

Community-based Informed Agents Selection for Flocking with a Virtual Leader

Nuwan Ganganath, Chi-Tsun Cheng, Xiaofan Wang, and Chi K. Tse

Abstract: It has been studied that a few informed individuals in a group of interacting dynamic agents can influence the majority to follow the position and velocity of a virtual leader. Previously it has been shown that a cluster-based selection of informed agents can drive more agents to follow the virtual leader compared to a random selection. However, a practical question is: How many informed agents to select? In order to address this, here we propose a novel method for selecting informed agents based on community structures in the initial spatial distribution of agents. The number of informed agents are decided based on the strongest community structure. We test and analyze the performance of the proposed method against random and cluster-based selections of informed agents using extensive computer simulations. Results of our study show that community-based selection can be useful in deciding an optimum number of informed agents such that a majority of the group can achieve their common objective.

Keywords: Flocking, informed agents, virtual leader, controllability, communities.

1. INTRODUCTION

Flocking can be identified as a form of collective behavior within a large group of *dynamic agents* which uses local information and simple rules to achieve common objectives. Such group behaviors are common in nature, e.g.: bird flocks, fish schools, mammal herds, bacteria swarms, and so on [1–4]. In one of the early attempts of modeling natural flocks, Reynold [5] introduced three heuristic rules which enabled agents to stay close to nearby flockmates, match their velocities, and avoid collisions. However, Reynold's rules fail to model some important and complex behaviors within flocks such as obstacle avoidance [6,7], terrain adaptation [8,9], and target tracking/goal seeking behavior [10, 11]. In this paper, our focus is on the target tracking behavior of groups of dynamic agents. Such a target or a common objective is commonly represented by a *virtual leader* in literature [11–14], which can be either dynamic or static.

Leonard et al. [15] studied the coordination control in multi-agent systems with a virtual leader using an artificial potential function method. A computational and theoretical framework for design and analysis of flocking algorithms was proposed by Olfati-Saber [16]. Among the three flocking algorithms in [16], the second algorithm is for flocking in free space, in which a dynamic virtual leader was used to guide a group of agents in a desired velocity using a *navigational feedback* mechanism. All the agents are treated as *informed agents* who has continuous access to the information of the virtual leader. Although having all the agents being informed can guarantee all of them to keep track of the virtual leader, such an argument is quite impractical in many engineering applications and uncommon in natural flocks.

Ren [17] studied the consensus problem in multi-vehicle systems with a virtual leader. Vehicles were modeled by single integrator dynamics and only a fraction of the vehicles in the system had been granted access to the virtual leader. The consensus of multi-agent dynamic systems with general nonlinear coupling was studied by Chen et al. [18] based on multiple Lyapunov functions and contraction analysis. Su et al. [12] modified the Olfati-Saber's second flocking algorithm by providing navigational feedback only for a few informed agents that are selected randomly. Their algorithm enabled some of *uninformed agents* which do not have direct access to the information of the virtual leader, to move with the same velocity if they can be influenced by the informed agents from time to time. Their simulation results conclude that the larger the group size, the smaller the proportion of the agents need to have the information of the virtual leader in order to guide a given fraction of agents with the desired velocity. In [19], we showed that selecting informed agents based on initial clusters in the spatial distribution of agents can lead to a

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higher percentage of following agents compared with the traditional random selection.

In this paper, we further investigate how informed agents selections can affect proportions of agents to achieve the common objectives. We perform this by varying spatial distribution of agents for different initial densities. This enables us to study how initial connections between informed agents and the rest of the group affect the fraction of the agents achieving the common objective. Our studies also lead to an observation that a desirable selection of informed agents based on their initial connections can help to drive majority of the agents to the group objective effectively. Based on this observation, we proposed a novel method for informed agents selection in a group of dynamic agents using their initial community structure. Analyses of the proposed method show that it can be very useful in finding an optimum number of informed agents to drive a majority of the group to its objective.

The rest of the paper is organized as follows. Relevant background materials that lead to the problem considered in this paper are recalled in Section 2. We propose community-based method for effective selection of informed agents in flocking in Section 3. Results of the proposed method are presented in Section 4. Concluding remarks are given in Section 5.

2. BACKGROUND

We consider a group of N dynamic agents moving in n dimensional Euclidean space with double integrator dynamics $\dot{q}_i = p_i$ and $\dot{p}_i = u_i$, where $i = 1,2,\ldots,N$ and q_i,p_i, $u_i \in \mathbb{R}^n$ are the position, velocity, and control input of the agent i, respectively. The configuration of all the agents is expressed as $q = [q_1,q_2,\ldots,q_N]^{\mathrm{T}} \in \mathbb{R}^{nN}$. The interaction range between two agents, $r > 0$, determines their spatial neighbors. The set of spatial neighbors of agent i at time t is denoted by $\mathcal{N}_i(t) = \{j : \|q_i - q_j\| < r, j = 1,2,\ldots,N, j \neq i\}$, where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. We assume an identical interaction range and communication capability for all the agents.

2.1. Flocking of Multi-agents with a Virtual Leader

A distributed control scheme for multi-agent dynamic systems, which only utilizes the information of the neighboring agents and the virtual leader, is proposed by the second flocking algorithm in [16]. In such algorithm, each agent applies a control input that consists of three terms:

$$u_i = f_i^g + f_i^d + f_i^\gamma. \tag{1}$$

In (1), the gradient-based term $f_i^g = -\nabla_{q_i} V(q)$, is used to control the position of agent i in its neighbourhood. A smooth collective potential function $V(q)$ is given by

$$V(q) = \sum_{j \in \mathcal{N}_i(t)} \psi_\alpha(\|q_j - q_i\|_\sigma). \tag{2}$$

The σ-norm is defined as $\|z\|_\sigma = \frac{1}{\varepsilon}\left(\sqrt{1+\varepsilon\|z\|^2} - 1\right)$, where $\varepsilon > 0$. In (2), $\psi_\alpha(z)$ is a smooth pairwise attractive/repulsive potential which is defined as

$$\psi_\alpha(z) = \int_{\|d\|_\sigma}^{z} \phi_\alpha(s)ds.$$

Here, $\psi_\alpha(z)$ reaches its global minimum at a desired distance $z = \|d\|_\sigma < \|r\|_\sigma$ and global maximum at $z = 0$ with a finite cut-off at $z = \|r\|_\sigma$. It is constant for $\|z\|_\sigma \geq \|r\|_\sigma$. An action function is defined as

$$\phi_\alpha(z) = p_h\left(\frac{z}{\|r\|_\sigma}\right)\phi(z - \|d\|_\sigma),$$

where $\phi(z) = (1/2)[(a+b)\sigma_1(z+c) + (a-b)]$. Here, $0 < a \leq b$, $c = |a-b|/\sqrt{4ab}$, and $\sigma_1(z) = z/\sqrt{1+z^2}$. One possible choice for a bump function $p_h(z)$ is

$$p_h(z) = \begin{cases} 1, & \text{if } z \in [0,h) \\ \frac{1}{2}\left[1 + \cos\left(\pi\frac{z-h}{1-h}\right)\right], & \text{if } z \in [h,1] \\ 0, & \text{otherwise} \end{cases}$$

where $h \in (0,1)$ [16].

The velocity consensus term in (1) is defined as $f_i^d = \sum_{j\in\mathcal{N}_i(t)} a_{ij}(q)(p_j - p_i)$. Here, terms of the adjacency matrix is defined as

$$a_{ij}(q) = \begin{cases} 0, & \text{if } j = i \\ p_h\left(\frac{\|q_j - q_i\|_\sigma}{\|r\|_\sigma}\right), & \text{otherwise.} \end{cases} \tag{3}$$

The third term in (1), $f_i^\gamma = -c_1(q_i - q_\gamma) - c_2(p_i - p_\gamma)$ is a navigational feedback due to mutual objective which drives agent i to track the virtual leader with double integrator dynamics $\dot{q}_\gamma = p_\gamma$ and $\dot{p}_\gamma = f_\gamma(q_\gamma, p_\gamma)$. Here c_1, c_2 are positive constants and $q_\gamma, p_\gamma, f_\gamma \in \mathbb{R}^n$ are the position, velocity, and acceleration (control input) of the virtual leader, respectively. One should note that this control protocol considers all the agents as informed agents which have the knowledge of (q_γ, p_γ).

2.2. Random Selection of Informed Agents

In [12], Su et al. modified (1) by assuming that only some of the agents are given the knowledge of (q_γ, p_γ):

$$\begin{aligned} u_i = &-\sum_{j\in\mathcal{N}_i(t)} \nabla_{q_i}\psi_\alpha(\|q_j - q_i\|_\sigma) + \sum_{j\in\mathcal{N}_i(t)} a_{ij}(q)(p_j - p_i) \\ &-h_i[c_1(q_i - q_\gamma) + c_2(p_i - p_\gamma)], \qquad c_1, c_2 > 0. \end{aligned} \tag{4}$$

Here, $h_i = 1$ if the agent i is informed, otherwise $h_i = 0$. M_0 agents are randomly selected as informed agents. They also simplified dynamics of the virtual leader as $\dot{q}_\gamma = p_d$ and $q_\gamma(0) = q_d$. Here, p_d is a desired constant velocity and q_d is an initial position of the virtual leader.

Fig. 1 illustrates random assignment of 5 informed agents in a network of 25 agents as proposed in [12]. Filled circles represent the informed agents and the rest is uninformed. As seen from Fig. 1, the random selection of

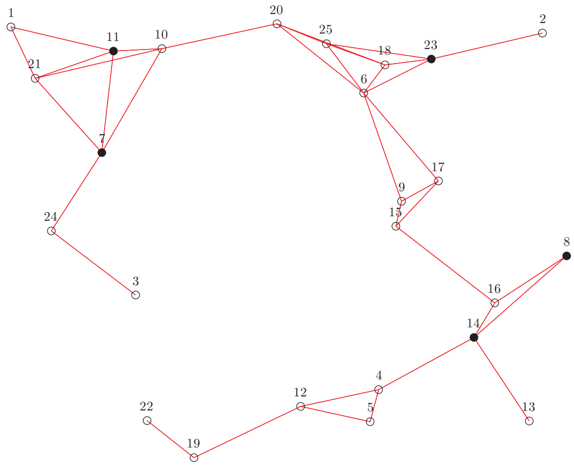

Fig 1: Random selection of 5 informed agents in a network of 25 agents. Solid lines represent the neighboring relations. Filled circles represent the informed agents and empty circles represent uninformed agents.

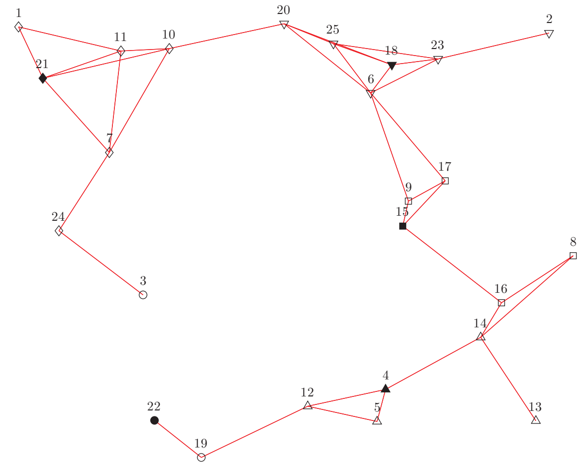

Fig 2: Clusters detected using the *k*-means algorithm for the network given in Fig. 1. Agents that are represented by same shape and color belong to the same cluster. Solid lines represent the neighboring relations. Filled markers represent the informed agents and empty markers represent uninformed agents.

informed agents does not guarantee an even spatial distribution of informed agents, thus leaving certain parts of the network highly influenced by the virtual leader while the rest is not.

2.3. Cluster-based Selection of Informed Agents

The random selection of informed agents does not guarantee an even spatial distribution of informed agents, thus leaving certain parts of the network highly influenced by the virtual leader while the rest is not. In order to achieve an even distribution of informed agents, a cluster-based selection method is proposed in [19]. It first groups the agents into M_0 clusters such that $\sum_{j=1}^{M_0}\sum_{\forall i \in C_j}\|q_i - \mu_j\|^2$ is minimized. The centroid of a cluster C_j is denoted as $\mu_j \in \mathbb{R}^n$. An agent that is closest to the centroid of a cluster is selected as an informed agent.

Fig. 2 illustrates cluster-based assignment of 5 informed agents for the same network of 25 agents given in Fig. 1. Agents that are represented by same shape belong to the same cluster. Filled markers represent the informed agents and the rest is uninformed. As it is observed from Fig. 2, the cluster-based method enables a fair distribution of the informed agents according the initial spatial density of the agents. According to the results presented in [19], cluster-based informed agent selection can considerably increase the number of following agents compared to the random selection of informed agents. However, one clear drawback in this method is that it does not considers the neighboring connections between agents when they are clustered. Therefore, agents which are not directly connected to each other might be included in the same cluster. For example, agents $\{3, 19, 22\}$ in Fig. 2 belong to the same cluster even though there are no direct connections exist between $\{3\}$ and $\{19, 22\}$. Also, the number of clusters is not an optimized value for each distribution of agents, but a user defined arbitrary value.

3. PROPOSED COMMUNITY-BASED SELECTION OF INFORMED AGENTS

In order to avoid the drawbacks of cluster-based method, in this paper, we propose a community-based method for selection of informed agents for flocking with a virtual leader. Community structures present in many networks where vertices within the same community are more likely to be connected to each other than vertices in other communities. Properties and dynamics of these networks can be understood by uncovering such communities. In groups of dynamic agents, community structures reveal the organization and interactions among the agents. Recently, a considerable number of attempts have been devoted for detecting the community structures in networks [20–22]. Many community detecting methods are based on a benefit function called *modularity* [23]. Higher values of the modularity correspond to the existence of stronger community structures. In this paper, we utilize Newman's fast algorithm [24] for community detection in the groups of agents.

The modularity for quantifying communities in a network of dynamic agents is based on the edge density in a subnetwork of the network with compared to a *null-model*. The null-model can be defined as a subnetwork with the same number of vertices, edges, and degree distribution as the original subnetwork, but the edges are randomly placed. The probability of having vertex i connected to vertex j in the null-model can be given by $P_{ij} = k_i k_j / 4m^2$,

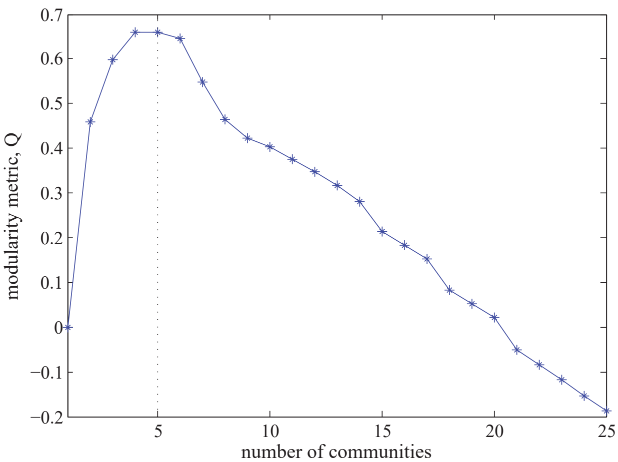

Fig 3: The modularity for the hierarchical community structure detected on the network given in Fig. 1. The modularity metric reach is maximum of 0.6588 when the number of communities are equal to 5.

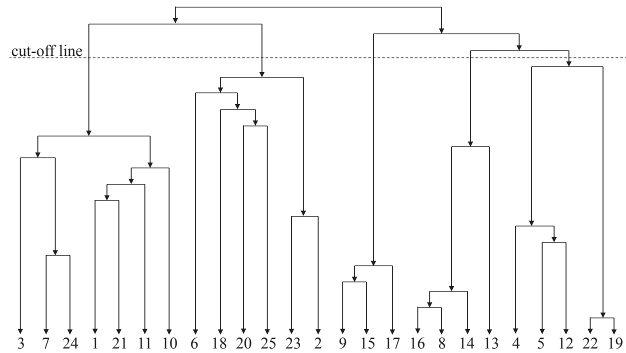

Fig 4: A dendrogram illustrating the hierarchical community structure of the network given in Fig. 1. The cut-off line can be decided based on the highest modularity value.

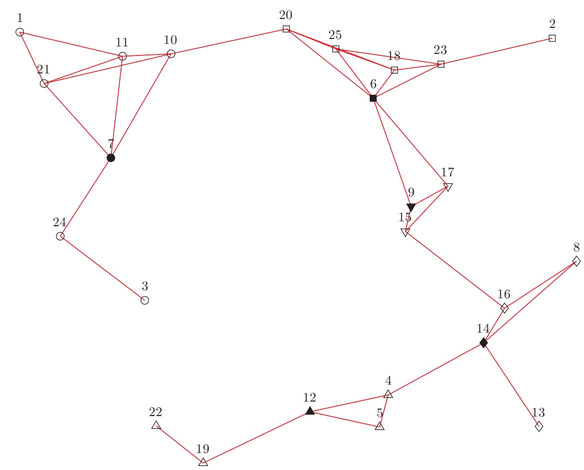

Fig 5: Community structures detected using the modularity metric for the network given in Fig. 1. Solid lines represent the neighboring relations. Filled markers represent the informed agents and empty markers represent uninformed agents.

where m is the total number of edges in the network. Degrees of vertices i and j are denoted by k_i and k_j, respectively. The probability of having vertex i connected to vertex j in the original subnetwork can be given by $a_{ij}/2m$. Therefore, the Newman's modularity is defined as

$$Q = \frac{1}{2m}\sum_{i,j\in V}\left(a_{ij} - \frac{k_i k_j}{2m}\right)\delta(\check{C}_i,\check{C}_j), \tag{5}$$

where vertices i and j respectively belong to communities $\check{C}_i$ and $\check{C}_j$. If $\check{C}_i = \check{C}_j$ then $\delta = 1$, otherwise $\delta = 0$. If Q is close to 1, a strong community structure can be observed in the network. The modularity metric Q tends to 0 as the number of edges within a community gets close to random. Also, $Q = 0$ if all the vertices in a network belong to a single community.

The Newman's fast algorithm is an agglomerative algorithm which takes the sole vertices as inputs. These vertices are considered as the initial communities. Then it iteratively joins these initial communities together in pairs such that it results in greatest increase or smallest decrease in Q. Fig. 3 illustrates the resulted modularity metric for the network of 25 agents given in Fig. 1 by executing the fast algorithm on it. Hierarchical results of this iterative process of community merging can be visualized by using a dendrogram as shown in Fig. 4. A cut-off level of the dendrogram is decided according to the maximum modularity value as it corresponds to the strongest community structure of the network. Therefore, modularity can be used as an effective measure of the number of communities. This is a added advantage over cluster-based method in which number of clusters is an arbitrary value. In this example, Q reaches its maximum of 0.6588 when the number of communities are equal to 5.

The strongest community structure obtained by using the fast algorithm for the network given in Fig. 1 is shown in Fig. 5. Agents that are represented by same shape belong to the same community. Filled markers represent the informed agents and empty markers represent uninformed agents. The highest degree vertex in each community is selected as an informed agent such that

$$h_i = \begin{cases} 1, & \text{if } i = \operatorname*{argmax}_{s\in\check{C}_j}(k_s). \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Thus, we can summarize the informed agent selection process as below:

1. Identify the location of each agent.
2. Calculate the adjacency matrix of the group of agents using (3).
3. Determine the best community structure using the adjacency matrix.

4. Select the highest degree agent from each community as an informed agent (6).

As explained in the previous section, initial clusters obtained using k-means algorithm may contain disconnected components. Therefore, an informed agent assigned for such a cluster may not be able to influence all the agents within the cluster to follow the virtual leader, which may cause to degrade the effectiveness of cluster-based selection method. However, this is not surprising since connections between agents are ignored during the clustering process. In contrast, while using the proposed community-based informed agent selection method, the following proposition holds for any initial distribution of the agents.

Proposition 1: All communities in the strongest community structure that is obtained by optimizing the modularity, are connected components unless a vertex itself is a community.

Proof: The modularity defined in (5) can be rewritten as

$$Q=\frac{1}{2m}\sum_{n}\sum_{i\in\check{C}_n,j\in\check{C}_n}\left(a_{ij}-\frac{k_ik_j}{2m}\right).$$

Without loss of generality, let us assume a network consists of two disconnected communities, $\check{C}_1$ and $\check{C}_2$. The corresponding modularity value to these two communities can be obtained as

$$Q_1=\frac{1}{2m}\sum_{n=1}^{2}\sum_{i\in\check{C}_n,j\in\check{C}_n}\left(a_{ij}-\frac{k_ik_j}{2m}\right).$$

If $\check{C}_1$ and $\check{C}_2$ are merged into a single community $\check{C}$, the modularity value is updated as

$$Q_2=\frac{1}{2m}\sum_{i\in\check{C},j\in\check{C}}\left(a_{ij}-\frac{k_ik_j}{2m}\right),$$

and the change in modularity can be obtained as

$$\begin{aligned}\Delta Q&=Q_2-Q_1,\\&=\frac{1}{2m}\Bigg[\sum_{i\in\check{C},j\in\check{C}}\left(a_{ij}-\frac{k_ik_j}{2m}\right)-\sum_{i\in\check{C}_1,j\in\check{C}_1}\left(a_{ij}-\frac{k_ik_j}{2m}\right)\\&\quad-\sum_{i\in\check{C}_2,j\in\check{C}_2}\left(a_{ij}-\frac{k_ik_j}{2m}\right)\Bigg].\end{aligned}\tag{7}$$

Since $\check{C}_1$ and $\check{C}_2$ are disconnected communities, one has

$$\sum_{i\in\check{C}}\sum_{j\in\check{C}}a_{ij}=\sum_{i\in\check{C}_1}\sum_{j\in\check{C}_1}a_{ij}+\sum_{i\in\check{C}_2}\sum_{j\in\check{C}_2}a_{ij}.\tag{8}$$

Using (7) and (8), the change in modularity can be reduced to

$$\Delta Q=-\frac{1}{4m^2}\left[\sum_{i\in\check{C}}\sum_{j\in\check{C}}k_ik_j-\sum_{i\in\check{C}_1}\sum_{j\in\check{C}_1}k_ik_j-\sum_{i\in\check{C}_2}\sum_{j\in\check{C}_2}k_ik_j\right].\tag{9}$$

However, $\check{C}=\check{C}_1\cup\check{C}_2$, hence, one has

$$\sum_{i\in\check{C}}\sum_{j\in\check{C}}k_ik_j=\sum_{i\in\check{C}_1}\sum_{j\in\check{C}_1}k_ik_j+\sum_{i\in\check{C}_2}\sum_{j\in\check{C}_2}k_ik_j+2\sum_{i\in\check{C}_1}\sum_{j\in\check{C}_2}k_ik_j.\tag{10}$$

Using (9) and (10), ΔQ can be further reduced to

$$\Delta Q=-\frac{1}{2m^2}\sum_{i\in\check{C}}\sum_{j\in\check{C}}k_ik_j,\tag{11}$$

which is always a negative quantity unless $\check{C}_1$ and $\check{C}_2$ are communities with sole vertices. Now, let us assume that strongest community structure which corresponds to maximum modularity contains a community with disconnected components. However, according to (11), the value of modularity reduces when disconnected components merged into a single community. The modularity cannot reached its maximum in such an occurrence, thus, the assumption of the strongest community structure contains a community with disconnected components cannot hold. Therefore, the proposition is proved. □

With the proven connectivity in each community corresponding to maximum community structure, the informed agents selected by using the proposed community-based method, have a higher possibility to influence uninformed agents to follow the virtual leader, compared to random and cluster-based selection methods.

4. SIMULATION STUDY

We evaluate and analyze the performances of the proposed community-based informed agents selection method against the random [12] and cluster-based [19] selection methods using computer simulations. Simulation settings and simulation results are presented in this section. The main objectives of our simulation study are to understand the applicability and performances of the proposed informed agents selection method under different conditions. Moreover, we try to understand the relationship of the fraction of agents that eventually move with the desired velocity η with the fraction of informed agents $\delta=M_0/N$ and the initial density of the informed agents ρ. Here, we are not interested in studying the effect on η by changing N, which has already been studied in previous research work. According to [3, 12], the larger the group size N, the smaller the fraction of informed agents δ needed to guide the group with a given fraction η.

The first set of simulations were performed on 100 agents ($N=100$) moving in a 2-dimensional ($n=2$) space under the influence of the control protocol (4). Initial positions and velocities of the 100 agents were randomly chosen from the boxes $[0,50]\times[0,50]$ and $[-4,4]\times[-4,4]$, respectively. The initial position and velocity of the virtual leader were set at $q_\gamma(0)=[25,25]^{\mathrm{T}}$ and $q_\gamma(0)=[1,1]^{\mathrm{T}}$ correspondingly. The interaction range was chosen as $r=$

4.8, the desired distance $d = 4$, $\varepsilon = 0.1$ for the σ-norm, $h = 0.7$ for the bump function, $a = 1$ and $b = 2$ for the action function, and $c_1 = 0.1$ and $c_2 = 0.4$ for the control protocol. The number of informed agents M_0 is decided using the number of communities which returns the maximum modularity value in the hierarchical community structure. For the current setup, the modularity metric reaches its maximum of 0.8793 when the number of communities is equal to 20, *i.e.* $M_0 = 20$.

Simulation results for community- and cluster-based selection methods are respectively shown in Figs. 6 and 7. Solid lines in the figures represent neighboring relations, arrowheads represent velocities of the agents, and hexagrams represent positions of the virtual leaders. The informed agents are marked with circles. The agents represented in same color arrowheads belong to the same initial community and cluster in Figs. 6 and 7, respectively. The 20 communities identified at the initial frame ($t = 0$) are well-defined as none of the disconnected components belong to the same community. When the informed agents are selected based on the initial community structure, 84 agents move with the desired velocity ($\eta = 0.84$) at $t = 50$. For the same setup, if the informed agents are selected based on 20 initial clusters, $\eta = 0.76$ (see the frame at $t = 50$ in Fig. 7). In this particular case, the community-based selection method outperformed the cluster-based selection method.

To further substantiate the results shown in Figs. 6 and 7, more simulations were performed by evaluating η against $\rho \in [0.01, 0.1]$. The simulation results given in Fig. 8 are average results over 50 simulations each. For a fair comparison, the following parameters remained fixed throughout all simulations: $N = 100$, $r = 4$, $d = 3.3$, $\varepsilon = 0.1$, $a = 1$, $b = 2$, $h = 0.6$, $c_1 = 0.1$, and $c_2 = 0.4$. Initial positions and velocities of the agents were randomly selected from a $[0, L] \times [0, L]$ box ($\rho = N/L^2$) and a $[-0.5, 0.5] \times [-0.5, 0.5]$ box, respectively. The initial position and velocity of the virtual leader were set at $q_\gamma(0) = [L/2, L/2]^{\mathrm{T}}$ and $p_\gamma(0) = [2, 2]^{\mathrm{T}}$, respectively. Here, δ is decided using the number of communities which returns the maximum modularity value in the hierarchical community structure. The average number of communities for each initial density is indicated by using the blue dashed line in Fig. 8.

According to the results given in Fig. 8, when the number of informed agents are selected based on the best community structure, the community-based selection method outperformed the cluster-based selection method in terms of η for a majority of ρ values. In the given results, the only time it is outperformed by the cluster-based selection method is when $\rho = 0.01$. At $\rho = 0.01$, $\langle k \rangle = 0.2284$. *i.e.* most of the agents are isolated. In such sparse networks, obviously, community detection is not very effective. The simulation results show that the community-based informed agents selection method can drive more agents to follow a virtual leader compared to other methods under test when the best community structure is selected and the highest connection degree nodes in each community are elected as informed agents.

5. CONCLUSION

In this paper, we proposed a community-based informed agents selection method for flocking with a virtual leader. Extensive simulations were performed to test and evaluate the performances of the proposed method against random and cluster-based informed agents selection methods which have been utilized in previous work. According to the results of the simulations, the cluster-based and community-based selection methods show superior results over the random selection method. Interestingly, if the number of informed agents are selected based on the best community structure which maximizes the modularity, then the community-based selection method outperforms the cluster-based selection method. Therefore, modularity optimization can be useful in optimizing the number of informed agents to drive majority of the agents to their common objective.

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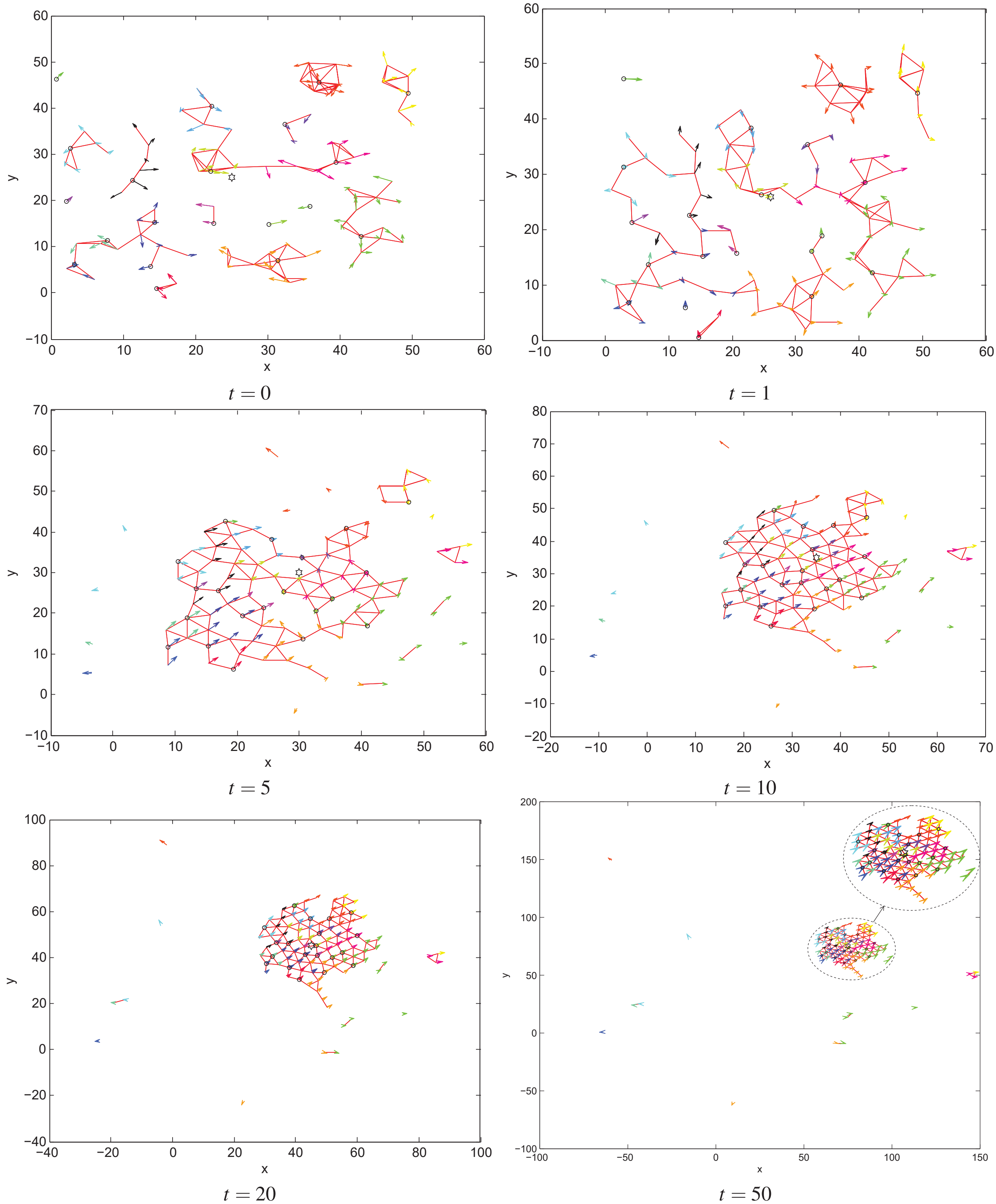

Fig 6: Community-based selection of informed agents with number of informed agents are selected on the best initial community structure, which is 20 in this case. At $t = 50$, there are 84 agents move with desired velocity.

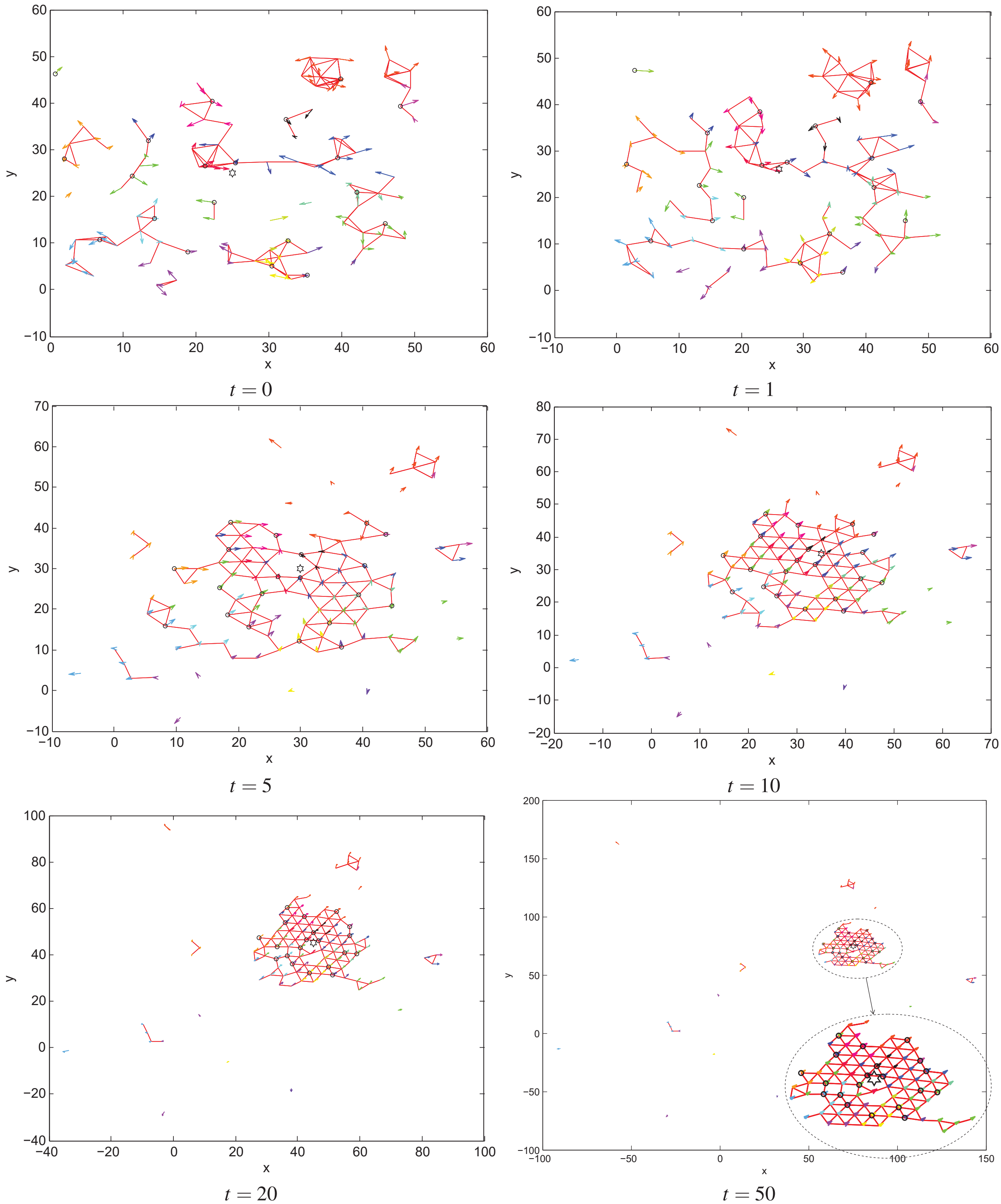

Fig 7: Cluster-based selection of informed agents with number of informed agents are selected on the best initial community structure, which is 20 in this case. All the parameters remain same as in Fig. 6. At $t = 50$, there are 76 agents move with desired velocity.

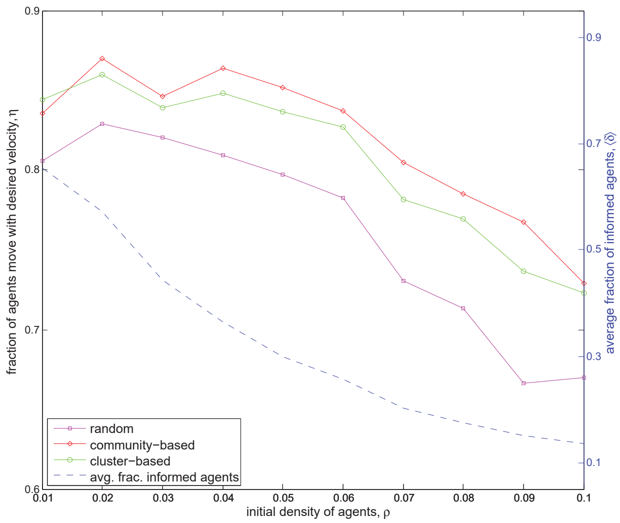

Fig 8: Fraction of agents with desired velocity as a function of the initial density of agents. All estimates are the results of averaging over 50 realizations with $N = 100$. The number of informed agents are selected based on the best community structure.

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