CHENG, Qi; WANG, Shengwei; YAN, Chengchu. Robust optimal design of chilled water systems in buildings with quantified uncertainty and reliability for minimized life-cycle cost. Energy and Buildings, 2016, 126: 159-169.

https://doi.org/10.1016/j.enbuild.2016.05.032 This is the Pre-Published Version.

Robust Optimal Design of Chilled Water Systems in Buildings with Quantified Uncertainty and Reliability for Minimized Life-Cycle Cost

Qi Cheng, Shengwei Wang*, Chengchu Yan

Department of Building Services Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong

Abstract: Conventional design of chilled water systems is typically based on the peak cooling loads of buildings, while the cooling load reaches its peak level for only a small proportion of time in a year. This results in that design flow of chilled water system could be significantly oversized in actual operation and it thus causes significant energy wastes. In this paper, a robust optimal design based on minimized life-cycle cost is proposed to optimize the design of chilled water pump systems while concerning the uncertainties of design inputs and models as well as the component reliability in operation. Monte Carlo simulation is used to generate the cooling load distribution and hydraulic resistance distribution by quantifying the uncertainties. Markov method is used to obtain the probability distribution of the system state. Under different control methods, this proposed design method minimizes the annual total cost. A case study on a building in Hong Kong is conducted to demonstrate the design process and validate the robust optimal design method. Results show that the system could operate at a relatively high efficiency and the minimum total life-cycle cost could be achieved.

Keywords: robust optimal design; chilled water system; uncertainty; reliability; total life-cycle cost

* Corresponding author: Shengwei Wang, Email: beswwang@polyu.edu.hk, Phone: (852) 27665858, Fax: (852) 27746146

1. Introduction

The building sector is the largest energy consumer in most countries and regions worldwide, especially in the metropolis such as Hong Kong [1, 2]. Building central chilled water systems, which are the major sub-systems in heating, ventilation and air-conditioning (HVAC) systems, accounts for a significant proportion to the total electricity used in buildings [3, 4].

1.1 Conventional design of chilled water system

The sizing and selection of chilled water pump systems is one of the most important aspects in determining the energy performance of the HVAC systems [5, 6]. The conventional design of chilled water pump systems, proposed by ASHRAE Handbook [7], mainly concerns the design flow required and design pressure head required. The intersection of the required head and flow on the pump curve should occur close to or perhaps a little to the left of best efficiency point (BEP), which may maintain the pumps operating at high efficiency and thus minimize the electricity cost of operating the pumps [8]. Considering that pumps are only manufactured in certain sizes, selection range between 66% and 115% of design flow at the BEP are suggested [9]. In a central air-conditioning system, the designer tends to use identical pumps in parallel to share the system flow [9]. In addition, a standby or backup pump of equal capacity and pressure installed in parallel to the main pumps is recommended to operate to ensure continuous operation when a pump fails to operate or needs to be maintained [10].

Oversizing of chilled water pump systems, which is a common problem in HVAC fields [11], may result in high capital cost, high operation cost, and increased maintenance problems over the system life-cycle when compared to properly sized systems [8]. Oversizing of pump systems contain the oversizing of design flow and oversizing of design pressure head. Due to the inevitable uncertainty of input parameters (e.g., weather condition, occupancy) on cooling load calculation [12], designers tend to select a larger design cooling capacity than the peak duty (e.g., multiply a safety factor) in order that the design cooling capacity can fulfil the cooling demand for safety [13, 14]. This may result in significant oversizing of cooling capacity and thus the design flow [15]. Based on the design flow rate and actual design information of the chilled water loop, component pressure drop information is utilized to calculate the assumed pump head. Then additional design safety factors are added on the assumed pump head to get the design pump head to allow the changes of system load and to cover unknown or unforeseen pressure drop factors [11]. Sometimes an artificial aging factor (e.g., an extra 15%) is included to account for the decrease in pipe diameter as deposits build up on the inside surfaces of the pipes due to aging [8]. Since part load conditions frequently occur throughout the entire cooling season [16], some engineers think they can grossly oversize a pump system and then use variable speed drives to maintain high efficiency and reduce operation cost during the part load period [17]. However, the capital cost and operation cost are still high while the variable speed drives are used.

1.2 Uncertainty and reliability study on building energy system

Conventional optimal design of building energy systems are typically based on the annual cooling load under the predefined conditions, which is commonly subject to a deterministic model-based simulation [12, 14]. However, many researchers had taken the impacts of uncertainties into account when calculating cooling loads and evaluating the performance of building energy systems [18, 19]. The peak cooling load distribution was studied by Domínguez-Muñoz et al. [20] considering the uncertainties in the building material, heat transfer coefficients of external and internal wall, internal sources, etc. Eisenhower et al. [21] conducted an uncertainty study in the intermediate processes by performing decomposition, aiming to find the most important subsystem in modelling. Sun et al. [14] proposed a design method to size building energy systems considering uncertainties in weather conditions, building envelope and operation. Cheng [22] proposed a probabilistic approach for uncertainty-based optimal design to size the chiller plant considering uncertainties of input parameters, which ensures that the chiller plant operate at a high efficiency and the minimum annual total cost could be achieved under various possible cooling load conditions.

Reliability can be defined as the probability of successful operation or performance of systems and their related equipment, with minimum risk of loss or disaster [23]. Reliability analysis or assessment is necessary to avoid/reduce losses caused by both the normal situations and abnormal situations such as the failure of some components [24]. Myrefelt [25] used actual data collected from buildings of seven large real estate operators to analyze the reliability of the HVAC systems. Peruzzi et al. [26] emphasized the importance of the reliability parameters considering financial (reduction of energy and maintenances costs), environmental and resources managing (both concerning the energy and staff) profits. Au-Yong et al. [27] investigated the maintenance characteristics of HVAC systems that affect occupants' satisfaction, subsequently established a relationship between the characteristics and occupants' satisfaction through questionnaire surveys and interviews and finally develop a regression model for prediction purpose. Gang [28, 29] proposed a robust optimal design of cooling systems considering uncertainties of inputs and system reliability, which could obtain the optimal cooling systems with low cost and high robustness and provide a promising means for designers to make their best design decisions.

In order to achieve more flexible, resilient and cost effective design of the chilled water pump systems, a life-cycle based robust optimal design method is proposed in this paper. It can ensure that the chilled water system could operate at high energy performance and the minimum total life-cycle cost could be achieved under various possible cooling load conditions considering the uncertainties of design inputs and reliability of the components. A series of so-called uncertainty “scenarios” generated by Monte Carlo simulation, is used for obtaining the accurate cooling load distribution and accurate hydraulic resistance distribution. Markov method is used to obtain the steady probability distribution of each state of the system considering the reliability. Besides, pump models and three different control methods are used for evaluating the proposed design method. In order to achieve the minimum total cost, trials of simulations on different design flows and different nominal flows are conducted to obtain the optimum chilled water pump system. Section 2 describes the concept of robust optimal design in the HVAC domain. Section 3 presents the method of the robust optimal design for chilled water pump systems. Section 4 shows a case study on the proposed robust optimal design of the chilled water pump system of a building in Hong Kong. The last section draws the conclusions.

2. Concept of robust optimal design

The objective of the robust optimal design is to achieve an optimal design option of minimized life-cycle cost, which provides the system with the capability to operate at relatively high efficiency at various possible conditions considering the uncertainties of design inputs and system reliability in operation. This proposed method takes into account the uncertainty and reliability compared with the conventional/optimal design method, which has the following features:

- Uncertainty being considered: the designed system has enough tolerance towards the deviation between the actual condition and the predefined information, associated to the design inputs such as weather conditions and number of occupants.
- Reliability being considered: the designed system has the capability to fulfill the cooling demands of users under the normal situations and abnormal situations (i.e., the failure of systems), associated to the uncertain heath situations of equipment.

The fundamental difference between the robust optimal design method and other design methods is illustrated in Ref. [28]. *Conventional optimal design in HVAC field* guarantees the optimization over predefined conditions (without considering the uncertainties and reliability) [30]. It can be seen that the conventional method determines the HVAC system without quantitative uncertainty and reliability analysis.

Uncertainty-based method determines the size of the systems [14] or investigates the building performance [31] considering uncertainties in design. Reliability-based method ensures the system capability by minimizing the effect of sources of design parameters or process variables, which is rarely studied in HVAC field [28]. Robust optimal design method concerns quantitative uncertainty and reliability analysis as well as quantitative performance optimization simultaneously.

Conventional design and uncertainty-based optimal design usually contain one state (all the equipment are healthy) and a standby or backup pump of equal capacity and pressure head is used in case that a pump fails to operate or needs to be maintained. Using the robust optimal design, chilled water pump system is usually regarded as a multi-state system. Multi-state pump system contains the situations: no failure of pump, failure of one pump and up to failure of all pumps. Failure of the pumps may result in that the chilled water system cannot deliver sufficient chilled water to the users and the cooling demand of users cannot be satisfied. Therefore, the reliability analysis is required to be conducted at the design stage.

3. Robust optimal design method for chilled water pump systems

There are two main advanced chilled water systems in which the chilled water can be distributed from the chiller to the terminal users (air handling unit)-i.e. primary-secondary pump system and primary only pump system [32]. For primary-secondary pump system, constant-speed pumps are usually used to circulate the water in the primary loop whereas variable-speed pumps are usually employed for varying the water circulation in the secondary loop depending upon the cooling demands of terminal users. For a primary only pump system, variable-speed pumps are used to circulate the chilled water through the entire system, and the chilled water flow rate varies corresponding to the load.

In this study, primary only pump system is used for the design, as shown in Fig.1. Identical variable-speed pumps are used to circulate the chilled water through the entire system and the chilled water flow rate varies corresponding to the load. Bypass is used to maintain the minimum flow rate for safety, and the cooling demands of three terminal users are similar.

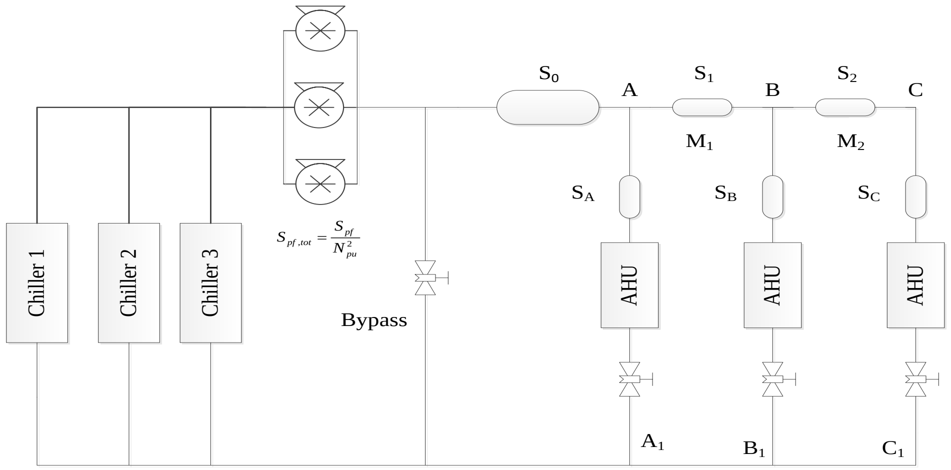

Fig.1 Scheme of primary only pump system

3.1 Objective of robust optimal design

The objective the proposed method is to ensure that the system operates at high efficiency over the entire cooling season and achieve the minimum annual total cost considering uncertainties and system reliability. The annual total cost (TC_n) consists of three parts: annualized capital cost (CC_n), annual operation cost (OC_n) and annual availability risk cost (RC_n). Annualized capital cost includes the expense in purchasing/installing the pumps and associated components (equipment cost) and the spaces for accommodating them (space cost), which is determined by the number and size of pumps. Annual operational cost is the cost electricity consumed by the pumps in operation, which is mainly associated to the annual cooling load distribution and the pump energy efficiency. Availability risk cost is the “expense” or service sacrifice which should be considered when the cooling demands cannot be fulfilled. Fig.2 illustrates the conceptual relationship between the costs and system total capacity under the optimized pump head. It is well-known that large system capacity means higher system reliability. The capital cost increases as the system capacity increases. Under the optimal configuration of chilled water pumps system, the operation cost may change slightly as the system capacity increases. On the other hand, the availability risk cost decreases as the system total capacity increases. The total life-cycle cost is comprised of the capital cost, operation cost and availability risk cost, as shown in Equation (1). According to Fig.2, there should be a comprised system capacity to achieve the minimum total life-cycle cost, at which a comprised level of reliability is achieved [33].

$$TC_n = CC_n + OC_n + RC_n \tag{1}$$

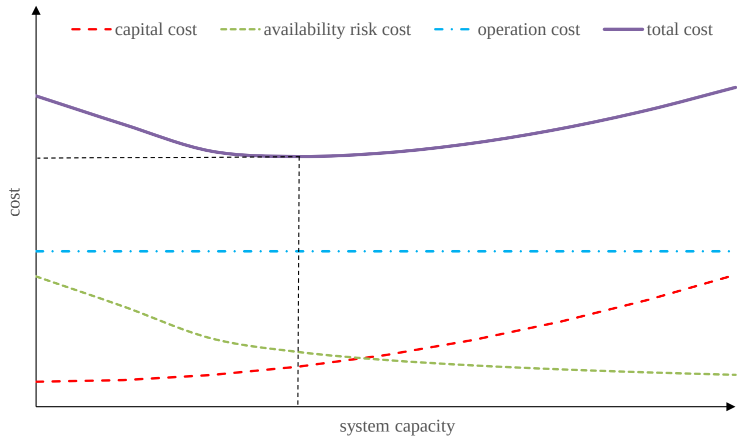

Fig.2 Total cost *vs* system capacity

3.2 Procedure of the proposed robust optimal design

The robust optimal design is performed by four steps as shown in Fig.3. Details of the four steps are explained as follows.

- *Uncertainty quantification:* generate the cooling load distribution involving uncertainties and determine the design flow; then generate the hydraulic resistance distribution and determine the design pressure head;
- *Reliability quantification:* obtain the probability distribution of each state of chilled water pumps;
- *Modeling and control methods of chilled water pumps:* obtain the pump models on the calculation of electricity consumption; determine the basic, medium and advanced control methods of chilled water pumps.
- *Trials of simulations on the total flow and nominal flow:* determine the

searching range of total pump flow capacity and conduct trials on each design flow step by step; obtain the operation cost, capital cost and availability risk cost under different pump numbers on each design flow; obtain the optimal chilled water pumps under each design flow.

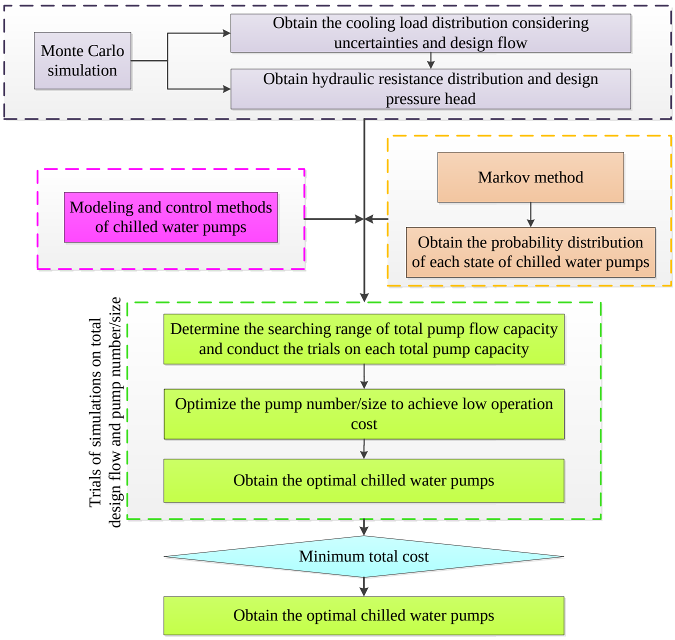

Fig.3 Procedure of the proposed robust optimal design

3.2.1 Modeling of chilled water pumps and control methods

The electricity consumption of chilled water system, also regarded as the operation cost, is one of the most important aspects in selecting the optimum chilled water pumps. The electricity consumption of the system (also called operation cost OC_{pu}) depends on the pressure drop (H_{pu}), the water flow rate (m_w), pump efficiency (η_{pu}) and VFD (variable frequency drive) efficiency (η_{VFD}), which can be computed by Equation (2) [34].

$$OC_{pu} = \frac{m_w H_{pu}}{102\eta_{pu}\eta_{VFD}} \quad (2)$$

The three efficiencies of variable speed pumps was modeled using a series of polynomial approximations [35]. The characteristics of pump efficiency and VFD efficiency are based on the manufacturers' data at the full speed operation and extended to the variable speed operation using the pump affinity laws. Pump efficiency is modeled using Equation (3), which is a function of the fraction of the nominal flow [36]. VFD efficiency is modeled using Equation (4), which is a function of the fraction of the nominal speed [37]. The coefficients in these polynomials can be regressed using the pump performance data or performance curves and VFD efficiency curve provided by the manufacturers.

$$\eta_{pump} = \eta_{design} \cdot (d_0 + d_1 x + d_2 x^2 + d_3 x^3) \tag{3}$$

$$\eta_{VFD} = e_0 + e_1 y + e_2 y^2 + e_3 y^3 \tag{4}$$

where, η_{design} is design pump efficiency, x is the fraction of nominal flow, y is the fraction of the nameplate brake horsepower or the nominal speed, d_0–d_3 and e_0–e_3 are coefficients.

The pressure head, which also determines the electricity consumption of chilled water pump systems, depends on the control method. In this study, three levels of control optimization methods, i.e. basic level, medium level and advanced level, are proposed for determining the pressure set-point in operation and then the operation cost of systems, as shown in Fig.4. Users can selected them based on the expected level of the control optimization of the system to be optimized. The basic level method is that the pressure set-point of the chilled water loop $\Delta p_{set,b}$ (a major part of the pump pressure head) in the building is assumed to be a constant value regardless of the water flow rate as shown in Fig.4 and Equation (5). The medium level method is that the pressure set-point of the chilled water loop $\Delta p_{set,m}$ in the building is assumed to be linear to the water flow rate (m_w) as shown in Fig.4 and Equation (6). The advanced level method is that the pressure set-point of the chilled water loop $\Delta p_{set,a}$ in the building is assumed to be square to the water flow rate (m_w) as shown in Fig.4 and Equation (7). The minimum pressure set-point is assumed to be 30% of design pump pressure head. The chilled water flow rate in the building is assumed to be linear to cooling load and the minimum water flow rate is assumed to be 20% of design flow rate.

$$\Delta p_{set,b} = p_D \tag{5}$$

$$\Delta p_{set,m} = p_{\min} + \frac{\alpha \cdot m_w}{m_D} \tag{6}$$

$$\Delta p_{set,a} = p_{\min} + \beta \left(\frac{m_w}{m_D} \right)^2 \tag{7}$$

where, p_D is design pressure head, m_D is design flow rate, α and β are coefficients.

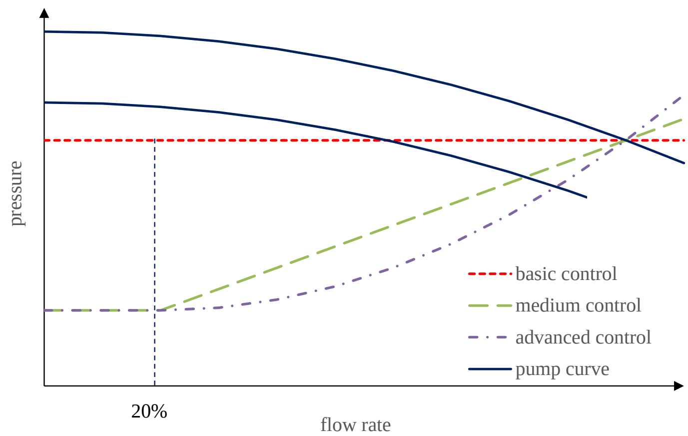

Fig.4 Pressure set-point of chilled water loop *vs* flow rate

3.2.2 Determine the design flow and pressure head involving uncertainties

To conduct the proposed robust optimal design, the first step is to obtain the cooling load distribution involving uncertainties and then determine the design flow, the second step is to obtain the hydraulic resistance distribution involving uncertainties and determine the design pressure head.

Module 1 – Obtain the cooling load distribution and design flow

In order to generate the cooling load distribution considering uncertainties and then obtain the design flow, Monte Carlo simulation is employed. In this study, the uncertainties of the design inputs are computed by Matlab. Three types of distributions (including normal distribution, tri-angular distribution and uniform distribution) are commonly used to describe the uncertainties of inputs. Table 1 shows an example of the settings of uncertainties of the inputs. Combining the output uncertainties from Matlab, the TRNSYS building model is used to generate the building cooling load distribution considering the uncertainties based on the determined simulation number. After conducting the required trials of Monte Carlo simulations, the cooling load distribution involving uncertainties is determined. In this study, about 780 times of Monte Carlo simulations are used to generate the cooling load distribution [22]. Then, the designers can determine the design cooling capacity based on their specific requirements.

Table 1 Distributions of stochastic inputs

Parameters	Distributions
Outdoor temperature (℃)	$N(0,1)$
Relative Humidity (%)	$N(0,1.35)$
Number of Occupants	$T(0.3,1.2,0.9)$
Infiltration rate (m^3/s)	$U(2.7, 3.3)$
Equipment rejection load (kW)	$U(376, 464)$
Remarks: N(μ, σ) - normal distribution with mean value μ and standard deviation σ; U(a, b) - uniform distribution between a and b; T (a, b, c) - triangular distribution with lower limit a, upper limit b and mode c.	

Besides, the design flow is determined by the temperature difference and design cooling load. In practice, the designers tend to choose a constant temperature difference in the design of cooling systems (i.e., the supply chilled water temperature is 7℃ and the return chilled water temperature is 12℃). The flow required is then calculated by Equation (8). Where, m_D is the design flow, CL_D is the design cooling load, c_p is the specific heat of chilled water and Δt is the temperature difference.

$$m_D = \frac{CL_D}{c_p \cdot \Delta t} \tag{8}$$

<u>*Module 2 – Obtain the hydraulic resistance distribution and pressure head*</u>

In practice, the pressure head is determined by the overall pressure drop of “worst case circuit". Fig.1 presents the simplified structure of the pressure-flow balance model for the chilled water network at the primary pump only system, in which only three terminal units are included as examples. The bypass is used to maintain the minimum flow rate of chilled water through chillers (i.e., 60% of the design flow rate of an individual chiller). The overall pressure drop of this system, i.e., along the sub-branch C-C_1, can be mathematically described as in Equation (9), which includes the pressure drop on the chillers, the pressure drop on the fittings around pumps (including the pressure drop on the headers that direct the flow into and from each pump and the pressure drop on the valves in the pump headers), the pressure drops on main supply and return pipelines, the pressure drop across the sub-branch (i.e., C-C_1) and the pressure drops on the pipeline sections of A-B and B-C.

$$\Delta p_{pump} = \frac{S_{ch}}{N_{ch}} m_0^2 + \frac{S_{pu}}{N_{pu}} m_0^2 + S_0 k_c m_0^2 + S_1 k_c m_1^2 + (S_2 k_c + S_C) m_2^2 \quad (9)$$

where, ΔP_{pump} is the pressure drop of the entire chilled water loop. S_{ch} and S_{pu} are the coefficients of chillers and pumps. S_0, S_1 and S_2 are the coefficients of pipeline. S_C is the coefficient of AHU. m_0, m_1 and m_2 are the flow rate of chilled water.

The pump pressure head is then determined by the hydraulic resistance coefficients, chilled water distribution in each terminal unit and aging factor of the pipelines as well as the fluctuation of the chilled water flow. For a given design cooling load, the design chilled water flow are influenced by the fluctuation (i.e. uncertainty) of the difference between return and supply chilled water temperatures. The flow of chilled water in each terminal unit usually fluctuates around the design flow considering the uncertainty of its heat transfer performance. The flow of chilled water in each terminal unit is assumed to be subject to normal distribution. Uniform distribution is used to describe the uncertainties of the hydraulic resistances of components. In addition, an artificial aging factor is adopted to account for a decrease in pipe diameter as the system ages. According to Equation (9), the distribution of pressure head can be generated and the design pressure head is assumed to be 99.6 percentile of the distribution.

3.2.3 Probability distribution of each state considering system reliability

Markov method is used in this study because of its wide application in reliability analysis of multi-state systems [38]. The aim of using Markov method is to obtain the probability of each state of a multi-state system at a specific period and then the performance of the system and capability can be estimated. It is assumed that the state probabilities at a future instant do not depend on the states occurred in the past. The system either keeps current state or transfer to other states at the next time step. Several steps are required using Markov method [28], including:

- List all the possible states of the chilled water pump system;
- Determine the state transition density matrix;
- Obtain how much is required to reach the steady state;
- Obtain the probability of each state of the system;
- Calculate the mean steady performance and capability under each state.

A system is comprised of n chilled water pumps. It is assumed that each pump has two states only: normal (0) and failure (1). Totally the system has n states (i.e., each states contains several situations) considering the reliability of pumps, as shown in Fig.5 [39]. It can be observed that state 0 symbolizes that no pump fail and state k symbolizes that k ($1 \leq k \leq n$) pumps fail. From state 0 to state n, the failure rate λ is used to represent the probability from one state to another. From state n to state 0, the repair rate μ is used to represent the probability from one state to another. The transition probability is determined by a state transition density matrix A (Equation (10)), which only involves the repair rate and failure rate of pumps [40]. Probability distribution of the system at each state at time t can be represented with a vector $P(t)$ (Equation (11)). It can be deduced from the initial state by Equation (12) and Equation (13). When the time approaches to infinity, $P(\infty)$ will keep stable (Equation (14)). Then the steady state probabilities can be obtained by solving the linear algebraic equations (Equation (15) and Equation (16)). The mean steady system performance under each state thus can be calculated. In addition, it is worth noticing that how much time is required to reach the steady condition is significant in the method. If the time is very long such as two years, it should be considered in the reliability assessment. If the time is very short such as one month, then it will not be considered in this method. Probability of state 0 is selected as the standard to assess the time achieving the steady state, as shown in Fig.6.

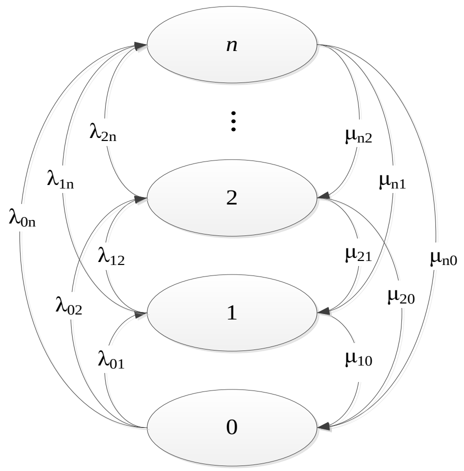

Fig.5 States of a *n*-pump system and possible transitions

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} & \dots & a_{0n} \\ a_{10} & a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n0} & a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \tag{10}$$

$$P(t) = \left[p_0(t),\ p_1(t),\ \dots,\ p_n(t) \right] \tag{11}$$

$$P(0) = [1, 0, 0, \dots, 0] \tag{12}$$

$$P(n) = P(n-1)A = P(0)A^n \tag{13}$$

$$P(\infty) = \lim_{n\to\infty} P(n) = \lim_{n\to\infty} P(0)A^n \tag{14}$$

$$P(\infty) = P(\infty - 1)A = P(\infty)A \tag{15}$$

$$\begin{cases} p(0) = a_{00}p(0) + a_{10}p(1) + \cdots + a_{n0}p(n) \\ p(1) = a_{01}p(0) + a_{11}p(1) + \cdots + a_{n1}p(n) \\ \quad\vdots \qquad\qquad\qquad \vdots \\ p(n) = a_{0n}p(0) + a_{1n}p(1) + \cdots a_{nn}p(n) \\ \sum_{i=0}^{n} p(i) = 1 \end{cases} \tag{16}$$

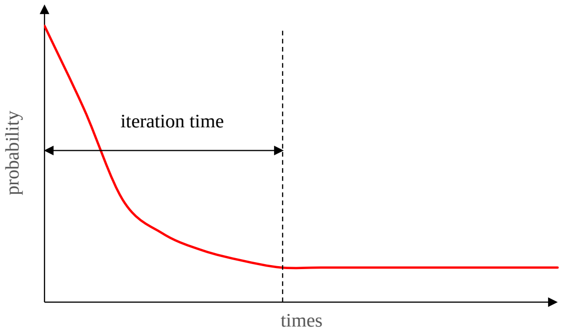

Fig.6 Scheme of the time achieving the steady condition

However, the key issue for using Markov method is to determine the transition density matrix A. As mentioned above, transition density matrix is related to the failure rate and repair rate of pump only. When the repair rate and failure rate are regarded as time-independent, these two variables can be obtained by Equation (17) and (18) [41]. Considering that each state (i.e. same number of failure pumps) may contain several situations (i.e. different combinations of failure pumps), the probability that the situations in a state transfer to those in another state in various possible conditions should be obeyed to the law of combinations, as shown in Equation (19).

$$\lambda = 1/MTTF \tag{17}$$

$$\mu = 1/MTTR \tag{18}$$

$$a_{ij} = \begin{cases} C_n^i \cdot C_{n-i}^{j-i}\lambda & , \quad i < j \\ 1 - a_{i0} - \cdots - a_{i(j-1)} - a_{i(j+1)} - \cdots - a_{in} & , \quad i = j \\ C_n^i \cdot C_{n-i}^{i-j}\mu & , \quad i > j \end{cases} \tag{19}$$

where, λ is failure rate, μ is repair rate, *MTTF* is mean time to failure, *MTTR* is mean time to repair, a_{ij} is the probability from state i to state j.

3.2.4 Trials of simulations on the total pump flow capacities and pump sizes

Different from previous research which is mainly based on the BEP, in this study, the cooling load distribution are selected to determine the optimal chilled water pumps, which could improve the total operating efficiency and reduce the operation cost. Considering that pumps are only manufactured in certain size, trials of simulations on different total pump flow capacities and different and discrete pumps sizes are conducted to select the optimal pump system. At start, the searching range of total pump flow capacity should be determined to facilitate the trials on design flow. In this study, the searching range of total pump flow capacity is assumed to be about 1~2 times of the design flow and the interval of total pump flow capacity is 2.5% of the design flow.

After the searching range and interval of total pump flow capacity are determined, the trials of simulations on each total pump flow capacity based on the cooling load distribution can be implemented as shown in Fig.7. The option which has the lowest total cost under each total pump flow capacity is selected. Eventually, among the options corresponding to various total pump flow capacities, the option which has the minimum total cost is selected as the optimum design for a building.

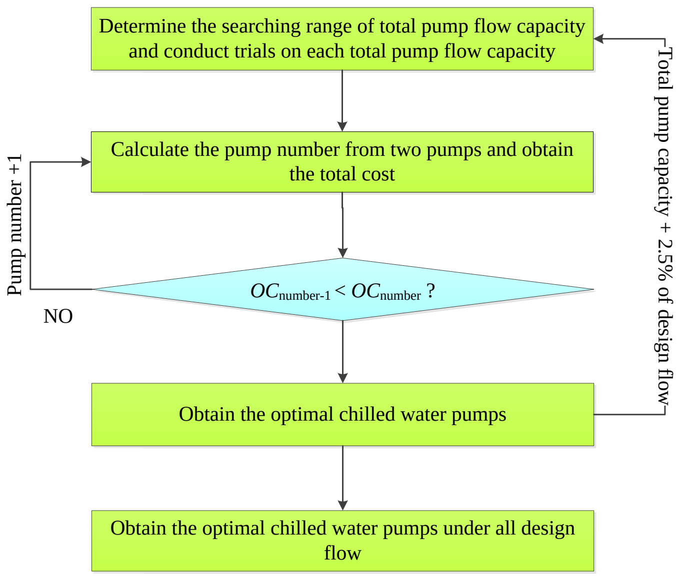

Fig.7 Trials of simulation to select optimum pump design

The main step in this searching process is "Calculate the pump number/size from two pumps and obtain the total cost". Under each total pump flow capacity, simulation trials start from two pumps (minimum two is assumed concerning the basis requirement for reliability and maintenance) until the operation cost begins to increase. At the same time, the capital cost and availability risk cost are determined. Identical variable-speed pumps are assumed in this study, which is typical particularly when chillers of identical capacity are selected.

The overall efficiency of pump systems is determined by the pump efficiency and VFD efficiency, as shown in Equation (20). It is well known that for a given building, if the number of pumps used is larger, the nominal flow of individual pumps is lower, the design pump efficiency is lower [42], and the VFD efficiency and load ratio of pumps are larger in operation because they can operate near their full load. Fig.8 presents the relationship between the rated pump efficiency and pump capacity in this study. It can be observed that the rated pump efficiency increases when the pump capacity increase.

$$\eta_e = \eta_{pu} \cdot \eta_{VFD} \tag{20}$$

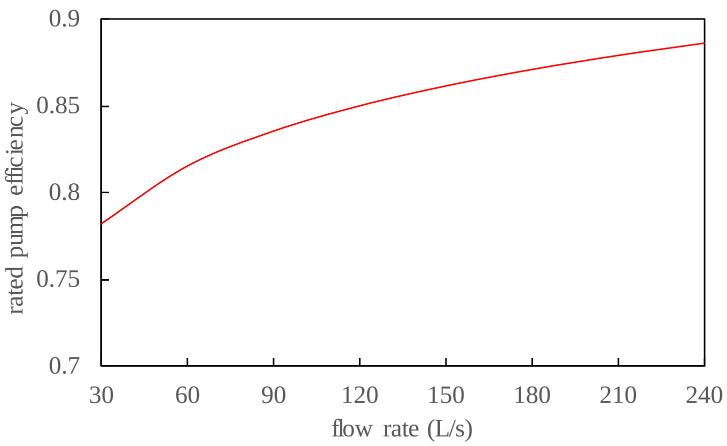

Fig.8 Rated pump efficiency vs. pump capacity

Fig.9 shows the conceptual relationship between the number of pumps and overall pumps efficiency. When the number of pumps is small, the increase of pump number may result in the increase of overall efficiency because the VFD efficiency and load ratio of pumps increase significantly along with the decrease of the design efficiency of pumps. When the number of pumps is large, the increase of pump number may result in the decrease of overall efficiency because the VFD efficiency and load ratio of pumps have no obvious further improvement. Since at least two pumps are assumed, the number of pumps is tested starting from two until the operation cost begins to increase. Equation (21) presents the typical pump efficiency profiles of a variable-speed pump (120L/s) according to the data from a pump manufacturer. In this study, under the same part load ratio, the pump efficiency of variable-speed pumps is assumed to be proportional to their capacity. The operation cost is calculated using Equation (22), the capital cost and availability risk cost are calculated using Equation (23) and (24). Where, $p_i(t)$ and OC_i are the probability and operation cost under the cooling load CL_i. n is the number of pumps, EC_{ind} is the equipment cost of individual pump, and SC_{ind} is the space cost of accommodating an individual pump. RC_i is the availability risk cost under the cooling load CL_i.

$$\eta_{pu} = 0.85 * \left[0.2455 + 1.509 * \left(\frac{m_w}{m_D} \right) - 1.671 * \left(\frac{m_w}{m_D} \right)^2 + 1.275 * \left(\frac{m_w}{m_D} \right)^3 \right] \tag{21}$$

$$OC = \sum_{i=0}^{n} p_i(t) \cdot OC_i \tag{22}$$

$$CC = n \cdot (EC_{ind} + SC_{ind}) \tag{23}$$

$$RC = \sum_{i=0}^{n} p_i(t) \cdot RC_i \tag{24}$$

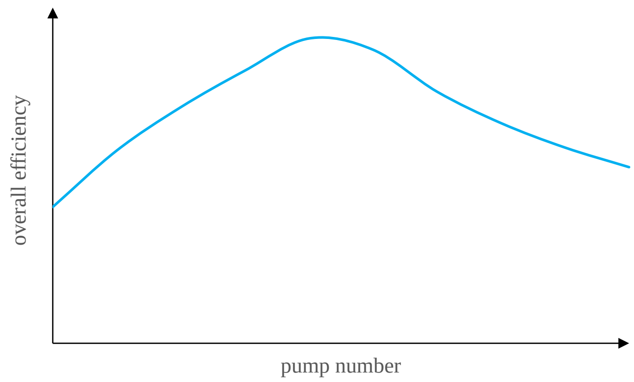

Fig.9 pump number *vs.* overall efficiency

4. Case study and results

A case study on the chilled water pump system design for a building in Hong Kong is conducted to test and evaluate the proposed robust optimal design. At first, Monte Carlo simulation is used to generate the cooling load distribution profile and the design flow is determined based on the cooling load distribution. Then, the hydraulic resistance distribution and design flow are determined according to the cooling load distribution profile. Reliability assessment of the pump system is conducted to obtain the probability of each state under different pump number. Combining the pump models and control methods, the trials of simulations on different total pump flow capacities and pump sizes are conducted to select the optimum chilled water pump system which has the minimum total cost.

4.1 Determine the design flow and pump head involving uncertainties

4.1.1 Obtain the cooling load distribution and design chilled water flow

To conduct the Monte Carlo simulations in order to obtain the cooling load distribution, it is essential to select the parameters of uncertainties of the design inputs [22]. Combining the output uncertainties from Matlab, the TRNSYS building model is used to generate the building cooling load involving the uncertainties. After conducting 780 times of Monte Carlo simulations [22], the cooling load distribution is obtained, as shown in Fig.10. The reference case is the normal cooling load distribution without considering the uncertainties. It can be seen that the cooling load distribution profile of 780 simulation trials is smoother than that of reference case because more cooling load conditions are considered. The design cooling capacity can be sized based on the load of 5100 kW according to the design standard “50 unmet hours” [22].

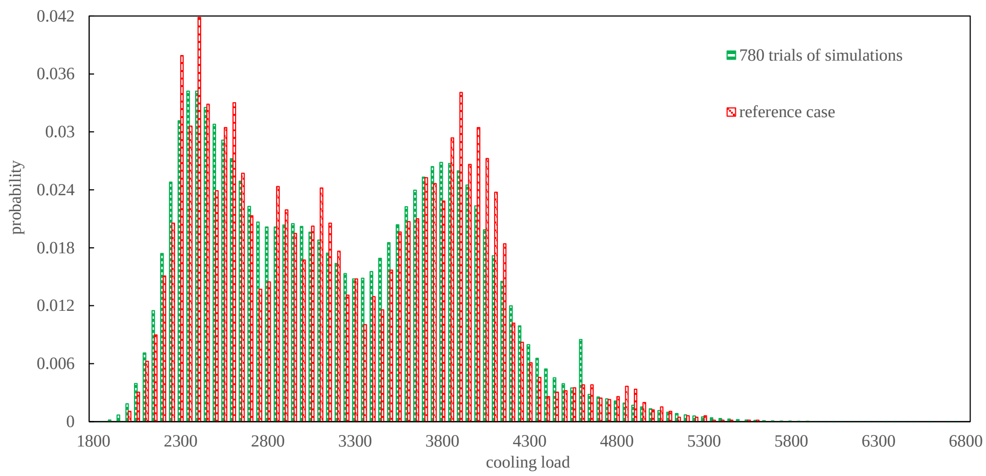

Fig.10 Distribution of cooling load considering uncertainties

The design chilled water flow is determined by the design capacity and temperature difference. In practice, designers tend to assume that chilled water supply temperature is 7℃, chilled water return temperature is 12℃ and then the temperature difference is 5℃. According to Equation (1), the design chilled water flow is 240L/s.

4.1.2 Obtain the probability distribution of pump pressure head

The pump pressure head is determined by chilled water flow rate, hydraulic resistance coefficient, chilled water distribution in each terminal unit and aging factor of the pipeline. Table 2 shows the pressure drops of components, water distribution in each terminal unit and aging factor [36]. In this study, three AHUs are used to serve three zones of the same cooling demand and the aging factor of pipes is assumed to be 15% as a constant. According to Equation (2), the distribution of pump pressure head can be generated as shown in Fig.11. The design pump pressure head is assumed to be about 26m, which is equivalent to 99.6% of the distribution of hydraulic resistance.

Table 2 Settings of hydraulic parameters

Parameters	Pressure drop of fittings (m)	Uncertainty
Chiller	5.8	*U* (0.9,1.1)
Pump	7.2	*U* (0.9,1.1)
Pipe (main)	5.7	*U* (0.9,1.1)
Pipe (branch)	2.9	*U* (0.9,1.1)
AHU	4.5	*U* (0.9,1.1)
Valve	4.4	*U* (0.9,1.1)
Chilled water flow rate	-	1+*N* (0,0.05)
Chilled water flow in each branch	1/3 of the total	1+*N* (0,0.05)
Aging factor of pipes	15%	-

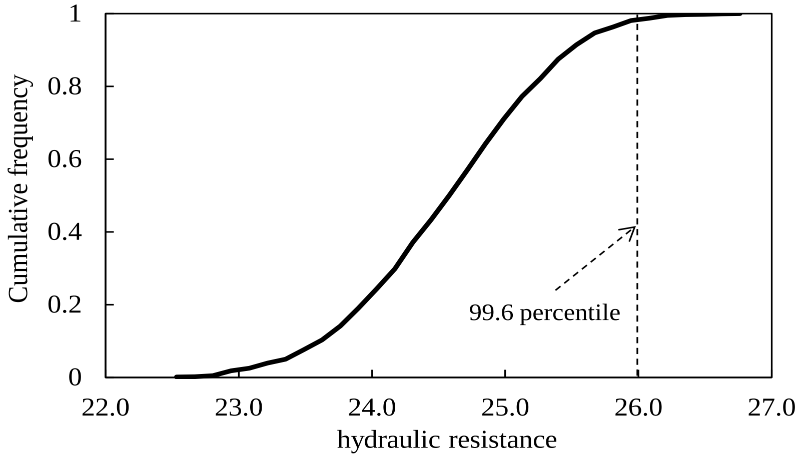

Fig.11 Accumulative probability distribution of the overall hydraulic resistance

4.2 Determine the probability distribution of each state of chilled water pump systems

As mentioned above, Markov method is used to obtain the probability of each (health) state of the pump system and to calculate the mean steady performance and capability under each state. In this study, the system is assumed to be comprised of about 2~8 chilled water pumps and the system there have 2~8 states accordingly. The failure rate is assumed to be 0.0001/hour [43], the repair rate is assumed to be 0.002/hour [28] and the ratio of repair rate/failure rate is 20. Fig.12 shows the iteration time achieving the steady state 0 under different pump numbers. It is worth noticing that the system comprised of less pumps needs more time to achieve the steady state 0. The probabilities of state 0 under 2, 3, 4, 5 and 6 pumps are 0.9222, 0.8906, 0.8494, 0.7951 and 0.7289 respectively. For the system comprised of two pumps, about 1500 hours (i.e., 83 days if the system works 18 hours daily) is required to achieve the steady state 0, which could be ignored during the life cycle of system.

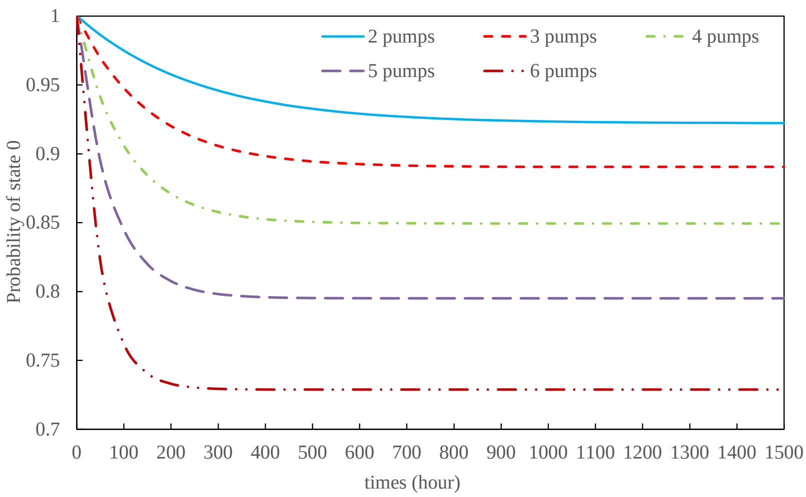

Fig.12 Iteration time achieving the steady state 0 under different pump numbers

It is worth noticing that the failure rate and repair rate may have significant impact on the iteration time. When the repair rate varies from 0.001 to 0.005, the ratio of repair rate to failure rate (i.e. repair rate/failure rate) varies from 10 to 50. Fig.13 shows the impact of the ratio of repair rate to failure rate on the iteration time under the system comprised of three pumps. It can be seen that the iteration time is shorter when the ratio of repair rate to failure rate increases. When the ratio increases from 10 to 50, about 1750, 900, 750, 600 and 400 hours are required to reach the steady state 0 respectively. At the same time, the probabilities of state 0 are 0.79, 0.88, 0.93, 0.95 and 0.96 respectively when the ratio increases from 10 to 50. Therefore, when the ratio of repair rate to failure rate increases, less iteration time is required to reach the steady state, and the probability of state 0 increases to higher level. In this study, the ratio is assumed to be 20, which has high robustness concerning the system reliability.

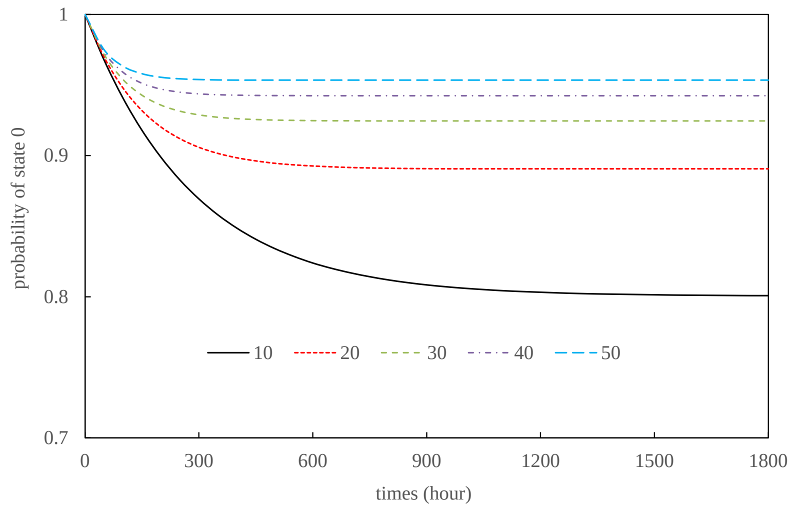

Fig.13 Iteration time under different ratios of repair rate to failure rate

Then, it is essential to obtain the probability distribution of each steady state under various pump numbers. Table 3 shows the probability distribution of each steady state under different pump numbers. It can be observed that the probability of state 0 decreases as the increase of pump number.

Table 3 Probability distribution of steady states of pumps

state	pumps						
	2	3	4	5	6	7	8
0	0.9222	0.8906	0.8494	0.7951	0.7289	0.6575	0.5711
1	0.0605	0.0809	0.1114	0.1517	0.1973	0.2381	0.2913
2	0.0173	0.02	0.0247	0.0327	0.0462	0.0666	0.0865
3	-	0.0085	0.0096	0.0115	0.0146	0.0198	0.0273
4	-	-	0.0049	0.0057	0.0068	0.0084	0.0106
5	-	-	-	0.0033	0.0039	0.0047	0.0057
6	-	-	-	-	0.0024	0.0029	0.0036
7	-	-	-	-	-	0.0019	0.0023
8	-	-	-	-	-	-	0.0016

4.3 Trials of simulations on the total pump flow capacity and pump size

At the previous steps, the design chilled water flow and pump pressure head are determined to be 240L/s and 26m respectively. To conduct the trials of simulation on the total pump flow capacity and pump size, it is essential to determine the searching range of total pump flow capacity. As mentioned above, the searching range of total pump flow capacity is assumed to be 1~2 times of design flow and the searching interval is selected to be 2.5% in this study. Trials of simulations are conducted on the 41 total pump flow capacities respectively (i.e. 240L/s, 246L/s, …, 474L/s and 480L/s).

For example, it is assumed that total pump flow capacity is 336L/s. According to Fig.9, the overall efficiency increases when the number of pumps increases in certain range and it decreases when the number of pumps increases further. According to Section 3.2.4, the evaluation of the number of pumps on the operation cost is conducted. The electricity price used in this study is 1 HKD/kW, which is the typical rate in Hong Kong. The results are shown in Table 4. It can be observed that the operation costs under basic and medium levels of control optimization decrease when the pump number increases from 2 to 7 and they increase when the pump number increases to 8. Under the advanced level of control optimization, the operation cost does not have obvious change when the number of pumps is 3 or more. In this paper, the detailed results the case study under medium level of control optimization are presented to demonstrate the design process. The design option comprised of 7 pumps has the lowest operation cost compared with the other options.

Table 4 Annualized capital cost of different design options

Option (size (L/s)× number)	Operation cost (10^3HKD)			*EC* (10^3HKD)	*SC* (10^3HKD)	*CC* (10^3HKD)
	Basic	Medium	Advanced			
168×2	579	452	380	60	10	70
112×3	547	439	364	77	15	92

84×4	517	435	366	92	20	112
67×5	511	433	366	105	25	130
56×6	493	431	362	117	30	147
48×7	485	424	366	128	35	163
42×8	483	431	366	139	40	179
Remarks: EC- equipment cost, *SC*- space cost, *CC*- capital cost						

Annualized capital cost contains the equipment cost and space cost. The life cycle of the chilled water pump system is assumed to be 10 years. Equipment cost of variable-speed pump (26m, 60L/s) is 150×10^3HKD, referring to the data from a manufacture. As for the equipment cost of other variable-speed pumps, they are estimated using Equation (25) [44, 45].

$$EC = EC_0 \cdot \left(C / C_0\right)^{\alpha} \quad (25)$$

where, EC_0 is the equipment cost of a reference pump with the capacity C_0. EC is equipment cost of pump with the capacity C. α is the coefficient, which set to be 0.15 in this study [46, 47]. The annualized capital costs under the different design options are estimated using Equation (22) and presented in Table 4. From Table 4, the annualized capital cost increases when pump number increases at a given design flow.

Availability risk cost is the “expense” or service sacrifice which should be considered when the cooling demands cannot be fulfilled. Table 5 shows the annual availability risk costs and total costs of different pump numbers under three penalty ratios (i.e., 1, 10 and 100 HKD/kW). It can be seen that, when the pumps number is small, the annual availability risk cost decreases rapidly when the pump number increases. The annual availability risk cost of options having 2, 3, 4 and 5 pumps is very sensitive to the penalty ratio when the pump number is small, but it is not sensitive any more when the number of pumps is 6 or more. It can also be observed that the total cost decreases when the pump number increases in certain range and it increases when the pump number increases further. Since the availability risk cost is high when the pump number is small and the capital cost is high when the pump number is large, there is a comprised pump number/size which has the minimum total cost. In this study, the penalty ratio is assumed to be 10HKD/kW. Among these options, the option 56L/s×6 pumps has the minimum total cost 586×10^3HKD, which is not sensitive to the penalty ratio. Therefore, it can be considered as the best option under the total pump capacity 336L/s. If the penalty ratio is 1HKD/kW, the best option under the total pump capacity is 67L/s×5 pumps. The designers can select the best option based on their specific requirement of penalty ratio.

Table 5 Annual availability risk cost (10^3HKD) and total cost (10^3HKD) of different pump design options

Penalty ratio (HKD/kW)	1		10		100	
Option (size(L/s)×number)	*RC*	*TC*	*RC*	*TC*	*RC*	*TC*
168×2	1,377	1,900	13,770	14,294	13,7700	13,8240
112×3	442	973	4,420	4,951	4,4200	44,729
84×4	96	643	963	1,510	9,630	10,179
67×5	10	573	100	662	1000	1,559
56×6	1	578	**9**	**586**	92	669
48×7	0	587	0	587	0	587
42×8	0	609	0	609	0	609
Remarks: RC- availability risk cost, *TC*- total cost.						

After conducting the trials on other total pump flow capacities, the minimum total costs are computed and presented in Table 6. It can be observed that the options comprised of more pumps may have lower operation cost compared with those design options comprised of less pumps. When the total pump flow capacity increases from 240L/s to 336L/s, the availability risk costs of the best options decrease rapidly and the total costs are also reduced. When the total pump flow capacity is over 384 L/s (i.e. 384 L/s to 480L/s), the availability risk cost of the best option is almost equal to 0.

When the total pump capacity is low, more number of pumps is required to reduce the availability cost resulting in high capital cost. When the total pump capacity is large, less number of pumps is sufficient to keep low availability risk cost while the capital cost is low. It can be seen that the option with 78 L/s×5 pumps has the minimum total cost 569×10^3 HKD compared with other options. It means that the selected option has better robustness to uncertainties and system reliability.

Table 6 Best pump design options under different total pump flow capacities and optimal options using different design methods (penalty ratio:10HKD/kW)

	Total capacity (L/s)	Best option (size (L/s) × number)	Availability risk cost (10^3HKD)	Operation cost (10^3HKD)	Total cost (10^3HKD)
Robust optimal design*	240	30×8	588	431	1,181
	264	33×8	142	432	740
	288	36×8	25	431	626
	312	45×7	11	428	598
	336	48×7	0	424	587
	360	72×5	24	430	587
	384	77×5	3	431	574
	390*	***78×5***	***1***	***432***	***569***
	408	82×5	0	438	572
	432	86×5	0	438	579
	456	114×4	5	446	574
	480	120×4	0	456	581
Uncertainty-based design	280	40×7 (including 1 standby pump)	116	431	700
Conventional design	400	100×4 (including 1 standby pump)	116	461	698

Table 6 also shows the results of uncertainty-based design and conventional design. It can be seen that the total cost under conventional design (698×10^3HKD) is close to that (700×10^3HKD) under uncertainty-based optimal design. Compared with conventional design and uncertainty-based optimal design, the total cost under robust optimal design (569×10^3HKD) is reduced by about 18.6% when the penalty ratio is 10 HKD/kW. To achieve the minimum annual total cost, the option with 5 variable-speed pumps (78L/s) can be selected as the optimum selection for the design. Compared with conventional design (OC=461×10^3HKD) and uncertainty-based design (OC=431×10^3HKD), the proposed robust optimal design (OC=432×10^3HKD) could achieve a relatively low operation cost. This best option also has the minimum total cost (569×10^3HKD), which may indicate that it has good robustness considering the uncertainties of design inputs and reliability of system components.

Table 7 shows the results under all the three levels of control methods. The optimal design option is 50 L/s×7 pumps under the basic level of control and its total cost is 643×10^3 HKD. The optimal design option is 79 L/s×5 pumps under the advanced level of control and its total cost is 501×10^3HKD. The users can choose the preferred option based on their specific level of control methods.

Table 7 Minimum total cost under different levels of control methods

	Control level	Best option (size (L/s) × number)	Total capacity (L/s)	Operation cost (10^3 HKD)	Total cost (10^3 HKD)
Robust optimal design	Basic	50×7	350	478	643
	Medium	78×5	390	432	569
	Advanced	79×5	395	365	501
Uncertainty-based design	Basic	40×7	280	480	749
	Medium	40×7	280	431	700
	Advanced	40×7	280	378	647
Conventional	Basic	100×4	400	527	761

design	Medium	100×4	400	461	698
	Advanced	100×4	400	368	602

Conclusion

This paper presented a robust optimal design method which is based on a minimized life-cycle cost to ensure the high performance of chilled water pump systems and achieve the minimum annual total cost considering uncertainties of inputs and system reliability. It is realized by optimizing the pump pressure head, the total pump flow capacity and number of chilled water pumps. A case study is given as an example to test and demonstrate the proposed method. Conclusions can be made as follows:

- Annual average cooling load varies largely when considering uncertainties. If the sizing of design cooling capacity is based on the cooling load without considering uncertainties, the design cooling capacity and design chilled water flow will be very likely oversized. If the pump head is determined without considering the uncertainties of hydraulic resistance and water flow distribution, the oversize of pump head will be greatly increased.

- Markov method can be effectively used to obtain the probability distribution of system state (health) for high accuracy and fast computation time. In this study, the iteration time under different repair rate and failure rate is obtained. Results show that the iteration time is less when more pumps are used. The results also show that the iteration time is less when the ratio of repair rate to failure rate is small.

- In this study, the design cooling capacity is that corresponding to the capacity under “50 unmet hours”. According to this design capacity, the design chilled water flow and searching range of total pump flow capacity are determined. If another design cooling capacity is selected, the design chilled water flow and the searching range of total pump flow capacity will change accordingly. Then, the optimal option may also change.

- The design option of the chilled water pump system can be selected by achieving the minimum total cost when considering uncertainties and system reliability. The selected pump system can perform well under various possible cooling load conditions and have the good robustness towards the system reliability. The results of the case study show that the total cost of optimized pump system can be reduced significantly (totally 18.6%) compared with the conventional design and uncertainty-based optimal design.

The robust optimization is conducted by separating optimizing trials into two steps, i.e. the determination of design chilled water flow and pump head, the optimization of total pump flow capacity and number/size of pumps. It is worth noticing that the optimization output may be slightly different from the best one in principle as not all options/combinations are tested due to the chosen test interval in computation and available pumps sizes in practice.

Acknowledgements

The research presented in this paper is financially supported a general research grant (5267/13E) of the Research Grant Council of Hong Kong SAR and a grant of the MTR Corporation Limited.

Reference

1. Census and Statistics Department. Hong Kong energy statistics, annual Report. Hong Kong Government 1998.
2. Hong Kong energy end-use data 2012, Hong Kong Electrical & Mechanical Services Department, 2012.
3. Z.J. Ma, S.W. Wang. Enhancing the performance of large primary-secondary chilled water systems by using bypass check valve. Energy 2011, 36(1): 268-276.
4. Y.C. Chang, W.H. Chen. Optimal chilled water temperature calculation of multiple chiller systems using Hopfield neural network for saving energy. Energy 2009;

34(4):448-456.
5. Tirmizi S A, Gandhidasan P, Zubair S M. Performance analysis of a chilled water system with various pumping schemes. Applied Energy 2012; 100: 238-248.
6. Nolte D. Why efficiency matters. World Pumps 2004; 2004(448): 27-29.
7. ASHRAE Handbook. HVAC systems and equipment, Atlanta, GA, 2012.
8. Ahlgren R C E. Why did I buy such an oversized pump?. ASHRAE Transactions 2001; 107: 566.
9. ASHRAE Handbook. HVAC systems and equipment, Atlanta, GA, 2000.
10. ASHRAE Handbook. HVAC systems and equipment, Atlanta, GA, 2004.
11. Mansfield S K. Oversized pumps--a look at practical field experiences. ASHRAE Transactions 2001; 107: 573.
12. Ashouri A, Petrini F, Bornatico R, Ben MJ. Sensitivity analysis for robust design of building energy systems. Energy 2014, 76: 264-275.
13. Domínguez-Muñoz F, Cejudo-López J M, Carrillo-Andrés A. Uncertainty in peak cooling load calculations. Energy and Buildings 2010; 42(7): 1010-1018.
14. Sun Y, Gu L, Wu C F, Augenbroe G. Exploring HVAC system sizing under uncertainty. Energy and Buildings 2014; 81: 243-252.
15. Yik F W H, Lee W L, Burnett J. Chiller plant sizing by cooling load simulation as a means to avoid oversized plant. HKIE Transactions 1999; 6(2): 19-25.
16. Gorter J L. HVAC Equipment right-sizing: Occupant comfort and energy savings potential. Energy Engineering 2012; 109(1): 59-75.
17. Walski T, Zimmerman K, Dudinyak M, Dileepkumar P. Some surprises in estimating the efficiency of variable speed pumps with the pump affinity laws. Proc. World Water and Environmental Resources Congress 2003. 2003: 1-10.
18. Li DHW, Wong SL, Lam JC. Climatic effects on cooling load determination in subtropical regions. Energy Conversion and Management 2003; 44(11):1831-1843
19. Yıldız Y, Arsan Z D. Identification of the building parameters that influence heating and cooling energy loads for apartment buildings in hot-humid climates. Energy 2011; 36(7):542 4287-4296.

20. Domínguez-Muñoz F, Cejudo-López JM, Carrillo-Andrés A. Uncertainty in peak cooling load calculations. Energy and Buildings 2010; 42(7):1010-1018.

21. Eisenhower B, O'Neill Z, Fonoberov VA, Mezić 547 I. Uncertainty and sensitivity decomposition of building energy models. Journal of Building Performance Simulation 2011; 5(3):171-184.

22. Q Cheng, S.W. Wang, C.C. Yan, Linda Xiao. Probabilistic approach for uncertainty-based optimal design of chiller plants in buildings. Applied Energy 2015; http://doi:10.1016/j.apenergy.2015.10.097.

23. Stapelberg RF. Handbook of reliability, availability, maintainability and safety in engineering design. Springer; 2009.

24. Vanderhaegen F. A non-probabilistic prospective and retrospective human reliability analysis method—application to railway system. Reliability Engineering & System Safety 2001; 71(1): 1-13.

25. Myrefelt S. The reliability and availability of heating, ventilation and air conditioning systems. Energy and Buildings 2004; 36(10): 1035-1048.

26. Peruzzi L, Salata F, De Lieto Vollaro A, De Lieto Vollaro R. The reliability of technological systems with high energy efficiency in residential buildings. Energy and Buildings 2014; 68: 19-24.

27. Au-Yong C P, Ali A S, Ahmad F. Improving occupants' satisfaction with effective maintenance management of HVAC system in office buildings. Automation in Construction 2014; 43: 31-37.

28. W.J. Gang, S.W. Wang, F Xiao, D.C. Gao. Robust optimal design of building cooling systems considering cooling load uncertainty and equipment reliability. Applied Energy 2015; 159: 265-275.

29. W.J. Gang, S.W. Wang, C.C. Yan, F Xiao. Robust optimal design of building cooling systems concerning uncertainties using mini-max regret theory. Science and Technology for the Built Environment 2015, 21(6): 789-799.

30. Sun Y, Gu L, Wu C F J, Augenbroe G. Sizing HVAC systems under uncertainty. ASHRAE/IBPSA-USA Building Simulation Conference 2014, Atlanta, GA,

September 10-12.

31. Hopfe C J, Hensen J L M. Uncertainty analysis in building performance simulation for design support. Energy and Buildings 2011; 43(10): 2798-2805.
32. Tirmizi S A, Gandhidasan P, Zubair S M. Performance analysis of a chilled water system with various pumping schemes. Applied Energy 2012; 100: 238-248.
33. Billinton R, Zhang W. Cost related reliability evaluation of bulk power systems. International journal of electrical power & energy systems 2001; 23(2): 99-112.
34. S.W. Wang, J. Burnett. Online adaptive control for optimizing variable-speed pumps of indirect water-cooled chilling systems. Applied Thermal Engineering 2001; 21 (11): 1083–1103.
35. W.P. Bahnfleth, E. Peyer. Energy use and economic comparison of chilled-water pumping systems alternatives, ASHRAE Transaction 2006; 112 (2): 198–208.
36. Rishel JB, Durkin TH, Kincaid BL. HVAC pump handbook. NY: McGraw-Hill Professional; 2006.
37. E.G. Hansen. Parallel operation of variable speed pumps in chilled water systems. ASHRAE Journal 1995; 37 (10): 34–38.
38. Lisnianski A, Levitin G. Multi-state System Reliability: Assessment, Optimization and Applications. World Scientific; 2003.
39. Lisnianski A, Elmakias D, Laredo D, Haim HB. A multi-state Markov model for a short-term reliability analysis of a power generating unit. Reliability Engineering & System Safety 2012; 98(1): 1-6.
40. Lisnianski A. Extended block diagram method for a multi-state system reliability assessment. Reliability Engineering & System Safety 2007; 92(12): 1601-1607.
41. Tian Z, Levitin G, Zuo M J. A joint reliability–redundancy optimization approach for multi-state series–parallel systems. Reliability Engineering & System Safety 2009; 94(10): 1568-1576.
42. Harvey D. A handbook on low-energy buildings and district-energy systems: fundamentals, techniques and examples. Routledge, 2012.
43. Blanchard A, Roy B.N. Savannah River Site Generic Data Base development

Savannah WSRC-TR-93-262. REV.1. Westinghouse Safety Management. Aiken (SC). 1998.

44. Taal M, Bulatov I, Klemeš J, StehlíK P. Cost estimation and energy price forecasts for economic evaluation of retrofit projects. Applied Thermal Engineering 2003; 23(14): 1819-1835.
45. K.M. Guthrie. Capital cost estimating. Chemical Engineering 1969; 3:114–142.
46. Biegler L T, Grossmann I E, Westerberg A W. Systematic methods for chemical process design. 1997.
47. Seider W D, Seader J D, Lewin D R. Product & process design principles: synthesis, analysis and evaluation. John Wiley & Sons 2009.