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Density Forecasting for Tourism Demand

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The contribution of the tourism industry to the Hong Kong economy is far-reaching. Hence, it is necessary to provide an accurate and informative forecast of the growth rates of visitor arrivals for both private and public bodies to help facilitate their business planning, and policymaking. However, most of the forecasting methodologies mainly focus on the mean forecast. Business plans based on such a point forecast measuring only its central tendency without considering its uncertainty, or its quality, can be risky. For example, relying on an overly optimistic forecast may end up with an overpriced investment. This research note attempts to alleviate this risk by introducing density forecast for tourism demand that accommodates both mean forecast and its uncertainty in the analysis.

Interval forecast represents an immediate response to the above criticism on point forecast as it specifies the probability that the actual outcome will fall within a stated interval. Using bootstrapping method, Song, Kim, and Yang (2010) provide interval forecasts for income and price elasticities of tourism demand while Otero-Giráldez, Álvarez-Díaz and González-Gómez (2012) obtain confidence intervals of the long-run effects of the major determinants in an autoregressive distributed lag ($ARDL$) model. Although interval forecast is undoubtedly more informative than a single point forecast, a particular interval corresponds only to part of a density. Whether intervals associated with other probabilities or intervals at the tail are correctly specified remains unsolved. In contrast, a correctly specified density forecast implies correct conditional interval forecasts at all confidence levels. Therefore, density forecast becomes more common in forecasting macroeconomic and financial time series (Tay & Wallis, 2000).

Our main objective is to highlight the importance of density forecast and the associated evaluation tool for the distributional assumption in tourism demand. Through analyzing the empirical results for the Hong Kong inbound tourism from mainland China and Macau, we found that both error distributions vary over time and a mix of t and normal distribution featuring heavy tail is more appropriate for China. The study starts with modeling quarterly data of standardized growth rates of tourist arrivals from mainland China and Macau during the period from 1996Q1 to 2015Q1, a total of 77 observations. It is then followed by a brief discussion of probability integral transform which was pioneered by Rosenblatt (1952) for evaluating the assumption underlying the density forecast. Lastly, the deficiency of traditional mean forecasts will be illustrated through the empirical analysis of the two territories.

Because income and price level are two major determinants for tourist arrivals, standardized year-on-year growth rates of both GDP ($growth$) and exchange rate-adjusted consumer price index ($infl$) are chosen as predictors for the standardized year-on-year growth rates of tourist arrivals (z_t) in an $ARDL(p, q, r)$ model[1]:

$$z_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i z_{t-i} + \sum_{i=1}^{q} \beta_i growth_{t-i} + \sum_{i=1}^{r} \gamma_i infl_{t-i} + u_t.$$

The forecast density is determined by the error distribution assumption. A rolling regression window of size $w = 30$ quarters is used to obtain a series of one-step-ahead forecasts yielding $n_{oos} = 47$ out-of-sample forecasts for the two territories. By setting maximum lag orders to 3 ($p_{max} = q_{max} = r_{max} = 3$), we choose $ARDL(1,1,1)$ for China and $ARDL(1,1,3)$ for Macau based on BIC[2].

Assuming u_t is standard normal, the forecast distribution at time $t + 1$ will be equal to $N(\hat{z}_{t+1}, s^2_{u,t+1})$ with mean forecast $\hat{z}_{t+1}$ and estimated variance $s^2_{u,t+1}$. Rosenblatt (1952) finds that if the model is correctly specified (i.e. the estimated one-step-ahead cumulative distribution function (F_{t+1}) coincides with the true one), then the probability integral transform at $t + 1$ ($pit_{t+1} = F_{t+1}(z_{t+1})$) will be independently, identically, and uniformly distributed $i.i.d.U(0,1)$. This crucial finding linking estimated distribution (F_{t+1}) to actual observation (z_{t+1}) lays the foundation for evaluating the model in a statistical way.

Diebold, Gunther, and Tay (1998) (DGT) propose a histogram-based evaluation technique for density forecasting based on the pit in the out-of-sample period. Suppose there are n_b equally sized non-overlapping bins over a unit interval. If the model is correctly specified (i.e. pit is uniformly distributed), then there is approximately $h = 1/n_b$ of pit in each bin with estimated variance $s^2_{pit} = h(1 - h)/n_{oos}$. Therefore, if $pit \sim i.i.d.U(0,1)$ at 1% level of significance (i.e. the estimated distribution F_{t+1} is appropriate), the relative frequencies $\hat{h}$ should lie within the 99% confidence interval $h \pm 2.58 s_{pit}$.

To highlight the importance of the distributional assumption, we start with normality assumption (i.e. $F_{t+1} = \Phi_{t+1}$ with mean $\hat{z}_{t+1}$ and standard error s_u) for the two time series. Then, we plot the histogram of 47 out-of-sample pit_{t+1} in Figure 1.

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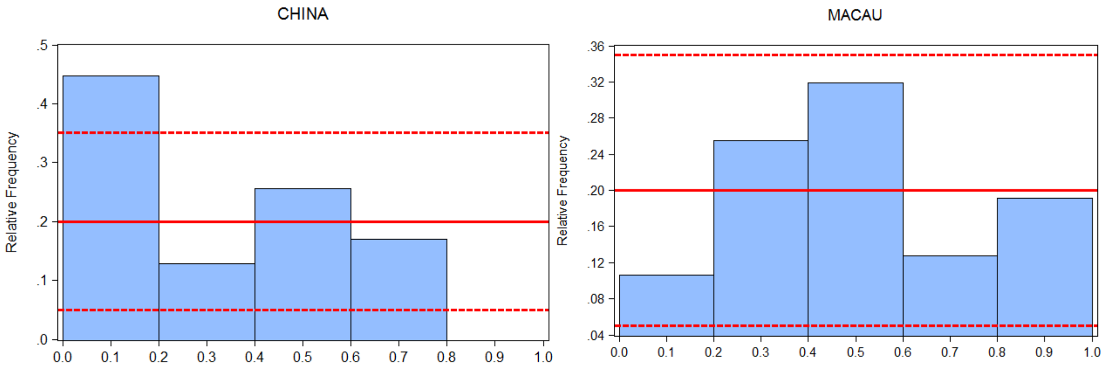
[1] The KPSS tests suggest that the three non-standardized growth rates are stationary. It justifies us to standardize them by subtracting their mean and dividing by their standard error.

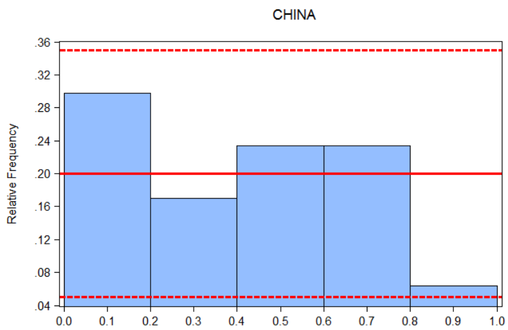
[2] Breusch Godfrey and BDS tests suggest that the predictive errors are serially uncorrelated and independent.

Figure 1



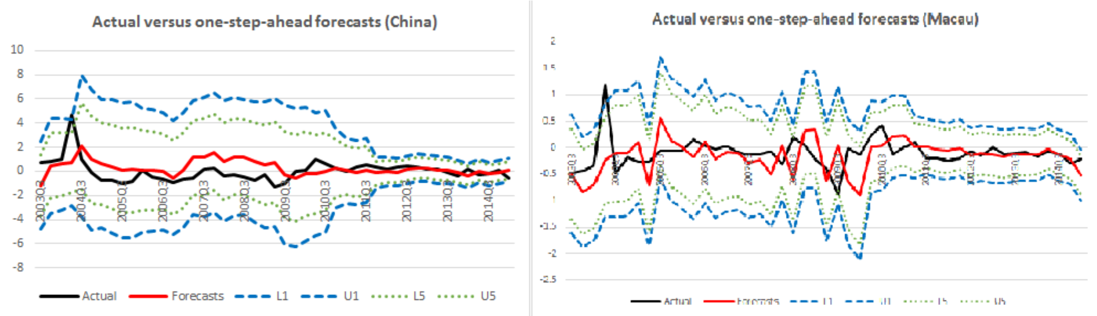
With $n_b = 5$, the solid and the two dashed red lines depict the mean ($h = 0.2$) and the 99% confidence interval of the uniform distribution. Because the pit of Macau is uniformly distributed, it suggests that the $ARDL$ model with normality assumption is appropriate. However, the first and the last bar of mainland China lie outside the boundaries indicating that its pit is not uniformly distributed. Therefore, we modify its distributional assumption as a Student- t distribution $t(v_{t+1})$ that can accommodate the time-varying thickness of the tail where the degree of freedom v_{t+1} is equal to $2s^2_{u,t+1}/\left(s^2_{u,t+1} - 1\right)$. If the estimated variance is less than 1, we assume that the density is standard normal. Under this new distributional assumption, the height of each bar in Figure 2 falls within the boundaries. It justifies the use of a mix of standard normal and t distribution for the growth rates of tourism demand from mainland China suggesting that its error distribution has a fat tail in some periods.

Figure 2



Once the model assumptions pass the evaluation test, we are ready to contrast the time-varying density forecasts with traditional point forecasts in Figure 3.

Figure 3



The black and red lines depict the actual and mean forecasts respectively while the two sets of dashed lines represent boundaries of the confidence interval at 95% (L5 and U5 in green) and 99% (L1 and U1 in blue). A traditional method of selecting a forecasting model is to compare the mean square forecast error in the out-of-sample period $\sum_{t=1}^{n_{oos}}(z_t - \hat{z}_t)^2 / n_{oos}$. However, the accuracy of the forecast at each time point is seldom mentioned. The plot of China demonstrates the deficiency of the mean forecast. The wide tunnel delineated by the blue boundaries before 2011Q3 casts doubt on the reliability of those forecasts. Without questioning the reliability of the mean forecast, one may miss a precious investment opportunity or suffer from huge loss even though the $ARDL$ model is appropriate for the whole period. For example, comparing with the 99% interval forecast under normality assumption [-2.14%, 3.63%], the much wider density forecast [-2.8%, 4.29%] in 2004Q2 suggests that the point forecast (0.74%) has a lower quality than what the interval forecast suggests. An investor without noticing this may fail to capitalize on the actual strong growth rate (4.69%) in the same period. Another scenario which may lead to investment loss occurs in 2008Q2 in which the forecast is 1.21% while the actual growth rate is a negative 0.25%. On the other hand, practitioners can be more confident about the forecasts after 2011Q3 as the band is so narrow that the forecast cannot differ much from the actual path. The time series for Macau has a similar but less obvious pattern. The U.S. subprime mortgage crisis in 2008, and the H1N1 in 2009 not only brought negative growth rates at some time points before 2011Q3, but also reduced the quality of the forecasts (increased volatility).

To conclude, traditional mean point forecasts can at best provide a general trend of the time series, and $MSFE$ can only summarize the model performance in the out-of-sample period. Without considering forecast quality, we cannot even assess the accuracy at each time point. Because it is an essential piece of information in risk management, it is necessary to include such a measurement in the empirical analysis for tourism demand forecasting so as to capitalize on investment opportunities or to avoid business planning mistakes. Density forecasts and the evaluation tool introduced in this research note offer practitioners a simple but more informative way in interpreting the point forecasts. Further research regarding the autocorrelation structure in the errors, shape of the error distributions such as asymmetric and leptokurtic distributions, and other evaluation statistics for the density assumptions can be carried out.

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