

Quantitative Evaluation of Residual Torque of A Loose Bolt Based on Wave Energy Dissipation and Vibro-Acoustic Modulation: *A Comparative Study*

Zhen ZHANG ^{a,b,‡}, Menglong LIU ^{a,‡}, Zhongqing SU ^{a,c,*} and Yi XIAO ^b

^a Department of Mechanical Engineering

The Hong Kong Polytechnic University, Hong Kong

^b School of Aerospace Engineering and Applied Mechanics

Tongji University, Shanghai, P. R. China

^c The Hong Kong Polytechnic University Shenzhen Research Institute

Shenzhen 518057, P. R. China

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[‡] PhD students

* To whom correspondence should be addressed. Tel.: +852-2766-7818, Fax: +852-2365-4703; Email: zhongqing.su@polyu.edu.hk (Prof. Zhongqing SU, *Ph.D.*)

Abstract

A wave energy dissipation (WED)-based linear acoustic approach and a vibro-acoustic modulation (VM)-based nonlinear method were developed comparatively, for detecting bolt loosening in bolted joints and subsequently evaluating the residual torque of the loose bolt. For WED-based, an analytical model residing on the Hertzian contact theory was established, whereby WED was linked to the residual torque of a loose bolt, contributing to a linear index. For VM-based, contact acoustic nonlinearity (CAN) engendered at the joining interface, when a pumping excitation **perturbs** a probing wave, was interrogated, and the nonlinear contact stiffness was described in terms of a **Taylor** series, on which basis a nonlinear index was constructed to associate spectral features with the residual torque. Based respectively on a linear and a nonlinear premise, the two indices were validated experimentally, and the results well coincided with theoretical predication. Quantitative comparison of the two indices surmises that the VM-based nonlinear method outperforms the WED-based linear approach in terms of sensitivity and accuracy, and particularly when the bolt loosening is in its embryo stage. In addition, the detectability of the nonlinear index is not restricted by the type of the joint, against a high dependence of its linear counterpart on the joint type.

Keywords: bolt loosening; wave energy dissipation; vibro-acoustic modulation; bolted joint; residual torque; structural health monitoring.

1 **1. Introduction**

2 Bolted joints are ubiquitous in engineering assets, performing a critical role in transferring
3 loads among interconnecting components [1]. The past decades have witnessed an escalation
4 of using bolted joints as a basic building block in complex engineering structures [2].
5 Throughout the service life of a bolted joint, a wide array of diatheses, including mainly
6 cyclic loads, adverse working conditions, mechanical or chemical attack, inappropriate
7 manipulation and structural ageing, can initiate and accelerate relaxation of the pre-load of
8 a bolt in the joint [3, 4], potentially leading to catastrophic consequences without timely
9 awareness. To put it into a perspective, approximately 20% of failure cases of mechanical
10 systems every year worldwide are reportedly owing to the self-loosening of threaded bolts
11 or fasteners, and about 10% of the lifetime of a machine is associated with the detection and
12 rectification of bolt loosening or related risks [5]. In the railway industry, wheel detachments
13 cause at least 20 reported accidents per week in the USA alone, and 45~50% of them are
14 found to be caused by loose wheel nuts [6]. Among hundreds of examples, the train derailling
15 in February 2007 near Lambrigg was a direct consequence of loosening of a wheel nut [7].
16 The vast hazard committed by bolt loosening, and those in critical load-bearing joints in
17 particular [8], has posed an impending need for periodic inspection and continuous
18 monitoring of the tension leftover of bolts, to assure an adequate level of integrity and
19 durability of joints, whose importance cannot be overemphasized. Effectual inspection and
20 monitoring, not only during installation of bolts but also throughout the intended service life,
21 warrants a reliable service of the joints, and the risk of system failure might accordingly be
22 weakened.

23

24 Nevertheless, the prevailing inspection techniques today to serve this purpose are still
25 represented by those that are somewhat primary (*e.g.*, visual and tapping inspection) and

1 performed at regularly scheduled intervals, or those that scrutinize changes in captured
2 global vibration signatures (*e.g.*, eigen-frequency, mode shape, strain energy or damping
3 properties [9]). Such a maintenance philosophy is highly subjective, fairly dependent on the
4 knowledge and experience of manipulation personnel. The sensitivity of these techniques
5 are usually inferior and sometimes questionable, until loosening of the bolt reaching a
6 significant level – this is because a loose bolt at its early stage would not induce perceivable
7 changes in global vibration signatures. For implementation, it is often a prerequisite to
8 suspend the normal operation of the inspected system temporarily, or in some occasions to
9 dismantle the bolted components from the system. Such an interruption or removal processes
10 can be time-consuming and labor-intensive, incurring exorbitant cost.

11

12 To circumvent the said deficiencies, a great deal of effort has been directed to the
13 development of alternative methodologies making good use of linear or nonlinear features
14 of acoustic waves, based on a promise that a loose bolt, showing somewhat similar behaviors
15 like damage, scatters incident acoustic waves propagating in the joint and introduces changes
16 in either linear signal features (*e.g.*, energy dissipation [10], delay in time-of-flight (ToF)
17 [11], reflection/transmission [12], and mode conversion[13]) or nonlinear signal features
18 (*e.g.*, high-order harmonic generation, and shift in resonance frequency [14-19]).
19 Representatively, Yang and Chang [20, 21] investigated the energy dissipation of Lamb
20 waves when traversing a bolted joint in a carbon-carbon composite thermal protection device.
21 Within a certain range of the pre-load, the transmitted energy was found to decrease with the
22 increase in the leftover torque of a loose bolt and vice versa. Estebana and Rogersb [10]
23 implemented, numerically and experimentally, an inspection using a similar principle to
24 determine the energy dissipation of incident waves at a higher frequency upon the wave
25 passing through a loose bolt in a single-lap aluminum connection, to observe an increase in

1 energy dissipation as the torque decreasing, this, however, contradicting the conclusion from
2 [20, 21]. Such an opposite trend of variation in wave energy dissipation can be attributed to
3 the fact that different types of joint were concerned (a cross-lap bolted joint in [20] and a
4 single-lap bolted joint in [10]). This high dependence of energy dissipation of acoustic waves
5 on joint type arises immense difficulty in evaluating the leftover torque of a bolt if the joint
6 type is unknown beforehand. On top of this, bolt loosening at an embryo stage, analogous to
7 an undersized damage, is not inclined to induce perceivable wave dissipation, which further
8 lowers the effectiveness of approaches taking advantage of liner signal features of acoustic
9 waves [11, 22-24].

10

11 In recognition of the above limitations that linear signal features may possess, contact
12 acoustic nonlinearity (CAN)-based modulation, sharing a similar principle in detecting
13 “breathing” fatigue cracks [25-27], has been spotlighted. In the approach, two excitation
14 sources at different frequencies, namely a low-frequency (LF) pumping vibration and a high-
15 frequency (HF) probing wave, are mixed and applied on the structure under inspection, to
16 interact with the “imperfect” contact due to damage or a loose bolt. The pumping vibration
17 drives the “imperfect” contact interface (due to damage or loose bolt) to close and open
18 periodically (“breathing” motion), and introduces a perturbation to the probing wave and
19 appearance of nonlinear signal features such as additional sidebands in signal spectra. The
20 magnitude of a sideband (either left sideband (LS) or right sideband (RS)) betokens the
21 occurrence of damage or bolt loosening [28]. The frequencies of the pumping vibration and
22 the probing wave are to be selected prudentially, so that damage- or loose bolt-related
23 nonlinearity can be strengthened, while the nonlinearity from other sources (*e.g.*,
24 environment or measurement equipment) be minimized [29, 30]. In this connection, impact
25 modulation (IM) and vibro-acoustic modulation (VM) are two major implementations of

1 CAN-based modulation, with a major difference being that the former adopts an impact force
2 to excite the natural vibration modes of the inspected structure, while the latter applies a
3 stable vibration to the structure using a harmonic force. Effectiveness of both
4 implementations in detecting damage (presence only) have been well demonstrated.
5 Extending such success, Meyer and Adams [31] attempted an IM-based method to detect
6 bolt loosening in a single-lap bolted aluminum joint, and found it was a challenging task to
7 identify the natural modes of the joint, this entailing extra efforts in processing data. Amerini
8 and Meo [32] employed both linear acoustic wave-based and modulation-based nonlinear
9 methods to detect bolt loosening, ending up with good results. It is relevant to note that a
10 single-lap bolted joint was considered in [31] and [32]. As commented earlier, the detection
11 results might be contradictive, depending on the type of a joint [10, 20].

12

13 In this study, an inspection framework was developed, for early detection of bolt loosening
14 in a bolted joint, and quantitative evaluation of residual torque of a loose bolt (the “residual
15 torque” herein is defined as the amount of tension leftover in a bolt). In the framework, a
16 wave energy dissipation (WED)-based linear acoustic method and a vibro-acoustic
17 modulation (VM)-based nonlinear approach were developed, and compared in terms of
18 respective efficiency and sensitivity to bolt loosening. In the former, an analytical model,
19 based on the Hertzian contact theory, was established, to correlate the residual torque with
20 WED, based on which a linear index was constructed; in the latter, CAN engendered at the
21 joining interface was explored to develop a nonlinear index, associating signal spectral
22 features (*e.g.*, sidebands) with the residual torque. Both approaches were validated
23 experimentally, in a comparative manner, by evaluating the residual torques of a loose bolt
24 in single-lap, cross-lap and hybrid-lap joints, respectively. It is noteworthy that the proposed
25 framework is not restricted by the joint type.

1 **2. Theory: *From Linear to Nonlinear***

2 **2.1. Linear Index: *WED-based***

3 Consider a bolted joint with two interconnecting components fastened via a bolt, as shown
4 schematically in Figure 1(a). Provided the bolt is fastened, the relation between the pre-load
5 P of the bolt and the torque T applied on the bolt can be described, in an elastic regime, as
6 [33]

$$7 \quad P = \frac{T}{Kd}. \quad (1)$$

8 where K is a thread coefficient subject to the friction between the nut and bolt, and d the bolt
9 diameter. From a microscopic perspective, the surface of a solid is rough and cratered, and
10 therefore the interface between the nut and bolt features an asperity contact (partial contact
11 at the interface), as illustrated schematically in the insert of Figure 1(a). The loosening of a
12 bolt, as a result of torque reduction, can lead to decrease in the contact pressure at the
13 interface and consequently the substantial contact area. To derive a quantitative correlation
14 between the leftover torque and the partial contact area, a simplified model for the two
15 interconnecting components (Assembly I and Assembly II) is developed. Within a small
16 contact area at the interface, the model assumes the two assemblies are spherical in geometry,
17 with radii of R_1 and R_2 , respectively, as seen in Figure 1(b). The radius r of the substantial
18 contact area – the region where a compressive stress exists – is given, according to the
19 Hertzian contact theory [34], as

$$20 \quad r = \left[\frac{3}{4} \pi (\gamma_1 + \gamma_2) \frac{R_1 R_2}{R_1 + R_2} \right]^{1/3} P^{1/3}. \quad (2)$$

$$21 \quad \gamma_1 = \frac{1 - \nu_1^2}{\pi E_1}, \text{ and } \gamma_2 = \frac{1 - \nu_2^2}{\pi E_2}. \quad (3)$$

22 where E denotes the Young's modulus and ν the Poisson's ratio, distinguished by subscripts
23 1 and 2 for Assembly I and Assembly II, respectively. The substantial contact area S can then

1 be calculated by

$$2 \quad S = \pi r^2 = \left[\frac{3}{4} \pi^{5/2} (\gamma_1 + \gamma_2) \frac{R_1 R_2}{R_1 + R_2} \right]^{2/3} P^{2/3}. \quad (4)$$

3 Under a given P , the total change (δ) of the two assemblies in the direction of P , in an elastic
4 region, can be given by [34]

$$5 \quad \delta = \left[\frac{9}{16} \pi^2 (\gamma_1 + \gamma_2)^2 \frac{R_1 + R_2}{R_1 R_2} \right]^{1/3} P^{2/3}. \quad (5)$$

6 Thus, the distance between the centers of two spheres, L (see Figure 1(b)) is yielded as

$$7 \quad L = R_1 + R_2 - \delta. \quad (6)$$

8 Equation (5) infers the dependence of the contact stiffness (*i.e.*, the extent to which the
9 interface resists deformation in response to a contact force) on L and P .

10

11 Now consider an elastic wave propagating in the above joint. Provided two assemblies are
12 thin in their thickness (left of Figure 1(b)), the wave takes the formality of Lamb waves –
13 the wave guided by a plate- or shell-like structure with its planar dimensions being far greater
14 than that of its thickness and with the wavelength of the wave being of the order of the plate
15 thickness. Upon interacting with the bolt, the incident Lamb waves in Assembly I are split
16 into four components:

17 (i) the wave to be dissipated, owing to the friction at the interface (*i.e.*, evanescent
18 waves); the evanescent waves decay quickly;

19 (ii) the wave to be reflected and then propagate in Assembly I in a direction opposite to
20 incident wave;

21 (iii) the wave to continue its propagation in Assembly I after transmitting the bolt,
22 carrying energy $\Omega_{transmitted\ in\ I}$; and

23 (iv) the rest to be leaked to Assembly II via the bolt, carrying energy Ω_{leak} , and then
24 propagate in Assembly II.

1 Equation (4) indicates that the substantial contact area S is proportional to $P^{2/3}$, and it has

$$2 \quad \Omega_{leak} \propto S \propto P^{2/3} \propto T^{2/3}. \quad (7)$$

3 $\Omega_{transmitted\ in\ I}$ and Ω_{leak} can be captured with a transducer (such as a lead zirconate titanate
4 (PZT) in sequent experiment) surface-mounted on the top of Assembly I and Assembly II,

5 respectively. From Eq. (7), it is axiomatic that Ω_{leak} inclines to increase as T increases, and
6 in the meanwhile $\Omega_{transmitted\ in\ I}$ decreases provided the incident energy remains unchanged.

7 Based on this, $\Omega_{transmitted\ in\ I}$ and Ω_{leak} can serve as a damage index (denoted by η in what
8 follows) to quantitatively indicate the degree of bolt loosening. As η explores WED – a
9 linear signal feature of waves, this index is called *linear index* in this study.

10

11 **2.2. Nonlinear Index: VM-based**

12 Extending the above modeling from linear to a nonlinear regime, now consider that the joint
13 is subject to an external harmonic force introduced by a pumping vibration. Depending on
14 the waveform of the pumping vibration, P varies periodically and leads to a change (denoted
15 by X) to L (defined by Eq. (6)). To put it into a quantitative manner, P is defined in terms its
16 Taylor series, up to its second-order [35, 36], as

$$17 \quad P(L + X) = P(L) - K_1 X + K_2 X^2. \quad (8)$$

18 where K_1 and K_2 are the linear and nonlinear contact stiffnesses at the joining interface,
19 respectively, which read [35],

$$20 \quad K_1 = - \left. \frac{dP}{dX} \right|_{X=0} = CP^m \propto T^m, \quad (9a)$$

$$21 \quad K_2 = \frac{1}{2} \left. \frac{d^2 P}{dX^2} \right|_{X=0} = 0.5mC^2 P^{2m-1} \propto T^{2m-1}. \quad (9b)$$

22 C and m are two parameters depending on the contact properties of the two assemblies at the

1 interface. In most engineering context where $m < 0.5$, it can be demonstrated that K_1 increases
 2 as P augments and in the meantime K_2 decreases [35].

3

4 The linear and nonlinear contact stiffnesses reflect the degree of the residual torque of a
 5 loose bolt. To link them to spectral features of the response of the joint when a bolt is loose,
 6 the bolted joint shown in Figure 1(a) is simplified using a single-degree-of-freedom system,
 7 shown schematically in Figure 1(c). Subjected to a mixed excitation from LF pumping
 8 vibration (with an equivalent force $F_1 \cos \omega_1 t$) and HF probing wave (with an equivalent
 9 force $F_2 \cos \omega_2 t$) which are independent to each other, the equation of motion of the joint
 10 can be described as

$$11 \quad M\ddot{x} + K_1 x - \varepsilon K_2 x^2 = F_1 \cos \omega_1 t + F_2 \cos \omega_2 t. \quad (10)$$

12 where ω_1 and ω_2 are the frequencies of the pumping vibration and probing wave,
 13 respectively. M denotes the mass and t the time. The term with K_2 represents a second-order
 14 perturbation in which ε is a small quantity to scale the perturbation to be minute. Using the
 15 perturbation theory, the solution to Eq. (10) takes the following form

$$16 \quad x = x_1 + \varepsilon x_2. \quad (11)$$

17 x_1 represents the linear dynamic response of the joint to the mixed excitation from both
 18 $F_1 \cos \omega_1 t$ (LF pumping vibration) and $F_2 \cos \omega_2 t$ (HF probing wave), and x_2 the nonlinear
 19 response of the joint, as manifested as a series of sidebands and second-order harmonics in
 20 a signal spectrum. Substituting Eqs. (11) to (10) and forcing the coefficients of ε -related
 21 terms to be identical on the left and right sides of the equation, one has

$$22 \quad M\ddot{x}_1 + K_1 x_1 = F_1 \cos \omega_1 t + F_2 \cos \omega_2 t, \quad (12a)$$

$$23 \quad M\ddot{x}_2 + K_1 x_2 = K_2 x_1^2. \quad (12b)$$

24 Further, upon neglecting the transient components which are independent of the magnitudes

1 of the linear and nonlinear responses, x_1 can be obtained as

$$2 \quad x_1 = G_1 \cos \omega_1 t + G_2 \cos \omega_2 t, \quad (13)$$

3 where $G_1 = \frac{F_1}{K_1 - M \omega_1^2}$ and $G_2 = \frac{F_2}{K_1 - M \omega_2^2}$. Solving Eq. (12b) yields

$$4 \quad x_2 = x_{sidebands} + x_{second\ harmonics}. \quad (14)$$

5 Where

$$6 \quad x_{sidebands} = \frac{G_1 G_2}{K_1 - M(\omega_1 + \omega_2)^2} K_2 \cos(\omega_1 + \omega_2)t + \frac{G_1 G_2}{K_1 - M(\omega_1 - \omega_2)^2} K_2 \cos(\omega_1 - \omega_2)t, \quad (15a)$$

$$7 \quad x_{second\ harmonics} = \frac{0.5G_1^2}{K_1 - 4M\omega_1^2} K_2 \cos(2\omega_1 t) + \frac{0.5G_2^2}{K_1 - 4M\omega_2^2} K_2 \cos(2\omega_2 t). \quad (15b)$$

8 In Eq. (15a), the terms $(\omega_1 + \omega_2)$ and $(\omega_1 - \omega_2)$ jointly regulate characteristics of the
9 sidebands in spectrum, and in Eq. (15b), those terms involving $2\omega_1$ and $2\omega_2$ concern the
10 second-order harmonic responses of the joint.

11
12 From Eq. (15a) it can be noted that the magnitude of a sideband in the spectrum is
13 proportional linearly to the nonlinear contact stiffness K_2 which is dependent on the contact
14 properties at the contact interface under the mixed excitation, as well as the residual torque
15 T of the bolt. Consequently, based on Eq. (15a), an index, β , embracing the magnitudes of
16 the left (A_L , in the unit of dB), right (A_R) sidebands and the probing wave (A_{HF}) in spectral
17 is defined, given that the change of linear response subjected to the variation of pumping
18 vibration can be neglected, as

$$19 \quad \beta = \frac{(A_L + A_R)}{2} - A_{HF}. \quad (16)$$

20 β is called *nonlinear index*, in parallel with the linear index η defined earlier. Similar to η ,
21 β can also indicate the degree of bolt loosening.

1

2 It is noteworthy that the above derivation is not restricted by the type of a joint, and therefore
3 β is applicable to any joint type. Integrating the linear and nonlinear indices, an inspection
4 framework is further developed for detecting bolt loosening and evaluating the residual
5 torque of a loose bolt.

6

7 **3. Experimental Validation**

8 **3.1. Set-up**

9 The proposed inspection framework was validated experimentally. Three types of bolted
10 joint, including a single-lap (Type I), a cross-lap (Type II) and a hybrid (comprising both
11 single- and cross- laps) (Type III) bolted aluminum alloy joints, were considered, as shown
12 in Figures 2(a), (b) and (c), respectively. In particular, the hybrid joint (Type III) was aimed
13 at testifying the performance of the two indices towards multi-type joints. Dimensions of
14 each joint are indicated in the figure. The right end of each joint was fixed to form a cantilever.
15 M6 bolts were used in all the three connections. As an example, the experimental set-up for
16 the Type III joint is photographed in Figures 3(a) and (b), respectively, for WED- and VM-
17 based approaches.

18

19 **3.2. Linear Index: *WED-based***

20 Two PZT wafers were surface-mounted on each joint, 112.5 mm from the bolt, one serving
21 as the actuator (PZT₁) and the other as the sensor (PZT₂). A 14-cycle Hanning window-
22 modulated sinusoidal tone-burst signal at a central frequency of 310 kHz was generated
23 using a signal generator (NI[®] PXIe-1071), amplified with a high-frequency power amplifier
24 (Ciprian[®] US-TXP-3) to 220 V (peak-to-peak), and then applied on PZT₁, to introduce Lamb
25 waves into the joint. At 310 kHz, the zeroth-order symmetric Lamb wave mode (S₀)

1 dominates the signal energy, which is demonstrably sensitive to both surficial and internal
 2 defects. The wave propagation, upon traversing the bolt, was monitored by PZT₂ using an
 3 oscilloscope (Agilent® DSO9064A) at a sampling frequency of 20 MHz. Captured Lamb
 4 wave signals were averaged 64 times to remove random measurement noise.

5
 6 Fast Fourier Transform (FFT) was performed on captured signals, to ascertain distribution
 7 of wave energy in the frequency domain. As representative results, Figure 4(a) shows a raw
 8 signal captured from Type I joint, when T was 1 N·m under which the bolt was fully loose,
 9 and Figure 4(b) the FFT-processed spectrum of the signal in (a). A wide range of the energy
 10 distribution can be observed in the spectrum which can be attributed to the dispersive and
 11 multimodal traits of Lamb waves propagating in the joint. The majority of the wave energy
 12 is observed to be in a range from 270 to 350 kHz.

13
 14 To establish Ω_{leak} and $\Omega_{transmitted\ in\ I}$ (*i.e.*, the linear index η), the power spectral density (PSD)
 15 of each signal in the spectrum (denoted by $W(f)$), where f signifies frequency) was
 16 integrated within a frequency range $[f_1, f_2]$ over which the signal possesses the majority of
 17 its energy (in this case, $f_1 = 270\ kHz$ and $f_2 = 350\ kHz$ as mentioned in the above), as

$$18 \quad \Omega_{leak} = \int_{f_1}^{f_2} W(f) \cdot df, \quad (\text{for Type I, single-lap joint}) \quad (17a)$$

19 or

$$20 \quad \Omega_{transmitted\ in\ I} = \int_{f_1}^{f_2} W(f) \cdot df. \quad (\text{for Type II, cross-lap joint}) \quad (17b)$$

21
 22 Taking into account the yielding strength of the selected aluminum alloy and the allowable
 23 tensile load of the bolt, it was calculated that a torque (T) of 13 N·m, at which a compressive
 24 stress of about 300 MPa would be applied on the bolt, guarantees a full tightening of the bolt.

1 Based on this, a series of scenarios were considered in experiment, with the residual torque
2 T varying from 1 (full loosening) to 13 N·m (full tightening) with a step of 1 N·m. Note that
3 in Type III (hybrid joint), the torque values applied on both bolts were identical for the
4 convenience of discussion.

5

6 For Type I joint, the obtained correlation between the linear index (*i.e.*, Ω_{leak} defined by Eq.
7 (17a)) and the degree of bolt loosening (represented by T) is shown in Figure 5(a), from
8 which it is noticed that the linear index Ω_{leak} increases drastically when T augmenting from
9 1 (full loosening) to 3 N·m, followed with a moderate increase until T reaches 7 N·m and
10 then a saturation afterwards. Such a monotonic relationship can be attributable to the fact a
11 tighter bolting naturally leads to a greater interfacial area at the interface, and consequently
12 a greater amount of leak of incident wave energy from Assembly I to II that is then captured
13 by PZT₂; when T continues the increase to 7 N·m afterwards, no further increase in the
14 substantial contact area could be created, echoing the observed saturation of the index
15 beyond 7 N·m. This experimental observation well matches the tendency in the variation of
16 transmitted wave energy as predicted theoretically using Eq. (7).

17

18 For Type II joint, an opposite tendency in the linear index ($\Omega_{transmitted\ in\ I}$ defined by Eq. (17b))
19 subject to T , is found, shown in Figure 5(b). This observation is also coincident with the
20 theoretical prediction using Eq. (7). It is interesting to note that the results in Figures 5(a)
21 and (b) indeed corroborate, respectively, the conclusions drawn from previous studies [20,
22 21] and [10] as detailed in Section 1, in which an opposite trend in variation of WED had
23 been observed for different types of joint.

24

25 For Type III joint, the trend of the linear index is displayed in Figure 5(c), revealing a non-

1 monotonic change which fluctuates against T . This is a consequence of the mixture of the
2 variation of Ω_{leak} in the single-lap joint (Type I) and the variation of $\Omega_{transmitted\ in\ I}$ in the
3 cross-lap joint (Type II), which are contrary over one another.

4

5 Conclusion can thus be drawn that evaluation of bolt loosening, purely based on the WED-
6 based linear method, may present good results only if a single type of joint is involved; it
7 may not show consistent prediction provided multi-types of joint concerned – highlighting
8 a bottleneck of the linear approach.

9

10 **3.3. Nonlinear Index: VM-based**

11 The same specimens with unchanged boundary conditions as those used in Section 3.2 were
12 recalled, in Figure 6, to validate the VM-based approach and the nonlinear index β . A piezo
13 stack actuator (PI[®], P-885.11) was surface-mounted on each joint to generate HF probing
14 waves, while a shaker (B&K[®], Model type: 4809) was used to introduce a point-force-like
15 LF pumping vibration to the joint. For Type I and Type II, the stack actuator was 40 mm
16 from the bolt and shaker was 40 mm from the free end. For Type III, the actuator and shaker
17 were collocated left and right to the bolt with an identical distance of 40 mm. Two sinusoidal
18 signals at different frequencies were generated by a waveform generator (HIOKI[®], Model
19 Type: 7075), to supply the stack actuator and the shaker, respectively. A power amplifier
20 (B&K[®], Model Type: 2706) intensified the pumping vibration. The response signal of the
21 joint under the mixed excitation was captured with an accelerometer (B&K[®], Model Type:
22 4393) and registered using an oscilloscope (Agilent[®] DSO9064A) at a sampling frequency
23 of 200 kHz.

24

25

1 To maximize the vibro-acoustic effect under the mixed excitation, frequencies of the
2 pumping vibration and probing wave were selected prudentially. Under the LF pumping
3 vibration, the contact interface between two assemblies presents complex motion patterns,
4 introducing different degrees of nonlinearity in structural responses at different nature modes
5 of the joint. To ascertain the most appropriate frequency for the pumping vibration, an impact
6 excitation test was conducted on three types of joint, to obtain their respective spectra, from
7 which the natural frequencies were identified for each joint. Figure 7(a) shows an exemplary
8 spectrum for Type I joint (when $T=1$ N·m), in which the three identified natural frequencies
9 are highlighted. The generation of second-order harmonics and sidebands is a direct
10 consequence of the occurrence of CAN at the interface, according to Eqs. (15a) and (15b).
11 Consequently, the identified natural frequencies of each joint were used to excite the joint
12 via the shaker, under which the nonlinear effect due to CAN induced by the loose bolt can
13 be maximized. The magnitude of the second-order harmonic in the response, under each
14 natural frequency, were obtained from the spectrum, in Figure 7(b), to find that at the natural
15 frequency of 992 Hz, it produces the second harmonic generation with the strongest
16 magnitude, indicating a high degree of nonlinearity in structural responses at this particular
17 natural frequency compared with the others. It is therefore 992 Hz was selected as the
18 frequency for pumping vibration for Type I joint. In an analogous manner, 1145 Hz and 1303
19 Hz were selected for Type II and Type III joints as the frequency of LF pumping vibration,
20 respectively.

21

22 To determine the frequency of the HF probing wave, three types of joint were excited using
23 a white noise via a PZT wafer, respectively, to obtain their respective spectra from which the
24 strongest response of each joint, termed *local defect resonance* (LDR), was ascertained. At
25 LDR, the nonlinearity due to CAN is expected to reach its local maximum. For illustration,

1 Figure 7(c) shows the identified LDR at 14.240 kHz for Type I joint. Following a similar
2 procedure, 14.777 kHz and 15.049 kHz were selected for Type II and Type III joints as the
3 frequency of HF probing wave, respectively.

4

5 As representative results, raw time-series signals captured by the accelerometer from Type I
6 joint, under the **mixed** excitation, is displayed in Figure 8, for two extreme situations: 1 N·m
7 (full loosening) and 13 N·m (full tightening). The corresponding spectra were then obtained
8 using FFT. Figure 9 shows the spectra for three types of joint at the above two extreme
9 situations, to observe the occurrence of a strong sideband when the bolt is fully loose (figures
10 on the left), regardless of the type of the joint, while a fairly weak sideband when the bolt is
11 fully fastened (figures on the right). Conclusion can therefore be drawn that the magnitude
12 of sideband can serve as an indicator to calibrate the degree of bolt loosening.

13

14 To put the analysis into a quantitative manner, a correlation between the magnitude of CAN-
15 induced sideband and the degree of bolt loosening was established, using the nonlinear index
16 β . Upon conducting the test at different degrees of bolt loosening, β , subject to residual
17 torque T , is displayed in Figure 10, for three types of joint. A consistent, monotonic tendency
18 in β can be clearly noted through all joint types, which increases rapidly when the bolt is
19 fully loose ($T=1$ N·m) from a transit condition ($T=2$ N·m). The nonlinear index β thus
20 renders a capability of quantitatively evaluating the residual torque of a loose bolt, in spite
21 of the type of the joint.

22

23 **4. Discussion: *Linear vs. Nonlinear***

24 Comparing Figures 5 (for WED-based linear approach) and Figure 10 (for VM-based
25 nonlinear method), it is apparent that the latter outperforms the former holistically, evidenced

1 by the following observations:

- 2 (i) both the linear and nonlinear indices present monotonic tendency, subject to the
3 residual torque, for single-lap and cross-lap joints. However such a monotonic
4 tendency does not exist for linear index when applied to a hybrid-lap joint, and thus
5 the linear method may fail to predict the residual torque; on the contrary, the nonlinear
6 approach is capable of evaluating the torque, regardless of the joint type;
- 7 (ii) observing Figures 5(a) and (b), it can be found that the linear index shows the best
8 sensitivity to the bolt loosening in a limited range. For example, for Type I, this range
9 is from 7 N·m down to 1 N·m (reflected as the steepest slope of the index in this range),
10 and from 4 N·m down to 1 N·m for the cross-lap joint; on the other hand, the sensitivity
11 of the nonlinear index persists throughout the whole range from fully fastened to fully
12 loose, showing enhanced sensitivity than its linear counterpart; and
- 13 (iii) particularly, when the bolt loosening is at an early stage (when T is not less than 11
14 N·m in this study), the nonlinear index exhibits much higher sensitivity over the linear
15 index. That is because the linear index tends to be saturated when T is over 11 N·m, as
16 seen in Figures 5 (a) and (b), which is, however, not the case for the nonlinear index
17 (Figure 10). This can also be interpreted by the fact that in the linear index, the
18 substantial contact area remains unchanged after T exceeds the elastic range of the
19 material, and as a result the linear index would not show prominent change at the early
20 stage of bolt loosening. This highlights good detectability of the nonlinear approach in
21 evaluating early bolt loosening.

22

23 **5. Concluding Remarks**

24 Identification of bolt loosening for three types of joint was attempted, both theoretically and
25 experimentally, using two approaches in a comparative manner: a wave energy dissipation

1 (WED)-based linear acoustic approach and a vibro-acoustic modulation (VM)-based
2 nonlinear method. The linear index, defined in terms of leaked or transmitted energy of
3 incident waves upon passing through the bolt, is found to increase as the applied torque
4 increases in a single-lap bolted joint, but decreases in a cross-lap bolted joint, showing high
5 dependence on the joint type. The bolt loosening at the early stage would not introduce a
6 phenomenal change in the substantial contact area, as a result the detectability is limited
7 when using the linear method. In the nonlinear approach, sidebands in signal spectra induced
8 due to contact acoustic nonlinearity (CAN), when the joint is subjected to the **mixed**
9 excitation, are found quantitatively related to the degree of bolt loosening. The theoretical
10 prediction and experimental validation are in a good agreement. The nonlinear VM-based
11 method shows enhanced sensitivity to bolt loosening than a linear WED-based approach,
12 with additional merit to detect multi-type joints and early stage of bolt loosening. In addition,
13 with a capability of built-in sensing using miniaturized sensors (PZT in this study), the
14 approach can be implemented in a real-time, automatic, and prompt manner, thus rendering
15 a potential to be extended to online health monitoring (SHM) for bolted structures.

16

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21

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