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On the Relationship Between Conditional Jump Intensity and Diffusive Volatility

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Abstract

In standard options pricing models that include jump components to capture large price changes, the conditional jump intensity is typically specified as an increasing function of the diffusive volatility. We conduct model-free estimation and tests of the relationship between jump intensity and diffusive volatility. Simulation analysis confirms that the tests have power to reject the null hypothesis of no relationship if data are generated with the relationship. Applying the method to a few stock indexes and individual stocks, however, we find little evidence that jump intensity positively depends on diffusive volatility as a general property of the jump intensity. The findings of the paper give impetus to improving the specification of jump dynamics in options pricing models.

1. Introduction

It is now standard to include jump components in models of underlying asset prices in order to evaluate options written on them. While jumps are rare, they have significant impacts on the welfare of investors. And precisely because they are rare, their properties are difficult to analyze. As a result, there is no consensus on how jumps should be modeled in terms of the conditional jump intensity and the conditional jump size. In this paper, we address the issue of modeling conditional jump intensity. More specifically, we focus on the relation between the conditional jump intensity and the diffusive volatility. In the development of options pricing models, diffusive volatility, also known as stochastic volatility, represents a major breakthrough after the Black-Scholes model of options pricing with a constant volatility. Stochastic volatility has been treated as the most important state variable, beside the price of the underlying asset, in determining options prices. It is thus very natural to specify the conditional jump intensity as an increasing function of the diffusive volatility of the underlying asset when jumps are added to the stochastic volatility models. In popular affine jump diffusion models, for example, the conditional jump intensity is specified as an affine function of the diffusive volatility of the underlying asset with positive slope coefficients (in non-trivial cases). It also seems natural to think that the probability of a jump is high when the diffusive volatility is also high. After all, both diffusive volatility and jump intensity are measures of the magnitude of possible future price changes.

In examining affine options pricing models, the relation between the conditional jump intensity and the diffusive volatility is estimated as part of the model. For example, Pan (2002) and Eraker (2004) find a significant increasing relation between the conditional jump intensity and the diffusive volatility of the underlying asset, using options data. Two issues involved with using options data make the results difficult to interpret. First, jumps in options prices can result from jumps in state variables, rather than from those in the underlying asset price. In potentially misspecified models, especially those that force the underlying asset price and other state variables to jump together, a relation between the jump intensity of the underlying asset price and diffusive volatility can be found spuriously. Second, since options pricing is conducted under the risk-neutral probability which involves risk premium, for models that implicitly assume no risk premium associated with jump intensity, a relation between the jump intensity and diffusive volatility under the risk-neutral probability may be attributed to that under the actual probability spuriously. Bates (2000) explicitly tests the difference in jump intensity-diffusive variance relation under the actual and risk-neutral probabilities and finds no relation under the former, but a positive relation under the latter. Using S&P 500 index return data only without options prices, Andersen, Benzoni and Lund (2002) find an insignificant relation between the conditional jump intensity and the diffusive volatility. In short, the issues of whether there is a relation between jump intensity of the underlying asset price and its diffusive volatility, whether the relation is positive, and whether the relation is monotonic are unsettled.[1]

The results of parametric analysis of jump intensity can also be sensitive to the model assumptions. Recent studies have shown that many standard options pricing models are mis-specified. Jones (2003) finds that the square-root stochastic volatility model is incapable of generating realistic return behavior and the data are better represented by a stochastic volatility model in the constant-elasticity-of-variance class or a model with a time-varying leverage effect. Christoffersen, Jacobs and Mimouni (2010) find that a stochastic volatility model with a linear diffusion term is more consistent with the data on the underlying asset and options than a stochastic volatility model with a square-root diffusion term is. Li and Zhang (2013) show that the affine drift of the diffusive volatility model is mis-specified because the mean reversion is particularly strong at the

[1]Eraker (2004) also has estimation results without using options data in his Table 4, in which he does not report the relation between jump intensity and diffusive volatility and does not explain why not. Andersen et al. (2002) attribute the insignificance result to the approximation used to calculate standard errors, and a multicollinearity type problem. As shown by Andersen et al. (2002), when the conditional jump intensity is specified as an affine function of diffusive variance, instead of a constant, the standard errors of the parameters related to jump intensity are about 20 times larger.

high end of volatility. These results suggest that the standard options pricing model with the square-root volatility process falls short of generating sharp increases and decreases in volatility when the level of volatility is high. As a result, when there are in fact spikes in the volatility, the model with a square-root volatility process attributes large changes in asset price to jumps and produces a spurious positive relation between the conditional jump intensity and diffusive volatility.

In this paper, we address the issue of whether the conditional jump intensity is an increasing function of the diffusive volatility under the actual probability. Since an exhaustive analysis of all possible combinations of specifications of jumps and diffusive volatility is impossible, we adopt nonparametric and semi-nonparametric approaches, which can reduce the chances of making erroneous inferences from mis-specification of parametric models.[2] We consider a simple method with many variations to identify jumps and choose among these variations through simulation to reduce the potential bias in examining the jump intensity-diffusive volatility relation, caused by the error in the estimated diffusive volatility. We then examine the relation between the conditional intensity of detected jumps and the estimated diffusive volatility for several stock indexes and individual stocks. Our results are based on intraday returns which enhance the powers of the jump detection and the test of jump intensity-diffusive volatility relation, compared with early studies based on daily returns. Using our more robust approaches, we arrive at the conclusion that the conditional jump intensity of most individual stocks and stock indexes we examine is unrelated to their diffusive volatility.

The finding that the conditional jump intensity is unrelated to the diffusive volatility for many indexes and individual stocks, contrary to what the standard jump-diffusion models assume, is important for understanding the dynamics governing underlying asset prices and deriving corresponding options pricing models. It prompts the study of what

[2]Nonparametric methods are applied to options pricing in Hutchinson, Lo, and Poggio (1994), Aït-Sahalia (1996), Aït-Sahalia and Lo (1998), Broadie, Detemple, Ghysels, and Torres (2000a, 2000b), Aït-Sahalia, Wang, and Yared (2001), Aït-Sahalia and Duarte (2003), Li and Zhao (2009), and Li and Zhang (2010), among others.

state variables really determine the conditional jump intensity and how to better model the relationship between these state variables and the jump intensity.[3]

The rest of the paper is organized as follows. Section 2 discusses the nonparametric jump detection methods. Section 3 conducts simulation analysis to examine the performance of the tests of the relation between conditional jump intensity and diffusive volatility. Section 4 presents the empirical analysis based on stock indexes and individual stocks. Section 5 concludes the paper.

2. The Jump Detection Test and Diffusive Volatility Estimators

Consider the log price of an asset, S_u, which follows a stochastic process,

$$dS_u = \mu_u du + \sqrt{v_u} dW_u + Z_u dN_u, \tag{1}$$

where μ_u is the drift, $\sqrt{v_u}$ is the diffusive volatility, N_u is a counting process with time-varying and finite intensity, Λ_u, and Z_u is the jump size. (1) is a very general process because no restrictions are imposed on the functional forms of μ_u, v_u, Z_u and N_u other than standard regularity conditions that guarantee the solutions of the process. The issue to be examined in this paper is the relation between Λ_u and $\sqrt{v_u}$. In the popular affine jump-diffusion models, the relation is specified as

$$\Lambda_u = \lambda_0 + \lambda_1 v_u, \tag{2}$$

where $\lambda_1 \geq 0$ because both Λ_u and v_u are nonnegative and v_u is unbounded.

The methodology of our analysis can be summarized as follows. First, we use nonparametric methods to estimate the diffusive variance, v_u. Second, we detect jumps by

[3]There is a separate line of research which models the conditional jump intensity as a function of realized past jumps and past conditional jump intensity, based on the observations that large price changes occur in clusters. Prominent works include Chan and Maheu (2002), Maheu and McCurdy (2004), Yu (2004), Santa-Clara and Yan (2010), Christoffersen, Jacobs and Ornthanalai (2012), Maheu, McCurdy and Zhao (2013), and Aït-Sahalia, Cacho-Diaz and Laeven (2015). These studies address issues different from ours and use approaches different from ours.

comparing the magnitude of returns with the estimate of the diffusive volatility. Third, we examine the relation between Λ_u and v_u by regressing the indicators of detected jumps on the estimated diffusive variance. This three-step approach may be less efficient than one-step tests under a specific model. However, the approach is largely model-free and robust to the specification of the stochastic volatility process. The simulation analysis in the next section shows that our approach is powerful enough to identify the true relation, so the gain from robustness overweighs the loss of efficiency. In this section, we present the nonparametric jump detection test, the potential bias issue, and various diffusive volatility estimators.

2.1. The Jump Detection Test

We examine a class of test statistics as a detector of possible jumps in stock prices. The test statistics we examine take the form of

$$L_{t+1} = \frac{r_{t+1}}{\sqrt{\hat{v}_t}}, \tag{3}$$

where $r_{t+1} = S_{t+1} - S_t$ is the log return at an intraday time $t+1$, $\hat{v}_t$ is an estimate of v_t, using past returns on and before t. The rationale of the test statistics is that, on an intraday time interval without jumps, r_{t+1} is driven by $\mu_t + \sqrt{v_t}\Delta W_{t+1}$ in discrete-time approximation, where $\Delta W_{t+1} \equiv W_{t+1} - W_t$ is normally distributed with mean equal to zero and variance equal to one. The drift term over small intervals is known to be negligible. See, for example, Merton (1980). As sampling interval (i.e., daily, hourly, every fifteen minutes, every minute, etc.) used to estimate v_t goes to zero, and $\hat{v}_t$ converges to v_t, L_{t+1} follows asymptotically the standard normal distribution. A jump on $t+1$ is detected if L_{t+1} exceeds a critical value determined by the standard normal distribution.[4] A jump indicator, J_{t+1}, takes the value of one in that case and zero otherwise. That the jump

[4]There are many other nonparametric jump detection tests proposed in the recent literature, such as Barndorff-Nielsen and Shephard (2004, 2006), Jiang and Oomen (2008), and Aït-Sahalia and Jacod (2009), which can be applied to a period using higher frequency returns within the period to test whether there is a jump during the period. These tests, however, do not suit our purpose of identifying intraday jumps using intraday returns.

detection test depends on the size of the return relative to its current diffusive volatility makes sense. When the diffusive volatility is large, a return with a large absolute value may not be due to a jump because the diffusive component may already be volatile enough to generate such a return. Likewise, when the diffusive volatility is small, a return with not-so-large an absolute value may indeed be due to a jump because the diffusive component is unlikely to generate such a return.

A tricky issue is to estimate v_t using past returns which may contain jumps. The test statistics we examine in this paper differ in their way of estimating v_t. They are all nonparametric as no parametric assumptions on the dynamics of the diffusive volatility and jumps are imposed.

A potential problem is that, since $\hat{v}_t$ is based on a limited number of past returns, it contains an estimation error, which causes the true distribution of L_{t+1} to deviate from its asymptotic one. Specifically, when v_t is underestimated by $\hat{v}_t$, $|L_{t+1}|$ tends to be greater than what it should be and a jump may be detected when there is in fact none. On the other hand, when v_t is overestimated by $\hat{v}_t$, $|L_{t+1}|$ tends to be smaller than what it should be and a jump may not be detected when there is one. Therefore, beside the usual type I and type II errors of the test when v_t is perfectly known, the tests based on an estimated diffusive volatility contains an additional small-sample error. This error may not be serious for all applications involving jumps, but it is for the relation between conditional jump intensity and diffusive volatility. It should be intuitively clear that this error tends to cause a bias towards finding no relation when the true relation is positive (i.e., increasing) and finding a negative (i.e., decreasing) relation when the true relation is no relation.

Because of the potential bias, we will adopt econometric methods that deal with the errors-in-variables problem and find estimators of v_t that reduce the bias. In Section 3, we use simulation to tackle these problems. The remainder of this section is devoted to introducing some possible estimators of v_t that have appeared in the literature.

2.2. Diffusive Variance Estimators

The diffusive variance, i.e., the squared volatility, is the instantaneous variance of asset returns at a particular point in time and does not include the variation due to jumps. We consider three types of estimators. The first estimator of the diffusive variance is the bipower variation proposed by Barndorff-Nielsen and Shephard (2004, 2006), BV_t, defined as

$$\mathrm{BV}_t = \frac{\pi}{2(K-1)} \sum_{i=1}^{K-1} |r_{t-i}||r_{t-i+1}|. \tag{4}$$

This is a localized version of the jump-robust quadratic variation estimator where the window-size is controlled by K. As K shrinks, BV_t measures the diffusive variance at a point in time. We will discuss the choice of K in the next subsection. This estimator used to be the most popular one in the literature and is used by Andersen, Bollerslev, and Dobrev (2007) and Lee and Mykland (2008) in the jump detection test in the same setting.

The second estimator is the quantile-based realized variance proposed by Christensen, Oomen and Podolskij (2010). The authors have shown that this measure is superior to the bipower variation, the truncated realized variance of Jacob (2008) and Mancini (2009), and the median truncated realized variance of Andersen, Dobrev and Schaumburg (2012) as the estimator of the jump-robust quadratic variance. We consider a version of the quantile-based realized variance which is more suitable for estimating the instantaneous diffusive variance, by allowing more weights for more recent return observations. The estimator is termed the exponentially weighted quantile realized variance, EQRV_t, defined as,

$$\mathrm{EQRV}_t = \sum_{j=1}^{d} a_j \frac{\sum_{i=0}^{K-b} \psi^i q_{t-i}(b, l_j)}{\nu(b, l_j) \sum_{i=0}^{K-b} \psi^i}, \tag{5}$$

where $q_t(b, l_j) = g^2_{bl_j}(D_{t,b}r) + g^2_{b-bl_j+1}(D_{t,b}r)$, $D_{t,b}r = (r_n)_{t-b+1 \leq n \leq t}$, and $g_p(x)$ is the pth order statistics of $x = (x_1, \cdots, x_m)$. $\nu(b, l_j)$ is the normalizing constant that measures the expectation of $q_t(b, l_j)$ under the standard normal distribution, and a_j is the optimal weight for l_j quantile, both given in Christensen, Oomen and Podolskij (2010). When the smoothing constant $\psi = 1$, this estimator reduces to the subsampling version of the quantile-based realized variance, which is more efficient asymptotically and in finite samples in many cases than the blocked version as shown in their paper. The estimator is robust to jumps because for a given block of b return observations, those greater than the highest quantile l_d or less than the lowest quantile $1 - l_d$ are excluded from the diffusive variance calculation. The choices of the number of quantiles, d, the quantiles, l_j with $l_j \in (0.5, 1)$ for $j = 1, \cdots, d$, the block size, b, and the smoothing constant, ψ, will be discussed in detail in the next subsection.

The third estimator, termed the exponentially weighted moving average with truncation, EMT_t, is defined as,

$$\text{EMT}_t = \frac{\sum_{i=0}^{K-1} \psi^i r_{t-i}^2 1_{\{\beta \leq |r_{t-i}| \leq \gamma\}}}{\sum_{i=0}^{K-1} \psi^i 1_{\{\beta \leq |r_{t-i}| \leq \gamma\}}}. \tag{6}$$

This estimator is similar to the commonly used exponentially weighted moving average estimator of volatility except that both the high and low ends are truncated. The upper truncation makes the estimator robust to infinite activity jumps and consecutive jumps, whereas the lower truncation ensures that the expected diffusive variance is the same as the one without truncations. The truncation levels will be discussed in detail in the next subsection.[5]

An issue with using this type of jump detection test in intraday returns is the deterministic intraday volatility pattern, where the volatility tends to be higher just after the market open and before the market close during a trading day. We need to adjust intraday returns so that the detected intraday jumps are not driven by such a volatility pattern. The intraday volatility level of a given intraday interval is estimated by the average squared return in that intraday interval across the full sample. Then, the relative intraday volatility level is estimated as the intraday volatility level divided by the

[5]This approach can be regarded as a kernel-based nonparametric estimation of the diffusive volatility of Kristensen (2010) with an exponential type of kernel.

average of the intraday volatility levels across all intraday intervals. The volatility pattern-adjusted intraday returns are calculated by dividing the raw returns by the square-root of the relative intraday volatility levels for the corresponding intraday intervals. For the results presented below, the diffusive volatility estimation and the jump detection are performed on the adjusted returns.

2.3. Choices of Parameters in the Diffusive Volatility Estimators

For all the diffusive volatility estimators, the window-size K needs to be specified. On the one hand, K should be large enough so that the distribution of the jump test statistic, L_{t+1}, is close to the asymptotic distribution. On the other hand, the return observations used to estimate the diffusive volatility need to be close to t to be able to capture the local information. EQRV_t and EMT_t are not as sensitive to K as BV_t is because the weights of past return observations used to calculate EQRV_t and EMT_t mainly depend on ψ.

For EQRV_t, the quantiles used to calculate the diffusive volatility, l_j for $j = 1, \cdots, d$, need to be specified. On the one hand, the quantiles at tails, i.e., high l_j and low $1-l_j$, are more informative about the level of the diffusive volatility. On the other hand, too high l_j or too low $1-l_j$ may include jumps and contaminate the diffusive volatility estimation. The block size, b, should be small because the volatility within the block is assumed to be constant. On the other hand, b should be large enough so that sufficient number of returns with large values in magnitude in the block can be discarded to remove the effects of jumps on the diffusive volatility estimation. To balance these, we choose $b = 10$, $d = 2$, $l_1 = 0.7$ and $l_2 = 0.8$. The choice makes the diffusive volatility estimator robust to two jumps in each tail within a relative small block of return observations.

For EMT_t, the upper truncation level, γ, needs to be specified to remove the effects of jumps on the estimation of the diffusive volatility. A too large value of γ may cause a potential jump component included in the estimation, leading to a upward bias, while a too small value of γ may cause relatively large diffusive components excluded in the estimation, leading to a downward bias. We consider several choices of γ written as multipliers of estimated diffusive volatility, $\gamma = c\sqrt{\mathrm{BV}}$, where BV is the bipower variation based on the entire sample, which estimates the unconditional mean of the diffusive variance. The lower truncation level, β, needs to be specified as well. For a given value of γ, β is set to be such a value that $E(X^2) = E(X^2|\beta^2 \leq X^2 \leq \gamma^2)$, where X follows the normal distribution with mean zero and variance BV. The lower truncation corrects for the downward bias from the upper truncation which may otherwise throw out large return observations due to the diffusive component in the return process.

Since the optimal values of K and c depend on actual stochastic processes of the stock returns, we rely on the simulation analysis, which is explained in detail in Section 3. In particular, we choose K for BV_t and EQRV_t or both K and c for EMT_t by minimizing the root-mean-square error (RMSE), defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\sqrt{v_t} - \sqrt{\hat{v}_t})^2}, \tag{7}$$

where T is the number of observations in the simulated data, $\sqrt{v_t}$ is the simulated diffusive volatility and $\sqrt{\hat{v}_t}$ is the estimated diffusive volatility from simulated returns. In the simulation, we consider different parameter values of jump sizes and of the relation between the conditional jump intensity and diffusive volatility. Larger jump sizes and stronger positive relations between the conditional jump intensity and diffusive volatility require lower values of c to counter the effects of the upward bias of the bipower variation in the presence of jumps. Our simulation analysis indicates that for EMT_t, the optimal c across different parameter values is close to be 8 and, for various jump detection tests and parameter values, the optimal K is close to 100. In Section 3 below, we report the results for $c = 8$ and $K = 100$.[6]

The smoothing constant, ψ, is chosen such that the quasi-log-likelihood functions,

[6] Choosing these parameters by the simulation corresponds to the so-called plug-in method as developed in the literature of the nonparametric estimation where a parametric model is used to guide the choice of turning parameters.

$\sum_{t=K+1}^{T}(-\log \mathrm{EQRV}_{t-1} - r_t^2/\mathrm{EQRV}_{t-1})1_{\{|r_t|\leq\gamma\}}$ and $\sum_{t=K+1}^{T}(-\log \mathrm{EMT}_{t-1} - r_t^2/\mathrm{EMT}_{t-1})1_{\{|r_t|\leq\gamma\}}$ are maximized for EQRV_t and EMT_t, respectively, where T is the number of return observations in the sample.[7]

When the significance level of the jump detection tests, α, increases, both the power and size of the tests increase. The choice of α also affects the results of testing the relationship between the jump intensity and the diffusive volatility. Our unreported simulation results show that the test results are not very sensitive to α for α between 0.001% and 0.1%, but, when $\alpha = 1\%$, tests tend to fail to find a significant positive relationship when there is one in simulated data because many detected jumps are spurious. To be on the conservative side, we use $\alpha = 0.01\%$ for tests on the actual stock indexes and individual stocks in Section 4, as well as in the reported simulation results in Section 3. Such a choice is also in line with other studies in the literature.[8]

3. Simulation Analysis

In this section, we conduct a simulation analysis to examine the performance of jump detection tests based on various diffusive volatility estimators and how accurately the relation between the conditional jump intensity and diffusive volatility can be identified.

[7]From the model (1) assumption, when jumps are removed, r_t follows a normal distribution approximately, its mean is close to zero, its variance is v_t, and it is serially independent. $1_{\{|r_t|\leq\gamma\}}$ removes observations that likely contain jumps. The distribution is approximately normal because of the time-varying v_t. In the actual returns of stocks and stock indexes with jumps removed, there are only 6.9% and 6.8% of days for which the normality is rejected at 1% level for EQRV_t and EMT_t, respectively, based on the Kolmogorov-Smirnov test, suggesting that the approximate normality is justified.

[8]Methods have been proposed in the literature to reduce the spurious jump detections by controlling the size of multiple jump detection tests. Lee and Mykland (2008) suggest a rejection region based on the distribution of the maximum of N i.i.d. $|L_{t+1}|$ statistics using the extreme value theory. In the case of 26 intraday returns a day, i.e., $N = 26$, the rejection threshold for a significance level of 1% based on this approach is 3.899, very close to 3.891 used in this paper, which is based on the significance level of 0.01% without using the maximum. Andersen, Bollerslev, and Dobrev (2007) use a Šidák correction to control for the number of spurious jumps detected per day. Under this approach, the rejection threshold is given by the $1 - [1 - (1 - \alpha^*)^{(1/N)}]/2$ quantile of the standard normal distribution, where α^* is the size of the jump detection test at the daily level. They suggest $\alpha^* = 10^{-5}$, which gives the rejection threshold of 5.076 for $N = 26$.

The data are simulated from the discretized version of the model

$$dS_u = \left(\mu - \frac{1}{2}v_u\right)du + \sqrt{v_u}dW_{1,u} + Z_u dN_u \tag{8}$$

$$d\ln v_u = (\theta - \kappa \ln v_u)du + \eta dW_{2,u}, \tag{9}$$

where S_u is the log asset price, v_u is the diffusive variance, $W_{1,u}$ and $W_{2,u}$ are standard Brownian motions with correlation ρ, N_u is a counting process, and Z_u is the jump size.[9] For notational convenience, the unit of time is chosen to be consistent with the frequency of the data used. The simulation results reported in the section are based on 15-minute returns.

3.1. Parameters Selection

The parameters used in the simulation are estimated based on the intraday return data of the S&P500 index from 1986 to 2012 by matching the realized Laplace transform of volatility, proposed by Todorov and Tauchen (2012), with the counterpart implied by the model. Specifically, for any $s \geq 0$, the realized Laplace transform of volatility for year j is defined as

$$\mathrm{TV}_j(s) = \frac{1}{T_d} \sum_{t=T_d(j-1)+1}^{T_d j} \cos(\sqrt{2sT_d}r_t), \tag{10}$$

where T_d is the number of intraday returns in a year. For $s_1 \geq 0$, $s_2 \geq 0$, and any positive integer l, define

$$\widehat{\mathrm{MV}}(s) = \frac{1}{T_y} \sum_{j=1}^{T_y} \mathrm{TV}_j(s) \tag{11}$$

$$\widehat{\mathrm{CV}}(s_1, s_2; l) = \frac{1}{T_y - l} \sum_{j=l+1}^{T_y} \mathrm{TV}_j(s_1)\mathrm{TV}_{j-l}(s_2), \tag{12}$$

where T_y is the number of years. Todorov and Tauchen (2012) show that as $T_y/T_d \to 0$, $\widehat{\mathrm{MV}}(s)$ and $\widehat{\mathrm{CV}}(s_1, s_2; l)$ converge to $\mathrm{MV}(s)$ and $\mathrm{CV}(s_1, s_2; l)$ in probability, respectively,

[9] We also examine data simulated from other popular models in the literature, for example, affine jump-diffusion models. Our results are fairly robust to the model chosen and are, therefore, not repeatedly reported.

where

$$\begin{aligned}
\mathrm{MV}(s) &= E\left(e^{-s\bar{v}_{\bar{u}}}\right) &(13)\\
\mathrm{CV}(s_1, s_2; l) &= E\left(\int_l^{l+1} e^{-s_1\bar{v}_{\bar{u}}} d\bar{u} \int_0^1 e^{-s_2\bar{v}_{\bar{u}}} d\bar{u}\right), &(14)
\end{aligned}$$

and $\bar{u} = u/T_d$ and $\bar{v}_{\bar{u}} = v_u T_d$ correspond to the one-year time unit adopted by Todorov and Tauchen (2012). We estimate the parameters in (9) by matching the quantities (11) and (12) calculated from the actual data with (13) and (14) inferred from the model. Since there is no closed-form expression for (13) and (14) under model (9), they are evaluated via simulations. For a given set of parameters, we simulate 5000 samples where each sample contains 30 year's worth of data with 252 days per year. 26 steps are simulated for each day so that each step corresponds to a 15-minute interval for 6.5 trading hours. For each simulated sample, we remove data for the first 3 years to minimize the effects of the initial values. Then, the averages across the 5000 samples are used to calculate (13) and (14). Like Todorov, Tauchen and Grynkiv (2011) and Todorov and Tauchen (2012), we calculate (11) and (13) for $s = 0.1, 4, 8$, a low, median, and high value, respectively. To calculate (12) and (14), we set $l = 1$ and $s_1 = s_2$ which takes the same value as s mentioned above. Then, parameters θ, κ, and η are estimated by minimizing $(\widehat{\mathbf{LV}}' - \mathbf{LV}')(\widehat{\mathbf{LV}} - \mathbf{LV})$, where

$$\begin{aligned}
\widehat{\mathbf{LV}} &= (\widehat{\mathrm{MV}}(0.1), \widehat{\mathrm{MV}}(4), \widehat{\mathrm{MV}}(8), \widehat{\mathrm{CV}}(0.1, 0.1; 1), \widehat{\mathrm{CV}}(4, 4; 1), \widehat{\mathrm{CV}}(8, 8; 1))',\\
\mathbf{LV} &= (\mathrm{MV}(0.1), \mathrm{MV}(4), \mathrm{MV}(8), \mathrm{CV}(0.1, 0.1; 1), \mathrm{CV}(4, 4; 1), \mathrm{CV}(8, 8; 1))'.
\end{aligned}$$

The parameter estimates are $\theta = -0.00855$, $\kappa = 0.00103$, and $\eta = 0.0607$. Using other values of s, s_1, s_2 and l gives similar parameter estimates, and does not alter our conclusion. Since our results are not sensitive to the values of μ and ρ, we simply set $\mu = 0.05/(252 \times 26)$ and $\rho = -0.5$ in the simulation.[10]

The merit of using this approach is that parameters in (9) can be estimated without making assumptions on the dynamics in (8). A misspecified model of jumps in the process

[10]Note that the unit of time of the simulated data and the parameters is for 15-minute intraday intervals assuming that each trading day has 26 15-minute intraday intervals and each year has 252 trading days.

of underlying asset returns can affect the estimation of the volatility process. This is particularly important in our application. Since the purpose of our study is to identify the relation between the conditional jump intensity and diffusive volatility, no assumptions should be made on this relation, nor the other properties of jumps when estimating the volatility process.

For the jump component, we fix the jump size at $m\sqrt{v_u}$, where $m = 4$ or 6. Jumps of too small sizes are meaningless as they cannot be distinguished from diffusive returns. The conditional jump intensity, Λ_u, is specified as $\lambda_0 + \lambda_1 v_u$, as suggested by the popular affine jump-diffusion models in options pricing. For the first set of parameters, $\lambda_0 = 20/T_d, \lambda_1 = 0$, the conditional jump intensity is unrelated to the diffusive variance, where $T_d = 252 \times 26$, the number of intraday returns in a year. For the second set of parameters, $\lambda_0 = 10/T_d, \lambda_1 = 400$, the conditional jump intensity is specified as an affine function of the diffusive variance. The empirical analysis in the next section suggests that for some stocks and stock indexes we consider, the relation between conditional jump intensity and diffusive variance is negative. Because the diffusive variance is unbounded, the decreasing and affine relation is impossible. Therefore, we consider another specification, the exponential affine conditional jump intensity, taking the form of $\exp(\lambda_0^e + \lambda_1^e v_u)$, where $\lambda_0^e = 3.5 - \ln(T_d), \lambda_1^e = -35$. [11] In addition to the typical Poisson type of jumps, we also consider the case of consecutive jumps, where two jumps occur in the adjacent 15-minutes returns. This case highlights the important differences among the diffusive variance estimators we consider as shown below.

3.2. Simulation Results

For the results reported below, 20-year worth of 15-minute intraday returns are simulated to match with the sample periods of most stocks and stock indexes in the empirical analysis

[11] The three sets of parameters are chosen to give about 20 jumps per year on average with an average of annualized v_t equal to 0.0265 in the simulated data. This is in line with the empirical results in Section 4 for a set of stocks and stock indexes.

of Section 4. To examine the performance of various diffusive volatility estimators, we first calculate the RMSE, defined as

$$\text{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\sqrt{v_t} - \sqrt{\hat{v}_t})^2}, \tag{15}$$

where T is the number of observations. Table 1 reports the median and interquartile range of the RMSE of the diffusive volatility estimated from the 5000 simulated samples. For the ease of interpretation, the numbers are annualized and in percentage. The median RMSE can be compared with the actual average annualized diffusive volatility of 13.06%. The RMSE of BV_t increases with the jump size, because BV_t has an upward bias when jumps are present and the magnitude of the bias increases with the size of jumps. The RMSE of BV_t is higher for the case of the positive relation between the conditional jump intensity and diffusive variance than for the case of the negative relation. This is so because jumps of larger sizes are more likely to occur when the relation is positive in the simulation. In Panel B, for the consecutive jumps, the RMSE for BV_t is larger and patterns of RMSE across jump sizes and conditional intensity specifications are stronger. These suggest that the upward bias in BV_t is stronger for the case of consecutive jumps. The RMSE of EQRV_t is smaller than that of BV_t for all the cases, especially for the case of consecutive jumps. The RMSE of EQRV_t is not sensitive to the jump size or the specification of the conditional jump intensity. EQRV_t performs equally well for the case of consecutive jumps because EQRV_t is robust to two positive and two negative jumps. EMT_t tends to have even smaller errors than EQRV_t does. However, the RMSE of EMT_t is more sensitive to the jump size, and the RMSE of EMT_t is larger than that of EQRV_t in some cases of consecutive jumps and large jump size. The superior performance of EQRV_t and EMT_t relative to BV_t is due to higher weights applied to more recent return observations to capture local information.

Table 1 here

We then examine the size and power properties of the jump detection tests based on these diffusive volatility estimators. Table 2 reports the sizes. Since jumps are infrequent in reality, the oversize problem can be a serious issue for jump detection tests. Suppose that the actual jump intensity is 20 per year, i.e., about 0.3% of 15-minute intraday returns contain jumps. An oversize of 0.03% means that an additional 10% of identified jumps are false. Thus, a good size property is important for jump detection tests. The results show that all the jump detection tests are oversized because of the errors in the estimation of diffusive volatility from finite samples. However, there are large variations in the size properties across these volatility estimators. The jump detection test based on EMT_t has the best size property for all the specifications of jump size and conditional jump intensity. The BV_t is ranked the second, and EQRV_t has a slightly larger size than BV_t does. For a given diffusive variance estimator, the size property is not sensitive to the specifications of jump size and conditional jump intensity, or the cases of Poisson or consecutive jumps.

Table 2 here

Table 3 reports the powers of the jump detections tests. As expected, the power increases with the jump size for all the diffusive volatility estimators. In Panel A, for Poisson jumps, BV_t has the lowest power among the diffusive volatility estimators except for the case of exponential affine conditional jump intensity and large jump size. The power of BV_t is even lower for the case of consecutive jumps in Panel B, consistent with its performance measured by the RMSE. EQRV_t has the highest power among the three diffusive variance estimators, and works equally well for both cases of Poisson jumps and consecutive jumps. EMT_t is generally more powerful than BV_t for the case of Poisson jumps. Combining with a better size property as shown in Table 2, the result suggests that EMT_t has a better size-and-power tradeoff than BV_t does. However, for the case of consecutive jumps, the power of EMT_t is the lowest because the second jump in the consecutive jumps is less likely to be detected when the first jump return observation is included in the diffusive volatility estimation with the highest weight, leading to an upward bias.

$\boxed{\text{Table 3 here}}$

Many studies use daily returns rather than intraday returns to detect jumps because daily returns are available for a longer time span. Therefore, we also report the powers and sizes of these jump detection tests based on daily returns in Table 4. The specifications of the conditional jump intensity are the same as for the case of intraday returns except that the parameters are different to match the annual jump intensity of 3, typically observed from the daily data. The jump sizes are fixed at 4 and 6 times of the daily diffusive volatility for the small jump and large jump, respectively. For the case of consecutive jumps, jumps occur in two adjacent daily returns. The significance level of the jump detection tests is 0.01%, same as the one for the case of intraday returns. The results suggest that all the tests are oversized, and the problem is more serious than for the case of intraday returns. EMT_t still has the best size property, followed by BV_t and EQRV_t. The powers are also lower than those for the case of intraday returns. EQRV_t is still the most powerful. The powers of EMT_t and BV_t are comparable for the case of Poisson jumps, but EMT_t is less powerful than BV_t for the case of consecutive jumps for the same reason discussed before.

$\boxed{\text{Table 4 here}}$

Positive errors in the estimated diffusive volatility lead to failure of identifying true jumps, whereas negative errors lead to false identification of jumps. These errors also lead to a downward bias in the estimated relation between the conditional jump intensity and diffusive volatility. The degree of bias depends on the magnitude of the errors in the diffusive volatility estimators, as well as the size and power properties of the jump detection tests based on these estimators.

We examine the magnitude of the bias in the estimated relation between the conditional jump intensity and diffusive volatility in regressions. Let $J_t = 1$ if there is a jump detected at t, and $J_t = 0$ otherwise. We examine the relation between $\hat{v}_t$ and the conditional jump intensity by running the following regressions,

$$J_{t+1} = \lambda_0 + \lambda_1 \hat{v}_t + \varepsilon_{t+1}, \tag{16}$$

for the cases of constant and affine conditional jump intensity, and

$$J_{t+1} = \exp(\lambda_0^e + \lambda_1^e \hat{v}_t) + \varepsilon_{t+1}^e, \tag{17}$$

for the case of the exponential affine conditional jump intensity. The affine specification is motivated by the affine jump-diffusion model in the options pricing literature, whereas the exponential affine specification is motivated by the empirical results in the next section. To mitigate the problem arising from the negative correlation between J_{t+1} and $\hat{v}_t$, we use a constant and $\hat{v}_{t-K}$ as instrumental variables to estimate (16) and (17), where $K = 100$.

To investigate how well $\hat{v}_{t-K}$ serves as a instrumental variable in the regression models (16) and (17), we report the median and the interquartile range of correlations between $\hat{v}_t$ and $\hat{v}_{t-K}$ across the 5000 simulated samples in Panel A1 and A2 of Table 5. The correlations are above 70% except for one case. The result suggests that for these diffusive volatility estimators, $\hat{v}_{t-K}$ is a good instrumental variable for $\hat{v}_t$.

Table 5 here

The median and the interquartile range of the estimated λ_1, denoted as $\hat{\lambda}_1$, across the 5000 simulated samples, are reported in Panel B1 and B2 of Table 5. The magnitude of bias varies substantially across these measures of diffusive volatility. For the case of constant jump intensity, i.e., $\lambda_1 = 0$, EQRV_t causes a negative bias to some degree, and the negative bias for BV_t is greater, evidenced by $\hat{\lambda}_1 < 0$. For the case of Poisson jumps and EMT_t, $\hat{\lambda}_1$ is close to zero, which suggests that using EMT_t does not cause a bias towards finding a negative relation between the conditional jump intensity and diffusive volatility when there is in fact no relation. This is consistent with the results in Table 1 and Table 2 that EMT_t has small errors and good size properties. However, for the case of consecutive jumps, there is an upward bias. Since the truncation parameters in EMT_t are fixed and the jump size is proportion to the diffusive volatility, EMT_t tends to fail to detect small jumps when the diffusive volatility is low, and the loss of power is more serious for the case of consecutive jumps as discussed in Table 3. This explains the upward bias in the estimated jump intensity-diffusive variance relation. For the case of affine conditional jump intensity, the negative bias is apparent in all the diffusive volatility estimators, evidenced by $\hat{\lambda}_1 < \lambda_1$. However, the magnitude of bias in $\hat{\lambda}_1$ tends to be lower when the jump size is larger because larger jumps are easier to detect. EMT_t still has the lowest biases. For the case of exponential affine conditional jump intensity, where the jump intensity-diffusive variance relation is negative, all the diffusive variance estimators cause a positive bias, evidenced by $\hat{\lambda}_1^e > \lambda_1^e$. This is mainly due to the errors-in-variables problem with $\hat{v}_t$ causing a bias toward finding no relation when there is one. Because of the relative low power of EMT_t to detect small jumps, the positive bias is the highest for EMT_t, especially for the case of consecutive jumps. The bias caused by EQRV_t is slightly greater than that caused by BV_t for the case of small jumps. BV_t performs slightly worse for the case of consecutive jumps than for the case of Poisson jumps, but the difference does not exist for EQRV_t.

The median t-statistic for $\hat{\lambda}_1$ and $\hat{\lambda}_1^e$, adjusted for heteroscedasticity and serial correlation based on the procedure of Newey and West (1987) with 500 lags, and the interquartile range are reported in Panel C1 and C2 of Table 4. For the case of constant jump intensity, the median t-statistic for EMT_t is close to zero, suggesting that the bias caused by EMT_t is statistically insignificant. The negative bias caused by BV_t tends to be significant on average, but becomes insignificant as the jump size increases. For both cases of positive and negative jump intensity-diffusive variance relations, the tests based on these three diffusive volatility estimators can identify the relations on average. EMT_t has the highest power of detecting the positive relation, followed by EQRV_t, whereas BV_t has the highest power of detecting the negative relation, followed by EQRV_t. The powers of detecting the jump intensity-diffusive relation are generally lower for the case of consecutive jumps, but EQRV_t is more robust to consecutive jumps than the other two. The results also show that the interquartile ranges of these t-statistics are large, suggesting a large variation in the t-statistics across the 5000 simulated samples. This indicates that these diffusive volatility estimators that do not cause bias on average may cause bias in individual samples.

To further investigate the bias in the diffusive volatility estimators, in Panel D1 and D2 of Table 5, we report, out of the 5000 simulated samples, the proportion of t-statistics less than -2.326, which corresponds to the 1st percentile of the standard normal distribution. For the case of constant jump intensity, the proportion is the smallest for EMT_t, followed by EQRV_t. The proportion for this case can also be regarded as the size for testing no relation against the negative relation between the conditional jump intensity and diffusive volatility. The test based on EMT_t has the best size property, and the test based on BV_t is the most oversized. The size property improves with the jump size. The proportion for the case of exponential affine jump intensity can be regarded as the power for testing no relation against the negative relation. The tests based on BV_t and EQRV_t are more powerful than that based on EMT_t. Since BV_t is much more oversized than EQRV_t, but they have comparable powers, EQRV_t is preferred for testing the negative relation. For completeness, the proportion for the case of affine jump intensity is also reported. Panel E1 and E2 of Table 5 report the proportion of t-statistics greater than 2.326, which corresponds to the 99th percentile of the standard normal distribution. The numbers reported in the columns of constant and affine jump intensities can be regarded as the size and power of testing no relation against the positive jump intensity-diffusive volatility relation, respectively. The results suggest that EMT_t is the most powerful in detecting the positive relation, followed by EQRV_t. The test based on EMT_t is slightly oversized in some cases, whereas tests based on the other diffusive variance estimators are undersized in all the cases. The test based on EMT_t performs the best for identifying the positive relation. For completeness, the proportion for the case of exponential affine jump intensity is also reported. Overall, the simulation analysis suggests that among the three diffusive variance estimators we consider, EQRV_t works the best for identifying the negative jump intensity-diffusive volatility relation, whereas EMT_t works the best at identifying the positive relation. Therefore, we use both EQRV_t and EMT_t as complements in the empirical analysis that follows.

4. Empirical Analysis

In this section, we analyze the relation between the conditional jump intensity and diffusive volatility of actual stock indexes and individual stocks. Since the relation is most relevant to options pricing, we choose 30 stocks with the highest dollar options trading volumes from 1996 to 2012, the S&P 500 index (SPX), and the NASDAQ composite index (NDX). The intraday levels of SPX and NDX are from the Thomson Reuters Tick History and Pi Trading Inc. For stocks, we use the average of bid and ask quotes from the primary exchange of each stock from the Trade and Quote database (TAQ). We sample the index/stock prices every 15-minute from 9:00am to 16:00pm, from which we calculate 26 intraday returns and one overnight return for each day. The previous studies suggest that 15-minute frequency is high enough to achieve a sufficient power in the jump detection and results are not greatly affected by the microstructure noise. The overnight returns of stocks are adjusted for dividends, which are from the Center for Research in Security Prices (CRSP). The diffusive volatility estimation and the jump detection are performed on the returns adjusted for the intraday volatility pattern as described in Section 2. The selected stocks and stock indexes and sample periods are shown in Table 6.

Table 6 here

4.1. Parametric Regression Analysis

We adopt the same regression specifications with instrumental variables in the actual indexes and stocks as those used in the simulated data. We use EQRV_t and EMT_t as the estimators of the diffusive volatility to test the negative and positive jump intensity-diffusive variance relations, respectively, because they work the best in different situations, as shown in the simulation analysis.

The results based on EQRV_t and EMT_t are reported in Panel A and B of Table 6, respectively. The third column shows the time-series average of the estimated diffusive volatility. SPX has a lower diffusive volatility than NDX does, and the diffusive volatilities of individual stocks are higher than those of indexes in general. The diffusive volatility estimated based on EQRV_t is slightly lower than that based on EMT_t in general. There is a large cross-sectional variation in volatility in the selected sample of stocks and indexes. The average annualized diffusive volatilities range from 13% to 56% based on EQRV_t, and from 14% to 62% based on EMT_t. The fourth column shows the standard deviation of the estimated diffusive volatility. The standard deviation of diffusive volatility tends to be higher for the stock or index with a higher average diffusive volatility. The next three columns show the annual jump intensities. On average, there are about 44 and 23 jumps per year in our sample of stock indexes and individual stocks based on EQRV_t and EMT_t, respectively. The higher number of jumps identified based on EQRV_t is due to the higher power and larger size of the jump detection test based on it. For indexes, the majority of jumps are negative, and for individual stocks, positive jumps and negative jumps are equally likely on average. The next column reports the correlation between $\hat{v}_t$ and $\hat{v}_{t-K}$. The average correlation is about 62% and 65%, for EQRV_t and EMT_t, respectively, which is lower than those reported in the simulation analysis. Nevertheless, the correlations are quite high, suggesting that $\hat{v}_{t-K}$ is a good instrument for $\hat{v}_t$. The instrumental variable regression is expected to be able to identify the relation between the conditional jump intensity and diffusive volatility if there is one. The next two columns show the coefficient estimate $\hat{\lambda}_1$ and the p-value for testing $\hat{\lambda}_1 = 0$ against $\hat{\lambda}_1 > 0$. Based on EQRV_t, 2 stocks are identified with a significant positive relation at 1% level, and based on EMT_t, 4 stocks are significant at 1% level. As indicated in the simulation analysis, the test based on EQRV_t is not as powerful as that based on EMT_t in identifying the positive relation. The results suggest that the conditional jump intensity as an increasing and affine function of diffusive variance is not supported by the data in general. The last two columns show the coefficient estimate $\hat{\lambda}_1^e$ and the p-value for testing $\hat{\lambda}_1^e = 0$ against $\hat{\lambda}_1^e < 0$. The test based on EQRV_t identifies 8 stocks and stock indexes with significant negative relations at 1% level, whereas the test based on EMT_t identifies only 3. The difference is due to the higher power of the test based on EQRV_t in identifying the negative relation than that based on EMT_t, as indicated in the simulation analysis. Most stocks do not show significant relations.

Table 7 here

4.2. Nonparametric Regression Analysis

The analysis based on the parametric regressions in the previous subsection suggests that for a few stocks or stock indexes, there is a significant relation between the conditional jump intensity and diffusive volatility. While these parametric regressions can be used to test for a simple increasing or decreasing relation, the affine or exponential affine relation may be mis-specified. In this subsection, we investigate the functional form of the relation using nonparametric regressions.

We apply the Nadaraya-Watson kernel regression to estimate the conditional jump intensity $\Lambda(\hat{v}_t)$ in

$$J_{t+1} = \Lambda(\hat{v}_t) + \eta_{t+1}. \tag{18}$$

Specifically, the function $\Lambda(v)$ is estimated by

$$\hat{\Lambda}(v) = \frac{\sum_{t=1}^{T-1} \phi(\frac{\hat{v}_t - v}{h(v)}) J_{t+1}}{\sum_{t=1}^{T-1} \phi(\frac{\hat{v}_t - v}{h(v)})}, \tag{19}$$

where $\phi(\cdot)$ is a kernel function, $h(\cdot)$ is the bandwidth, and T is the number of observations. We choose the second-order Gaussian kernel $\phi(z) = (1/\sqrt{2\pi})e^{-z^2/2}$. The bandwidth takes the form $h(v) = b\sigma[f(v)T]^{-1/5}$, where σ is the standard deviation of $\hat{v}_t$, and $f(v)$ is the density of $\hat{v}_t$. Since the distribution of $\hat{v}_t$ is extremely skewed to the right, a bandwidth varying with the value of the density function allows a smoother function to be estimated than would a constant bandwidth. The density function $f(v)$ is also estimated nonparametrically by a kernel density estimator. The constant b is determined by the cross-validation method to minimize the following objective function,

$$\mathrm{CV}(b) = \frac{1}{T-1}\sum_{t=1}^{T-1}[J_{t+1} - \hat{\Lambda}_{-(t+1),b}(\hat{v}_t)]^2, \tag{20}$$

where $\hat{\Lambda}_{-(t+1),b}(\hat{v}_t)$ is the kernel estimator of $\hat{\Lambda}(\hat{v}_t)$ without using the observation J_{t+1}.

The top two panels in Figure 1 show the nonparametric estimation of the conditional jump intensity of SPX and APPL, which are among the most significant negative relations in Panel A of Table 7. The solid line is the mean estimate, and the dashed lines show the 90 percent confidence interval.[12] For SPX, the significant negative relation comes from the low level of diffusive variance, where the confidence interval is narrow. The relation becomes flat at the medium to high levels of diffusive variance. For APPL, the relation is generally negative, and the negative relation is stronger at the low level of diffusive variance. The bottom two panels show the nonparametric estimation of the conditional jump intensity of TWX and GM, the two stocks with the most significant positive linear relations in Panel A of Table 7. The positive relation is the strongest at the low level of the diffusive variance. At the high level of diffusive variance, the confidence interval is wide. There is also a region where the relation is negative. These results suggest that the positive relation between the conditional jump intensity and diffusive variance is rather weak.

Figure 1 here

[12]The confidence interval is calculated based on the block bootstrap method of Kunsch (1989) to account for the time-series dependence of the observations.

The results based on EMT_t are shown in Figure 2. The top two panels show the nonparametric estimation for NDX and GOOG, which have the most significant negative relations in Panel B of Table 7. For NDX, the significant negative relation comes from the low to medium levels of diffusive variance with a narrow confidence interval. The relation becomes positive at the high level of diffusive variance, showing a convex shape. However, the positive relation is weak evidenced by the wide confidence interval. The pattern of the jump intensity-diffusive variance relation for GOOG resembles that for NDX. The bottom two panels show the nonparametric estimation of stocks with the most significant positive relations in Panel B of Table 7. For C, the significant positive relation is at the medium to high levels of diffusive variance, and the relation is slightly negative for the low level of diffusive variance. GM shows up in both Figure 1 and Figure 2 as the one with a significant positive relation, however, the positive relation is weak as suggested by the wide confidence interval.

Figure 2 here

5. Conclusion

Jumps are essential components in asset price dynamics. The jump intensity is typically assumed to be positively related to the diffusive variance, as in the standard options pricing models. We examine this relation empirically using nonparametric and semi-nonparametric approaches. Simulation analysis shows that various nonparametric jump detection tests are powerful enough to detect the relation between the conditional jump intensity and diffusive variance if the relation exists. However, the empirical results show that, for most of the stocks and stock indexes we examine, the conditional jump intensity is unrelated to the diffusive variance. When there is a relation for some stocks, the relation tends to be nonlinear and non-monotonic. The results presented in this paper suggest that normal changes in asset prices, i.e. the diffusive component of asset returns, and extreme changes, i.e. jumps, are driven by separate state variables. In particular, this finding is at odds with the affine jump-diffusion models of options pricing in which the relation between the conditional jump intensity and diffusive variance is assumed to be increasing and linear.

The findings in this paper contribute to the literature in the following sense. Existing tests based on specific models and options data suffer from two problems. First, jumps in options prices can result from jumps in other state variables rather than those in the underlying asset price. In this case, the positive relation obtained using options data does not have a clear interpretation of the relation between the jump intensity of underlying asset price and diffusive variance. Second, models that do not clearly differentiate relations under the actual probability and the risk-neutral probability may mistaken a relation under the risk-neutral probability as a relation under the actual probability. Our methodology is model-free, robust, and powerful enough to detect the relation if it indeed exists. Our results indicate that a commonly assumed positive relation between the jump intensity and the diffusive volatility is not bourn out by data and that the actual relation can be nonlinear and non-monotonic. These findings suggest that identifying state variables governing the dynamics of jump intensity is important for understanding the evolution of asset prices and for improving models of options pricing.

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Table 1
Errors in the Estimated Diffusive Volatility: Simulation Results
This table reports the root-mean-square error (RMSE) of the estimated diffusive volatility, defined as RMSE $= \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\sqrt{v_t}-\sqrt{\hat{v}_t})^2}$, multiplied by $100\sqrt{T_d}$, where $T_d = 252 \times 26$ is the number of intraday returns in a year, v_t is the actual diffusive variance from the simulation, $\hat{v}_t$ is its estimate, and T is the total number of observations. Median and interquartile range (in parentheses) from 5000 simulated samples are shown. The RMSE can be compared with the mean of actual annualized diffusive volatility 13.06%. In the simulation, the jump intensity is specified as $\Lambda_t = \lambda_0 + \lambda_1 v_t$, where $\lambda_0 = 20/T_d, \lambda_1 = 0$ for the constant jump intensity case and $\lambda_0 = 10/T_d, \lambda_1 = 400$ for the affine jump intensity case, and $\Lambda_t = \exp(\lambda_0^e + \lambda_1^e v_t)$ with $\lambda_0^e = 3.5 - \ln(T_d), \lambda_1^e = -35$ for the exponential affine jump intensity case. The jump size is fixed at $4\sqrt{v_t}$ for small jumps (SJ) and $6\sqrt{v_t}$ for large jumps (LJ). The diffusive variance estimators include: the bipower variation, BV_t, the exponentially weighted quantile realized variance, EQRV_t, and the exponentially weighted moving average with truncation, EMT_t. Panel A is for the case of normal jumps, and Panel B is for the case of consecutive jumps.

A. Jumps	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	3.109	3.168	3.292	3.549	3.074	3.091
	(0.43)	(0.44)	(0.64)	(0.89)	(0.43)	(0.42)
EQRV_t	2.817	2.818	2.903	2.946	2.812	2.813
	(0.38)	(0.38)	(0.50)	(0.69)	(0.37)	(0.37)
EMT_t	2.569	2.739	2.591	2.742	2.531	2.702
	(0.45)	(0.46)	(0.47)	(0.48)	(0.43)	(0.43)
B. Consecutive Jumps	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	3.255	3.690	3.739	4.999	3.127	3.296
	(0.45)	(0.53)	(0.94)	(1.77)	(0.42)	(0.40)
EQRV_t	2.827	2.827	2.879	2.888	2.817	2.817
	(0.38)	(0.38)	(0.44)	(0.47)	(0.37)	(0.37)
EMT_t	2.626	2.867	2.666	2.902	2.573	2.815
	(0.45)	(0.46)	(0.47)	(0.51)	(0.43)	(0.41)

Table 2
Sizes of the Jump Detection Tests: Simulation Results
This table reports the sizes of the jump detection tests, multiplied by 100. Median and interquartile range (in parentheses) from 5000 simulated samples are shown. The significance level of the jump detection tests is 0.01%. In the simulation, the jump intensity is specified as $\Lambda_t = \lambda_0 + \lambda_1 v_t$, where $\lambda_0 = 20/T_d, \lambda_1 = 0$ for the constant jump intensity case and $\lambda_0 = 10/T_d, \lambda_1 = 400$ for the affine jump intensity case, and $\Lambda_t = \exp(\lambda_0^e + \lambda_1^e v_t)$ with $\lambda_0^e = 3.5 - \ln(T_d), \lambda_1^e = -35$ for the exponential affine jump intensity case. $T_d = 252 \times 26$ is the number of intraday returns in a year. The jump size is fixed at $4\sqrt{v_t}$ for small jumps (SJ) and $6\sqrt{v_t}$ for large jumps (LJ). The diffusive variance estimators include: the bipower variation, BV_t, the exponentially weighted quantile realized variance, EQRV_t, and the exponentially weighted moving average with truncation, EMT_t. Panel A is for the case of normal jumps, and Panel B is for the case of consecutive jumps.

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
A. Jumps						
BV_t	0.074	0.071	0.074	0.072	0.074	0.071
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
EQRV_t	0.078	0.078	0.078	0.078	0.078	0.078
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
EMT_t	0.047	0.045	0.047	0.046	0.046	0.044
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
B. Consecutive Jumps						
	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	0.073	0.071	0.074	0.071	0.073	0.070
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
EQRV_t	0.080	0.080	0.080	0.080	0.080	0.080
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
EMT_t	0.048	0.047	0.048	0.048	0.048	0.047
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)

Table 3
Powers of the Jump Dectection Tests: Simulation Results
This table reports the powers of the jump detection tests, multiplied by 100. Median and interquartile range (in parentheses) from 5000 simulated samples are shown. The significance level of the jump detection tests is 0.01%. In the simulation, the jump intensity is specified as $\Lambda_t = \lambda_0 + \lambda_1 v_t$, where $\lambda_0 = 20/T_d$, $\lambda_1 = 0$ for the constant jump intensity case and $\lambda_0 = 10/T_d$, $\lambda_1 = 400$ for the affine jump intensity case, and $\Lambda_t = \exp(\lambda_0^e + \lambda_1^e v_t)$ with $\lambda_0^e = 3.5 - \ln(T_d)$, $\lambda_1^e = -35$ for the exponential affine jump intensity case. $T_d = 252 \times 26$ is the number of intraday returns in a year. The jump size is fixed at $4\sqrt{v_t}$ for small jumps (SJ) and $6\sqrt{v_t}$ for large jumps (LJ). The diffusive variance estimators include: the bipower variation, BV_t, the exponentially weighted quantile realized variance, EQRV_t, and the exponentially weighted moving average with truncation, EMT_t. Panel A is for the case of normal jumps, and Panel B is for the case of consecutive jumps.

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
A. Jumps						
BV_t	49.648	92.803	50.969	92.716	47.294	91.865
	(3.60)	(1.76)	(3.48)	(2.17)	(3.36)	(1.94)
EQRV_t	52.244	95.012	53.110	95.028	50.822	94.559
	(3.51)	(1.40)	(3.72)	(1.67)	(3.63)	(1.47)
EMT_t	49.757	93.399	52.778	95.053	47.678	91.797
	(3.39)	(1.76)	(3.72)	(1.45)	(3.28)	(1.76)
B. Consecutive Jumps						
	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	47.469	90.051	47.962	88.315	45.046	88.679
	(3.75)	(2.36)	(4.10)	(3.60)	(3.51)	(2.60)
EQRV_t	52.000	94.977	52.877	95.182	50.948	94.560
	(3.99)	(1.62)	(4.05)	(1.63)	(3.61)	(1.60)
EMT_t	36.692	75.056	41.304	80.440	34.224	70.940
	(3.39)	(2.67)	(3.66)	(2.56)	(2.97)	(2.76)

Table 4
Sizes and Powers of the Jump Dectection Tests: Simulation Results (Daily Frequency)
This table reports the sizes and powers of the jump detection tests, multiplied by 100, based on daily returns. Median and interquartile range (in parentheses) from 5000 simulated samples are shown. The significance level of the jump detection tests is 0.01%. In the simulation, the jump intensity is specified as $\Lambda_t = \lambda_0 + \lambda_1 v_t$, where $\lambda_0 = 3/T_d, \lambda_1 = 0$ for the constant jump intensity case and $\lambda_0 = 1.5/T_d, \lambda_1 = 60$ for the affine jump intensity case, and $\Lambda_t = \exp(\lambda_0^e + \lambda_1^e v_t)$ with $\lambda_0^e = 1.5 - \ln(T_d), \lambda_1^e = -25$ for the exponential affine jump intensity case. $T_d = 252$ is the number of daily returns in a year. The jump size is fixed at $4\sqrt{v_t}$ for small jumps (SJ) and $6\sqrt{v_t}$ for large jumps (LJ). The diffusive variance estimators include: the bipower variation, BV_t, the exponentially weighted quantile realized variance, EQRV_t, and the exponentially weighted moving average with truncation, EMT_t. Panel A1 and B1 report the sizes and powers for the case of normal jumps, respectively, and Panel A2 and B2 report the sizes and powers for the case of consecutive jumps, respectively.

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
A1. Sizes for Jumps						
BV_t	0.707	0.668	0.706	0.686	0.708	0.686
	(0.16)	(0.18)	(0.16)	(0.18)	(0.18)	(0.18)
EQRV_t	1.092	1.092	1.091	1.091	1.108	1.094
	(0.23)	(0.23)	(0.24)	(0.24)	(0.24)	(0.25)
EMT_t	0.504	0.485	0.485	0.484	0.485	0.484
	(0.14)	(0.14)	(0.13)	(0.12)	(0.14)	(0.14)
A2. Sizes for Consecutive Jumps						
	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	0.708	0.690	0.724	0.707	0.708	0.688
	(0.16)	(0.18)	(0.18)	(0.18)	(0.16)	(0.16)
EQRV_t	1.113	1.113	1.111	1.111	1.112	1.112
	(0.22)	(0.22)	(0.24)	(0.24)	(0.23)	(0.23)
EMT_t	0.506	0.504	0.504	0.485	0.506	0.504
	(0.14)	(0.12)	(0.12)	(0.12)	(0.14)	(0.14)

Table 4 (Cont'd)

B1. Powers for Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	43.478	72.059	51.667	76.517	36.667	66.071
	(8.74)	(8.00)	(8.39)	(7.90)	(8.37)	(8.52)
EQRV_t	48.181	77.193	56.431	81.678	41.935	72.414
	(8.57)	(7.38)	(8.49)	(7.11)	(8.65)	(8.26)
EMT_t	42.857	73.611	55.000	82.692	35.185	66.000
	(8.40)	(8.09)	(8.71)	(7.15)	(8.81)	(8.53)

B2. Powers for Consecutive Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	39.639	68.086	46.571	71.212	33.784	62.121
	(10.25)	(10.70)	(10.45)	(9.86)	(9.99)	(10.35)
EQRV_t	48.214	77.778	56.343	82.813	42.308	73.333
	(11.32)	(10.13)	(10.47)	(7.99)	(11.11)	(9.81)
EMT_t	32.353	58.000	44.643	68.182	25.000	50.000
	(9.46)	(9.96)	(10.53)	(8.85)	(8.29)	(9.82)

Table 5
Relation Between Conditional Jump Intensity and Diffusive Variance: Simulation Results
This table reports the results of the following regressions from the simulated data,

$$J_{t+1} = \lambda_0 + \lambda_1 \hat{v}_t + \varepsilon_{t+1},$$

for the cases of constant and affine jump intensity, and

$$J_{t+1} = \exp(\lambda_0^e + \lambda_1^e \hat{v}_t) + \varepsilon_{t+1}^e,$$

for the case of exponential affine jump intensity, with the constant and $\hat{v}_{t-K}$ as instrumental variables, where $K = 100$, $J_t = 1$ indicates a detected jump at t, zero otherwise, and $\hat{v}_t$ is the estimate of the diffusive variance. The significance level of the jump detection tests is 0.01%. In the simulation, the jump intensity is specified as $\Lambda_t = \lambda_0 + \lambda_1 v_t$, where $\lambda_0 = 20/T_d, \lambda_1 = 0$ for the constant jump intensity case and $\lambda_0 = 10/T_d, \lambda_1 = 400$ for the affine jump intensity case, and $\Lambda_t = \exp(\lambda_0^e + \lambda_1^e v_t)$ with $\lambda_0^e = 3.5 - \ln(T_d), \lambda_1^e = -35$ for the exponential affine jump intensity case. $T_d = 252 \times 26$ is the number of intraday returns in a year. The jump size is fixed at $4\sqrt{v_t}$ for small jumps (SJ) and $6\sqrt{v_t}$ for large jumps (LJ). The diffusive variance estimators include: the bipower variation, BV_t, the exponentially weighted quantile realized variance, EQRV_t, and the exponentially weighted moving average with truncation, EMT_t. Panel A1 and A2 report the correlation between $\hat{v}_t$ and $\hat{v}_{t-K}$. Panel B1 and B2 report the coefficient estimate of λ_1 or λ_1^e, $\hat{\lambda}_1$ or $\hat{\lambda}_1^e$. Panel C1 and C2 report the heteroscedasticity and serial correlation consistent t-statistic based on the Newey-West procedure with 500 lags, $t(\hat{\lambda}_1)$ or $t(\hat{\lambda}_1^e)$. Median and interquartile range (in parentheses) from 5000 simulated samples are shown. Panels D1 and D2 (E1 and E2) report the proportions of $t(\hat{\lambda}_1) < -2.326$ or $t(\hat{\lambda}_1^e) < -2.326$ ($t(\hat{\lambda}_1) > 2.326$ or $t(\hat{\lambda}_1^e) > 2.326$), from the 5000 simulated samples.

A1. Correlation Between $\hat{v}_t$ and $\hat{v}_{t-K}$ for Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	0.834	0.831	0.810	0.787	0.836	0.836
	(0.04)	(0.04)	(0.05)	(0.06)	(0.04)	(0.04)
EQRV_t	0.783	0.783	0.758	0.744	0.784	0.784
	(0.04)	(0.04)	(0.06)	(0.12)	(0.04)	(0.04)
EMT_t	0.831	0.825	0.829	0.823	0.832	0.827
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)

Table 5 (Cont'd)

A2. Correlation Between $\hat{v}_t$ and $\hat{v}_{t-K}$ for Consecutive Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	0.823	0.782	0.767	0.683	0.835	0.829
	(0.04)	(0.06)	(0.06)	(0.08)	(0.04)	(0.04)
$EQRV_t$	0.783	0.783	0.769	0.767	0.784	0.784
	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)
EMT_t	0.827	0.817	0.822	0.809	0.830	0.822
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)

B1. $\hat{\lambda}_1$ or $\hat{\lambda}_1^e$ for Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	-34.543	-25.613	84.552	201.049	-23.581	-23.334
	(35.28)	(35.95)	(42.28)	(68.87)	(11.00)	(7.39)
$EQRV_t$	-24.479	-17.224	130.745	281.621	-20.893	-22.789
	(33.41)	(37.64)	(55.87)	(86.67)	(9.90)	(7.54)
EMT_t	-1.883	4.634	234.099	409.457	-18.010	-20.951
	(42.12)	(46.95)	(105.16)	(99.28)	(13.02)	(9.44)

B2. $\hat{\lambda}_1$ or $\hat{\lambda}_1^e$ for Consecutive Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	-32.407	-26.092	54.263	114.358	-22.909	-22.650
	(35.41)	(42.42)	(38.42)	(72.99)	(11.79)	(8.44)
$EQRV_t$	-24.636	-18.972	137.122	297.999	-20.862	-22.804
	(36.62)	(48.43)	(65.92)	(100.52)	(10.42)	(8.77)
EMT_t	8.555	24.263	219.912	395.943	-14.894	-18.325
	(41.74)	(56.37)	(103.60)	(112.75)	(12.84)	(10.57)

C1. $t(\hat{\lambda}_1)$ or $t(\hat{\lambda}_1^e)$ for Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	-1.957	-1.218	3.329	5.897	-3.682	-5.075
	(1.56)	(1.56)	(1.96)	(2.89)	(1.56)	(1.67)
$EQRV_t$	-1.376	-0.789	4.145	6.477	-3.444	-4.662
	(1.69)	(1.68)	(2.23)	(3.32)	(1.48)	(1.62)
EMT_t	-0.075	0.162	4.452	7.003	-2.815	-4.152
	(1.74)	(1.56)	(1.85)	(2.93)	(2.02)	(2.39)

Table 5 (Cont'd)

C2. $t(\hat{\lambda}_1)$ or $t(\hat{\lambda}_1^e)$ for Consecutive Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	-1.772	-1.043	2.482	3.716	-3.386	-4.091
	(1.75)	(1.70)	(1.59)	(2.15)	(1.35)	(1.24)
$EQRV_t$	-1.268	-0.675	3.550	5.286	-3.208	-3.964
	(1.85)	(1.67)	(1.91)	(2.95)	(1.24)	(1.23)
EMT_t	0.314	0.676	3.813	5.638	-2.217	-3.014
	(1.54)	(1.39)	(1.53)	(2.37)	(1.65)	(1.59)

D1. Proportion of $t(\hat{\lambda}_1) < -2.326$ or $t(\hat{\lambda}_1^e) < -2.326$ for Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	0.393	0.185	0.000	0.000	0.864	0.962
$EQRV_t$	0.227	0.123	0.000	0.000	0.836	0.957
EMT_t	0.064	0.019	0.000	0.000	0.620	0.808

D2. Proportion of $t(\hat{\lambda}_1) < -2.326$ or $t(\hat{\lambda}_1^e) < -2.326$ for Consecutive Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	0.354	0.182	0.000	0.000	0.833	0.946
$EQRV_t$	0.228	0.120	0.000	0.000	0.814	0.937
EMT_t	0.037	0.009	0.000	0.000	0.467	0.712

E1. Proportion of $t(\hat{\lambda}_1) > 2.326$ or $t(\hat{\lambda}_1^e) > 2.326$ for Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	0.000	0.001	0.785	0.967	0.000	0.000
$EQRV_t$	0.000	0.004	0.900	0.976	0.000	0.000
EMT_t	0.010	0.027	0.985	1.000	0.008	0.004

E2. Proportion of $t(\hat{\lambda}_1) > 2.326$ or $t(\hat{\lambda}_1^e) > 2.326$ for Consecutive Jumps

	Constant		Affine		Exponential Affine	
	SJ	LJ	SJ	LJ	SJ	LJ
BV_t	0.000	0.000	0.555	0.839	0.000	0.000
$EQRV_t$	0.000	0.002	0.849	0.979	0.000	0.000
EMT_t	0.019	0.057	0.953	0.997	0.017	0.007

Table 6
List of Stock Indexes and Individual Stocks
This table lists two stock indexes and 30 individual stocks used in the empirical analysis. The ticker, full name, start and end dates of the sample data of each index/stock are shown.

No.	Ticker	Name	Start Date	End Date
1	SPX	S&P 500	19860102	20121231
2	NDX	Nasdaq Composite	19960102	20121231
3	AAPL	Apple Inc	19940308	20121231
4	GOOG	Google Inc	20040819	20121231
5	MSFT	Microsoft Corp	19940308	20121231
6	YHOO	Yahoo Inc	19960412	20121231
7	C	Citigroup Inc	19930104	20121231
8	INTC	Intel Corp	19940308	20121231
9	MO	Altria Group Inc	19930104	20121231
10	GS	Goldman Sachs Group Inc	19990504	20121231
11	IBM	International Business Machs Co	19930104	20121231
12	RIMM	Research In Motion Ltd	19990204	20121231
13	AMZN	Amazon Com Inc	19970515	20121231
14	CSCO	Cisco Systems Inc	19940308	20121231
15	BAC	Bank Of America Corp	19930104	20121231
16	JPM	JPMorgan Chase & Co	19930104	20121231
17	QCOM	Qualcomm Inc	19940308	20121231
18	TWX	Time Warner Inc	19940308	20121231
19	FCX	Freeport McMoRan Copper & Gold	19951113	20121231
20	GE	General Electric Co	19930104	20121231
21	XOM	Exxon Mobil Corp	19991201	20121231
22	DELL	Dell Inc	19940308	20121231
23	WMT	Wal Mart Stores Inc	19930104	20121231
24	AIG	American International Group Inc	19930104	20121231
25	CAT	Caterpillar Inc	19930104	20121231
26	PFE	Prizer Inc	19930104	20121231
27	T	AT&T Inc	19930104	20121231
28	VZ	Verizon Communications Inc	19930104	20121231
29	GM	General Motors Corp	19930104	20090601
30	PG	Procter & Gamble Co	19930104	20121231
31	MER	Merrill Lynch & Co Inc	19930104	20081231
32	OXY	Occidental Petroleum Corp	19930104	20121231

Table 7
Testing the Relation Between Conditional Jump Intensity and Diffusive Variance

This table reports the results of testing a linear and a nonlinear relation between conditional jump intensity and diffusive variance for the sample of stock indexes and individual stocks. For the linear specification, the regression specification is

$$J_{t+1} = \lambda_0 + \lambda_1 \hat{v}_t + \varepsilon_{t+1},$$

and for the nonlinear specification, it is

$$J_{t+1} = \exp(\lambda_0^e + \lambda_1^e \hat{v}_t) + \varepsilon_{t+1}^e,$$

with the constant and $\hat{v}_{t-K}$ as instrumental variables, where $K = 100$, $J_t = 1$ indicates a detected jump at t, zero otherwise, and $\hat{v}_t$ is the estimated diffusive variance. In Panel A, $\hat{v}_t$ is the exponentially weighted quantile realized variance, EQRV_t, and in Panel B, $\hat{v}_t$ is the exponentially weighted moving average with truncation, EMT_t. The significance level of the jump detection tests is 0.01%. The estimate of λ_1 multiplied by 0.01 or λ_1^e, $\hat{\lambda}_1$ or $\hat{\lambda}_1^e$, and the heteroscedasticity and serial correlation consistent p-value for testing $\hat{\lambda}_1 = 0$ against $\hat{\lambda}_1 > 0$ or testing $\hat{\lambda}_1^e = 0$ against $\hat{\lambda}_1^e < 0$ based on the Newey-West procedure with 500 lags, $p^+(\hat{\lambda}_1)$ or $p^-(\hat{\lambda}_1^e)$, are shown. The table also reports the time-series average of the estimated diffusive volatility multiplied by $100\sqrt{T_d}$, $m(\sqrt{\hat{v}_t})$, and the standard deviation of the estimated diffusive volatility multiplied by $100\sqrt{T_d}$, $s(\sqrt{\hat{v}_t})$, where $T_d = 252 \times 26$ is the number of intraday returns in a year. $\bar{J}$, $\bar{J}^-$, and $\bar{J}^+$ are annual intensities of all jumps, negative jumps, and positive jumps, respectively. corr is the correlation between $\hat{v}_t$ and $\hat{v}_{t-K}$.

Table 7 (Cont'd)

A. EQRV

No.	Ticker	$m(\sqrt{\hat{v}_t})$	$s(\sqrt{\hat{v}_t})$	$\bar{J}$	$\bar{J}^-$	$\bar{J}^+$	corr	$\hat{\lambda}_1$	$p^+(\hat{\lambda}_1)$	$\hat{\lambda}_1^e$	$p^-(\hat{\lambda}_1^e)$
1	SPX	13.27	8.78	57.39	34.55	22.84	0.55	-2.78	1.00	-20.12	0.00
2	NDX	20.19	12.80	30.86	20.02	10.84	0.59	-1.19	1.00	-9.41	0.00
3	AAPL	42.24	21.21	56.85	28.34	28.51	0.56	-0.40	1.00	-0.90	0.00
4	GOOG	27.90	15.32	53.04	26.70	26.34	0.77	-0.94	1.00	-4.81	0.00
5	MSFT	28.18	14.05	33.62	16.51	17.11	0.68	-0.55	1.00	-2.37	0.00
6	YHOO	48.96	30.43	50.03	22.94	27.10	0.61	0.02	0.42	0.04	0.58
7	C	38.11	29.80	48.07	24.09	23.99	0.62	-0.07	0.97	-0.23	0.10
8	INTC	35.92	18.15	28.66	15.12	13.55	0.67	-0.34	1.00	-1.67	0.00
9	MO	23.77	12.04	51.24	28.11	23.13	0.53	-0.14	0.65	-0.28	0.36
10	GS	32.53	22.29	35.07	17.83	17.24	0.42	0.01	0.46	0.03	0.54
11	IBM	25.06	12.65	42.38	20.97	21.42	0.69	-0.38	0.96	-1.09	0.07
12	RIMM	56.49	33.14	60.85	28.10	32.75	0.65	0.00	0.48	0.01	0.52
13	AMZN	52.17	32.95	53.25	23.67	29.58	0.63	-0.05	0.80	-0.10	0.21
14	CSCO	37.59	20.60	32.83	16.74	16.09	0.58	-0.27	1.00	-1.08	0.01
15	BAC	32.46	26.72	38.82	20.97	17.85	0.72	-0.08	1.00	-0.34	0.04
16	JPM	32.53	19.99	40.07	20.61	19.46	0.66	-0.15	0.97	-0.47	0.06
17	QCOM	43.21	25.14	54.02	26.74	27.28	0.68	-0.19	0.99	-0.44	0.02
18	TWX	38.54	23.30	47.35	21.92	25.42	0.54	0.38	0.01	0.61	1.00
19	FCX	41.49	20.11	44.47	21.03	23.44	0.79	-0.12	0.99	-0.39	0.07
20	GE	26.45	15.98	37.96	19.11	18.85	0.75	-0.20	0.93	-0.73	0.14
21	XOM	22.31	12.69	32.18	18.09	14.09	0.65	-0.08	0.68	-0.29	0.34
22	DELL	39.85	20.59	39.74	19.98	19.76	0.57	0.05	0.37	0.11	0.63
23	WMT	26.35	13.42	32.94	17.00	15.94	0.62	-0.50	0.95	-1.80	0.07
24	AIG	39.24	44.36	63.36	28.96	34.39	0.27	-0.01	0.86	-0.03	0.19
25	CAT	29.41	14.02	42.48	20.41	22.07	0.70	-0.39	0.99	-1.33	0.04
26	PFE	25.46	10.79	39.47	20.86	18.60	0.58	-0.16	0.67	-0.41	0.34
27	T	24.78	12.59	37.41	20.01	17.40	0.73	-0.10	0.72	-0.30	0.29
28	VZ	24.35	12.23	42.89	21.82	21.07	0.66	-0.49	0.99	-1.53	0.02
29	GM	38.55	32.86	61.96	30.71	31.26	0.55	0.11	0.00	0.09	1.00
30	PG	21.30	10.40	39.97	19.41	20.56	0.60	0.11	0.38	0.27	0.62
31	MER	36.32	23.53	40.67	19.36	21.31	0.58	-0.08	0.89	-0.26	0.18
32	OXY	31.10	15.04	31.43	17.15	14.28	0.78	-0.39	1.00	-4.44	0.00

Table 7 (Cont'd)

B. EMT

No.	Ticker	$m(\sqrt{\hat{v}_t})$	$s(\sqrt{\hat{v}_t})$	$\bar{J}$	$\bar{J}^-$	$\bar{J}^+$	corr	$\hat{\lambda}_1$	$p^+(\hat{\lambda}_1)$	$\hat{\lambda}_1^e$	$p^-(\hat{\lambda}_1^e)$
1	SPX	13.88	7.64	34.48	20.60	13.87	0.73	-1.28	0.97	-5.51	0.07
2	NDX	20.18	11.81	22.22	14.66	7.57	0.72	-0.83	1.00	-6.49	0.00
3	AAPL	47.08	23.69	30.03	15.67	14.36	0.54	-0.20	0.99	-0.83	0.02
4	GOOG	29.52	14.98	34.76	16.84	17.92	0.73	-0.40	1.00	-1.68	0.01
5	MSFT	30.49	14.60	18.95	9.37	9.58	0.64	-0.21	0.98	-1.34	0.04
6	YHOO	53.63	32.34	25.26	11.19	14.07	0.62	0.04	0.18	0.13	0.83
7	C	41.72	26.88	21.62	10.86	10.76	0.69	0.23	0.00	0.53	1.00
8	INTC	38.07	18.15	15.12	8.07	7.04	0.66	-0.15	1.00	-1.33	0.02
9	MO	26.26	12.73	26.60	15.09	11.52	0.53	0.56	0.04	1.68	0.99
10	GS	33.58	19.79	23.14	11.27	11.86	0.73	0.12	0.21	0.43	0.85
11	IBM	27.01	13.05	23.78	11.51	12.27	0.65	0.10	0.29	0.41	0.73
12	RIMM	61.76	34.71	35.00	16.12	18.88	0.64	0.03	0.22	0.08	0.79
13	AMZN	56.36	34.82	29.58	13.10	16.47	0.66	0.01	0.38	0.04	0.62
14	CSCO	40.33	21.09	16.96	9.75	7.21	0.61	-0.16	1.00	-1.24	0.01
15	BAC	34.73	23.41	21.02	11.71	9.30	0.79	0.19	0.00	0.53	1.00
16	JPM	34.86	19.47	23.18	12.32	10.86	0.67	0.04	0.32	0.15	0.69
17	QCOM	48.10	27.35	25.27	12.39	12.88	0.64	-0.03	0.78	-0.14	0.23
18	TWX	42.45	25.11	22.95	11.10	11.85	0.60	0.32	0.00	1.00	1.00
19	FCX	46.29	20.89	20.27	9.75	10.52	0.68	0.02	0.41	0.08	0.60
20	GE	29.03	16.37	19.86	9.85	10.01	0.58	0.11	0.17	0.48	0.88
21	XOM	23.93	11.69	16.70	9.85	6.85	0.74	0.23	0.05	1.04	0.98
22	DELL	43.73	21.80	19.60	10.94	8.66	0.62	-0.07	0.85	-0.38	0.18
23	WMT	29.40	14.57	18.15	9.86	8.30	0.70	-0.24	0.92	-1.46	0.10
24	AIG	42.00	33.74	32.43	15.34	17.10	0.83	0.17	0.06	0.25	1.00
25	CAT	31.99	14.20	21.42	11.06	10.36	0.65	0.09	0.26	0.39	0.76
26	PFE	27.98	11.56	21.12	11.56	9.55	0.52	0.08	0.35	0.36	0.65
27	T	27.23	13.02	18.20	9.65	8.55	0.67	0.31	0.02	1.32	0.99
28	VZ	27.46	14.43	22.07	11.92	10.16	0.47	0.13	0.16	0.47	0.89
29	GM	41.82	27.67	32.97	16.43	16.55	0.70	0.46	0.00	0.58	1.00
30	PG	23.64	11.44	19.81	9.85	9.95	0.58	0.00	0.50	0.01	0.50
31	MER	39.63	23.30	22.75	10.18	12.57	0.52	0.15	0.09	0.40	0.99
32	OXY	34.90	14.98	14.38	7.79	6.59	0.72	-0.19	0.89	-2.01	0.16

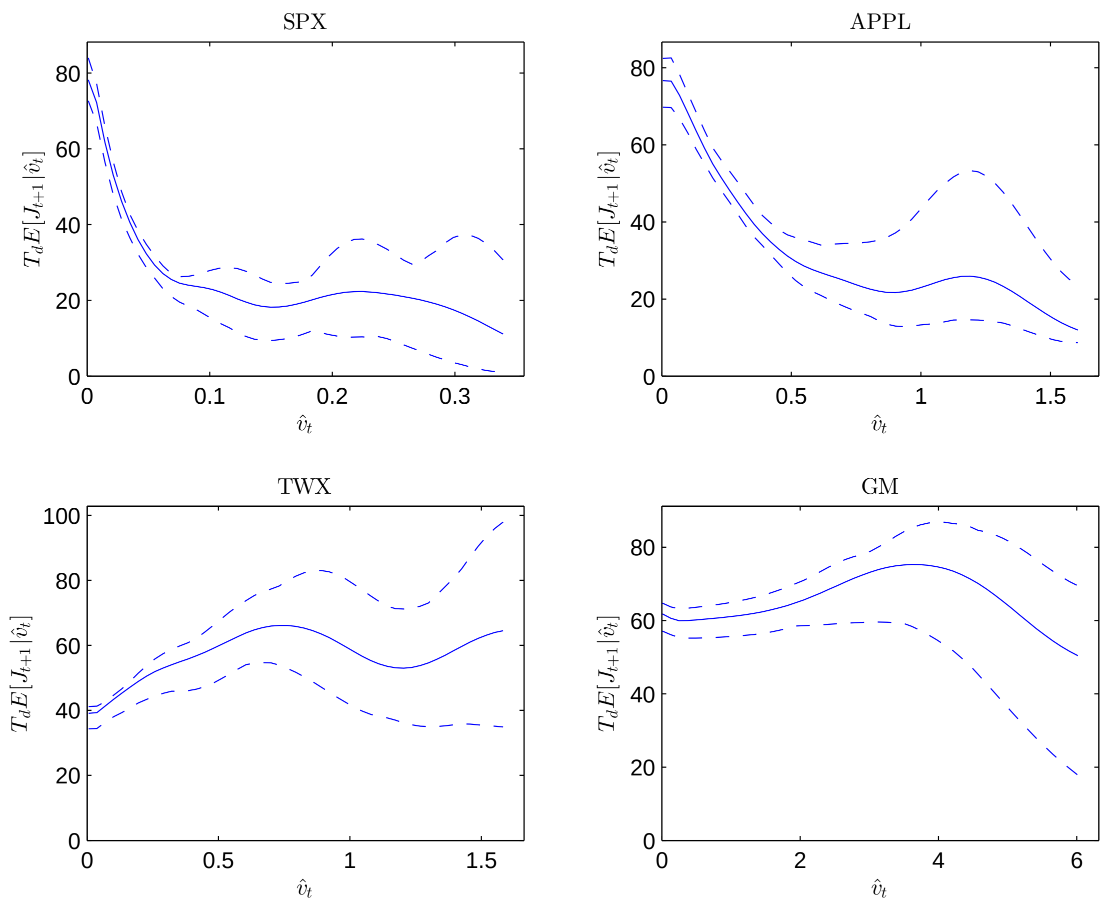

Figure 1. Conditional Jump Intensity and Diffusive Variance (EQRV$_t$)

This figure shows the nonparametric estimation of the annual jump intensity conditional on the diffusive variance, $T_dE[J_{t+1}|\hat{v}_t]$, for a few stocks and the S&P 500 index, where $J_t = 1$ indicates a detected jump at t, zero otherwise, $\hat{v}_t$ is the estimated diffusive variance, and T_d is the number of intraday returns in a year. The solid line is the fitted curve. The dashed lines cover the 90% confidence interval. The diffusive variance is estimated by the exponentially weighted quantile realized variance, EQRV$_t$.

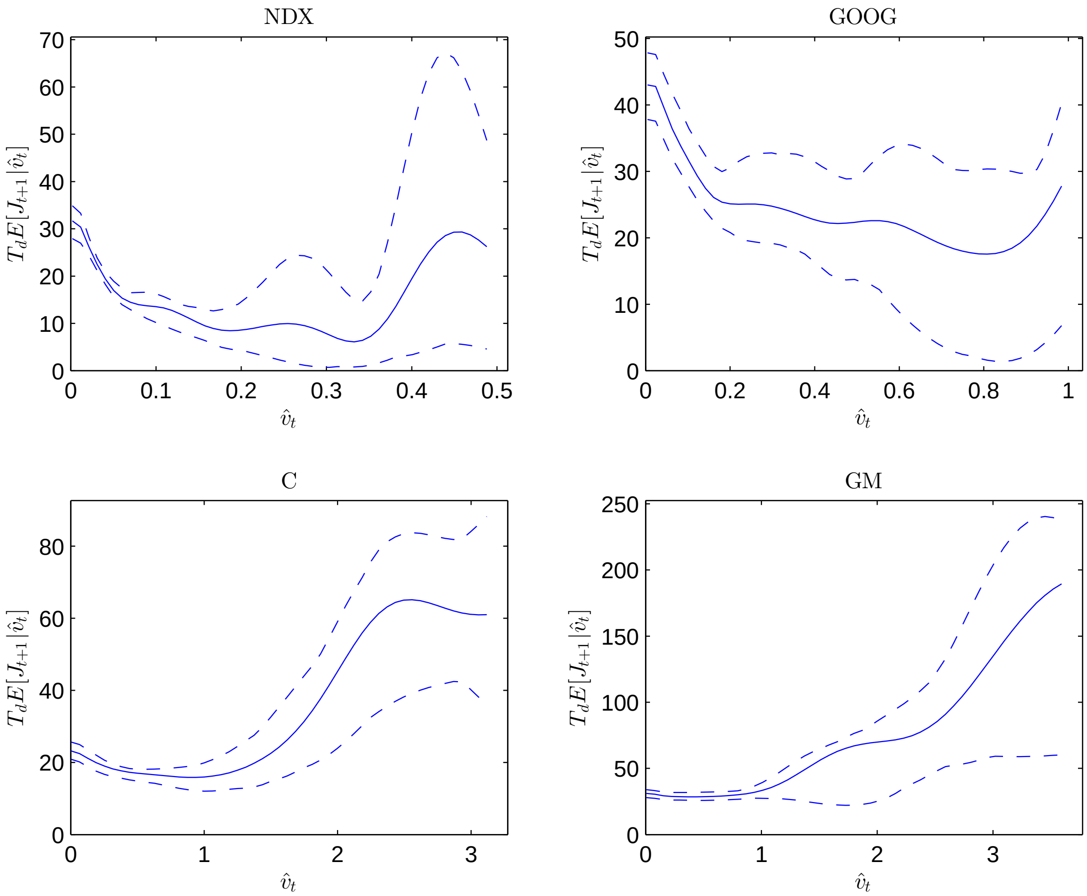

Figure 2. Conditional Jump Intensity and Diffusive Variance ($\mathbf{EMT}_t$)

This figure shows the nonparametric estimation of the annual jump intensity conditional on the diffusive variance, $T_dE[J_{t+1}|\hat{v}_t]$, for a few stocks and the NASDAQ index, where $J_t = 1$ indicates a detected jump at t, zero otherwise, $\hat{v}_t$ is the estimated diffusive variance, and T_d is the number of intraday returns in a year. The solid line is the fitted curve. The dashed lines cover the 90% confidence interval. The diffusive variance is estimated by the exponentially weighted moving average with truncation, EMT_t.