ghidaoui et al., 2005), with the pipeline discretized into 200 sections. The Darcy-Weisbach formula with friction factor f =0.015 and the weighting function based unsteady friction model by Vardy and Brown (1996) are used for representing the steady and unsteady components of wall shear during the transient events. The time domain head traces collected at the upstream face of the valve are converted into frequency response functions using the technique described in Lee et al. (2006 and 2008). The frequencies of the resonant peaks are then extracted from the resultant spectrum, normalized by the fundamental frequency of the original uniform pipe and plotted in Figs. 4 and 5 for all cases. Note that Fig. 4 shows the impact of small blockage severities (with ε_A<0.5) whereas Fig. 5 shows the impact of large blockage severities (ε_A>0.5). The results are labeled as "numerical - MOC" in the figures and they display shifting patterns for the resonant frequencies that are consistent with the periodic form of Eq. (19).

To examine the validity of Eq. (19), the extended blockage properties are determined by inversely fitting the observed resonant peak shifts with the analytical form of Eq. (19). The fitted result for each case is plotted and labeled as "predicted - Eq. (19)". The relative error of the estimated blockage parameters is calculated for each case and listed in Table 2. The relative error of prediction in Table 2, γ, is defined as the difference between the predicted result and the real value normalized by the real value.

The results from Table 2 show that Eq. (19) can be used in an inverse procedure to accurately locate and size the blockage in the pipeline for a wide range of ε_A, with the maximum prediction errors for the blockage location, length and severity at 0.3%, 2.3% and 4.9% respectively. It is also important to note that the computation efficiency has been increased in this procedure using Eq. (19) compared to original Eq. (2). The usage of Eq. (19) only requires 10% of the computation time needed when Eq. (2) is used and yet Figs. 4 and 5 demonstrate that the Eq. (19) can correctly reproduce the patterns of frequency shift.

The relationship between the amplitude of the frequency shift patterns and the severity of the blockage constriction, ε_A, is plotted in Fig. 6 for values of ε_A from mild blockages (e.g.,

~0.0) to severe blockages (e.g., ~1.0). The results show that the shift pattern amplitudes of all six cases are monotonously increasing with the value of ε_A and is consistent with Eq. (19) where the maximum frequency shift size (i.e., $\max\left(\left|\frac{\Delta\omega_{rf}}{\omega_{th0}}\right|\right)$) is increasing with a factor of $\frac{\varepsilon_A}{2-\varepsilon_A}$.

Another impact from an extended blockage on the pressure response is the change in the pipe wave celerity within the blockage section. The variation of wavespeed in water piping system from pipe deterioration has been studied in the field and up to 37% reduction in wavespeed is observed in extreme cases (Stephens et al., 2005). From this point of view, the ε_a values for most practical cases are expected to be significantly smaller than 1 (i.e., ε_a<<1) and the impact of a large wavespeed variation is studied to test the accuracy of Eq. (19) under extreme conditions. Two different cases (cases *N*2 and *N*5 in Table 1) are used for this illustration. The wavespeed of the blockage section for case *N*2 is changed by 20% and 50% for case *N*5. Other parameters of the blockage are kept the same as in Table 1. The size of the wavespeed change covers the range of extreme cases presented in the literature (Stephens et al., 2005, Stephens, 2008, Hachem et al., 2011, Hachem et al., 2012a and Hachem et al., 2012b). The results are shown in Fig. 7. Compared to the previous results with $\varepsilon_a = 0$, Fig. 7 shows that despite a small difference in the magnitude of the shifting pattern, the variation of ε_a value (from 0 to 0.5) caused by the extended blockage has little influence on the accuracy of Eq. (19). This result also validates the first order simplification with regard to ε_a in deriving Eq. (19).

6. Experimental Verification and Discussion

The experimental data used for the verification of Eq. (19) are retrieved from Duan et al. (2013) and Tuck et al. (2012, 2013). Details of the experimental system information are shown in Table

3 and in total 6 experimental tests are considered, labeled as cases no. *E*1 to *E*6. Detailed descriptions of the experimental system and measurement procedures can be found in the original publications. The results of the resonant frequency shifts for cases *E*1 to *E*3 are shown in Fig. 8 and cases *E*4 to *E*6 are shown in Fig. 9.

The predicted results from Eq. (19) are also plotted for comparison. The results show that Eq. (19) can accurately capture the phases of the frequency shift patterns but significant errors exist for capturing the amplitudes of some resonant peaks. This error is due to the violation of the linear approximation by the large blockage severities used in these experimental cases (e.g., ε_A = 0.91 and 0.56) and the inability of the current models for capturing the frequency dependent behaviors at higher modes.

Table 4 shows the accuracy for predicting the location and length of an extended blockage (l_1~l_3) is higher than the accuracy for predicting the constriction severity (ε_A). The maximum errors in predicting the blockage lengths and locations are 5.8% and 7.9%, while this value increases for blockage severity to 22.6%. In other words, the prediction of blockage severity is easily affected by the model uncertainties and experimental errors. Moreover, the prediction accuracy was found to decrease with the blockage length and is likely caused by the highly unsteady and multiple dimensional turbulence effect at the blockage junctions. These effects, which are currently approximated as quasi-steady in 1-D models, play a more significant role as the blockage length decreases (Zhao et al., 2013).

The experimental tests also show that a high number of resonant frequency peaks is required for the application of the proposed method. For example, to obtain the frequency shift pattern for these experimental cases, at least 6 peak points have to be retrieved from the frequency response data (e.g., case *E*2). The number of resonant peaks observed in the data is a function of the speed of the original valve operation and the results indicate that rapid maneuvers, with broad frequency content is most appropriate for the application of the blockage detection technique.

7. Conclusions

This paper investigates the effect of transient wave-blockage interaction on the system frequency responses and confirms the possibility of using the changes (shifts) of resonant frequencies for extended blockage detection in pipes. Particularly, the wave-blockage interaction process and physical insights for resonant frequency shifting in blockage pipe system are inspected using wave perturbation analysis in the paper. Further analytical analysis is conducted in this study to produce a simplified relationship between the blockage parameters and the nature of the shifting pattern. Both numerical and experimental tests with a wide range of blockage constriction severities are used to validate the analytical results. These numerical and experimental tests confirm the ability of the proposed method to be used to detect extended blockages provided a sufficient number of resonant responses are obtained from the test data.

It is also necessary to point out that the study is conducted on a single extended blockage of uniform characteristics. While the findings of this study are validated by numerical tests and preliminary laboratory experiments in this paper, more verification work will be required in the future to identify practical issues associated with the application of this technique in the field, such as the problem of data measurement and pre-processing, the detection of multiple extended blockages and the impact of other system complexities.

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Appendix – Analytical Derivation for Eq. (19)

By applying Eq. (13) and Eq. (14), Eq. (2) can be rewritten as,

$$\begin{aligned}&(2+\varepsilon_Y)(2+\varepsilon_Y)\cos\left[(1+\varepsilon_\lambda)\lambda_0\omega_{rfb}\right]+\varepsilon_Y(2+\varepsilon_Y)\cos\left[\left(1+\varepsilon_\lambda-2\frac{\lambda_1}{\lambda_0}\right)\lambda_0\omega_{rfb}\right]\\&-\varepsilon_Y(2+\varepsilon_Y)\cos\left[\left(1+\varepsilon_\lambda-2\frac{\lambda_3}{\lambda_0}\right)\lambda_0\omega_{rfb}\right]-(\varepsilon_Y)^2\cos\left[\left(1+\varepsilon_\lambda-2\frac{\lambda_2}{\lambda_0}\right)\lambda_0\omega_{rfb}\right]=0\end{aligned}, \quad \text{(A1)}$$

where $\omega_{rfb}=\omega_{rf0}+\Delta\omega_{rf}$ with $\omega_{rfb},\omega_{rf0}$ = resonant frequency of the blocked section and the uniform pipeline respectively, $\Delta\omega_{rf}$ = magnitude of the resonant frequency shift between the uniform and blocked cases, ε_Y, ε_λ = extended blockage induced variations of characteristic impedance and wave propagation coefficients in the pipeline.

By considering the first order approximation using the Taylor series expansion, the co-sinusoidal functions in Eq. (A1) can be expanded about ω_{rf0} in general form as,

$$\cos[\alpha\omega_{rfb}]=\cos[\alpha\omega_{rf0}]-\alpha\sin[\alpha\omega_{rf0}]\Delta\omega_{rf}+O\left[(\Delta\omega_{rf})^2\right]+\cdots, \quad \text{(A2)}$$

where α = coefficient of the resonant frequency terms in the co-sinusoidal functions of Eq. (A1). Expanding the co-sinusoidal function related terms and substituting Eq. (14) through Eq. (16), and carrying out trigonometric transformations and rearrangements give,

$$\Delta\omega_{rf}=\frac{C_u}{C_d}, \quad \text{(A3)}$$

where C_u and C_d are coefficients relating to the extended blockage parameters, and

$$C_u = \left(2-\varepsilon_A-\varepsilon_a\right)^2 \sin\left[\left(\varepsilon_L - \frac{\varepsilon_L}{1-\varepsilon_a}\right)\lambda_0\omega_{rf0}\right] + \left(\varepsilon_A-\varepsilon_a\right)\left(2-\varepsilon_A-\varepsilon_a\right)\sin\left[\left(\varepsilon_L - \frac{\varepsilon_L}{1-\varepsilon_a}+\frac{2\lambda_1}{\lambda_0}\right)\lambda_0\omega_{rf0}\right]$$
$$-\left(\varepsilon_A-\varepsilon_a\right)\left(2-\varepsilon_A-\varepsilon_a\right)\sin\left[\left(\varepsilon_L - \frac{\varepsilon_L}{1-\varepsilon_a}+\frac{2\lambda_3}{\lambda_0}\right)\lambda_0\omega_{rf0}\right] - \left(\varepsilon_A-\varepsilon_a\right)^2 \sin\left[\left(3\varepsilon_L - \frac{\varepsilon_L}{1-\varepsilon_a}\right)\lambda_0\omega_{rf0}\right],$$

$$C_d = \lambda_0 \begin{bmatrix} \left(2-\varepsilon_A-\varepsilon_a\right)^2\left(1-\varepsilon_L+\frac{\varepsilon_L}{1-\varepsilon_a}\right)\cos\left[\left(\varepsilon_L-\frac{\varepsilon_L}{1-\varepsilon_a}\right)\lambda_0\omega_{rf0}\right] \\ +\left(\varepsilon_A-\varepsilon_a\right)\left(2-\varepsilon_A-\varepsilon_a\right)\left[1-\varepsilon_L+\frac{\varepsilon_L}{1-\varepsilon_a}-\frac{2\lambda_1}{\lambda_0}\right]\cos\left[\left(\varepsilon_L-\frac{\varepsilon_L}{1-\varepsilon_a}+\frac{2\lambda_1}{\lambda_0}\right)\lambda_0\omega_{rf0}\right] \\ -\left(\varepsilon_A-\varepsilon_a\right)\left(2-\varepsilon_A-\varepsilon_a\right)\left(1-\varepsilon_L+\frac{\varepsilon_L}{1-\varepsilon_a}-\frac{2\lambda_3}{\lambda_0}\right)\cos\left[\left(\varepsilon_L-\frac{\varepsilon_L}{1-\varepsilon_a}+\frac{2\lambda_3}{\lambda_0}\right)\lambda_0\omega_{rf0}\right] \\ -\left(\varepsilon_A-\varepsilon_a\right)^2\left(3\varepsilon_L-\frac{\varepsilon_L}{1-\varepsilon_a}\right)\cos\left[\left(3\varepsilon_L-\frac{\varepsilon_L}{1-\varepsilon_a}\right)\lambda_0\omega_{rf0}\right] \end{bmatrix}. \quad \text{(A4)}$$

Published literature has shown that the variation of wavespeed (ε_a) imposed by an extended blockage in most practical pipelines is usually much smaller than 1. Therefore, the coefficients of C_u and C_d in Eq. (A4) can be simplified by assuming ε_a <<1. By considering ε_a <<1 in Eq. (4A), C_u and C_d becomes,

$$C_u = \varepsilon_A\left(2-\varepsilon_A\right)\sin\left(2\lambda_1\omega_{rf0}\right) - \varepsilon_A\left(2-\varepsilon_A\right)\sin\left(2\lambda_3\omega_{rf0}\right) - \left(\varepsilon_A\right)^2\sin\left(2\varepsilon_L\lambda_0\omega_{rf0}\right),$$

$$C_d = \left(2-\varepsilon_A\right)^2 + \varepsilon_A\left(2-\varepsilon_A\right)\left[\lambda_0-2\lambda_1\right]\cos\left(2\lambda_1\omega_{rf0}\right) - \varepsilon_A\left(2-\varepsilon_A\right)\left(\lambda_0-2\lambda_3\right)\cos\left(2\lambda_3\omega_{rf0}\right)$$
$$-2\varepsilon_L\left(\varepsilon_A\right)^2\cos\left(2\varepsilon_L\lambda_0\omega_{rf0}\right). \quad \text{(A5)}$$

Result of Eq. (15) shows that the frequency shift size ($\Delta\omega_{rf}$) is dependent on the blockage constriction severity (ε_A), blockage length (ε_L) and blockage location (λ). Furthermore, since ε_A<1, λ_1<λ_0 and λ_3 <λ_0, C_d in Eq. (A5) can be approximated as,

$$C_d = \lambda_0\left(2-\varepsilon_A\right)^2. \quad \text{(A6)}$$

Therefore, the resultant frequency shift size in dimensionless form (i.e., normalized by the fundamental frequency of blockage free case, ω_{th0}) becomes,

$$\frac{\Delta\omega_{rf}}{\omega_{th0}} \approx \frac{2}{\pi}\frac{\varepsilon_A}{2-\varepsilon_A}\left[\sin(2\lambda_1\omega_{rf0}) - \sin(2\lambda_3\omega_{rf0}) - \frac{\varepsilon_A}{2-\varepsilon_A}\sin(2\varepsilon_L\lambda_0\omega_{rf0})\right]. \tag{A7}$$

Or in terms of the resonant peak number (m) as,

$$\begin{aligned}\frac{\Delta\omega_{rf}}{\omega_{th0}} &\approx \frac{2}{\pi}\frac{\varepsilon_A}{2-\varepsilon_A}\left[\sin[(4m-2)\lambda_1\omega_{th0}] - \sin[(4m-2)\lambda_3\omega_{th0}] - \frac{\varepsilon_A}{2-\varepsilon_A}\sin[(4m-2)\varepsilon_L\lambda_0\omega_{th0}]\right] \\ &= (-1)^{k+1}\frac{4}{\pi}\frac{\varepsilon_A}{2-\varepsilon_A}\sin[(2m-1)\varepsilon_L\lambda_0\omega_{th0}]\left(\sin[(2m-1)(\lambda_1-\lambda_3)\omega_{th0}] - \frac{\varepsilon_A}{2-\varepsilon_A}\cos[(2m-1)\varepsilon_L\lambda_0\omega_{th0}]\right)\end{aligned}$$

.

(A8)

Notations

The following symbols have been used in the paper:

a = wavespeed;

A = pipe cross-sectional area;

C^* = shear decay coefficient in unsteady friction model;

C_n = amplitude of boundary wave;

C_p = amplitude of incident pressure wave;

D = pipe diameter;

f = friction factor;

g = gravitational acceleration;

i = imaginary unit for complex number;

I = amplitude of incident wave;

k = wave number;

L = pipe length;

m = integer number;

P = pressure head;

Q = discharge;

R = amplitude of reflection wave;

R_f = friction damping factor;

R_e = Reynolds number;

t = time;

x = longitudinal distance;

Y = characteristic impedance of pipeline;

γ = relative error of prediction;

δ = operator of variation;

ε_a = change of wavespeed;

ε_A = change of pipe cross-sectional area;

ε_L = longitudinal blockage range;

ε_Y = change of characteristic impedance;

ε_λ = change of wave propagation coefficient;

θ = wave number;

λ = wave propagation coefficient;

ν = kinematic viscosity;

ξ = coefficient relating to the blockage size;

ρ = density of fluid;

ω_{rf} = resonant frequency of pipe transients;

ω_{th} = theoretical frequency of a pipe system;

$\Delta\omega_{rf}$ = shift size of resonant peak frequency;

Subscripts and Superscripts

$_b$ = quantity of blockage pipe case;

$_{S,U}$ = quantity of steady and unsteady state;

$_0$ = quantity of uniform pipe case;

$_{1,2,3}$ = section indexes of blockage pipeline.

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