# A Note on Mean-variance Analysis of the Newsvendor Model with Stockout Cost

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**Abstract**

We apply the mean-variance approach to analyze the risk-averse newsvendor problem with stockout cost. We first derive an explicit form and some new properties of the variance of the profit function. Then, under the assumption that demand follows the power distribution, and its special case the uniform distribution, we obtain the set of optimal ordering quantities. We also give a counterexample to one result presented in the literature. Contrary to the traditional result in the literature that the risk-averse newsvendor always orders less than the risk-neutral newsvendor, our findings show that this may not be the case when stockout cost is considered because the newsvendor may order more than the risk-neutral newsvendor order quantity under a stockout situation with mean-variance tradeoff.

**Keywords**: supply chain, newsvendor problem, mean-variance analysis, stockout cost

## 1 Introduction

As a fundamental problem in stochastic inventory control, the newsvendor problem has been studied for a long time and applied in a broad array of business settings with the objective of expected profit maximization or expected cost minimization. However, modern supply chains are very complex and increasingly becoming more vulnerable to uncertainties. What

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supply chain managers concern most is not only profit, but also risk or loss to their firms. The assumption of risk-neutrality seems to be inadequate for contemporary supply chain management. In view of this, a number of papers have been devoted to risk analysis of supply chain models. Recent studies include, but are not limited to, those by Lau [13], Bouakiz and Sobel [5], Choi et al. [9], Eeckhoudt et al. [10], Lau and Lau [14], Agrawal and Seshadri [1, 2], Chen and Federgruen [7], Buzacott et al.[6], Chen et al. [8], Wang and Webster [18], Wu et al. [19], Bogataj and Bogataj [4], He and Zhang [11], Sounderpandian, Prasad and Madan [16], and Agrawal and Ganeshan [3]. For an extensive review of the literature on supply chain risk management or extension of different objectives on newsvendor problem, the reader is referred to Khouja [12], Tang [17] and Wu et al. [20].

In the newsvendor problem, if there is not enough stock to satisfy all the demand occurring in the selling season, the newsvendor may incur a stockout cost. Besides making a loss in marginal profit, the stockout cost may include such adverse effects on a firm as tarnishing the firm's reputation and jeopardizing the loyalty of the firm's customers, which can greatly impair the firm's performance and profitability. For example, the *Wall Street Journal* [21] reported that IBM, as a result of under-producing its Aptiva PC line, lost more than $100 million in potential revenue in 1994.

Although stockout cost plays an important role in the practice of supply chain management, it is often ignored or has not been studied in depth in risk analysis of supply chain models. Chen and Federgruen [7] studied the newsvendor problem using the mean-variance framework. Without stockout cost, the variance function of the stochastic profit is a monotone increasing function of order quantity, so the mean-variance tradeoff can be carried out efficiently. However, if stockout cost is considered, the variance function will lose this monotonicity property and the mean-variance tradeoff becomes much more complicated. Buzacott et al. [6] studied a class of commitment-option supply contracts under the mean-variance framework. They used the mean-variance criterion as the objective function, which is a newsvendor type of problem without stockout cost, and obtained a similar monotone increasing property of the variance function. They further emphasized that monotonicity is a fundamental result in this type of stochastic planning models. Choi et al. [9] investigated the issues of channel coordination in a supply chain when individual supply chain decision makers take the mean-variance objective. Eeckhoudt et al. [10] examined the effects of risk and risk aversion on a risk-averse and prudent newsvendor without considering the stockout cost. They pointed out that risk aversion will lead to a reduced initial newspaper order. Lau [13] considered the risk-averse newsvendor problem with mean-standard deviation tradeoff under two cases: one is without stockout cost; the other is with stockout cost. He proved that the risk-neutral newsvendor order quantity is an upper bound on the optimal order quantity of the risk-averse newsvendor without stockout cost. He also stated without proof that a similar result still holds when stockout cost is considered.

Our study is most related to [18], but with several major differences. Both studies consider the newsvendor problem with stockout cost based upon objectives different from profit maximization and find some results different from the risk-neutral newsvendor model. The significant differences are: (1) Wang and Webster [18] used loss aversion to model the newsvendor problem, while we use mean-variance tradeoff. Loss aversion belongs to a

class of utility function, while mean-variance tradeoff belongs to the return-risk framework. Generally speaking, utility maximization is mainly used in theoretical study, while mean-variance tradeoff is widely applied both in theoretical study and in practice. We select mean-variance tradeoff mainly based on the following two considerations. First, since there are various utility functions, it is not easy to construct a proper one convenient for analysis. Second, return-risk models usually have a much more intuitive explanation than the utility maximization approach. Here we must point out that, unlike many other utility functions, the loss aversion used in [18] is also intuitively appealing. (2) Wang and Webster [18] studied the risk-averse newsvendor problem within the loss aversion framework, while our paper is motivated mainly from previous studies. Thus, we also carry out comparisons with previous risk-averse newsvendor problems and present a counterexample to one result presented in the literature. (3) Besides studying the impact of stockout cost on the ordering quantity, we also study its impact on the newsvendor's profit.

If stockout cost is considered in the newsvendor model, the properties of the variance function and the mean-variance tradeoff may be very different from those of the model without stockout cost. Moreover, some results obtained in the above literature may no longer be valid. Motivated by this observation, we study in this paper the risk-averse newsvendor model presented in Chen and Federgruen [7] but with stockout cost consideration. We derive an explicit form of the variance of the profit function and obtain its properties. We show that the variance of the profit function is no longer a monotone increasing function. Furthermore, under the assumption that the demand function follows the power distribution, we work out the set of optimal ordering quantities. Contrary to the traditional result in the literature that the risk-averse newsvendor (without stockout cost) always orders less than the risk-neutral newsvendor order quantity, our findings show that this may not be the case when stockout cost is considered because the newsvendor may order more than the risk-neutral newsvendor order quantity under a stockout situation with mean-variance tradeoff. We also give a counterexample to one result presented in Lau [13].

The rest of this paper is organized as follows. In Section 2 we analyse the newsvendor problem under study using the mean-variance approach. In Section 3, under the assumption that demand follows the power distribution, we derive the new properties and results due to the stockout cost. We give conclusions in Section 4.

# 2 Mean-variance analysis with stockout cost

Let $Q$ be the newsvendor's order quantity. Let $D$ be the future stochastic demand during the selling season. Let $F$ be the cumulative distribution function and $f$ the probability density function of demand, respectively. We assume that $F$ is a continuous and strictly increasing function and $f$ is a nonnegative function.

The purchasing cost of the product is $c$ per unit, the selling price is $r$ per unit, the salvage value of any unsold product is $s$ per unit, and the stockout cost of unsatisfied demand is $p$ per unit. To avoid unrealistic and trivial cases, we assume that $0 < s < c < r$ and $0 < p$. Throughout the paper, we use the following notation: for any numbers $a$ and $b$,

$a^+ = \max\{a, 0\}$, and $a \wedge b = \min\{a, b\}$.

Let $\pi(Q)$ be the newsvendor's random profit, namely

$$\pi(Q) = r(Q \wedge D) + s(Q - D)^+ - p(D - Q)^+ - cQ. \tag{1}$$

Let $\Pi(Q)$ be the mean profit, namely

$$\Pi(Q) = E[\pi(Q)] = -(r + p - s)\int_0^Q F(x)dx + (r + p - c)Q - pE[D], \tag{2}$$

where $E[D]$ is the mean of the random demand $D$.

The risk-neutral newsvendor problem is given by

$$\max_{Q \geq 0}\{E[\pi(Q)]\}. \tag{3}$$

The optimal solution $Q^*$ for problem (3) is called the *newsvendor order quantity.* It is straightforward to verify that the expected profit function is a concave function of $Q$. By using the first-order optimality conditions, we obtain the newsvendor order quantity $Q^*$ as follows

$$F(Q^*) = \frac{r + p - c}{r + p - s}. \tag{4}$$

The mean-variance analysis was first proposed by Markowitz [15] to measure the risk associated with the return of assets. It uses a parameter $\alpha$ ($\alpha \geq 0$) to characterize a decision maker's risk averseness, which is a quantitative balance between the mean profit and the risk associated with its variance. $\alpha = 0$ denotes the special case of maximizing the mean profit function only. An increase in $\alpha$ indicates the decision maker's increasing willingness to sacrifice the mean profit to avoid the risk of its variance. Note that, for any given $\alpha$, a solution is optimal in the sense that we cannot improve the mean profit without bearing more risk, or reduce the risk without decreasing the mean profit.

Under the mean-variance framework, the objective function of the newsvendor problem is given by

$$\max_{Q \geq 0}\{E[\pi(Q)] - \alpha Var[\pi(Q)]\}, \tag{5}$$

where $Q$ is the order quantity, $\alpha$ is the parameter denoting the decision maker's risk attitude, $\pi(Q)$ is the random profit given by Eq. (1), $E[\pi(Q)]$ is the mean of the random profit given by Eq. (2), and $Var[\pi(Q)]$ is the variance of the random profit given by Eq. (7).

Note that the variance of the random profit is given by

$$Var[\pi(Q)] = E\left[(\pi(Q))^2\right] - (E[\pi(Q)])^2. \tag{6}$$

Substituting Eqs. (1) and (2) into Eq. (6), the variance function of the newsvendor's profit can be written as

$$Var[\pi(Q)]$$

$$
\begin{aligned}
&= -(r+p-s)^2\left(\int_0^Q F(x)dx\right)^2 \\
&\quad +\Big\{2Q(r-s)(r+p-s)-2p(r+p-s)E[D]\Big\}\int_0^Q F(x)dx \\
&\quad -2(r+p-s)(r-p-s)\int_0^Q xF(x)dx + p^2Var[D], \qquad (7)
\end{aligned}
$$

where $Var[D]$ is the variance of the random demand $D$.

The first-order derivative of the variance function of Eq. (7) with respect to $Q$ is given by

$$
\begin{aligned}
&\frac{dVar\left[\pi(Q)\right]}{dQ} \\
&= -2(r+p-s)^2F(Q)\int_0^Q F(x)dx + 2p(r+p-s)QF(Q) \\
&\quad +2(r+p-s)(r-s)\int_0^Q F(x)dx - 2p(r+p-s)E[D]F(Q). \qquad (8)
\end{aligned}
$$

The second-order derivative of the variance function of Eq. (7) with respect to $Q$ is given by

$$
\begin{aligned}
&\frac{d^2Var[\pi(Q)]}{dQ^2} \\
&= 2(r+p-s)\Bigg\{f(Q)\Big[p(Q-\int_0^Q F(x)dx - E[D]) - (r-s)\int_0^Q F(x)dx\Big] \\
&\quad +(r+p-s)F(Q)(1-F(Q))\Bigg\}. \qquad (9)
\end{aligned}
$$

**Remark 2.1** *From Eqs. (7), (8) and (9), we see that the variance function with stockout cost is more complicated than that without stockout cost. Compared with the results without stockout cost presented in Chen and Federgruen [7], where the variance function is an increasing function of the order quantity, such results may no longer be valid when stockout cost is considered.*[1]

**Theorem 2.2** *The expected profit function is a concave function of $Q$ and asymptotically linear with a slope $(s-c)<0$.*

*Proof:* See Appendix A1.

**Remark 2.3** *When $Q \to +\infty$, no stockout could happen; so the expected profit function preserves the same property as that presented in Chen and Federgruen [7] without stockout cost. And one unit of overstocking will result in a unit loss of $|s-c|$.*

[1] We give an example in the next section to show that the variance function is not an increasing function of the order quantity when stockout cost is considered.

If the monotonicity property of the variance function does not hold, the tradeoff between return and risk becomes much more complicated. However, under the assumption that the random demand $D$ has a finite second moment, the following Theorem 2.4 guarantees the boundedness of the variance function.

**Theorem 2.4** *The variance function $Var[\pi(Q)]$ is a bounded function in $Q \in [0, +\infty)$. Moreover,*

$$\lim_{Q \to 0} Var[\pi(Q)] = p^2 Var[D], \tag{10}$$

$$\lim_{Q \to +\infty} Var[\pi(Q)] = (r - s)^2 Var[D]. \tag{11}$$

*Proof:* See Appendix A2.

**Remark 2.5** *The results of Theorem 2.4 characterize the relationship between profit fluctuation and demand fluctuation. In fact, profit fluctuation originates from demand fluctuation.*

**Remark 2.6** *In the extreme case of $Q \to 0$, i.e., no supply, the relationship between profit fluctuation and demand fluctuation is dependent on $p^2$. In the extreme case of $Q \to +\infty$, i.e., no stockout, the relationship between profit fluctuation and demand fluctuation is independent of the stockout cost.*

**Remark 2.7** *The concavity of the expected profit function and the boundedness of the variance function guarantee the existence of an optimal order quantity. We can obtain the optimal order quantity by applying a one-dimensional search algorithm when the closed-form solution cannot be obtained. Furthermore, the newsvendor order quantity can be used as an initial solution for the search algorithm.*

**Remark 2.8** *Compared with the results presented in Lau [13], the explicit form of the variance function expressed as Eq. (7) in this paper has computational advantages. In Lau [13], computation of the value of the variance function needs some central moments, their derivatives and partial moments. Our results show that we only need to compute $\int_0^Q F(x)dx$ and $\int_0^Q xF(x)dx$, which can be easily determined via numerical integration methods or the results may even have closed-form expressions.*

# 3 Special case

To derive structural results and generate managerial insights into the optimal decisions of the risk-averse newsvendor problem, we present in the following specific results for the case where demand follows the power distribution, and its special case the uniform distribution, as presented in Chen and Federgruen [7], and compare our results with theirs.

Without loss of generality, we choose the interval $[0,1]$ with probability distribution function $F_D(x)$ and probability density function $f_D(x)$ as follows:

$$F_D(x) = \begin{cases} 1, & x > 1, \\ x^k, & 0 \le x \le 1, \\ 0, & x < 0, \end{cases} \tag{12}$$

$$f_D(x) = \begin{cases} kx^{k-1}, & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

The mean and variance of the power distribution are $k/(k+1)$ and $k/[(k+2)(k+1)^2]$, respectively.

Based on the assumption that the demand function follows the power distribution, the mean function of Eq. (2) is given by

$$\Pi(Q) = \begin{cases} -\frac{(r+p-s)}{k+1}Q^{k+1} + (r+p-c)Q - p\frac{k}{k+1}, & 0 \le Q \le 1, \\ (s-c)Q + k\frac{r-s}{k+1}, & 1 < Q. \end{cases} \tag{14}$$

The first-order derivative of the mean profit function of Eq. (14) with respect to $Q$ is given by

$$\frac{d\Pi(Q)}{dQ} = \begin{cases} -(r+p-s)Q^k + (r+p-c), & 0 \le Q \le 1, \\ (s-c)Q, & 1 < Q. \end{cases} \tag{15}$$

The variance function of Eq. (7) is given by

$$Var[\pi(Q)] = \begin{cases} \frac{p^2k}{(k+2)(k+1)^2} - (r+p-s)^2\frac{Q^{2(k+1)}}{(k+1)^2} \\ + \left[2Q(r-s)(r+p-s) - \frac{2p(r+p-s)k}{k+1}\right]\frac{Q^{k+1}}{k+1} \\ -2(r+p-s)(r-p-s)\frac{Q^{k+2}}{k+2}, & 0 \le Q \le 1, \\ \frac{k(r-s)^2}{(k+1)^2(k+2)}, & 1 < Q. \end{cases} \tag{16}$$

The first-order derivative of the variance function of Eq. (16) with respect to $Q$ is given by

$$\frac{dVar[\pi(Q)]}{dQ} = \begin{cases} \frac{2(r+p-s)Q^k}{k+1}\Big\{-(r+p-s)Q^{k+1} \\ +[(k+1)p+(r-s)]Q - pk\Big\}, & 0 \le Q \le 1, \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

**Theorem 3.1** *There exists one unique minimizer $Q_P^0$ for $Var[\pi(Q)]$ on $(0,1)$, where $Var[\pi(Q)]$ is decreasing in $[0, Q_P^0]$ and increasing in $[Q_P^0, 1]$. Moreover, there exists a critical value $k^*$ with $0 < k^* < 1$ and the newsvendor's optimal order quantity is distinguished by three cases as follows:*
*(1) If $0 < k < k^*$, then $Q^* < Q_P^0$ and the optimal order quantity is in the interval $[Q^*, Q_P^0]$.*
*(2) If $k = k^*$, then $Q^* = Q_P^0$ and the optimal order quantity is exactly $Q^*$.*
*(3) If $k > k^*$, then $Q^* > Q_P^0$ and the optimal order quantity is in the interval $[Q_P^0, Q^*]$.*

*Proof:* See Appendix A3.

**Remark 3.2** *Lau [13] pointed out that the newsvendor solution is an upper bound on the optimal order quantity that maximizes a mean-variance type of objective function without and with stockout cost. However, Theorem 3.8 indicates that this may not always be the case, since the newsvendor solution may be larger than the optimal order quantity with stockout cost.*[2]

**Remark 3.3** *Contrary to the traditional result in the literature that the risk-averse newsvendor always orders less than the risk-neutral newsvendor order quantity, our findings in Theorem 3.8 show that this may not be the case when stockout cost is considered because the newsvendor may order more than the newsvendor order quantity under a stockout situation with mean-variance tradeoff.*

**Remark 3.4** *Besides making a loss in marginal profit, stockout cost may have other adverse effects on a firm's performance and profitability, and so it should not be ignored. Our findings presented above clearly characterize the significant impact of stockout cost on the newsvendor's optimal ordering decisions and give the set of optimal order strategies when demand follows the power function. Managers should find our results helpful in making ordering decisions in their practice of supply chain management under a risk-averse environment.*

When we restrict our analysis to the uniform distribution, i.e., $k = 1$ in the power distribution, we obtain the following results. The mean function of Eq. (2) and the variance function of Eq. (7) are respectively given by

$$\Pi(Q) = \begin{cases} -(r+p-s)\frac{Q^2}{2} + (r+p-c)Q - \frac{p}{2}, & 0 \le Q \le 1, \\ (s-c)Q + \frac{r-s}{2}, & 1 < Q. \end{cases} \tag{18}$$

$$Var[\pi(Q)] = \begin{cases} -(r+p-s)^2\frac{Q^4}{4} + [(r-s)(r+p-s) - \frac{2}{3}(r+p-s) \\ \cdot(r-p-s)]Q^3 - p(r+p-s)\frac{Q^2}{2} + \frac{p^2}{12}, & 0 \le Q \le 1, \\ \frac{(r-s)^2}{12}, & 1 < Q. \end{cases} \tag{19}$$

**Proposition 3.5** *There exists a critical number $\tilde{Q}$ given as follows:*

$$\tilde{Q} = F(Q^*) - \frac{\sqrt{r^2 + c^2 + pr - 2rc - 2pc + ps}}{r+p-s}. \tag{20}$$

*When $Q \in [0, \tilde{Q}]$, the newsvendor makes a negative profit; only when $Q > \tilde{Q}$ does he begin to make a positive profit.*

[2]The model setup in Lau [13] is the same as that in this paper except that he considered a mean-standard deviation tradeoff. The difference between a mean-standard deviation tradeoff and a mean-variance tradeoff has no significant impact on the results obtained in this paper since the monotonicity of the mean-standard deviation of order quantity is the same as that of mean-variance.

**Remark 3.6** *Proposition 3.5 shows that the newsvendor may not always make a positive profit when stockout cost is considered. Only when the newsvendor orders enough can he begin to earn a positive profit.*

**Proposition 3.7** *There exists a unique minimizer* $Q^0 \in [0,1]$ *of the variance function* $Var[\pi(Q)]$ *of Eq. (19) given by* $Q^0 = \frac{p}{p+r-s}$, *where the variance function* $Var[\pi(Q)]$ *is decreasing in* $[0, Q^0]$ *and increasing in* $[Q^0, 1]$.

**Theorem 3.8** *The newsvendor's optimal order quantity is in the interval* $[Q^0, Q^*]$.

**Remark 3.9** *Theorem 3.8 presents the interval in which the optimal order quantity lies. With the two critical values* $Q^0$ *and* $Q^*$, *the whole value set is divided into three subsets* $[0, Q^0]$, $[Q^0, Q^*]$ *and* $[Q^*, 1]$. *The mean profit function keeps increasing in* $[0, Q^*]$ *and decreasing in* $[Q^*, 1]$, *while the variance function keeps decreasing in* $[0, Q^0]$ *and increasing in* $[Q^0, 1]$. *To tradeoff between the mean and the variance, the values in* $[Q^0, Q^*]$ *are superior to those that lie in the other two subsets.*

# 4 Numerical results

To precisely compare our results with those presented in Chen and Federgruen [7], we conducted numerical experiments using the following parameters in the models: $r = 100, c = 70, s = 50$, and $\alpha = 0.1$ with the demand following the uniform distribution.

With the above parameters on hand, we studied the impact of stockout cost on the newsvendor's optimal decisions (see Tables 1-2). We let the stockout cost $p$ increase from 0 to 35 in steps of 5, while keeping all other parameters unchanged.

Table 1: Impact of stockout cost on the order quantity

| $p$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
|---|---|---|---|---|---|---|---|---|
| $Q^*_M$ | 0.6 | 0.636364 | 0.66667 | 0.692308 | 0.714286 | 0.73333 | 0.75 | 0.764706 |
| $Q^*_{MV}$ | 0.294333 | 0.335857 | 0.374521 | 0.410178 | 0.442864 | 0.47441 | 0.5 | 0.524897 |

The first row ($p$) of Table 1 is the stockout cost. The second row of Table 1 ($Q^*_M$) is the optimal order quantity with mean as the optimal criterion. The third row of Table 1 ($Q^*_{MV}$) is the optimal order quantity with mean-variance as the optimal criterion.

From the numerical results shown in Table 1, we conclude that

(1) The optimal order quantity $Q^*_M$ with respect to the expectation criterion increases as stockout cost increases.

(2) The optimal order quantity $Q^*_{MV}$ with respect to the mean-variance criterion increases as stockout cost increases.

(3) With the same stockout cost, all the optimal order quantities with respect to the expectation criterion are greater than the optimal order quantities with respect to the mean-variance criterion.

Table 2: Impact of stockout cost on the objective function

| $p$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
|---|---|---|---|---|---|---|---|---|
| $\Pi_M^*$ | 9 | 8.63636 | 8.3333 | 8.07692 | 7.85714 | 7.66667 | 7.5 | 7.35294 |
| $V_{MV}^*$ | 5.00837 | 4.29059 | 3.56366 | 2.84503 | 2.14626 | 1.47441 | 0.83333 | 0.224688 |

The first row ($p$) of Table 2 is the stockout cost. The second row of Table 2 ($\Pi_M^*$) is the optimal value with mean as the optimal criterion. The third row of Table 2 ($V_{MV}^*$) is the optimal value with mean-variance as the optimal criterion.

From the numerical results shown in Table 2, we conclude that

(1) The optimal value $\Pi_M^*$ with respect to the expectation criterion decreases as stockout cost increases.

(2) The optimal value $V_{MV}^*$ with respect to the mean-variance criterion decreases as stockout cost increases.

(3) With the same stockout cost, all the optimal values with respect to the expectation criterion are greater than the optimal values with respect to the mean-variance criterion.

# 5 Conclusions

Stockout cost is often ignored in most traditional supply chain literature. However, stockout cost could play an important role in both theoretical analysis and real-world inventory management. Motivated by this consideration, we studied the risk-averse newsvendor model with stockout cost and focused on analyzing the impact of stockout cost on the newsvendor's ordering decisions. We showed that stockout cost has a significant impact on the newsvendor's optimal ordering decisions with mean-variance tradeoff.

We derived an explicit form and some new properties of the variance of the profit function. Under the assumption that demand follows the power distribution, and its special case the uniform distribution, we obtained the range of the optimal ordering quantities. We also gave a counterexample to one result presented in the literature.

While most of the literature on the risk-averse newsvendor problem suggests that the newsvendor orders less than the risk-neutral newsvendor solution, our findings show that this may not be the case when stockout cost is considered because the newsvendor may order more than the newsvendor order quantity under a stockout situation with mean-variance tradeoff.

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# Appendices

**A1: Proof of Theorem 2.2**

*Proof:* First, we analyze the property of the mean profit function $\Pi(Q)$. The first-order derivative of $\Pi(Q)$ of Eq. (2) with respect to $Q$ is given by

$$\frac{d\Pi(Q)}{dQ} = -(r+p-s)F(Q) + (r+p-c). \tag{21}$$

The second-order derivative of $\Pi(Q)$ of Eq. (2) with respect to $Q$ is given by

$$\frac{d^2\Pi(Q)}{dQ^2} = -(r+p-s)f(Q). \tag{22}$$

From the assumptions in Section 2, we know that $f(Q) \geq 0$, $0 < s < r$ and $0 < p$; hence, $\frac{d^2\Pi(Q)}{dQ^2} \leq 0$. Therefore, $\Pi(Q)$ is a concave function of $Q$.

Second, when $Q \to +\infty$, we obtain $\lim_{Q\to+\infty} \frac{\Pi(Q)}{Q} = s - c < 0$.

From the above analysis, we see that the expected profit function is a concave function of $Q$ and is asymptotically linear with a slope $(s-c) < 0$. □

**A2: Proof of Theorem 2.4**

*Proof:* First, we study the case where $Q \to 0$. When $Q$ approaches 0, we obtain

$$\lim_{Q\to 0} Var[\pi(Q)] = Var[\pi(0)] = p^2 Var[D]. \tag{23}$$

So, Eq. (10) holds.

Second, we study the case where $Q \to +\infty$. Under the assumption that the random demand $D$ has a finite second moment, we have $\int_0^{+\infty} x^2 dF(Q) < +\infty$. So, we have $\lim_{Q\to+\infty} \int_Q^{+\infty} x^2 dF(x) = 0$. Note that $0 \le Q \int_Q^{+\infty} x dF(x) \le \int_Q^{+\infty} x^2 dF(x)$; hence, $\lim_{Q\to+\infty} Q \int_Q^{+\infty} x dF(x) = 0$.

From the above analysis, we obtain that

$$\begin{aligned}
& \lim_{Q\to+\infty} \left( Q^2 F(Q) - Q \int_0^Q F(x)dx - E[D] \int_0^Q F(x)dx \right) \\
= & \lim_{Q\to+\infty} \left( Q^2 F(Q) - Q \left[ QF(Q) - \int_0^Q x dF(x) \right] - E[D] \int_0^Q F(x)dx \right) \\
= & \lim_{Q\to+\infty} \left( Q \int_0^Q x dF(x) - E[D]QF(Q) + E[D] \int_0^Q x dF(x) \right) \\
= & \lim_{Q\to+\infty} \left( Q \int_0^Q x dF(x) - QE[D] + QE[D] - E[D]QF(Q) + E[D] \int_0^Q x dF(x) \right) \\
= & \lim_{Q\to+\infty} \left( -Q \int_Q^{+\infty} x dF(x) + QE[D]\,(1 - F(Q)) + E[D] \int_0^Q x dF(x) \right) \\
= & E^2[D].
\end{aligned} \tag{24}$$

Since $0 \le Q^2[1 - F(Q)] \le \int_Q^{+\infty} x^2 dF(x)$, we have

$$\lim_{Q\to+\infty} Q^2[1 - F(Q)] = 0. \tag{25}$$

When $Q$ approaches $+\infty$, we get that

$$\begin{aligned}
& \lim_{Q\to+\infty} Var[\pi(Q)] \\
= & \lim_{Q\to+\infty} \Bigg\{ p^2 Var[D] - (r+p-s)^2 \Bigg[ \left( Q - \int_0^Q F(x)dx \right)^2 - Q^2 \\
& +2Q \int_0^Q F(x)dx \Bigg] + \Big[ 2Q(r-s)(r+p-s) - 2p(r+p-s)E[D] \Big] \\
& \cdot \int_0^Q F(x)dx - (r+p-s)(r-p-s) \int_0^Q F(x)dx^2 \Bigg\} \\
= & \lim_{Q\to+\infty} \Bigg\{ p^2 Var[D] - (r+p-s)^2 \left( \int_0^Q (1 - F(x))dx) \right)^2 \\
& +(r+p-s)(r-p-s) \int_0^Q x^2 dF(x) + 2p(r+p-s)
\end{aligned}$$

$$\cdot\left(Q^2F(Q) - Q\int_0^Q F(x)dx - E[D]\int_0^Q F(x)dx\right)$$
$$+(r+p-s)^2Q^2(1-F(Q))\Bigg\}. \tag{26}$$

From Eqs. (24) and (25), we can re-write Eq. (26) as

$$\begin{aligned}
&\lim_{Q\to+\infty} Var[\pi(Q)] \\
=\ & p^2Var[D] - (r+p-s)^2E^2[D] + (r+p-s)(r-p-s)E[D^2] \\
& +2p(r+p-s)E^2[D] \\
=\ & p^2Var[D] + \left[(r-s)^2 - p^2\right]Var[D] \\
=\ & (r-s)^2Var[D]. \qquad (27)
\end{aligned}$$

So, Eq. (11) holds.

Note that the variance function is a continuous function. Also from Eqs. (10) and (11), we obtain that the variance function is a bounded function in $Q \in [0, +\infty)$. □

**A3: Proof of Theorem 3.1**

*Proof:* First, we define a function $W(Q)$ as follows:

$$W(Q) := -(r+p-s)Q^{k+1} + [(k+1)p + (r-s)]Q - pk. \tag{28}$$

Then, for $Q \in [0,1]$, the sign of $\frac{dVar[\pi(Q)]}{dQ}$ is the same as the sign of $W(Q)$.

Note that the first-order derivative function of $W(Q)$ of Eq. (28) with respect to $Q$ is given by

$$\frac{dW(Q)}{dQ} = -(k+1)(r+p-s)Q^k + (k+1)p + (r-s). \tag{29}$$

The second-order derivative function of $W(Q)$ of Eq. (28) with respect to $Q$ is given by

$$\frac{dW^2(Q)}{dQ^2} = -k(k+1)(r+p-s)Q^{k-1} \leq 0. \tag{30}$$

From Eq. (30), we know that $W(Q)$ is a concave function. Furthermore, we have

$$\begin{aligned}
&W(0) = -kp < 0, W(1) = 0, \\
&\frac{dW(Q)}{dQ}|_{Q=0} = (k+1)p + (r-s) > 0, \\
&\frac{dW(Q)}{dQ}|_{Q=1} = -k(r-s) < 0.
\end{aligned}$$

Thus, there must exist a unique root $Q_P^0 \in (0,1)$ that satisfies the equation $W(Q) = 0$. So, the signs of $W(Q)$ and $\frac{dVar[\pi(Q)]}{dQ}$ change exactly once in $Q_P^0$ and the sign changes are from negative to positive. Hence, $Q_P^0$ is the unique minimizer for $Var[\pi(Q)]$ in $Q \in [0,1]$, where

$Var[\pi(Q)]$ is decreasing in $[0, Q_P^0]$ and increasing in $[Q_P^0, 1]$. Therefore, the newsvendor's optimal order quantity is within an interval bounded by $Q^*$ and $Q_P^0$.

Note that $(Q^*)^k = \frac{r+p-c}{r+p-s}$, and

$$\begin{aligned} W(Q^*) &= -(r+p-c)Q^* + [(k+1)p + (r-s)]Q^* - pk \\ &= (kp+c-s)Q^* - pk. \end{aligned} \tag{31}$$

Therefore, we obtain the following results:

$$\begin{aligned} & Q^* < (=,>)Q_P^0 \\ \Longleftrightarrow\quad & W(Q^*) < (=,>)0 \\ \Longleftrightarrow\quad & Q^* < (=,>)\frac{kp}{kp+c-s} \\ \Longleftrightarrow\quad & \frac{r+p-c}{r+p-s} < (=,>)\frac{(kp)^k}{(kp+c-s)^k}. \end{aligned}$$

It is straightforward to verify that $\frac{d(\frac{kp}{kp+c-s})^k}{dk} = (\frac{kp}{kp+c-s})^k \left(\ln(1-\frac{c-s}{kp+c-s}) + \frac{c-s}{kp+c-s}\right) < 0$. Moreover, we have the following results:

$$\lim_{k\to 0}(\frac{kp}{kp+c-s})^k = 1 > \frac{r+p-c}{r+p-s}, \tag{32}$$

$$\left(\frac{kp}{kp+c-s}\right)^k |_{k=1} = \frac{p}{p+c-s} < \frac{r+p-c}{r+p-s}. \tag{33}$$

Thus, there exists a critical value $k^*$ with $0 < k^* < 1$ and the newsvendor's optimal order quantity is distinguished by three cases as follows:
(1) If $0 < k < k^*$, then $Q^* < Q_P^0$ and the optimal order quantity is in the interval $[Q^*, Q_P^0]$.
(2) If $k = k^*$, then $Q^* = Q_P^0$ and the optimal order quantity is exactly $Q^*$.
(3) If $k > k^*$, then $Q^* > Q_P^0$ and the optimal order quantity is in the interval $[Q_P^0, Q^*]$. □