

# Investigation of interferometric noise in fiber-optic gas sensors with use of wavelength modulation spectroscopy

W. Jin, Y. Z. Xu, M. S. Demokan, and G. Stewart

We report on interferometric noise limitation of fiber-optic gas sensors with highly coherent lasers and wavelength modulation spectroscopy. Interference between signal wave and reflected waves causes signal fluctuation in the output, which limits the performance of the sensing system. Sensor resolution limited by interferometric noise is calculated for a fiber-optic gas sensor with the  $Q(6)$  absorption line of methane gas at approximately 1650 nm. The results are useful for system designers of this particular type of gas sensor. © 1997 Optical Society of America

*Key words:* Fiber-optic sensors, gas sensors, interferometric noise.

## 1. Introduction

Optical gas sensors based on absorption of light by the vibrational-rotational energy levels of gas molecules at near-IR wavelength (1–1.8  $\mu\text{m}$ ) have attracted considerable attention during recent years.<sup>1–5</sup> The advantages of fiber sensors include remote detection capability, safety in hazardous environments, immunity to electromagnetic fields, and so forth. The possible gases that can be detected include methane, acetylene, hydrogen sulfide, carbon dioxide, and carbon monoxide. Like other types of fiber sensors, the performance of gas sensors is limited by various kinds of noise, e.g., source noise, shot noise, and thermal noise. We already reported the results of an investigation on the effect of source, shot, and thermal noises on the performance of a gas sensor that used a low-coherence optical source.<sup>6</sup>

Gas sensors that make use of highly coherent sources like distributed feedback (DFB) lasers and fiber lasers are advantageous for obtaining high sensitivity. For this type, however, interferometric

noise caused by interference between signal waves and reflected waves might be larger than the source and detector noise and might set a limit on the sensor performance.<sup>7,8</sup> Investigation on the interferometric noise in a fiber gas sensor that makes use of differential absorption spectroscopy is reported in a previous paper.<sup>8</sup> However, interferometric noise in a fiber gas sensor based on wavelength modulation spectroscopy, which is more popular than differential absorption spectroscopy, has not been reported yet to our knowledge.

Noise analysis for such a system is considerably more complicated than that for the differential spectroscopy technique because it involves continuous dynamic modulation of the wavelength and intensity and the detection of harmonic signals. However, such an investigation is necessary for understanding the difference between the wavelength modulation spectroscopy and the conventional differential absorption spectroscopy and for accurately estimating the noise performance of the gas sensors based on wavelength modulation spectroscopy. We report on the results of such an investigation: the magnitude of the interferometric noise caused by coherent reflections is estimated, and the results of our investigation can be used to design and to estimate the expected performance of this particular type of fiber-optic gas sensor.

## 2. Wavelength Modulation Spectroscopy Based on Current Modulation of a Distributed Feedback Laser

Figure 1 shows a diagram of a gas sensor with a DFB-type laser. The frequency (wavelength) of the

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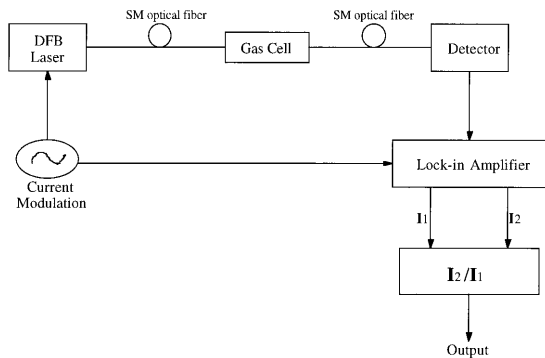


Fig. 1. Principle of wavelength modulation spectroscopy:  $I_1$  and  $I_2$ , amplitudes of the first- and the second-harmonic signals SM, single mode.

laser is modulated sinusoidally through the modulation of the injection current, while its average wavelength is locked at the center of the gas absorption line with a separated reference cell and feedback control electronics.<sup>2,3,5,7</sup> The modulated light signal from the source passes through a gas cell that contains a gas sample to be measured and is subsequently converted to an electric current through a photodiode. A lock-in amplifier is used to detect both the first- and the second-harmonic signals and either the second-harmonic signal or the ratio of the two harmonic signals may be used as a system output.

Although wavelength modulation spectroscopy and the ratio-detection technique have been used for fiber gas sensors for a considerable length of time, no detailed analysis on the relation between the system output and the key system parameters has been reported yet to our knowledge, especially when measurement errors are involved. In this section, we report on such an analysis and derive a relation between the measurement error in gas concentration and the errors in the measurement of the first and the second harmonics. The results obtained in this section are used to evaluate the influence of interferometric noise on the gas concentration measurement in later sections.

If we assume that the DFB laser line width is much narrower than the line width of the gas absorption line and that the laser driving current is modulated sinusoidally, i.e.,

$$i(t) = i_0 + i_m \sin \omega t, \quad (1)$$

the frequency and the intensity of the output light from the DFB laser may be expressed as

$$\nu = \nu_{L0} + \nu_{Lm} \sin \omega t, \quad (2)$$

$$I_i(t) = I_0[1 + \eta \sin \omega t], \quad (3)$$

where  $I_0$  and  $\nu_{L0}(=c/\lambda_{L0})$  represent, respectively, the average laser output power and the average laser frequency.  $\nu_{Lm}$  is the amplitude of the frequency modulation and  $\eta$  is an intensity modulation index. In  $\omega = 2\pi f$ ,  $f$  is the frequency of current modulation. For fiber gas sensor applications, the value of  $f$  is usually of the order of a few tens of kilohertz.<sup>2,3,5,7</sup>

When the modulated laser light is passing through a gas sample with an (amplitude) absorption coefficient  $\alpha(\nu)$ , the output light intensity may be written as

$$I(t) = I_0(1 + \eta \sin \omega t) \exp[-2\alpha(\nu_{L0} + \nu_{Lm} \sin \omega t)CL], \quad (4)$$

where  $C$  represents gas concentration and  $L$  is an interaction length that equals the length of the gas cell for a transmission-type sensor shown in Fig. 1.

In practical applications, we are most interested in the measurement of small gas concentrations, and the residual intensity modulation is usually very small also. Considering these facts, we may assume that  $2\alpha(\nu)CL \ll 1$  and  $\eta \ll 1$ .  $I(t)$  may be approximated as

$$I(t) \approx I_0[1 + \eta \sin \omega t - 2\alpha(\nu_{L0} + \nu_{Lm} \sin \omega t)CL], \quad (5)$$

where we have used the approximation

$$\exp[-2\alpha(\nu)CL] \approx 1 - 2\alpha(\nu)CL \quad (6)$$

and have neglected the higher-order term

$$\eta \sin \omega t \times 2\alpha(\nu_{L0} + \nu_{Lm} \sin \omega t)CL. \quad (7)$$

Under atmospheric pressure, the gas absorption line is collision broadened and the line shape is given by Lorentzian distribution, i.e.,

$$\alpha(\nu) = \frac{\alpha_0}{1 + \left(\frac{\nu - \nu_g}{\delta\nu}\right)^2}, \quad (8)$$

where  $\alpha_0$  is the absorption coefficient for pure gas at the center of the absorption line, and  $\nu_g$  and  $\delta\nu$  are the center frequency and the half-width of the absorption line. Equation (5) can be rewritten as

$$I(t) = I_0 \left[ 1 + \eta \sin \omega t - \frac{2\alpha_0 CL}{1 + \left(\frac{\nu_{L0} - \nu_g + \nu_{Lm} \sin \omega t}{\delta\nu}\right)^2} \right]. \quad (9)$$

Because the average wavelength of the laser is locked to the center of the gas line (i.e.,  $\nu_{L0} = \nu_g$ ) by the use of a reference loop and feedback control,<sup>2,3,5,7</sup>  $I(t)$  may be written as

$$I(t) = I_0 \left[ 1 + \eta \sin \omega t - \frac{2\alpha_0 CL}{1 + x^2 \sin^2 \omega t} \right], \quad (10)$$

where we defined that  $x = \nu_{Lm}/\delta\nu$ .

$I(t)$  as expressed in Eq. (10) can be expanded into a Fourier series with the magnitudes of the first and second harmonic expressed as

$$I_1 = I_0\eta, \quad (11)$$

$$I_2 = -2k\alpha_0 CLI_0, \quad (12)$$

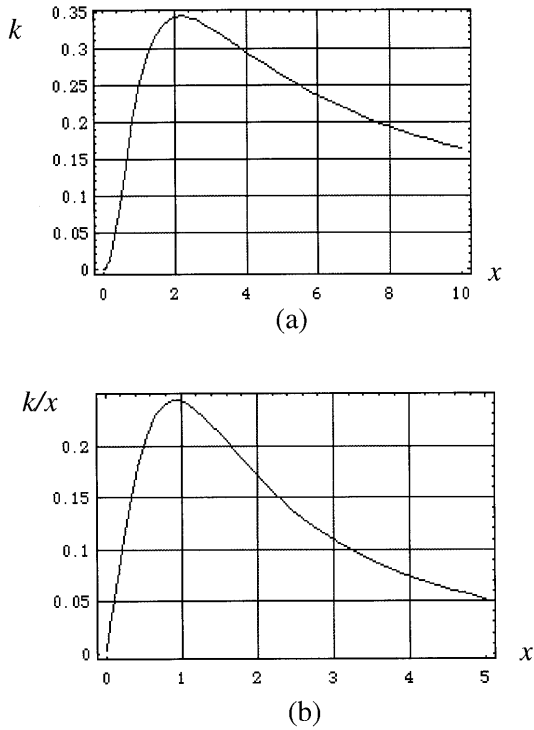


Fig. 2. Scale factor versus modulation index,  $x = v_{Lm}/\delta v$ : (a) for the second-harmonic detection technique,  $k$  versus  $x$ ; (b) for the ratio-detection technique,  $k/x$  versus  $x$ .

with

$$k = \frac{2[2 + x^2 - 2(1 + x^2)^{1/2}]}{x^2(1 + x^2)^{1/2}}. \quad (13)$$

$k$  as a function  $x$  is shown in Fig. 2(a). The value of  $x$  can be tuned to maximize the second-harmonic signal. The maximum occurs when  $dk/dx = 0$ , which gives a value of

$$x = (2 + 2\sqrt{2})^{1/2} \approx 2.2. \quad (14)$$

Under this optimal condition,  $k = 0.343$ . In most reported gas sensor systems, the value of  $x$  is limited to be less than 2,<sup>2,3,7</sup> and the system output is the ratio of the amplitude of the first harmonic and the second harmonic and may be expressed as

$$\frac{I_2}{I_1} = -\frac{2k}{\eta} \alpha_0 CL. \quad (15)$$

The ratio-detection method eliminates intensity fluctuation resulting from factors other than gas absorption. As  $\eta$  is proportional to injection current modulation  $i_m$ , which is proportional to  $x$ , we have  $\eta = \gamma x$ . Therefore the scale factor for the ratio-detection technique becomes  $2\alpha_0 CL/\gamma \cdot (k/x)$ .  $k/x$  as a function of  $x$  is plotted in Fig. 2(b). The scale factor is maximized at  $x = 0.93$ , which is smaller for the case of a pure second-harmonic detection scheme. In the following, we discuss mainly the ratio-detection technique.

If there are errors in the measurement of  $I_1$  and  $I_2$ , i.e.,  $I_{1m} = I_1 + \Delta I_1$  and  $I_{2m} = I_2 + \Delta I_2$ , or  $I_{1m} = I_1(1 + \delta_1)$  and  $I_{2m} = I_2(1 + \delta_2)$  [here  $\delta_1 (= \Delta I_1/I_1)$  and  $\delta_2 (= \Delta I_2/I_2)$  represent relative measurement errors], the measured value of gas concentration  $C_m$  will be different from the real value of gas concentration  $C$ .  $C_m$  and  $C$  may be related by the following equation:

$$\begin{aligned} \frac{I_{2m}}{I_{1m}} &= -\frac{2k}{\eta} \alpha_0 C_m L \\ &= \frac{I_2(1 + \delta_2)}{I_1(1 + \delta_1)} \approx -\frac{2k}{\eta} \alpha_0 CL(1 + \delta_2 - \delta_1), \end{aligned} \quad (16)$$

where  $m$  represents the measured values and we assume that  $\delta_1$  and  $\delta_2$  are small, so that

$$\frac{1 + \delta_2}{1 + \delta_1} \approx 1 + \delta_2 - \delta_1. \quad (17)$$

Rearranging Eq. (16), we have

$$\frac{C_m - C}{C} \approx \delta_2 - \delta_1. \quad (18)$$

The relative measurement error in gas concentration can then be expressed as

$$\frac{|\Delta C|}{C} = \frac{|C - C_m|}{C} = |\delta_2 - \delta_1|. \quad (19)$$

This formula is significantly different from that for the differential absorption techniques as reported in Ref. 8. Relative measurement errors  $\delta_1$  and  $\delta_2$  may result from various noise sources such as shot noise and thermal noise in the receiver and source noise in the laser, for example. In the following, we investigate only the influence of interferometric noise, which as we mentioned above is usually much larger than other noise sources.

### 3. Interferometric Noise in Wavelength Modulation Spectroscopy Sensors

To study interferometric noise, we need to use the electric field representation of laser light. Considering a DFB laser with sinusoidal current modulation, we can write its output electric field as

$$\begin{aligned} E_i(t) &= [I_0(1 + \eta \sin \omega t)]^{1/2} \\ &\times \exp \left[ j2\pi(v_{LO}t + v_{Lm} \int_0^t \sin \omega u du) \right]. \end{aligned} \quad (20)$$

When passing through the gas cell, the signal wave at the detector may be expressed as

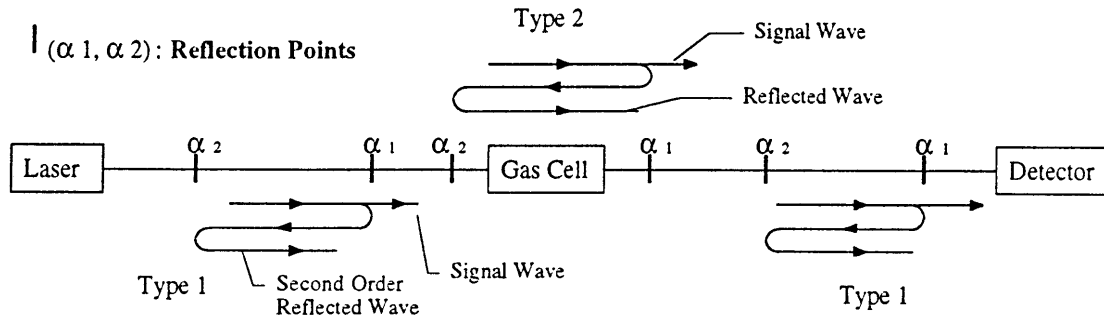


Fig. 3. Second-order reflection pairs in a transmission-type sensor.

$$\begin{aligned}
 E(t) = & [I_0(1 + \eta \sin \omega t)]^{1/2} \\
 & \times \exp[-\alpha(\nu_{LO} + \nu_{Lm} \sin \omega t)CL] \\
 & \times \exp\left[j2\pi\left(\nu_{LO}t + \nu_{Lm} \int_0^t \sin \omega u du\right) \right. \\
 & \left. + \phi(\nu_{LO} + \nu_{Lm} \sin \omega t)\right], \quad (21)
 \end{aligned}$$

where  $\phi(\nu)$  represents a modulation term resulting from changes of refractive index of the gas sample as a function of wavelength. The magnitude of  $\phi(\nu)$  increases with gas concentration and interaction length. The signal intensity can be calculated with  $I(t) = \langle |E(t)|^2 \rangle$  and is given by Eq. (4).

Apart from the signal wave, however, there may be reflected waves in the system. Reflections can occur at fiber connectors, fiber-cell joints (cell surfaces), and so forth. For a sensor configuration as shown in Fig. 3, while the first-order reflection is directly fed back into the source, we may assume reflected waves have no effect on system performance when a proper isolator is used at the laser output port. The second-order reflection (reflection caused by a pair of reflective points along the fiber, first backward then forward; see Fig. 3), however, can reach the photo detector and may affect the system performance. In the output of the system, in addition to a primary beam, many second-order waves resulting from second-order reflections may exist. For simplicity, we divide the reflections into two types as shown in Fig. 3.

#### A. Reflection Pairs Before or After the Cell (Type I)

For type I reflections, the reflected wave passes through the gas sample once (the same as the signal wave), the reflected wave may be written as

$$\begin{aligned}
 E_r(t) = & \alpha_1\alpha_2[I_0(1 + \eta \sin \omega(t - \tau))]^{1/2} \\
 & \times \exp\{-\alpha[\nu_{LO} + \nu_{Lm} \sin \omega(t - \tau)]CL\} \\
 & \times \exp\left\{j2\pi\left[\nu_{LO}(t - \tau) + \nu_{Lm} \int_0^{t-\tau} \sin \omega u du\right] \right. \\
 & \left. + \phi[\nu_{LO} + \nu_{Lm} \sin \omega(t - \tau)]\right\}, \quad (22)
 \end{aligned}$$

where  $\alpha_1$  and  $\alpha_2$  represent the amplitude reflection coefficients at two points as shown in Fig. 3 and  $\tau$  is a time delay between the primary wave and the reflected wave. The total light intensity may be written as

$$I(t) = \langle |E(t) + E_r(t)|^2 \rangle. \quad (23)$$

The total light intensity at the output detector,  $I(t)$ , may be divided into three parts: intensity of the signal wave  $\langle |E(t)|^2 \rangle$ , intensity of the reflected wave  $\langle |E_r(t)|^2 \rangle$ , and the mixing term between the signal wave and the reflected wave. The intensity of the reflected wave is of a higher order compared with the other two terms and may be neglected. We consider only two terms: signal term and the mixing term (noise term), which induces errors in the measurement. The signal intensity is given by Eq. (4) and the noise intensity may be expressed as

$$\begin{aligned}
 I_n(t) = & 2\text{Re}\langle E(t)E_r^*(t) \rangle \\
 = & 2\alpha_1\alpha_2I_0\{[1 + \eta \sin \omega(t - \tau)](1 + \eta \sin \omega t)\}^{1/2} \\
 & \times \exp(-\{\alpha[\nu_{LO} + \nu_{Lm} \sin \omega(t - \tau)] \\
 & + \alpha(\nu_{LO} + \nu_{Lm} \sin \omega t)\}CL) \cos\left\{2\pi\left(\nu_{LO}\tau + \nu_{Lm} \right. \right. \\
 & \left. \left. \times \int_{t-\tau}^t \sin \omega u du\right) + \phi(\nu_{LO} + \nu_{Lm} \sin \omega t) \right. \\
 & \left. - \phi[\nu_{LO} + \nu_{Lm} \sin \omega(t - \tau)]\right\}. \quad (24)
 \end{aligned}$$

Theoretically,  $I_n(t)$  and its harmonics can be calculated if  $\phi(\nu)$  is known. The process however, is a very complicated one. As we show in the following, for most practical applications the calculation can be simplified.

It is obvious that not all the reflected waves can interfere with the signal wave; only those that travel approximately the same distance with the signal wave (within a coherence length of the source) interfere with the signal wave and contribute significantly to the system noise. We can therefore use the following assumption: If the optical path difference between the signal and the reflected waves is less than the coherent length of the source, we regard them as totally coherent; if the optical path difference be-

tween them is larger than the coherent length of the source, the reflected waves will not interfere with the signal wave and will not introduce interferometric noise to the sensing system.

With the above assumption, the maximum distance between the two reflection points that may contribute to the interferometric noise is half the coherence length of the source. The implication is that the maximum time delay between two reflection points that contribute to interferometric noise is the coherence time of the source. For DFB lasers used for gas sensors, the typical line width is approximately 50 MHz, which gives a coherence time of 20 ns, or a coherence length of 4 m. For most gas sensors, the modulation frequency of the current modulation is in a range of a few tens of kilohertz,<sup>2,3,5,7</sup> the value of  $\omega\tau(2\pi/\tau)$  should therefore be much less than 1 ( $\omega\tau \ll 1$ ). Based on these facts, we may use the approximation

$$\{[1 + \eta \sin \omega(t - \tau)](1 + \eta \sin \omega t)\}^{1/2} \approx 1 + \eta \sin \omega t, \quad (25)$$

and for small gas concentration, i.e.,  $\alpha(v)CL \ll 1$ , we may use

$$\exp(-\{\alpha[v_{LO} + v_{Lm} \sin \omega(t - \tau)] + \alpha(v_{LO} + v_{Lm} \sin \omega t)\}CL) \approx 1, \quad (26)$$

and  $I_n(t)$  may be approximated as

$$I_n(t) \approx 2\alpha_1\alpha_2 I_0(1 + \eta \sin \omega t) \cos \left[ \psi + \zeta \sin \omega \left( t - \frac{\tau}{2} \right) \right]. \quad (27)$$

In the derivation of Eq. (27), we used

$$\psi = 2\pi v_{LO}\tau, \quad (28)$$

$$2\pi v_{Lm} \int_{t-\tau}^t \sin \omega u du = \frac{4\pi v_{Lm}}{\omega} \sin \left( \frac{\omega\tau}{2} \right) \sin \omega \left( t - \frac{\tau}{2} \right), \quad (29)$$

and we defined a phase modulation index

$$\zeta = \frac{4\pi v_{Lm}}{\omega} \sin \frac{\omega\tau}{2} \approx \frac{4\pi v_{Lm}}{\omega} \frac{\omega\tau}{2} = 2\pi v_{Lm}\tau. \quad (30)$$

We also neglected the phase modulation term

$$\phi(v_{LO} + v_{Lm} \sin \omega t) - \phi[v_{LO} + v_{Lm} \sin \omega(t - \tau)], \quad (31)$$

because it is very small (less than  $\alpha_0 CL$ ) compared with the other two terms (see Appendix A). Using

$$\begin{aligned} & \cos[\psi + \zeta \sin \omega(t - \tau/2)] \\ &= \cos \psi [J_0(\zeta) + 2J_2(\zeta) \cos 2\omega(t - \tau/2) \\ & \quad + 2J_4(\zeta) \cos 4\omega(t - \tau/2) + \dots] \\ & \quad - \sin \psi [2J_1(\zeta) \sin \omega(t - \tau/2) + 2J_3(\zeta) \\ & \quad \times \sin 3\omega(t - \tau/2) + \dots], \end{aligned} \quad (32)$$

and considering that  $\omega\tau \ll 1$ , we may approximate the first and the second harmonics of  $I_n(t)$  as

$$I_{1n}(t) \approx -2\alpha_1\alpha_2 I_0 \{ 2 \sin \psi J_1(\zeta) - \eta \times \cos \psi [J_0(\zeta) - J_2(\zeta)] \} \sin \omega t, \quad (33)$$

$$I_{2n}(t) \approx -2\alpha_1\alpha_2 I_0 \{ -2 \cos \psi J_2(\zeta) \sin 2\omega t - \eta \times \sin \psi [J_1(\zeta) - J_3(\zeta)] \} \cos 2\omega t. \quad (34)$$

The noise terms in approximations (33) and (34) may be divided into two classes: intensity variation resulting from direct conversion from laser frequency modulation with interferometric process [the first terms in approximations (33) and (34)] and a noise term that is related to residual intensity modulation [the second terms in approximations (33) and (34)].

Considering the fact that  $\eta \ll 1$ , the second term is very small compared with the first term, and approximations (33) and (34) may be further approximated as

$$I_{1n} \approx -4\alpha_1\alpha_2 I_0 \sin \psi J_1(\zeta), \quad (35)$$

$$I_{2n} \approx 4\alpha_1\alpha_2 I_0 \cos \psi J_2(\zeta). \quad (36)$$

The relative measurement error may then be calculated as

$$\delta_1 = \frac{I_{1n}}{I_1} = -\frac{4\alpha_1\alpha_2 \sin \psi J_1(\zeta)}{\eta}, \quad (37)$$

$$\delta_2 = \frac{I_{2n}}{I_2} = -\frac{4\alpha_1\alpha_2 \cos \psi J_2(\zeta)}{2k\alpha_0 CL}. \quad (38)$$

Equations (37) and (38) show that the measurement errors  $\delta_1$  and  $\delta_2$  depend on the reflection coefficients  $\alpha_1$  and  $\alpha_2$  and also on the phase modulation index  $\zeta$  that is given by Eq. (30). If the relative position of the reflection points is known (i.e., if  $\tau$  is known), either  $\delta_1$  and  $\delta_2$  can be minimized through the choice of proper amplitude of current modulation so that  $\zeta$  corresponds to one of the zeros of  $J_1(\zeta)$  or  $J_2(\zeta)$ . Note that the measurement error depends on the relative positions of the reflection points. Equation (30) holds when the two reflection points are situated so that the time delay ( $\tau$ ) between the reflected wave and the signal wave is less than the coherence time of the source. If the optical path difference between the two reflection points that form the pair is much longer than the coherence length of the source, it should have negligible contribution to the measurement errors.

With Eq. (19), and with the assumption  $\alpha_1 = \alpha_2$ , the relative measurement error may be written as

$$\frac{|\Delta C|}{C} = 4\alpha^2 \left| \frac{J_1(\zeta)}{\eta} \sin \psi - \frac{J_2(\zeta)}{2k\alpha_0 CL} \cos \psi \right|. \quad (39)$$

The first term on the right side of Eq. (39) is independent of gas concentration and introduces only a

small (of the order of  $\alpha^2$ ) relative error to the concentration measurement. The second term increases quickly as  $C$  decreases and sets a limit to the measurement of gas concentration. By considering only the second term and setting  $|\Delta C|/C = 1$ , we can obtain a minimum detectable gas concentration:

$$|C|_{\min,1} = 2\alpha^2 \frac{|J_2(\zeta)|}{k\alpha_0 L}. \quad (40)$$

For multiple pairs of reflection points, if we assume that the cross interference between the different secondary waves is small and can be neglected, the signal intensity remains approximately the same but the noise intensity is a summation of contributions from all the pairs of reflection points.

We can see that the  $|C|_{\min,1}$  has a  $J_2(\zeta)$  dependence and may be minimized by setting  $\zeta$  to be one of the zeros of  $J_2$ . In practical cases, however, there may be multiple reflection points and their positions may be unknown; we may then estimate the minimum detectable gas concentration with Eq. (40) and take the maximum possible value of  $J_2(\zeta)$ .

For one pair of reflection points with known positions, the magnitude of the modulation current may be adjusted so that  $J_2(\zeta) = 0$ . For this case, the  $|C|_{\min,1}$  calculated from Eq. (40) is zero and the estimation of the minimum detectable gas concentration should include the second terms in approximations (33) and (34). The relative measurement error under the condition of  $J_2(\zeta) = 0$  can be written as

$$\frac{|\Delta C|}{C} = 2\alpha^2 \left| 2 \sin \psi \frac{J_1(\zeta)}{\eta} - \cos \psi J_0(\zeta) - \eta \sin \psi \frac{J_1(\zeta) - J_3(\zeta)}{2k\alpha_0 CL} \right|. \quad (41)$$

Again, the first and the second terms on the right-hand side are independent of  $C$  and induce only a small relative error of the order of  $\alpha^2$ . The third term is inversely proportional to  $C$  and sets a limit to the measurement of gas concentration that can be obtained by setting  $|\Delta C|/C = 1$ , expressed as

$$|C|_{\min,2} = 2\alpha^2 \frac{|J_1(\zeta) - J_3(\zeta)|\eta}{2k\alpha_0 L}. \quad (42)$$

Because  $\eta$  is much smaller than 1, the contribution resulting from intensity modulation ( $|C|_{\min,2}$ ) is much smaller than that from the interferometric conversion of the frequency modulation ( $|C|_{\min,1}$ ) of the DFB laser.

#### B. Reflection Pairs Across the Cell (Type II)

For the case of a pair of reflection points, one after the cell and the other before the cell (see Fig. 3), the reflected wave passes through the gas cell two more

times than the signal wave. The reflected wave may be written as

$$E_r(t) = \alpha_1 \alpha_2 \{ I_0 [1 + \eta \sin \omega(t - \tau)] \}^{1/2} \times \exp\{-3\alpha[v_{LO} + v_{Lm} \sin \omega(t - \tau)]CL\} \times \exp\left\{ j2\pi \left[ v_{LO}(t - \tau) + v_{Lm} \times \int_0^{t-\tau} \sin \omega u du \right] + 3\phi[v_{LO} + v_{Lm} \sin \omega(t - \tau)] \right\}. \quad (43)$$

Under the condition that  $\alpha(v)CL \ll 1$ , the minimum detectable gas concentration can be calculated with a procedure the same as for the type I reflections and can be described again by Eqs. (40) and (42).

A particular example of type II reflection is the reflections from the surfaces of the gas cells:  $\alpha_1$  and  $\alpha_2$  are the reflection coefficients of the cell surfaces and  $\tau$  is the round-trip delay of the gas cell;  $\tau$  can be regarded as a constant and given by  $2L/c$ . We can minimize the minimum detectable gas concentration calculated by Eq. (40) by adjusting the magnitude of the current modulation so that  $\zeta = 2\pi v_{Lm} \tau$  is set to one of the zeros of  $J_2(\zeta)$ , and we can estimate the final limit of the gas detection by using Eq. (42).

Again, if there are multiple reflection points across the cell, the summation of contributions from all possible pairs should be considered. The total measurement error of the whole system is the summation of contributions from both the type I and the type II reflections.

#### 4. Performance Limit of Methane Gas Sensor by Interferometric Noise

In Subsections 3.A and 3.B we derived formulas for estimating sensor resolution as limited by coherent reflections. Now we estimate the practical achievable limit of a particular type of sensor with the ratio-detection technique. The performance limit can be estimated with Eq. (40) or Eq. (42) if system parameters such as  $\eta$ ,  $\alpha^2$ ,  $\alpha_0$ ,  $k$  and  $\zeta$  are known.

We consider a methane gas sensor based on absorption at a wavelength of 1665.5 nm [ $Q(6)$  line]. We assume that  $L = 10$  cm, which corresponds to a 10-cm-long gas cell. The absorption coefficient (at atmospheric pressure) at this wavelength is  $\alpha_0 \approx 0.1$   $\text{cm}^{-1}$ , giving  $\alpha_0 L = 1$ . The  $Q(6)$  line has a HWHM of approximately 2 GHz. The light source is assumed to be a DFB laser with a frequency tuning coefficient of  $-1$  GHz/1 mA and an intensity modulation coefficient of 5%  $I_0/\text{mA}$ .<sup>9</sup> This gives a value of  $\gamma = 0.1$  (or  $\eta \approx 0.1x$ ). For this particular sensor, Eq. (40) and Eq. (42) may be rewritten as

$$|C|_{\min,1} = 2\alpha^2 \frac{|J_2(\zeta)|}{k}, \quad (44)$$

$$|C|_{\min,2} = 0.1\alpha^2 \frac{|J_1(\zeta) - J_3(\zeta)|x}{k}. \quad (45)$$

We consider the following two cases:

(1)  $x = 0.93$ , which is the optimal modulation index for the ratio-detection technique. This gives a value of  $k = 0.25$ ,  $\nu_{Lm} \approx 1.90$  GHz, and  $\zeta = 2\pi\nu_{Lm}\tau \approx 3.8\pi \approx 12$  rad (the value of  $\tau$  for a 10-cm cell is approximately 1 ns). Around  $\zeta = 12$ , the maximum value of  $J_2$  may be estimated to be  $|J_n(\zeta)| \sim (2/\pi\zeta)^{1/2} = 0.23$ , which gives  $|\Delta C|_{\min,1} \approx 1.8\alpha^2$ . If  $\zeta$  is adjusted to make  $J_2(\zeta) = 0$  ( $\zeta = 11.62$ , third zero), we obtain  $|J_1(\zeta) - J_3(\zeta)| \approx 0.46$  and  $|\Delta C|_{\min,2} \approx 0.17\alpha^2$ .

(2)  $x = 2.2$ , which is the optimal modulation index for the second-harmonic detection technique. This gives a value of  $k = 0.34$ ,  $\nu_{Lm} \approx 4.4$  GHz, and  $\zeta = 2\pi\nu_{Lm}\tau \approx 17.6\pi \approx 55.3$  rad. Around  $\zeta = 55$ , the maximum value of  $J_2$  may be estimated to be  $|J_n(\zeta)| \sim (2/\pi\zeta)^{1/2} = 0.11$ , which gives  $|\Delta C|_{\min,1} \approx 0.65\alpha^2$ . If  $\zeta$  is adjusted to make  $J_2(\zeta) = 0$  ( $\zeta = 52.62$ , 15th zero), we obtain  $|J_1(\zeta) - J_3(\zeta)| \approx 0.13$  and  $|\Delta C|_{\min,2} \approx 0.08\alpha^2$ .

The minimum detectable gas concentration depends on the phase modulation index  $\zeta$ . The minimum detectable gas concentration without modulation index optimization ( $|C|_{\min,1}$ ) is of the order of  $\alpha^2$ , which is of the same order as that for a differential absorption case with similar parameters.<sup>8</sup> For a single pair of reflection points, if the position is known, the modulation index may be adjusted to make  $J_2(\zeta) = 0$ . Under this condition, the minimum detectable gas concentration ( $|C|_{\min,2}$ ) is of the order of  $0.1\alpha^2$ . This is much smaller than that for a differential absorption sensor.

For a conventional fiber cable with FC/PC connectors, the intensity reflection at the joint is of the order of  $-40$  dB ( $\alpha^2 = 10^{-4}$ ), and  $|\Delta C|_{\min,1}$  (for  $x = 0.95$ ) can be calculated as 180 ppm (parts per million) or 18 ppm.m [18 ppm per unit (1 m) interaction length]. If we require a resolution of 1 ppm.m, the required power reflection coefficient should be better than  $-53$  dB. This may be realized with FC/APC rather than FC/PC connectors.

We now look at a methane gas sensor with a gas cell formed with paired gradient index lenses (NSG Europe) of  $-30$  dB back reflectance; the calculated sensor resolution for  $x = 0.95$  without index optimization is  $|C|_{\min,1} = 1800$  ppm or 180 ppm.m. This result is in good agreement with our previous experimental results.<sup>7</sup> Because the positions of the reflection points for this case are known, the modulation current can be adjusted to minimize the measurement error. The final limit after modulation index optimization is calculated to be  $|C|_{\min,2} = 18$  ppm.m. To obtain high-accuracy measurement, the reflection coefficient from the cell surface should be reduced. For example, for obtaining  $|C|_{\min,2} = 10$  ppm (or 1 ppm.m) sensitivity, the reflection coefficient should be less than  $-43$  dB. This can be achieved with antireflection coating and with angled cell surfaces.

## 5. Summary

We investigated interferometric noise in optical fiber gas sensing systems based on wavelength modulation spectroscopy with coherent sources such as DFB lasers. We found that measurement error in gas concentration depends on the magnitude of the source frequency modulation and can be minimized with the choice of a proper modulation index. The minimum detectable gas concentration limited by interferometric noise without modulation index optimization is of the same order as a differential absorption sensor and we can reduce it 10 times by optimizing the modulation index.

### Appendix A: Phase Modulation Associated with Gas Absorption

When the frequency-modulated light passes through the sample gas, not only the output intensity but also the phase of the light will be modulated. If the laser frequency  $\nu$  is close enough to center frequency of the gas absorption line  $\nu_g$  to justify the approximation

$$|\nu - \nu_g| \ll \nu, \nu_g, \quad (\text{A1})$$

the electric field after the gas sample can be written as

$$E(t) = E_0 \exp[\alpha(\nu)CL] \exp[j\Phi(\nu)], \quad (\text{A2})$$

where  $\alpha(\nu)$  is given by Eq. (8),  $\Phi(\nu) = 2\pi(\nu/c)n_rL$ ,  $c$  is the speed of light in vacuum, and  $n_r$  is the refractive index of the gas given by<sup>10</sup>

$$n_r = 1 + \alpha_0 C \frac{c}{4\pi\nu} \frac{\nu - \nu_g}{\delta\nu} \frac{1}{1 + \left(\frac{\nu - \nu_g}{\delta\nu}\right)^2}. \quad (\text{A3})$$

$\Phi(\nu)$  can then be written as

$$\Phi(\nu) = 2\pi L \frac{\nu}{c} + 2\alpha_0 CL \frac{\nu - \nu_g}{\delta\nu} \frac{1}{1 + \left(\frac{\nu - \nu_g}{\delta\nu}\right)^2}. \quad (\text{A4})$$

For sinusoidal modulation of laser frequency,  $\nu = \nu_{L0} + \nu_{Lm} \sin \omega t$ , with  $\nu_{L0} = \nu_g$ , the above equation can be rewritten as

$$\begin{aligned} \Phi(\nu) = & \frac{2\pi\nu_{L0}L}{c} + \frac{2\pi\nu_{Lm}}{c} \int_0^{L/c} \sin \omega u du \\ & + 2\alpha_0 CL \frac{\nu_{Lm} \sin \omega t}{\delta\nu} \frac{1}{1 + \left(\frac{\nu_{Lm} \sin \omega t}{\delta\nu}\right)^2}. \end{aligned} \quad (\text{A5})$$

The three terms in Eq. (A5) are the same as the three phase terms in Eq. (22). The first term is a DC phase delay and the second term is a phase modula-

tion after length  $L$  resulting from laser frequency modulation. The third term is a phase modulation associated with the absorption process and is proportional to  $\alpha_0 CL$ .

Considering methane absorption at approximately 1665 nm [ $Q(6)$  line] as an example, we have  $\alpha_0 L = 0.1 \text{ cm}^{-1}$ ; if a 10-cm gas cell is used and if the gas concentration is less than 5%, the maximum value of the third term will be 5%. If the sensing length  $L$  is long and the gas concentration is large, however, the phase modulation resulting from the absorption process will be larger and may need to be considered in practice.

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