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R. Chen, C. Y. Wen, S. Lorente, and A. Bejan

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R. Chen,[1] C. Y. Wen,[1] S. Lorente,[2] and A. Bejan[3,a)]
[1]Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hung Hom, Hong Kong
[2]Université de Toulouse, INSA, 135 Avenue de Rangueil, 31077 Toulouse, France
[3]Department of Mechanical Engineering and Materials Science, Duke University, Durham, North Carolina 27708-0300, USA

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Here, we show that during their half-century history, helicopters have been evolving into geometrically similar architectures with surprisingly sharp correlations between dimensions, performance, and body size. For example, proportionalities emerge between body size, engine size, and the fuel load. Furthermore, the engine efficiency increases with the engine size, and the propeller radius is roughly the same as the length scale of the whole body. These trends are in accord with the constructal law, which accounts for the engine efficiency trend and the proportionality between "motor" size and body size in animals and vehicles. These body-size effects are qualitatively the same as those uncovered earlier for the evolution of aircraft. The present study adds to this theoretical body of research the evolutionary design of all technologies [A. Bejan, The Physics of Life: The Evolution of Everything (St. Martin's Press, New York, 2016)]. *Published by AIP Publishing.*
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I. INTRODUCTION

Earlier work with the constructal law has shown that it is possible to predict and correlate the speed-mass data of all animals (insects, birds, mammals, fish, and crustaceans), including airplanes, athletics, and inanimate flow systems.[1–5] All these designs of movement on the globe evolve. Airplanes do not evolve by themselves—they evolve as a *duo*, with the humans who design them and use them. Evolving along with the flying animals is the "human and machine species."

The history of airplanes illustrates in our lifetime the evolutionary design of all fliers, animal, and human made, as they move on earth: farther, faster, more efficiently, and with greater lasting power (sustainability). Recent work has shown that the evolution of airplanes is predictable from the constructal law of design and evolution in nature.[1,6] The main features of aircraft evolutionary design predicted from the constructal law are the speed, engine size, fuel load, range, and aspect ratios (wing span vs fuselage length, wing profile, fuselage profile). The same theory accounts for the alignment of 1950 aircraft data in Gabrielli and von Kármán's chart of specific power vs speed,[7,8] which along with the broader method of evolutionary design continues to be of interest in the aircraft literature.[9–14] The constructal law further predicts the time arrow of the change from propellers to jets, in the same way that for animal design it predicts the change (and the increase in movement complexity) from swimming to running and, finally, flying.

In this new article, we report a new domain where the constructal law manifests itself as the evolution of vehicle technology. We show that the classical alignment of helicopter designs can be anticipated based on the constructal law, and that it can be added to the grand evolutionary design of animal and vehicle movement on the globe.

[a)]Author to whom correspondence should be addressed. Electronic mail: abejan@duke.edu.

The current findings can also be applied to foreseeing evolution of the emerging Unmanned Aerial Vehicles (UAVs). Starting from the last decade, the UAVs are gaining rapid popularity, which is attributed to the rapid advance and maturing of information technologies and autonomous capabilities.[15,16] Many military and civil endeavors have served to showcase the potential of UAVs, such as aerial photography and selfie, border surveillance, highway traffic monitoring, wildfire management, agricultural chemical spraying, and other disaster response needs. An UAV, either rotorcraft or fixed-wing vehicle, is operated without pilots and does not carry any passengers. Nevertheless, the navigation is still the controlled body with the power source, which uses the dynamic lift and thrust based on fundamental aerodynamics.[17]

II. EVOLUTIONARY TRENDS

We start with the dimensions and performance data of helicopter models during their 60-year history (Table I). The data are collected from Ref. 18 and the Type Certificate Data Sheet of FAA and EASA. Figures 1–4 show at first glance that during the evolution of helicopter technology, very sharp correlations have emerged between design features and body size.

Each of Figures 1–4 display the helicopter data of Table I with two symbols. The black circles indicate military helicopters. The empty circles are for the rest of the data compiled in Table I. The purpose of this two-frame display of the body-size effect on evolutionary design is to show that the correlations that emerge are somewhat sharper when the military models are excluded (note the relatively larger R^2 values). This finding makes sense because the evolution of military models is driven by an objective (mission) that is not exactly the same as the objective of civilian helicopter models.

For conciseness, the analytical formulas that correlate the data (without the military data) are reported directly on

TABLE I. Helicopter models, and their dimensions and performance (m: military models).

Model	Year	Engine model	Number of engines	Engine mass (kg)	Maximum T-O weight (kg)	Radius of propeller (m)	SFC (lb/shp h)	Fuel capacity (l)
Alpi Syton AH 130	2008	Solar T62	1	64	580	3.82	N/A	N/A
Robinson R66	2010	RR 300	1	91	1225	5.03	N/A	282
Bell 206A	1966	RR 250-C18B	1	64	1360	N/A	0.65	287.7
MD 500E	1982	RR 250-C20B	1	71.7	1361	4.05	0.65	242
Bell 206B	1971	RR 250-C20	1	71.7	1451.5	N/A	0.65	287.7
MD 520N	1991	RR 250-C20R/2	1	76.7	1591	4.2	0.608	235
MD 530F	1985	RR 250-C30	1	115.1	1610	4.16	0.592	242.3
Airbus Helicopter SA 318C	1964	Turbomeca Astazou IIA	1	140	1650	5.1	0.623	580
Airbus Helicopter EC120B	1997	Turbomeca Arrius 2F	1	103.5	1680	5	N/A	410
Airbus Helicopter SA 341G	1972	Turboméca ASTAZOU IIIA	1	147.5	1800	5.25	N/A	457
Bell 206L	1975	RR 250-C20B	1	71.7	1814.4	N/A	0.65	371
MD 600N	1997	RR 250-C47M	1	126.3	1859	4.19	0.58	440
Bell 206L-1	1978	RR 250-C28	1	106	1882	N/A	0.606	371
Airbus Helicopter SA 342J	1976	Turboméca ASTAZOU XIV H	1	160	1900	5.25	N/A	457
Airbus Helicopter AS 350B	1977	Turbomeca Arriel 1B	1	120	1950	5.46	0.573	540
Airbus Helicopter SA 315B	1970	Turbomeca ARTOUSTE III B	1	173	1950	5.51	N/A	565
Bell 206L-3	1981	RR 250-C30P	1	112.4	2018	N/A	0.592	419
Airbus Helicopter AS 355E	1980	RR 250-C20F	2	71.7	2100	5.345	0.65	736
Airbus Helicopter SA 316B	1970	Turbomeca Artouste IIC	1	178	2200	5.5	N/A	565
Airbus Helicopter AS 350B3	1997	Turbomeca Arriel 2B	1	134	2250	5.35	N/A	540
Airbus Helicopter SA 316C	1971	Turboméca ARTOUSTE III D	1	178	2250	5.51	N/A	565
Airbus Helicopter SA 319B	1971	Turboméca ASTAZOU XIV B	1	160	2250	5.51	N/A	565
Bell 407	1996	RR 250-C47B	1	113.85	2268	5.33	0.58	483.7
Airbus Helicopter AS 355F	1981	RR 250-C20F	2	71.7	2300	5.345	0.65	736
Agusta A109	1971	RR 250-C20	2	71.7	2450	5.5	0.65	550
Agusta A109A	1976	RR 250-C20B	2	71.7	2600	5.5	0.65	550
Airbus Helicopter AS 355N	1989	Turbomeca Arrius 1A	2	101.3	2600	5.345	N/A	736
Airbus Helicopter EC135 T1	1996	Turbomeca Arrius 2B1	2	114	2630	5.1	N/A	680
Agusta A109C	1989	RR 250-C20R/1	2	78.5	2720	5.5	0.608	N/A
Airbus Helicopter EC135 P1	1996	PW 206B	2	118.9	2720	5.1	0.548	680
Airbus Helicopter EC135 P2	2001	PW 206B2	2	117.2	2835	5.1	N/A	680
Airbus Helicopter EC135 T2	2002	Turbomeca Arrius 2B2	2	114.3	2835	5.1	N/A	680
MD Explorer	1996	PW206A	2	108	2835	5.15	0.574	564
Agusta A109K2	1992	TURBOMECA Arriel 1K1	2	123	2850	5.5	N/A	468
AW119MKII	2007	PT6B-37A	1	184.8	2850	5.415	N/A	595
Bell 427	2000	PW207D	2	113.7	2970	N/A	0.555	770L
Agusta A109E Power	1996	Turbomeca Arrius 2K1	2	112.8	3000	5.5	N/A	595
Airbus Helicopter EC635 P3	2015	PW 206B3	2	116.9	3000	5.2	N/A	680
Agusta A109S	2005	PW207C	2	113.7	3175	5.415	N/A	563
Bell 429	2009	HTS 900	2	142.9	3175	N/A	0.54	821L
Airbus Helicopter BK117 A-4	1986	LTS 101-650B-1	1	127	3200	5.5	0.577	607.6
Airbus Helicopter BK117 B-2	1992	LTS 101-750B	1	123	3350	5.5	0.577	697
Bell 222	1983	LTS 101-650C-3/3A	2	109	3560	6.1	0.572	670
Airbus Helicopter BK117 C-2	2000	Turbomeca Arriel 1E2	1	125	3585	5.5	N/A	N/A
Airbus Helicopter EC145	2002	Turbomeca Arriel 1E2	2	125	3585	5.5	N/A	879
Mi-2	1965	PZL GTD-350W	2	140	3700	7.25	0.817	N/A
Bell 222B	1983	LTS 101-750C-1	2	111	3742	N/A	0.577	709
Bell 230	1992	RR 250-C30G2	2	117.93	3810	N/A	0.592	709
Bell 204B	1963	T5309A	1	220	3855	6.35	N/A	605
Bell 205A	1968	T5311A	1	225	3855	N/A	0.68	832.8
Airbus Helicopter SA 365N	1981	Turbomeca Arriel 1C	2	116	4000	5.965	N/A	1144.7
(m) Kawasaki OH-1	2000	TS1-M-10	2	152	4000	5.8	N/A	N/A
Airbus Helicopter SA 365N1	1983	Turbomeca Arriel 1C1	2	118	4100	5.972	N/A	1134.5
Bell 430	1999	RR 250-C40B	2	127	4218	6.4	0.57	710L
Airbus Helicopter AS 365N2	1989	Turbomeca Arriel 1C2	2	119	4250	5.972	N/A	1134.5
Airbus Helicopter AS 365N3	1997	Turbomeca Arriel 2C	2	131	4300	5.972	N/A	1134.5
Bell 205A-1	1968	T5313A	1	246.8	4309	N/A	0.58	832.8
(m) Bell UH-1H	1970	T53-L-13B	1	247	4309	N/A	0.6	789
(m) Bell AH-1F	1995	T53-L-703	1	247	4500	6.8	0.568	N/A

TABLE I. (*Continued.*)

Model	Year	Engine model	Number of engines	Engine mass (kg)	Maximum T-O weight (kg)	Radius of propeller (m)	SFC (lb/shp h)	Fuel capacity (l)
Mitsubishi MH 2000	1996	Mitsubishi MG5-110	2	154	4500	6.1	N/A	N/A
(m) US Helicopter AH-1S	1996	T53-L-703	1	247	4536	N/A	0.568	511
Sikorsky S-76A	1978	RR 250-C30	2	115.1	4762	N/A	0.592	1084
Bell 210	2005	T5317B	1	248	4762.7	N/A	N/A	780
Airbus Helicopter EC155B	1998	Turbomeca Arriel 2C1	2	129.2	4800	6.3	N/A	1256
Airbus Helicopter EC155B1	2002	Turbomeca Arriel 2C2	2	131.5	4920	6.3	N/A	1256
Bell 212	1971	PT6T-3B	1	299	5080	7.32	0.596	N/A
(m)AW Lynx	1990	RR Gem 42	2	183	5125	N/A	0.65	N/A
Sikorsky S-76B	1985	PT6B-36	2	169	5307	6.7	0.594	1084
Sikorsky S-76C	1991	Turbomeca Arriel 2S1	2	131.2	5307	N/A	N/A	1084
Sikorsky S-76D	2012	PW210S	2	162.4	5386	N/A	N/A	1128
Bell 412	1983	PT6T-3B	1	299	5397	7	0.596	N/A
Bell 412EP	1994	PT6T-3D	1	325	5397	N/A	0.601	1277.5
Kaman K-Max	1994	T5317A	1	256	5443	7.35	0.59	N/A
HAL Dhruv	2002	Turbomeca TM333-2B2	2	156	5500	6.6	0.529	N/A
Sikorsky S-58T	1972	PT6T-6	1	305	5897	N/A	0.592	1400
(m) Airbus Helicopters Tiger	1991	MTR 390	2	154	6000	N/A	N/A	N/A
AW 159	2009	CTS800-4N	2	173.7	6000	6.5	0.448	N/A
W-3 Sokol	1979	PZL-10W	2	141	6400	7.85	0.598	N/A
(m) Bell AH-1W	1980	T700-GE-401	2	197	6690	7.3	0.464	N/A
Kamov Ka-60	2010	RD-600 V	2	220	6750	6.75	N/A	N/A
Airbus Helicopters SA 330J	1976	Turboméca Turmo IV C	2	230	7400	7.95	0.629	1565
(m) Boeing–Sikorsky RAH66A	1996	T800-LHT-801	2	149.7	7896	5.95	0.462	N/A
Bell 214ST	1982	CT7-2A	2	212	7938	7.92	0.473	N/A
(m) Bell-UH-1Y	2008	T700-GE-401C	2	208	8391	7.44	0.459	1438
Airbus Helicopter AS 332L1	1984	Turbomeca Makila 1A1	2	241	8600	7.8	0.481	2043
Airbus Helicopters AS 332 L2	1986	Turboméca Makila 1A2	2	247	9300	8.1	N/A	2043
(m) Boeing AH 64D	1995	RTM 322-01/12	2	249	9525	7.3	0.45	N/A
(m) Sikorsky HH-60G	1991	T700-GE-700	2	198	9900	7.05	0.459	N/A
(m) NHIndustries NH90	2007	RTM 322-01/9	2	233	10 600	8.15	0.42	N/A
(m) NHIndustries NH90 NFH/TTH	2013	T700-GE-T6E	2	220	10 600	N/A	0.434	N/A
Airbus Helicopter EC225LP	2004	Turbomeca Makila 2A	2	279	11 000	8.1	N/A	2588
(m) Mi-35M	2005	TV3-117VMA	2	310	11 500	N/A	0.473	N/A
Airbus Helicopter EC725	2005	Turbomeca Makila 1A4	2	247	11 751	8.1	N/A	N/A
(m) Mi-24	1972	TV3-117V	2	285	12 000	8.65	0.485	N/A
Sikorsky S-92	2002	CT7-8A	2	246	12 020	8.58	0.452	2896
Airbus Helicopter SA 321F	1993	Turbomeca Turmo IIIC3	3	225	13 000	9.45	0.603	N/A
(m) Mi-17	1977	VK-2500	2	295	13 500	10.63	0.485	N/A
Mi-38	2003	TV7-117V	2	360	14 200	10.55	0.439	3942
AW EH101-500	1994	CT7-6	3	220	14 290	9.3	0.47	4235
(m) AW EH101-400	2003	RTM 322-02/8	3	248	14 600	9.3	0.45	N/A
(m) Mi-26	1983	Lotarev D-136	2	1050	56 000	16	0.456	N/A

each of the graphs of Figs. 1–4. Indicated is also the R^2 value of each correlation, which shows that the correlation is statistically meaningful. The corresponding correlations obtained by including the military data are indicated in the respective figure captions. In these empirical formulas, the masses (M, M_e, M_f) are expressed in kg, the propeller radius R_p is expressed in m, and the heating value of the fuel (H) is expressed in shp h/lb (or 5.9×10^6 J/kg), where shp means shaft horse power. The engine efficiency η is defined in Section III.

Figure 1 shows that the efficiencies (η) of helicopter engines have evolved such that η is proportional to the engine size (M_e) raised to a power that is less than 1. This is in accord with the prediction based on the constructal law,[4] according to which η should vary as M_e^{α}, where $\alpha < 1$.

In Fig. 2, we see the correlation of the engine size (M_e) versus vehicle size. The two frames, together, reveal an approximate proportionality between engine size and body size, and, in addition, a ratio M_e/M that is in the order of 1/10. This finding is the same as in the engine size versus body size scaling exhibited by the evolution of airplanes.[6]

Figure 3 shows that the engine size and the fuel load have emerged to be proportional over a size range that spans one full order of magnitude. The engine mass is roughly one third of the fuel load mass over this entire range. This too

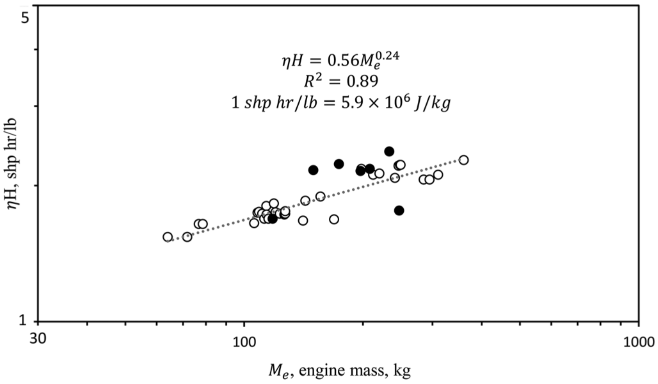


FIG. 1. Bigger engines are more efficient: the correlation between engine efficiency and engine size. In the indicated correlation, the military helicopter data (the black circles) were not included. If the military data are included, the correlation becomes $\eta H = 0.53\, M_e^{0.25}$, with $R^2 = 0.79$.

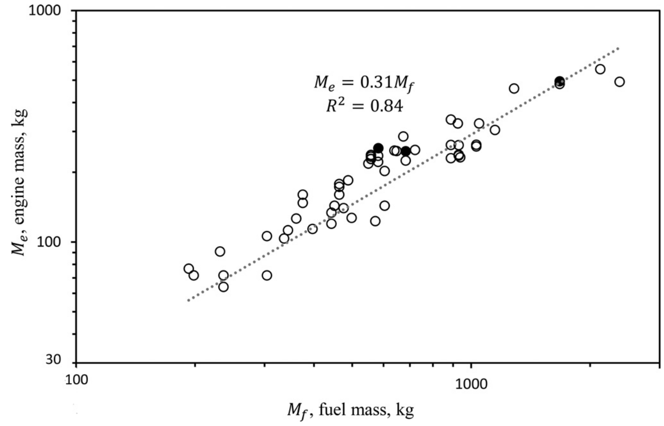


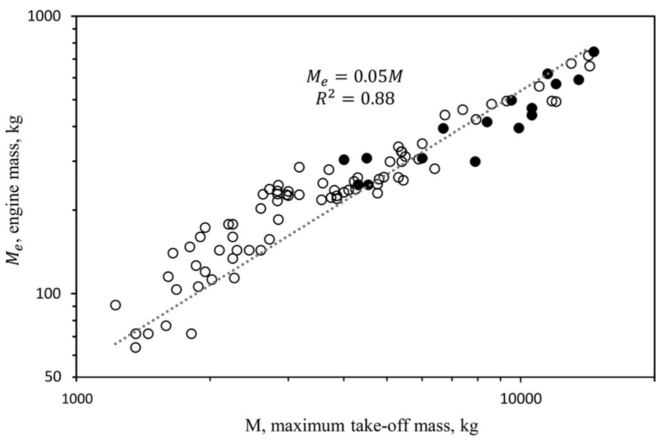
FIG. 3. The proportionality between fuel load and engine size. In the indicated correlation, the military helicopter data (the black circles) were not included. If the military data are included, the correlation becomes $M_e = 0.29\ M_f$, with $R^2 = 0.79$.

agrees with the trend uncovered for the evolution of aircraft.[6]

Figure 4 reveals the correlation that emerged between the helicopter propeller radius (R_p) and the vehicle size, which is represented by the maximum take-off mass (M). The figure shows that the propeller radius varies monotonically with the vehicle size, where R_p emerged as proportional to $M^{0.3}$. Because the vehicle size M is proportional to the vehicle length scale cubed (L^3), the proportionality between R_p and $M^{0.3}$ means that R_p is essentially proportional to L.

The geometric meaning of the body-size scaling revealed by Fig. 4 is that the propeller radius scales with the length scale of the vehicle, and that all helicopters (large and small, old and new) are geometrically similar. This conclusion is the same as the one reached in the study of the evolution of aircraft, where all aircraft evolve to be geometrically similar, with the wing span almost the same as the fuselage length.[6]

The geometric similarity of old and new helicopter models is evident in Fig. 5. Furthermore, the figure shows that during the past five decades the specific fuel consumption (SFC) has decreased to half of its original level. This too is in accord with the evolution of the specific fuel consumption of commercial aircraft (measured in liters of fuel spent for one seat and 100 km flown).[6] The specific fuel consumption plotted in Fig. 5 is defined in Section III.



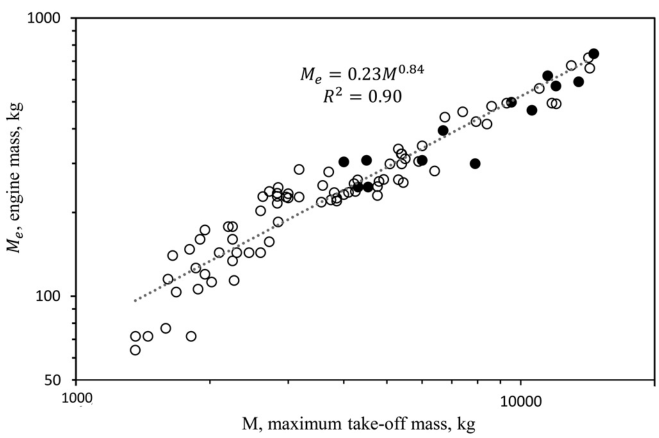


FIG. 2. Bigger engines belong on bigger helicopters: the proportionality between engine mass and helicopter mass. The first graph shows the linear correlation of the data of Table I; the second graph shows the power-law correlation. In the indicated correlations, the military helicopter data (the black circles) were not included. If the military data are included, the linear correlation becomes $M_e = 0.05$ M, $R^2 = 0.87$, and the power-law becomes $M_e = 0.24\ M^{0.83}$, $R^2 = 0.90$.

III. DISCUSSION

As shown in studies of the evolution of commercial aircraft and animals,[6,8,19–21] theory can deepen our understanding of the origin of body-size scaling. We start with the

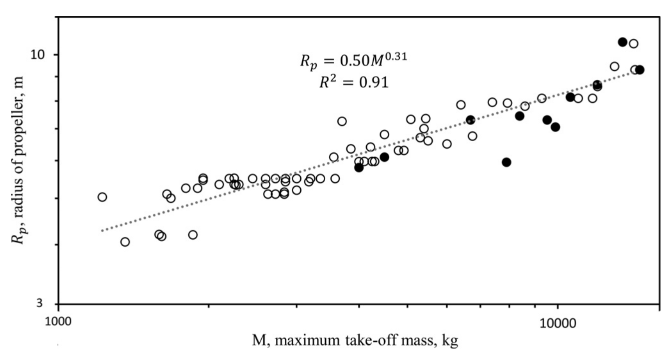


FIG. 4. Bigger propellers belong on bigger helicopters: the rough proportionality between propeller radius and helicopter length scale, or body mass raised to the power 1/3. In the indicated correlation, the military helicopter data (the black circles) were not included. If the military data are included, the correlation becomes $R_p = 0.47\ M^{0.31}$, with $R^2 = 0.88$.

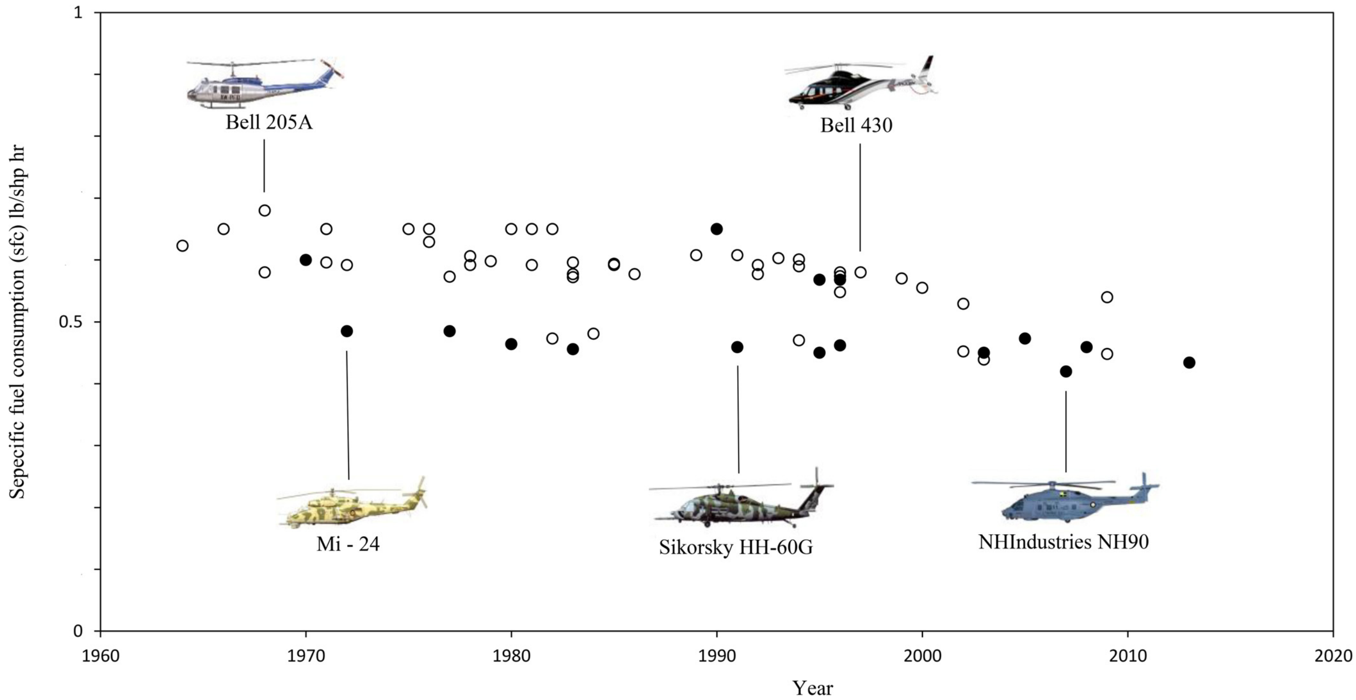


FIG. 5. The evolution of the specific fuel consumption of helicopters during the past five decades. The black circles indicate military helicopters (see "m" in Table I).

observation that a hovering aircraft such as a helicopter can move in all directions. Chief among these is the vertical direction: the main function of the aircraft is to hover, i.e., to maintain its altitude above ground. Secondary is the sliding movement in the horizontal direction. The simplest model is the one that retains the fewest and most important features of the actual physical system. This is why we begin with the assumption that the hovering body is stationary at its altitude, while consuming fuel to maintain itself in this position for the longest time possible.

Thermodynamics shows that larger flow systems function less irreversibly, because their flows encounter smaller obstacles, such as wider ducts and larger heat transfer areas in heat exchangers. The monotonic effect of size on efficiency was predicted in Ref. 4. The mathematical conclusion is that if the size of the engine is represented by its mass M_e, then the energy conversion efficiency of the engine evolves such that it increases monotonically with size

$$\eta = c_1\ M_e^{\alpha}, \tag{1}$$

where c_1 is a constant factor. The α exponent is comparable with 1, and must be less than 1 because the η curve must be concave with respect to M_e: this is because in accord with thermodynamics, the efficiency cannot surpass an ideal level, a ceiling. The more mature the engine technology, the higher the efficiency, the closer to the ideal level, and the smaller the α exponent. The engine efficiency is defined as

$$\eta = \frac{Pt}{HM_f}, \tag{2}$$

where P is the shaft power from engine, t is the time of hovering, H is the heating value of the fuel, and M_f is the mass of consumed fuel. The specific fuel consumption (SFC) is the quantity of fuel consumed in order to produce one unit of power in one unit of time[22]

$$SFC = \frac{M_f}{Pt}. \tag{3}$$

By comparing Eq. (3) with Eq. (2), we see that

$$\eta = \frac{1}{SFC \cdot H}, \tag{4}$$

or, $\eta H = 1/SFC$. By using the helicopter data compiled in Table I, we found the correlation shown in Fig. 1.

The rotor hovering efficiency (η_p) is defined as the ratio of the minimum possible power required to hover (induced power) to the actual power required to hover (shaft power). The total hover power is a value that can be obtained only by measurement. Unfortunately, we did not have access to measurements of performance. In any case, care must be taken when comparing rotors. Only rotors with the same disk loading should be compared. Testing is the only way to figure out the relationship between the η_p and the radius of the propeller. Noteworthy is the study[23] that reported the static testing of micro propellers. A load cell and a torque transducer were used to measure the thrust and torque created by the propeller. The results show that a larger-diameter propeller tends to be more efficient, which is in accord with the body-size effect anticipated with the constructal law.[1,4] At the design loading of the rotor, a value of $\eta_p = 0.55–0.60$ is typical. Because of this narrow range, in the following analysis we treat η_p as a constant.

The size of the hovering aircraft is represented by its total mass M, which accounts for everything that hovers, engine (M_e), propeller (M_p), fuel (M_f) and the rest of the body frame (M_b)

$$M = M_e + M_p + M_f + M_b. \quad (5)$$

Assume that the engine mass M_e varies, while the other masses do not vary. Consequently, the total mass changes with the engine mass. We explore the idea that there is a certain relationship between the engine mass and the total helicopter mass when considering that best performance means maximum hovering time for a given amount of fuel. To start, from Eq. (2) we find that the engine power output is

$$P = H\eta \dot{M}_f / t. \quad (6)$$

The engine power is responsible for the force (the thrust, T) that holds the hovering body at constant altitude. The relationship between P and T is

$$T = \frac{1}{V}\eta_p P, \quad (7)$$

where T and V are the thrust and the induced air velocity, respectively.[22] The induced velocity is $V = (T/2\rho A)^{1/2}$, where ρ is air density, and A is the rotor disk, i.e., the circular area swept by the blades of the rotor. The vertical equilibrium of the hovering body requires

$$T = Mg. \quad (8)$$

Combining Eqs. (6)–(8), where t is the duration of the hovering flight, we obtain

$$t = \frac{HM_f \eta \eta_p}{TV} = \frac{0.56 M_f M_e^{\alpha} \eta_p}{TV} = K\frac{M_e^{\alpha}}{M^{1.5}}, \quad (9)$$

where $K = 0.56\, M_f \eta_p g^{-3/2} (2\rho)^{1/2}$ is treated as constant, and M varies linearly with M_e as shown in Eq. (5). The maximum hovering time is obtained by maximizing t with respect to M_e, and the result is

$$\frac{M_e}{M} = \frac{2}{3}\alpha < 1. \quad (10)$$

In conclusion, the evolutionary designs should tend toward vehicles with a certain proportion between engine size and total body size. This is in accord with the empirical correlation found in Fig. 2 and is the same as the proportionality between muscle mass and total body mass in animal design[19–21] and the proportionality between engine mass and total mass in airplane design.[1,6]

In Fig. 4, we saw that a larger helicopter carries larger blades. A relation between R_p and M is[22]

$$C_T/\sigma = \frac{T}{\rho A (R\Omega)^2}\frac{\pi R_p}{Nc} = \frac{T}{\rho N C^2 R_p^3} = \text{constant}. \quad (11)$$

Under hovering conditions, C_T/σ can be thought of as the lift coefficient per blade. The number of blades is N. Here, it is assumed that the mean angle of attack of the blade is a constant. The thrust coefficient C_T is equal to $T/[\rho A(R\ \Omega)^2]$, where Ω is angular speed of the rotor, σ is the rotor solidity, which is equal to $Nc/(\pi R_p)$ where c is the chord of the blade. Equation (11) becomes

$$\frac{T}{\rho N c \Omega^2 R_p^3} = \text{constant}, \quad (12)$$

and, in view of $T = Mg$, we arrive at the proportionality

$$R_p \sim M^{\frac{1}{3}}. \quad (13)$$

This is in agreement with Fig. 4, which shows that if a power function is used for curve-fitting, then R_p emerges as proportional to $M^{0.3}$. This means that R_p is roughly proportional to the length scale of the helicopter, which is proportional to $M^{1/3}$. This agrees with a correlation of data reported in Ref. 24.

In summary, the evolution of helicopters adds itself to the universal phenomenon of evolution,[1,25] which is exhibited by all flow systems that are free to morph as they flow: animate, inanimate, and engineered. The latter are the technology evolutions responsible for empowered humans—the evolving human and machine species.[1–3] The application of the constructal law to the evolution and spreading of UAVs recommends itself as a subject for future investigation.

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