Applied Physics Letters

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Cite as: Appl. Phys. Lett. **106**, 121906 (2015); https://doi.org/10.1063/1.4916678
Submitted: 01 February 2015 . Accepted: 12 March 2015 . Published Online: 27 March 2015

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APPLIED PHYSICS LETTERS **106**, 121906 (2015)

Damage characterization in plates using singularity of scale mode shapes

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(Received 1 February 2015; accepted 12 March 2015; published online 27 March 2015)

Damage is a prevailing physical phenomenon in in-service structures; accumulation of damage can cause catastrophic structural failure. For damage identification in plates, the concept of scale mode shape with fractal singularity is formulated based on 2D Gabor wavelet transform incorporating fractal dimension analysis of measured mode shapes. With this concept, a scale fractal complexity spectrum is created to reveal mode shape singularities by eliminating noise and interference. The singularity manifests the abnormality of the mode shape, clearly indicating damage. This study develops a philosophy of fusing wavelets and fractals to detect singularities of physical fields in noisy conditions. *© 2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4916678]

Damage is a prevailing physical phenomenon in in-service structures;[1,2] the accumulation of damage can result in catastrophic structural failure.[3,4] Damage in a structure can cause various changes in structural responses such as frequency shifts, mode shape variations, changes in mode shape curvature, changes in modal flexibility, variations in strain mode shape, and modal strain energy fluctuations.[5,6] Damage detection has been widely investigated in various engineering fields. Damage can be interpreted as a local physical abnormality of structural material or of geometric properties.[7–9] Detection of damage evokes the subject of structural damage identification. In most cases, structural damage detection relies on singularity analysis of structural dynamic responses. In this regard, the philosophy is that the physical abnormality caused by damage destroys the local signal structure of dynamic responses, leading to a singularity of dynamic response. Various methods relying on theories such as the Fourier transform,[10] Hilbert-Huang transform,[11] or singular value decomposition[12,13] have been developed for revealing singularities of mode shapes of beams. However, few methods have been formulated to address the singularity of mode shapes of plates, due to the much greater complexity induced by increases in the dimensions of mode shapes.

Fractals[14,15] have advanced the development of complexity analysis of the space of 2D physical fields, leading to attractive methods such as the surface area method,[16] the projective covering method,[17] and the cubic covering method.[18] In general, these methods calculate the fractal dimension[19] of a signal to produce a statistical index that characterizes the complexity of the signal. Unfortunately, this statistical index is inadequate to reflect the singularity of a mode shape, the reasons for which are twofold: (i) the statistical index comes largely from noise and trends rather than from the local singular component of the mode shape and (ii) as a global quantity, the statistical index is incapable of portraying the local complexity of a mode shape. Damage alters local complexity, in turn inducing singularity of a mode shape; however, this singularity is unlikely to be detected by existing fractal-based methods.

To explore the use of the fractals in singularity analysis, this study proposes a scale fractal singular feature of the space of 2D physical fields. This feature is formulated using the 2D Gabor wavelet transform to enhance the capability of the fractal dimension in singularity analysis.[20,21] Use of this feature to identify singularities of mode shapes provides a method for detecting damage in plates.

A mode shape acquired from a plate containing damage can be represented by $W : (x_i, y_j, w_{i,j})$, with $w_{i,j}$ being the deflection amplitude at the grid point (x_i, y_j). Damage locally alters the stiffness of the plate, evoking singularity of $W : (x_i, y_j, w_{i,j})$. By virtue of this effect, damage identification in a plate can be converted into singularity analysis of its mode shape. Unfortunately, the singularity of $W : (x_i, y_j, w_{i,j})$ is commonly obscured by measurement noise and trend interference. Disclosure of the singularity is difficult for methods based on conventional theories such as Fourier transform,[10] Hilbert-Huang transform,[11] and singular value decomposition.[12,13]

Wavelets[22] have advanced the development of scale analysis of the space of 2D physical fields. To explore the singularity, $W : (x_i, y_j, w_{i,j})$ is transformed by 2D Gabor wavelets[23]

$$\varphi_s(x,y)=\frac{f_{\max}^2}{2^s\pi\gamma\eta}\exp\left(\frac{f_{\max}^2x'^2}{2^s\gamma^2}-\frac{f_{\max}^2y'^2}{2^s\gamma^2}\right)\exp\left(\frac{j2\pi f_{\max}^2x'}{\sqrt{2^s}}\right), \quad (1)$$

where s is the scale parameter of $\varphi_s(x,y)$ and $x' = x\cos\theta + y\sin\theta$ and $y' = -x\sin\theta + y\cos\theta$. The transform is represented by a convolution regime

$$W^s = W \otimes \varphi_s(x_i, y_j), \quad (2)$$

where $\otimes$ denotes the convolution and $W^s : (x_i, y_j, w^s_{i,j})$ is a scale mode shape conveying the s-scale component of

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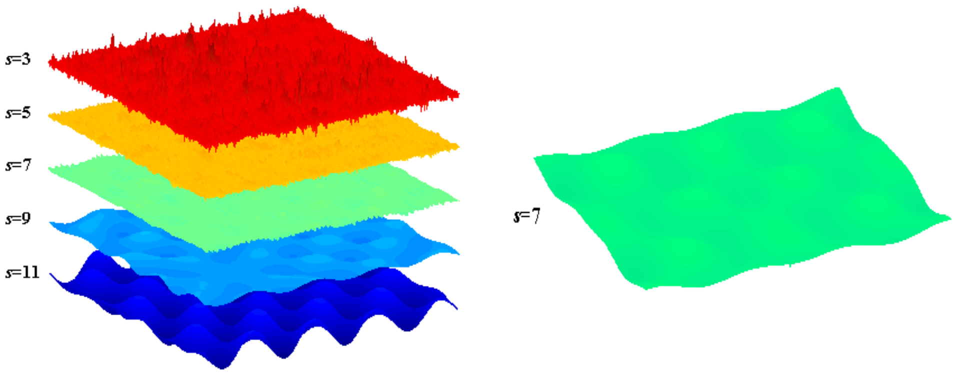


(a) (b)

FIG. 1. Array of scale mode shapes $W^s : (x_i, y_j, w^s_{i,j})$ (a) and the scale mode shape $W^7 : (x_i, y_j, w^7_{i,j})$ bearing the major damage component (b).

$W : (x_i, y_j, w_{i,j})$. In essence, $\varphi_s(x, y)$ is the scaled derivative of a 2D Gaussian function adjusted by a rotational angle θ and a dilation coefficients γ and η. $f_{\max}$ is the maximum frequency of the modulating sinusoidal plane wave. The set of parameter [θ, γ, η, and $f_{\max}$] endows $\varphi_s(x, y)$ with greater flexibility to characterize complex damage than other wavelets lacking these parameters.

Using Eq. (2), an array of scale mode shapes $W^s : (x_i, y_j, w^s_{i,j})$ of consecutively varied s can be derived from the mode shape $W : (x_i, y_j, w_{i,j})$ of a plate, as illustrated in Fig. 1. In the figure, the scales dominated by measurement noise are $s = 3$ and $s = 5$, and the scales dominated by trend interference are $s = 9$ and $s = 11$. Consequently, the scale mode shape carrying the damage component can be differentiated from the array of scale mode shapes as $s = 7$. Generally, a Shannon entropy[24] metric can provide a quantitative method to direct selection of the scale mode shape that carries the major damage component.

To reveal the singularity of $W^7 : (x_i, y_j, w^7_{i,j})$, a concept of fractal complexity is created based on the capacity dimension proposed by Kolmogorov:[25] assuming that a set Θ is covered by identically geometrical hypercubes with the side length r, the minimum number of hypercubes needed to cover set Θ is $N(r)$; if $N(r)$ is inversely proportional to $1/r$ as r tends towards zero, the expression

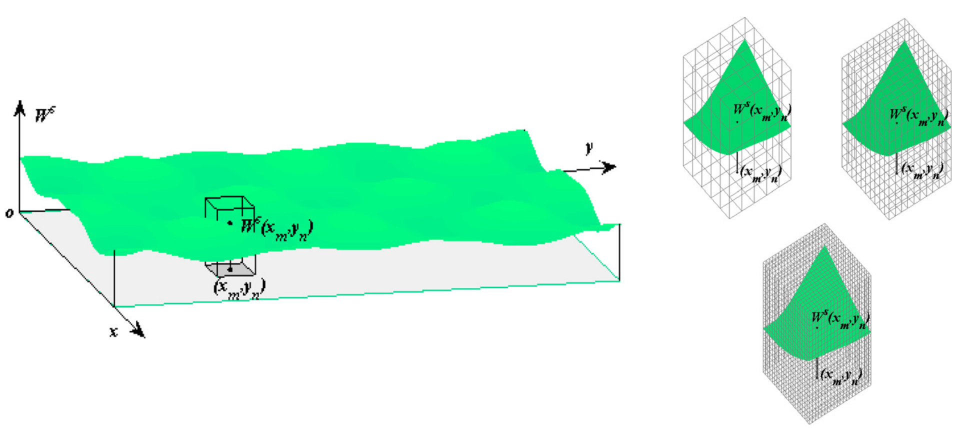
$$D_c(\Theta) = \lim_{r \to 0} \frac{\log(N(r))}{\log(1/r)} \tag{3}$$

defines the capacity dimension. $D_c(\Theta)$ provides a statistical index of the fractal complexity of set Θ.

A scale mode shape $W^s = W \otimes \varphi_s(x_i, y_j)$, $1 \le i \le P$, $1 \le j \le Q$, can be viewed as a cluster of sets $\Theta_{i,j}$ centered at (x_i, y_j). When capacity dimension analysis is consecutively performed on each set labeled by the central point (x_i, y_j), a set $D_c(\Theta_{i,j})$, $1 \le i \le P, 1 \le j \le Q$, termed the fractal complexity spectrum, can be produced to quantitatively represent the point-wise complexity of the scale mode shape. Damage alters the complexity of one or several $\Theta_{i,j}$ of $W^s : (x_i, y_j, w^s_{i,j})$, evoking singular values of the fractal complexity spectrum. Therefore, detection of singular values of the fractal complexity spectrum provides a method for identifying damage in plates.

Calculation of the capacity dimension is illustrated on an arbitrarily selected set $\Theta_{m,n}$ (Fig. 2(a)) centered at (x_m, y_n) of $W^s : (x_i, y_j, w^s_{i,j})$: a cuboid consisting of a number of hypercubes (hereinafter hypercubes refer to cubes), stemming from the x-y plane, intersects $W^s : (x_i, y_j, w^s_{i,j})$ at the intersectional region $\Theta_{m,n}$, such that $\Theta_{m,n}$ is covered by $N(r)$ hypercubes. To alternatively actualize the covering, the reduced r is correlated with the increased $N(r)$, as depicted in Fig. 2(b). When r gradually changes to zero, a sequence of $\{r_i, N(r_i)\}$ arises. From $\{r_i, N(r_i)\}$, $D_c(\Theta_{m,n})$ is calculated by Eq. (3) to quantify the fractal complexity of $\Theta_{m,n}$ at point (x_m, y_n). Iteration of this operation along each point of $W^s : (x_i, y_j, w^s_{i,j})$ produces its fractal complexity spectrum. Damage induces singular values of the fractal complexity spectrum, in turn signifying damage.

Damage identification by detecting singular values of a fractal complexity spectrum is examined on a glass fiber reinforced polymer (CFRP) plate. A CFRP plate of length



(a) (b)

FIG. 2. Illustration of calculating the capacity dimension. (a) A set $\Theta_{m,n}$ of $W^s : (x_i, y_j, w^s_{i,j})$ and (b) covering $\Theta_{m,n}$ with hypercubes of varying side length, i.e., r.

400 mm, width 400 mm, and thickness 3 mm in the x, y, and z directions (Fig. 3(a)) consists of four-ply unidirectional carbon fiber mats oriented in the x direction. A small damage zone is created by removing one layer of the carbon fiber in a local square region 20 mm × 20 mm. The plate is excited in the z direction under a natural frequency of 1139.06 Hz by a circular piezoelectric lead-zirconate-titanate actuator with diameter of 10 mm, located at the geometrical center of the plate. While the plate is vibrating, isochronous z-directional velocities of 374 × 374 measurement points evenly distributed on the intact surface are captured using a scanning laser vibrometer (Polytec PSV-400) to form the mode shape (Fig. 3(b)).

Representative scale mode shapes of W are illustrated by W^4, W^8, and W^{12} (Fig. 4(a)), respectively. Distinctly, W^4 is dominated by noise and W^{12} is governed by trend, each delivering little damage information. The intermediate transitional layer from W^4 to W^{12}, W^8, presents characteristics that are dissimilar from noise and trend, mostly conveying damage information, and it is therefore selected for use in damage detection. The fractal complexity spectrum $D_c(\Theta_{i,j})$, $1 \le i \le P, 1 \le j \le Q$ of W^8 (Fig. 4(b)) reveals a singular peak clearly standing out from its surroundings. This singular peak and its projection (Fig. 4(c)) on the x-y plane clearly indicate not only the location but also the shape of the damage region as depicted in Fig. 3(a).

To examine the function of the *scale* factor in portraying damage, the fractal complexity spectrum stemming from the original mode shape signal W in Fig. 3(b) is presented in Fig. 5(a), showing a cluster of ridge lines delineating the fluctuation of the primary peaks of the mode shape, but in which no singular feature responsible for damage can be identified. To further verify the advantage of the proposed method, an attempt was made to identify the damage in the plate using a damage identification technique developed by the classical curvature mode shape method.[26,27] Unfortunately, as demonstrated in Fig. 5(b), the features obtained from the curvature mode shape of W in Fig. 3(b) are too insignificant, incapable of characterizing such slight damage.

The fractal complexity spectrum calculated by pointwise fractal dimension estimations along the scale signal can be used to identify singularities by virtue of sudden changes

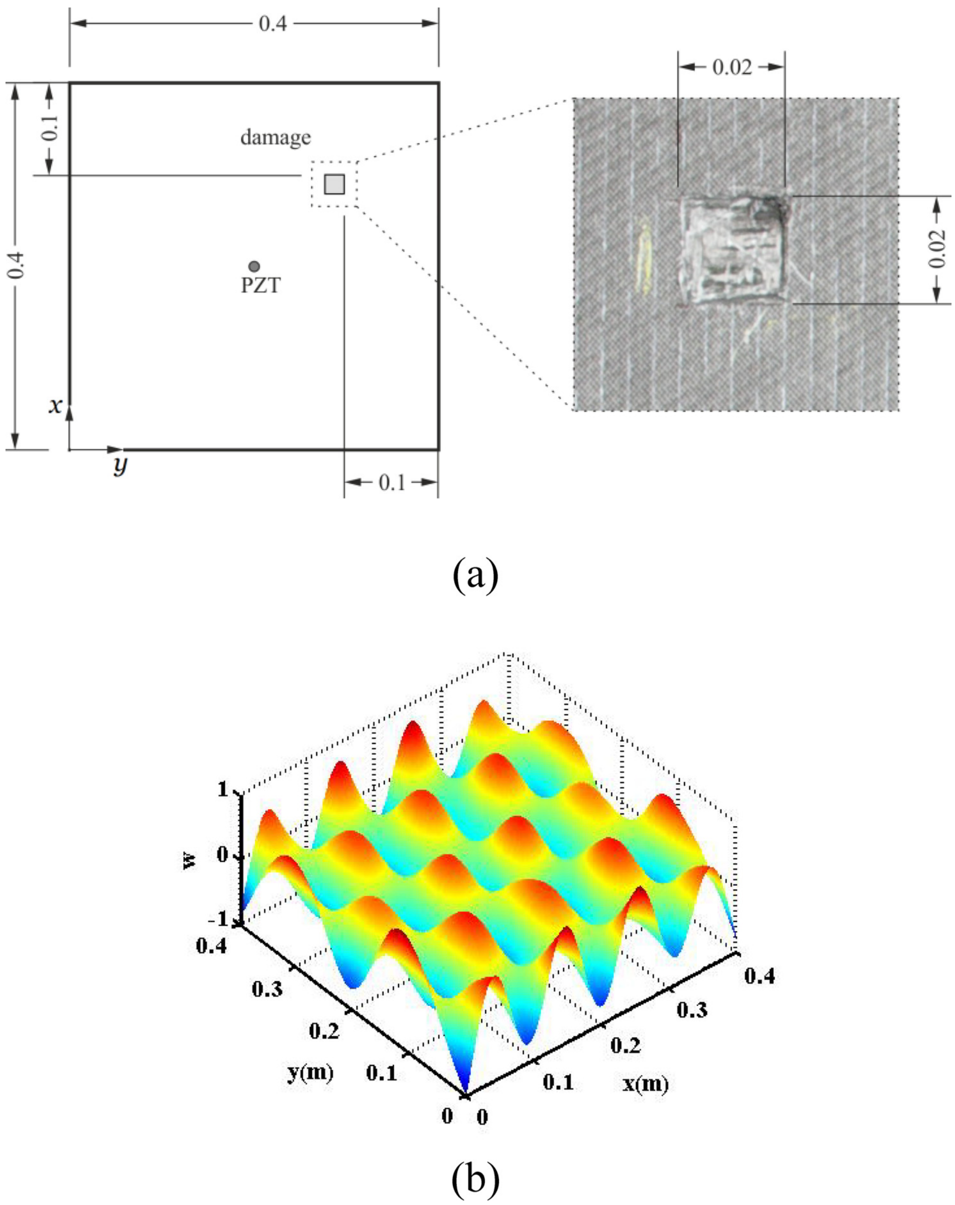


FIG. 3. Zoomed-in damaged zone of CFRP plate (unit: m) (a) and mode shape measured from its intact surface.

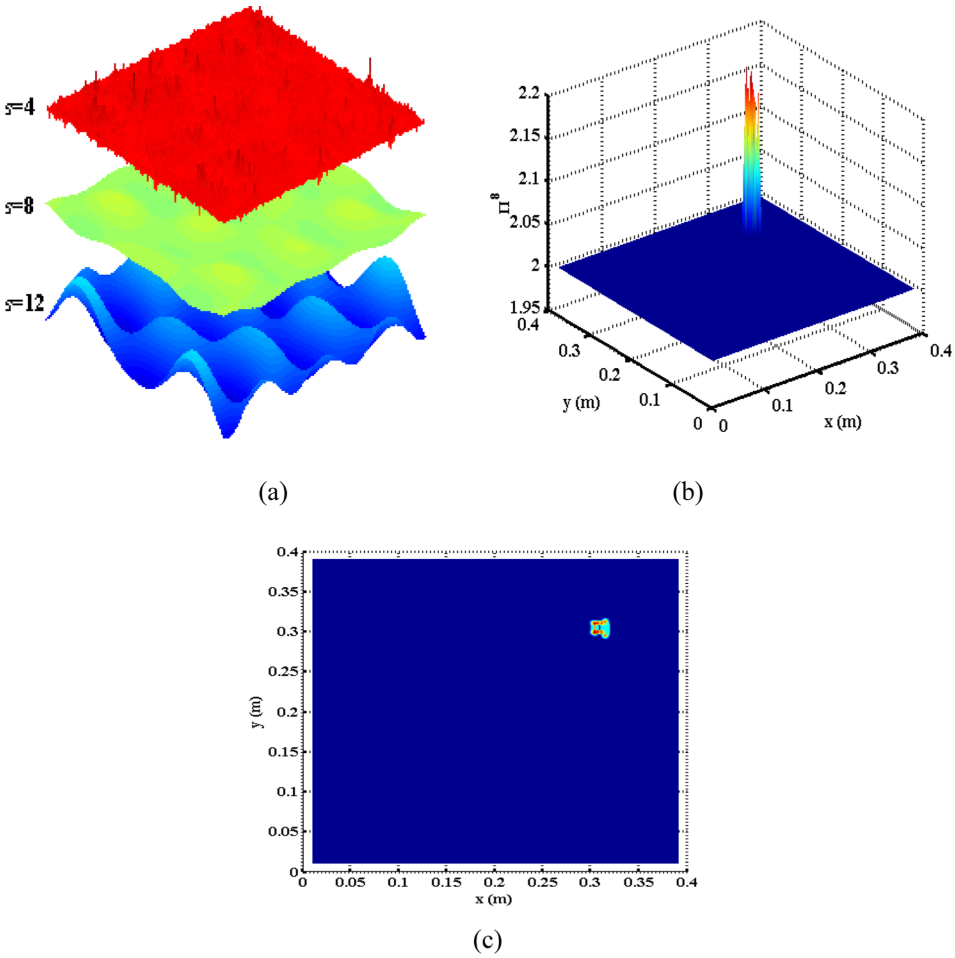


FIG. 4. (a) W^4, W^8, and W^{12} of W; (b) fractal complexity spectrum $D_c(\Theta_{i,j})$, $1 \le i \le P, 1 \le j \le Q$; and (c) planform of complexity spectrum.

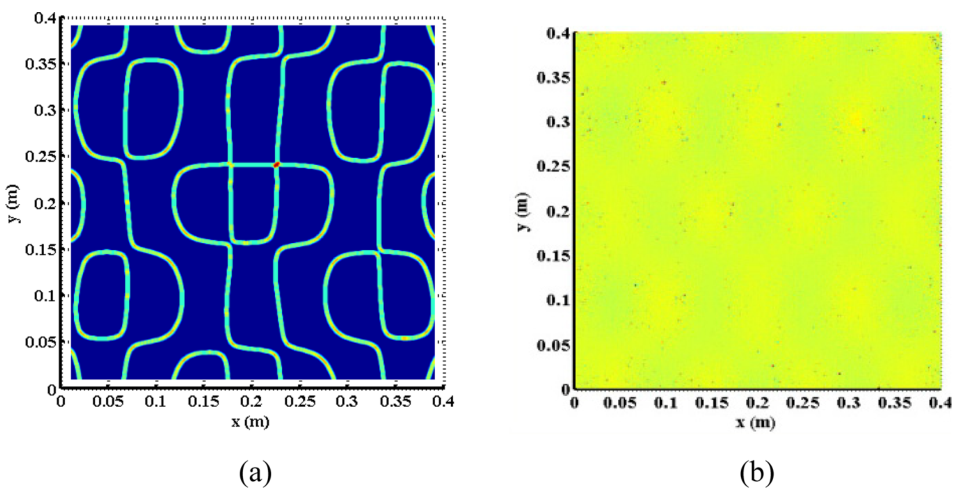


FIG. 5. Failure to identify damage by prevailing (a) fractal- and (b) curvature mode shape-based methods.

in its spatial variation. Compared with prevailing methods, the proposed approach demonstrates considerably improved precision of singularity identification, with much enhanced immunity to noise, as evidenced by our experiment detecting small damage contained in a composite plate using its mode shape.

As the results suggest, this investigation provides a philosophy of fusing advanced mathematical theories, wavelet, and fractal, to detect singularity in physical fields. It should be emphasized that besides damage identification, the proposed approach holds promise for other applications where the physical phenomena of singularity or abnormality need to be revealed in noisy conditions.

The authors gratefully acknowledge the financial support provided by the Natural Science Foundation of China (Grant No. 11172091), the China Postdoctoral Science Foundation (Grant No. 2014M560386), and the Natural Science Foundation of Shandong Province of China (Grant No. ZR2014EL034).

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