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Research Article

The Cuntz Comparison in the Standard C^*-Algebra

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The Cuntz comparison, introduced by Cuntz in early 1978, associates each C^*-algebra with an abelian semigroup which is an invariant for the classification of the nuclear C^*-algebras and called the Cuntz semigroup. In this paper, we study the Cuntz comparison in the standard C^*-algebra. We characterize the Cuntz comparison in terms of the dimension of the operator range. Also, we consider the structure of the semilinear map which preserves the Cuntz comparison.

1. Introduction and the Statement of Results

Throughout this paper, let $\mathscr{H}$ and $\mathscr{K}$ be complex Hilbert spaces, let $\mathscr{B}(\mathscr{H},\mathscr{K})$ be the algebra of all bounded linear operators from $\mathscr{H}$ into $\mathscr{K}$, and abbreviate $\mathscr{B}(\mathscr{H},\mathscr{H})$ to $\mathscr{B}(\mathscr{H})$. For an operator $T \in \mathscr{B}(\mathscr{H},\mathscr{K})$, by T^*, $\mathscr{N}(T)$, and $\mathscr{R}(T)$ we denote the adjoint, the null space, and the range of T, respectively. An operator $T \in \mathscr{B}(\mathscr{H})$ is said to have finite rank if $\mathscr{R}(T)$ is finite, dimensional, and, in this case, we write $\operatorname{rank}(T) = \dim(\mathscr{R}(T))$. Denote by $\mathscr{F}(\mathscr{H})$ the ideal of all finite rank operators in $\mathscr{B}(\mathscr{H})$. A standard C^*-algebra acting on the Hilbert space $\mathscr{H}$ is a C^*-subalgebra of $\mathscr{B}(\mathscr{H})$ which contains the identity I and the ideal $\mathscr{F}(\mathscr{H})$. In this paper, we always assume that $\mathscr{A}$ and $\mathscr{B}$ are standard C^*-algebras acting on $\mathscr{H}$ and $\mathscr{K}$, respectively. Furthermore, we denote by $\mathscr{A}_+$ the positive cone of all positive elements in $\mathscr{A}$.

In [1], Cuntz introduced a notion of the comparison for positive elements which extends the usual Murray-von Neumann comparison for projections in the C^*-algebra. This comparison that we will denote by $\precsim$ is nowadays called the Cuntz comparison.

Definition 1 (see [1]). Let $A, B \in \mathscr{A}_+$. One writes $A \precsim B$, if there exists a sequence $(X_n)_{n=1}^{\infty}$ of elements in $\mathscr{A}$ such that

$$A = \lim_{n\to\infty} X_n B X_n^*. \tag{1}$$

In this case, we say that A is Cuntz subequivalent to B. Furthermore, we say that A is Cuntz equivalent to B and write $A \sim B$, if $A \precsim B$ and $B \precsim A$.

The Cuntz comparison plays an important role in Elliott's program for the classification of the nuclear separable simple C^*-algebras. Indeed, the Cuntz comparison associates each C^*-algebra with an abelian semigroup which is an invariant for the classification of the nuclear C^*-algebras and called the Cuntz semigroup. Recently, it has been studied intensively by many authors (see [2–7]). In the present paper, we study the Cuntz comparison in the standard C^*-algebra.

In Section 2, we characterize the Cuntz comparison in terms of the dimension of the operator range. To classify C^*-algebras via their Cuntz semigroups, one will prove the uniqueness and existence theorem for homomorphisms between C^*-algebras. The uniqueness theorem says that if a semigroup map between the Cuntz semigroups of C^*-algebras $\mathscr{A}$ and $\mathscr{B}$ is induced by two homomorphisms α and β between C^*-algebras $\mathscr{A}$ and $\mathscr{B}$, then $\beta = Adu \circ \alpha$ for some unitary $u \in \mathscr{B}$. Motivated by the investigation of this uniqueness theorem and the extensive study of the preserver problems in matrix spaces or general operator algebras (see [8–15]), we discuss the structure of the semilinear map ϕ between $\mathscr{A}_+$ and $\mathscr{B}_+$ which preserves the Cuntz comparison. Recall that a map $\phi : \mathscr{A}_+ \to \mathscr{B}_+$ is said to be semilinear if ϕ is additive, and $\phi(\lambda A) = \lambda\phi(A)$ for all positive numbers

λ and $A \in \mathscr{A}_+$. Moreover, it is said to preserve the Cuntz comparison, if $\phi(A) \precsim \phi(B)$ whenever $A \precsim B$. In Section 3, we present some results for the semilinear map which preserves the Cuntz comparison.

2. The Cuntz Comparison in the Standard C*-Algebra

In this section, we characterize the Cuntz comparison in terms of the dimension of the operator range.

Lemma 2. *Let $A, B \in \mathscr{A}_+$ with $A \precsim B$ and B having finite rank. Then*

$$\operatorname{rank}(A) \leq \operatorname{rank}(B). \tag{2}$$

Proof. Let $\operatorname{rank}(B) = k$. Then one has $e_1, \ldots, e_k \in \mathscr{H}$ such that

$$B = \sum_{i=1}^{k} e_i \otimes e_i, \tag{3}$$

where $e_i \otimes e_i$ is the rank-1 operator satisfying $(e_i \otimes e_i)(h) = \langle h, e_i \rangle e_i$ for all $h \in \mathscr{H}$.

Since $A \precsim B$, there exists a sequence $\{X_n\}_{n=1}^{\infty}$ of elements in $\mathscr{A}$ such that

$$A = \lim_{n\to\infty} X_n B X_n^* \tag{4}$$

with respect to the norm topology on $\mathscr{B}(\mathscr{H})$. It follows that

$$A = \lim_{n\to\infty} X_n \left(\sum_{i=1}^{k} e_i \otimes e_i \right) X_n^* = \lim_{n\to\infty} \sum_{i=1}^{k} X_n(e_i) \otimes X_n(e_i) \tag{5}$$

with respect to the norm topology on $\mathscr{B}(\mathscr{H})$. So the sequence $\{\sum_{i=1}^{k} X_n(e_i) \otimes X_n(e_i)\}_{n=1}^{\infty}$ is bounded, and thus, for each i, the sequence $\{X_n(e_i)\}_{n=1}^{\infty}$ is bounded. Now, one has a subsequence $\{n_j\}_{j=1}^{\infty}$ and a sequence $\{\widetilde{e}_1, \widetilde{e}_2, \ldots, \widetilde{e}_k\}$ of elements in $\mathscr{H}$ such that, for each i,

$$\lim_{j\to\infty} X_{n_j}(e_i) = \widetilde{e}_i \tag{6}$$

with respect to the weak topology on $\mathscr{H}$. Consequently,

$$\lim_{j\to\infty} \sum_{i=1}^{k} X_{n_j}(e_i) \otimes X_{n_j}(e_i) = \sum_{i=1}^{k} \widetilde{e}_i \otimes \widetilde{e}_i \tag{7}$$

with respect to the weak operator topology on $\mathscr{B}(\mathscr{H})$. By formulas (5) and (7),

$$A = \sum_{i=1}^{k} \widetilde{e}_i \otimes \widetilde{e}_i. \tag{8}$$

Thus A has finite rank, and $\operatorname{rank}(A) \leq k = \operatorname{rank}(B)$. □

If $A, B \in \mathscr{A}_+$ and there exists an element $X \in \mathscr{A}$ such that $A = XBX^*$, then $A \precsim B$ by Definition 1. The converse statement is not true in the general case (see [2]). But, we have the following.

Theorem 3. *Let $A, B \in \mathscr{A}_+$ with at least one of them having finite rank. Then the following statements are equivalent:*

(a) $A \precsim B$,

(b) $\dim(\mathscr{R}(A)) \leq \dim(\mathscr{R}(B))$,

(c) $A = XBX^*$ *for some finite rank operator* $X \in \mathscr{F}(\mathscr{H})$.

Proof. It is clear that (c)⇒(a).

(a)⇒(b). Suppose that $A \precsim B$. By Lemma 2, the desired inequality clearly holds if B has finite rank. Now, we suppose that $\dim(\mathscr{R}(B))$ is infinite. Then A must have finite rank by the given assumption. Thus, the inequality holds too.

(b)⇒(c). Suppose that $\dim(\mathscr{R}(A)) \leq \dim(\mathscr{R}(B))$. Let A and so $A^{1/2}$ have finite rank k. As $B \geq 0$, $\dim(\mathscr{R}(B)) = \dim(\mathscr{R}(B^{1/2}))$ and thus we see that

$$\dim\left(\mathscr{R}\left(B^{1/2}\right)\right) \geq k. \tag{9}$$

Pick a k-dimensional subspace $\mathscr{H}_1$ of $\mathscr{R}(B^{1/2})$ and denote by $P_{\mathscr{H}_1}$ the orthogonal projection of $\mathscr{H}$ onto $\mathscr{H}_1$. Then $P_{\mathscr{H}_1} B^{1/2}$ has rank k. Consequently, there is an invertible element $U \in \mathscr{A}$ such that

$$A^{1/2} = U P_{\mathscr{H}_1} B^{1/2}. \tag{10}$$

It follows that

$$A = U P_{\mathscr{H}_1} B P_{\mathscr{H}_1} U^*. \tag{11}$$

And thus the rank-k operator $X = U P_{\mathscr{H}_1}$ does the job. □

Theorem 3 shows the relation between the Cuntz comparison and the dimension of the operator range. Moreover, we can characterize the rank-k positive operator in terms of the Cuntz equivalence as follows.

Corollary 4. *Let $A \in \mathscr{A}_+$. The following statements are equivalent:*

(a) $\operatorname{rank}(A) = k$,

(b) $A \sim X$ *for all* $X \in \mathscr{A}_+$ *with* $\operatorname{rank}(X) = k$,

(c) $A \sim X$ *for some* $X \in \mathscr{A}_+$ *with* $\operatorname{rank}(X) = k$.

Proof. It is clear that (b)⇒(c).

(a)⇒(b). If $X \in \mathscr{A}_+$ and $\operatorname{rank}(X) = k$, then $A \precsim X$ and $X \precsim A$ by Theorem 3, and so $A \sim X$ by Definition 1.

(c)⇒(a). Suppose that $A \sim X$ and $\operatorname{rank}(X) = k$. Then $A \precsim X$ and $X \precsim A$ by Definition 1. From Theorem 3, it follows that $\operatorname{rank}(A) \leq \operatorname{rank}(X)$ and $\operatorname{rank}(X) \leq \operatorname{rank}(A)$. Thus, $\operatorname{rank}(A) = \operatorname{rank}(X) = k$. □

3. Preserver of the Cuntz Comparison

In this section, we focus our attention on the semilinear map $\phi : \mathscr{A}_+ \to \mathscr{B}_+$ which preserves the Cuntz comparison. The map $\phi : \mathscr{A}_+ \to \mathscr{B}_+$ is said to be semilinear if ϕ is additive, and $\phi(\lambda A) = \lambda\phi(A)$ for all positive numbers λ and $A \in \mathscr{A}_+$. Moreover, it is said to preserve the Cuntz comparison, if $\phi(A) \precsim \phi(B)$ whenever $A \precsim B$. And it is said to preserve the Cuntz comparison in both directions when $\phi(A) \precsim \phi(B)$ if and only if $A \precsim B$. In a similar way, ϕ is said to preserve the Cuntz equivalence, if $\phi(A) \sim \phi(B)$ whenever $A \sim B$. And it is said to preserve the Cuntz equivalence in both directions when $\phi(A) \sim \phi(B)$ if and only if $A \sim B$.

For $A \in \mathscr{A}$, we denote by $\sigma_{\mathscr{A}}(A)$ the spectrum of A as an element in the C*-algebra $\mathscr{A}$.

Lemma 5 (see [16]). *Let $\mathscr{A}$ and $\mathscr{B}$ be unital C*-algebras with a common identity and norm such that $\mathscr{A} \subseteq \mathscr{B}$. If $A \in \mathscr{A}$, then $\sigma_{\mathscr{A}}(A) = \sigma_{\mathscr{B}}(A)$.*

Theorem 6. *Let $\phi : \mathscr{F}(\mathscr{H})_+ \rightarrow \mathscr{F}(\mathscr{K})_+$ be a semilinear surjective transformation. The following statements are equivalent.*

(a) *ϕ preserves the Cuntz comparison in both directions.*

(b) *ϕ preserves the Cuntz equivalence in both directions.*

(c) $\operatorname{rank}(\phi(A)) = \operatorname{rank}(A)$ *for all* $A \in \mathscr{F}(\mathscr{H})_+$.

Proof. (a)⇒(b). Let $A, B \in \mathscr{F}(\mathscr{H})_+$. Then $A \sim B$ if and only if $A \precsim B$ and $B \precsim A$, and, by (a), this is the case if and only if $\phi(A) \precsim \phi(B)$ and $\phi(B) \precsim \phi(A)$, or, equivalently, if and only if $\phi(A) \sim \phi(B)$.

(b)⇒(c). By (b) and Corollary 4, one can conclude that, for any $A, B \in \mathscr{F}(\mathscr{H})_+$,

$$\operatorname{rank}(A) = \operatorname{rank}(B) \Longleftrightarrow \operatorname{rank}(\phi(A)) = \operatorname{rank}(\phi(B)). \tag{12}$$

This induces an injective map f on N_0, the set of all nonnegative integers, such that, for any nonnegative integer $k \in N_0$,

$$\operatorname{rank}(A) = k \Longleftrightarrow \operatorname{rank}(\phi(A)) = f(k). \tag{13}$$

Furthermore, since ϕ is surjective, so is f. Thus f is a bijective map. We claim that f is indeed the identity map. Once the claim is proved, (c) of Theorem 6 clearly follows.

Suppose f is not the identity map. Since f is bijective, there exist k and l in N_0 such that

$$k < l, \qquad f(k) > f(l). \tag{14}$$

Now take an operator $B \in \mathscr{F}_l(\mathscr{H})_+$, where $\mathscr{F}_l(\mathscr{H})_+$ denotes the set of all rank l operators in $\mathscr{F}(\mathscr{H})_+$. One can always find two operators $B_1 \in \mathscr{F}_k(\mathscr{H})_+$ and $B_2 \in \mathscr{F}_{l-k}(\mathscr{H})_+$ such that

$$B = B_1 + B_2. \tag{15}$$

Since ϕ is additive and $\phi(B_2)$ is a positive operator, we have

$$\begin{aligned} f(l) &= \operatorname{rank}(\phi(B)) = \operatorname{rank}(\phi(B_1) + \phi(B_2)) \\ &\geq \operatorname{rank}(\phi(B_1)) = f(k) > f(l). \end{aligned} \tag{16}$$

Contradiction is reached.

(c)⇒(a). Let $A, B \in \mathscr{F}(\mathscr{H})_+$. Then $A \precsim B$ if and only if $\operatorname{rank}(A) \leq \operatorname{rank}(B)$ by Theorem 3, and this is the case if and only if $\operatorname{rank}(\phi(A)) \leq \operatorname{rank}(\phi(B))$ by (c), which again is the case if and only if $\phi(A) \precsim \phi(B)$ by Theorem 3. □

Now, we give an explicit version of the surjective transformation which preserves the Cuntz comparison in both directions.

Proposition 7. *Let $\phi : \mathscr{A}_+ \rightarrow \mathscr{B}_+$ be a surjective transformation. If $M \in \mathscr{B}(\mathscr{H}, \mathscr{K})$ is invertible and*

$$\phi(A) = MAM^* \tag{17}$$

for all $A \in \mathscr{A}_+$, then ϕ preserves the Cuntz comparison in both directions.

Proof. Let $A, B \in \mathscr{A}_+$ and $A \precsim B$. Then by Definition 1, one has a sequence $(X_n)_{n=1}^{\infty}$ of elements in $\mathscr{A}$ such that $A = \lim_{n\to\infty} X_n B X_n^*$. It follows that

$$\begin{aligned} \phi(A) &= MAM^* \\ &= \lim_{n\to\infty} MX_nM^{-1}MBM^*M^{*-1}X_n^*M^* \\ &= \lim_{n\to\infty} \left(MX_nM^{-1}\right)\phi(B)\left(MX_nM^{-1}\right)^*. \end{aligned} \tag{18}$$

To prove $\phi(A) \precsim \phi(B)$, it remains to show that $MX_nM^{-1} \in \mathscr{B}$ for all n.

Since $\mathscr{A}$ and $\mathscr{B}$ are the linear spans of $\mathscr{A}_+$ and $\mathscr{B}_+$, respectively, it is easy to see that $MX_nM^* \in \mathscr{B}$ for all n. Moreover, since MM^* is invertible in $\mathscr{B}(\mathscr{K})$ and $MM^* = \phi(I_{\mathscr{H}}) \in \mathscr{B}_+$, we conclude by Lemma 5 that $(MM^*)^{-1} \in \mathscr{B}_+$. Thus $MX_nM^{-1} = MX_nM^*(MM^*)^{-1} \in \mathscr{B}$ for all n.

Conversely, let $A, B \in \mathscr{A}_+$ with $\phi(A) \precsim \phi(B)$. Then by Definition 1, one has a sequence $(Y_n)_{n=1}^{\infty}$ of elements in $\mathscr{B}$ such that $\phi(A) = \lim_{n\to\infty} Y_n\phi(B)Y_n^*$. It follows that

$$\begin{aligned} A &= M^{-1}\phi(A)M^{*-1} \\ &= \lim_{n\to\infty} M^{-1}Y_n\phi(B)Y_n^*M^{*-1} \\ &= \lim_{n\to\infty} \left(M^{-1}Y_nM\right)B\left(M^{-1}Y_nM\right)^*. \end{aligned} \tag{19}$$

Now, to prove that $A \precsim B$, we show that $M^{-1}Y_nM \in \mathscr{A}$ for all n.

Since ϕ is surjective, $M^{-1}\mathscr{B}_+M^{*-1} = M^{-1}\phi(\mathscr{A}_+)M^{*-1} = \mathscr{A}_+$. Noting that $\mathscr{B}$ is the linear span of $\mathscr{B}_+$, $M^{-1}Y_nM^{*-1} \in \mathscr{A}$ for all n. Furthermore, since $M^{-1}M^{*-1}$ is invertible in $\mathscr{B}(\mathscr{H})$ and $M^{-1}M^{*-1} = M^{-1}I_{\mathscr{K}}M^{*-1} \in \mathscr{A}_+$, $M^*M = (M^{-1}M^{*-1})^{-1} \in \mathscr{A}_+$ by Lemma 5 again. So $M^{-1}Y_nM = (M^{-1}Y_nM^{*-1})(M^*M) \in \mathscr{A}$ for all n. □

Proposition 7 shows that if ϕ has the explicit form as formula (17), then ϕ preserves the Cuntz comparison in both directions. Unfortunately, the converse statement fails to hold. A counterexample, indebted to Professor Fangyan Lu, is presented as follows.

Example 8. Let $\mathscr{A} = M_2(\mathbb{C})$ and let ϕ be a map on $\mathscr{A}_+$ such that

$$\phi\left(\begin{pmatrix} a & b \\ \bar{b} & d \end{pmatrix}\right) = \begin{pmatrix} d & b \\ \bar{b} & a \end{pmatrix}. \tag{20}$$

Then it is easy to check that ϕ is a continuous semilinear surjective map and $\operatorname{rank}(\phi(A)) = \operatorname{rank}(A)$ for all $A \in \mathscr{A}_+$. From Theorem 6, it follows that ϕ preserves the Cuntz comparison in both directions.

Now, we show that ϕ does not have the explicit form as formula (17). Indeed, suppose on the contrary that there is a matrix $M = \begin{pmatrix} x & u \\ v & y \end{pmatrix}$ satisfying

$$\phi(A) = MAM^* \tag{21}$$

for all $A \in \mathscr{A}_+$. Then one has

$$\begin{pmatrix} x & u \\ v & y \end{pmatrix}\begin{pmatrix} a & b \\ \overline{b} & d \end{pmatrix}\begin{pmatrix} \overline{x} & \overline{v} \\ \overline{u} & \overline{y} \end{pmatrix} = \begin{pmatrix} d & b \\ \overline{b} & a \end{pmatrix} \tag{22}$$

for all $a, b, d \in \mathbb{C}$ with $a, d \geq 0$ and $ad \geq |b|^2$.

If we take $b = d = 0$ and $a = 1$ in (22), then we get $x = 0$ and $|v| = 1$. If we take $a = b = 0$ and $d = 1$ in (22), we get $y = 0$ and $|u| = 1$. Hence from formula (22), $u\overline{v}\overline{b} = b$ for all $b \in \mathbb{C}$, which is impossible.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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