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Evaluation of Voltage Collapse under Open Access Environment

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开放电力市场环境下的电压稳定性评估

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摘要：传统垄断结构下电力输电系统的设计和运行原则之一是运行点不常接近安全边界。然而在新的开放电力市场环境下，电力系统的运行点倾向于越来越接近于安全边界；在这一环境下，如果无功功率支持不足，就可能出现电压失稳。文章提出一种计算静态电压稳定极限点的崩溃点(Point of Collapse)算法。由于崩溃点算法特殊的优越性，该算法得到广泛的关注；然而，其方程的维数约为普通潮流方程的两倍，且不易采用稀疏矩阵技术，故其在大系统中的应用存在困难。针对这一问题提出一种算法，其程序可构筑在标准潮流程序基础上。以新英格兰 39 母线测试系统为例，验证了所提算法的有效性。计算结果表明该算法计算精确且计算高效。

关键词：电压稳定极限；无功功率支持；电力市场

Abstract: Transmission systems in regulated monopoly structure were designed and operated so that conditions in close proximity to security boundary were not frequently encountered. However, in the new open access environment, operating conditions tend to be much closer to security boundaries. Under this environment, voltage instability may result when reactive power support is insufficient. This paper proposes an algorithm for point of collapse method to compute the static voltage stability limit point. The point of collapse method, due to its special advantages, has attracted many attentions. However, as the dimension of its equations is almost twice the ordinary power flow equations and it is not easy to make use of the advantages of sparse matrix technique, it is difficult to apply the point of collapse method to large systems. An algorithm is formulated in this paper to tackle this problem. The program is built by modifying a standard power flow program. The New England 39 bus test system is used as examples to demonstrate the efficiency of the proposed algorithm. The performance of this method is found to be accurate and computational efficient.

Key words: voltage stability limit; reactive power support; power market

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Transmission systems in regulated monopoly structure were designed and operated so that conditions in close proximity to security boundary were not frequently encountered. One reason for this was the load patterns and consequently the flow directions were fairly predictable and not significantly different from that for which they were originally designed. Another reason is that companies could usually justify construction of new facilities that could alleviate operating constraints if they could show reliability would otherwise be compromised. However, in the new open access environment, operating conditions tend to be much closer to security boundaries. This is because transmission use is increasing in sudden and unpredictable directions. Transmission unbundling, coupled with other regulatory requirements, has made new transmission facility construction more difficult. So there is an acute need for R&D work in the new market structure, especially in the areas of voltage security and reactive power support.

With the evolvement of electric power market throughout the world, large scale economic power transmission is more and more prevail and transmission systems are heavily loaded more frequently. Under this environment, voltage instability may result when reactive power support is insufficient.

Point of collapse method[1~3], also known as direct method, is one of the methods (such as continuation power flow method, multiple power flow solution method, etc) to compute static voltage collapse point. The continuation power flow method is a rather reliable method that traces the PV curve of the system until the voltage collapse point is reached; however its major shortcomings include slow calculation speed and large computation burden. Multiple power flow solution can get the voltage collapse point quickly but it needs a suitable low voltage power flow solution in advance, which is quite difficult to obtain[4~5], especially in large systems. Contrast to the above methods, the concise and straightforward point of collapse method can obtain the desired voltage collapse point under specific stress direction by solving the following equations:

$$\begin{cases} \boldsymbol{f}(\boldsymbol{x}, \lambda) = 0 & (1a) \\ \boldsymbol{f}_x(\boldsymbol{x}, \lambda) \cdot \boldsymbol{v} = 0 & (1b) \\ v_k - 1 = 0 & (1c) \end{cases}$$

Equation (1a) represents a set of power flow equations, $\boldsymbol{x}$ is a vector of system state variables, such as bus voltage magnitudes and angles, $\lambda \in R^1$ is a parameter of load factor. Equation (1b) represents the power flow Jacobian matrix $\boldsymbol{f}_x$ is singular and has a zero eigenvector v corresponding to the zero eigenvalue. Equation (1c) is a normalization condition that shows the kth element of $\boldsymbol{v}$, v_k, is equal to 1, i. e., the eigenvector $\boldsymbol{v}$ is not a zero vector. The whole equation characterizes the conditions of the generic static voltage collapse point. If the total bus number is $n+1$, the dimension of equation (1) will be about $4n+1$ which is about twice that of power flow equations, and the size of Jacobian matrix is approximately four times larger than that of the power flow problem. Furthermore, when using the popular Newton method to solve equation (1), its specific form and characteristics of the Jacobian matrix make sparse matrix technique difficult to apply directly.

From the mathematical viewpoint, equation (1) describes the general conditions that its solution is a bifurcation point of one parameter $\lambda \in R^1$. This kind of bifurcation point has many types, for example, saddle node (SN) bifurcation point, transcritical bifurcation point, pitchfork bifurcation point, etc. Among them, SN bifurcation point is a generic one that means it will be encountered most frequently. The other types of one parameter bifurcation point will disappear under generic perturbations and degrade to SN bifurcation point[6]. This paper takes advantages of a generic property of SN bifurcation point to simply the solution procedures, which can reduce the memory requirements and can be implemented easily. This paper is organized as follows: In section 1, the theory and formulation of the algorithm are presented. Implementation issues and their solutions are discussed in section 2. Examples and conclusions are provided in sections 3 and 4, respectively.

1 Problem Formulation

Normally, the load factor λ can be decoupled from the state variables $\boldsymbol{x}$, i. e. equation (1) can be expressed as:

$$\begin{cases} \boldsymbol{f}(\boldsymbol{x}) + \lambda \cdot \boldsymbol{d} = 0 & (2) \\ \boldsymbol{f}_x(\boldsymbol{x}) \cdot \boldsymbol{v} = 0 & (3) \\ v_{2n} - 1 = 0 & (4) \end{cases}$$

where vector $\boldsymbol{d} \in R^{2n}$ is the system stress direction. Without loss of generality, the subscript of vector $\boldsymbol{v}$ in equation (4) is set to $2n$, because from the engineering viewpoint, the probability of v_{2n} equal to zero is negligible.

It is well known that, the static voltage collapse point is generally corresponding to a SN bifurcation point which has the properties of rank $(\boldsymbol{f}_x) = 2n-1$ *and* $\boldsymbol{f}_\lambda \notin$ range $(\boldsymbol{f}_x)$, i. e., $\boldsymbol{d} \notin$ range $(\boldsymbol{f}_x)$, which means $\boldsymbol{f}_\lambda$ *or* $\boldsymbol{d}$ is not belonging to the space spanned by the columns of $\boldsymbol{f}_x$[6], where $\boldsymbol{f}_\lambda$ is the partial derivative of $\boldsymbol{f}(\boldsymbol{x})$ with respect to λ. This means that at the voltage collapse point the rank of $\boldsymbol{f}_x$ is $2n-1$, and some columns of $\boldsymbol{f}_x$ are linear correlative. Note that $\boldsymbol{f}_\lambda$ cannot be expressed as a linear combination of column vectors of $\boldsymbol{f}_x$, and the augmented matrix $[\boldsymbol{f}_x \mid \boldsymbol{f}_\lambda]$ has the rank of $2n$. Further investigations of equation (4) show that the last column of $\boldsymbol{f}_x$ can be expressed as a linear combination of the rest columns of $\boldsymbol{f}_x$. After removing this column from $\boldsymbol{f}_x$, the remaining matrix still has the rank of $2n-1$. Synthesizing the above observations, it can be found that at the voltage collapse point, the square matrix resulting from substituting the last column of $\boldsymbol{f}_x$ by vector $\boldsymbol{f}_\lambda$ or $\boldsymbol{d}$ is full rank. We can use this property to simplify the solution procedures of equations (2) ~ (4) as follows:

To solve equations (2) ~ (4) by Newton

method, the following linear equations must be solved,

$$\begin{bmatrix} f_x & d & 0 \\ f_{xx}v & 0 & f_x \\ 0 & 0 & 0...01 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \\ \Delta v \end{bmatrix} = \begin{bmatrix} -f-\lambda\cdot d \\ -f_x\cdot v \\ 0 \end{bmatrix} \tag{5}$$

where, f_{xx} is Hessian matrix of vector $f(x)$. Equation (5) can result the following two expressions:

$$f_x\Delta x + d\Delta\lambda = -f-\lambda\cdot d \tag{6}$$

$$f_x\Delta v + f_{xx}v\cdot\Delta x = -f_x v \tag{7}$$

where $\Delta v_{2n}=0$ is known. We can firstly solve equation (6) and then use the results to solve equation (7). However in equation (6), the numbers of unknown variables are more than the equation numbers by one. In order to use the conditions $v_{2n}=1$ and $\Delta v_{2n}=0$, we assume Δx_{2n} is known, and replace the last column of f_x with a suitable vector. Reformulating equations (6) and (7) as:

$$f_x\ (\Delta x - \Delta x_{2n}v + d\Delta\lambda = -f-\lambda\cdot d - \Delta x_{2n}f_x v \tag{8}$$

$$f_x\Delta v + d\Delta\lambda = -f_x v - f_{xx}v\cdot\Delta x + d\Delta\lambda \tag{9}$$

Obviously the last element of the vector $(\Delta x - \Delta x_{2n}v)$ in equation (8) is zero, so we can replace this element by $\Delta\lambda$ and replace the last column of f_x by the vector d. Introducing the following notations:

$$A = [f_{x(1)},\ f_x\ (2),\ \cdots,\ f_{x(2n-1)} | d]$$

$$\Delta a = \begin{bmatrix} \Delta x_1 - \Delta x_{2n}v_1 \\ \vdots \\ \Delta x_{2n-1} - \Delta x_{2n}v_{2n-1} \\ \Delta\lambda \end{bmatrix}$$

$$\Delta b = \begin{bmatrix} \Delta v_1 \\ \vdots \\ \Delta v_{2n-1} \\ \Delta\lambda \end{bmatrix}$$

where $f_{x(1)}$ is the first column of matrix f_x, and $f_{x(2n-1)}$ is the $(2n-1)$ th column of the matrix f_x. Equations (8) and (9) can be written as:

$$A\cdot\Delta a = -f-\lambda\cdot d - \Delta x_{2n}f_x v \tag{10}$$

$$A\cdot\Delta b = -f_x v - f_{xx}v\cdot\Delta x + d\Delta\lambda \tag{11}$$

The left hand sides of equations (10) and (11) have the same matrix A. Introducing two additional unknown vectors c^1 and c^2, equation (10) can be split into the following two equations:

$$A\cdot c^{-1} = -f-\lambda\cdot d \tag{12}$$

$$A\cdot c^2 = -f_x v \tag{13}$$

Upon solving equations (12) and (13), the vector Δa can be expressed by

$$\Delta a = c^1 + \Delta x_{2n}c^2$$

That means

$$\Delta x_i = c_i^1 + \Delta x_{2n}\ (c_i^2 + v_i) \qquad i=1,\ \cdots,\ 2n-1 \tag{14}$$

$$\Delta\lambda = c_{2n}^1 + \Delta x_{2n}c_{2n}^2 \tag{15}$$

Substituting expressions (14) and (15) into equation (11), rearranging it into two vectors relative and irrelative to Δx_{2n} and introducing two unknown vectors c^3 and c^4, equation (13) can be split into the following two equations:

$$A\cdot c^3 = -f_x v - f_{xx}v \begin{bmatrix} c_1^1 \\ \vdots \\ c_{2n-1}^1 \\ 0 \end{bmatrix} + c_{2n}^1 d \tag{16}$$

$$A\cdot c^4 = -f_{xx}v \begin{bmatrix} c_1^2 + v_1 \\ \vdots \\ c_{2n-1}^2 + v_{2n-1} \\ 0 \end{bmatrix} + c_{2n}^2 d \tag{17}$$

Similarly, after solving equations (16) and (17), the vector Δb can be expressed as:

$$\Delta b = c^3 + \Delta x_{2n}c^4$$

which means,

$$\Delta v_i = c_i^3 + \Delta x_{2n}c_i^4 \qquad i=1,\ \cdots,\ 2n-1 \tag{18}$$

$$\Delta\lambda = c_{2n}^3 + \Delta x_{2n}c_{2n}^4 \tag{19}$$

It can be proved that equations (15) and (19) can be solved without numeric problem for SN bifurcation point[6]. As a result Δx_{2n} and $\Delta\lambda$ can be obtained. Δx_i and Δv_i $(i=1,\ldots,\ 2n-1)$ can then be calculated using equations (14) and (18).

At this stage all the corrective variables of Newton iteration, i. e., Δx, Δv, $\Delta\lambda$, are known. All variables are then updated in the usual way. The above iterative process is repeated until the solution converges.

From above, we can see the features of this algorithm. Firstly, it only needs to solve four equations (12), (13), (16) and (17) which have the same matrix A on the left hand side, and the only difference between them is the vectors on the right hand side. Hence matrix LU decomposition process only needs to be performed once. Secondly, the structure of the matrix A is the same as the load flow Jacobian f_x except the last column, both are of $2n\times 2n$ dimensions, so the sparse matrix technique can be applied to matrix $\boldsymbol{A}$ easily. Thirdly, the main computation burdens involve forming matrix $\boldsymbol{A}$ and its LU decomposition, calculating $\boldsymbol{f}_{xx}\boldsymbol{v}$, and some minor vector manipulations.

2 Implementation Issues

2.1 Initial value

The point of collapse method, similar to the Newton method applying to nonlinear equations, needs appropriate initial values. Normally, we can initiate system state variables $\boldsymbol{x}$ as the flat start method in Newton power flow calculation, or use converged power flow result as the initial values. The scalar variable λ can be simply initiated as zero. The initial value of vector $\boldsymbol{v}$ is a bit difficult to choose. The simplest method is to initiate the value of all its elements as one. A more efficient means is to initiate it as the eigenvector of the power flow Jacobian matrix corresponding to the minimal eigenvalue by using inverse power method[8] at the current operating point, i. e., the operating condition $\lambda = 0$. An alternative choice is to initiate $\boldsymbol{v}$ as a parallel vector of $\boldsymbol{f}_x^{-1}\boldsymbol{d}$ calculated at the current operating point, which can be interpreted as the tangent line of the PV curve at the current operating point. The above two initial values of $\boldsymbol{v}$ will normally converge to the actual value of $\boldsymbol{v}$ as the current operating point is approaching the voltage collapse point.

2.2 Critical subscript selection and data structure

The subscript in equation (4) is set to $2n$ from an engineering viewpoint. To be more practical, we can choose this subscript as $2m$ where m is the weakest PQ load bus for voltage collapse. Reference [7] shows that under this condition, the magnitude of v_{2m} is the largest among all the elements of $\boldsymbol{v}$, which means the $2m$th column is strongly relevant to the rest columns of $\boldsymbol{f}_x$. However, we do not know which one is the weakest bus for voltage collapse in the current operating conditions. As a result, we use the heaviest loaded bus or the lowest voltage load bus which should also be a member of the increased loading bus set.

This method avoids the arbitrariness in choosing the special subscript. However it may produce many fill-ins when performing LU decomposition of the resulted matrix $\boldsymbol{A}$ and hamper the applicability of sparse technique. To overcome this problem, we can adopt some techniques of data structure. More precisely, we move the $2m$th row and the $2m$th column of matrix $\boldsymbol{A}$ to the last row and the last column respectively, and rearrange the relevant entries of the equations correspondingly. As a result, the ultimate form is similar to power flow Jacobian matrix with exceptions in deleting one row and one column, and at the same time one row and one column are added which will be stored in two extra vectors. By doing so, we pertain the well known 2×2 basic element structure of power flow Jacobian matrix, almost without introducing any extra fill-ins.

3 Example

The New England 39-bus test system depicted in Fig. 1 is used to demonstrate the validity and effectiveness of the proposed algorithm. For simplicity, the limits of reactive power generation are not considered in these case studies. The algorithm can calculate the point of collapses of the above two systems within 10 iterations when the initial value selection methods described in section 2.1 are used.

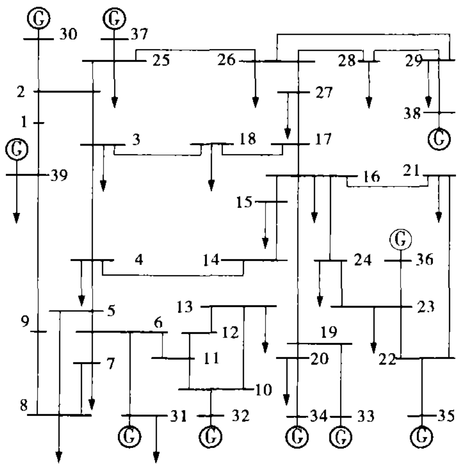

Fig.1 New England 39-bus test system

Fig.2 shows two PV curves of bus 8 in the New England system under two different system stress modes where λ is load increasing ratio. These PV curves are obtained by the continuation power flow method. The collapse points calculated by the continuation method are served as a comparison to the result of our proposed algorithm. The results from the two methods are very close, and they are within the convergence tolerance. The stress mode of the dashed curve (called mode 1) is that the loadings of all pure load buses (i.e., buses only have loads but without generators) are increased proportional to its original real power with power factor of increased power kept at 0.9. Another stress mode (mode 2) is that only the loadings of buses 4, 8, 20 are increased with the ratio of 1:2:3 and the original power factor remains unchanged. From Fig.2, we can see that the relative voltage stability margin ($VSM_{relative}$) of mode 1 when expressed in percentage is 29 %, which is calculated as:

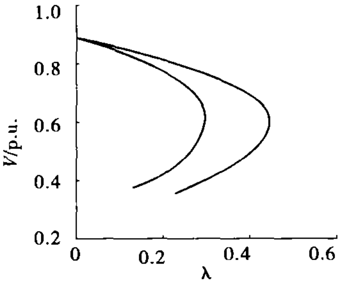

Fig.2 PV curves of bus 8

$$VSM_{relative}(\%) = (P_{mas} - P_0)/P_0 \times 100(\%) \qquad (20)$$

where P_{max} is total loadings of the loads at the voltage collapse point. The relative voltage stability margin of mode 2 is 43 %.

For mode 1, the real and reactive loads increase by 1 421 MW and 663 MVar respectively from the current operating point to the voltage collapse point, while the system generations increase by 1 494 MW and 4 366 MVar. Compared with mode 2 which has 1 424 MW/351 MVar increased loads and 1 503 MW/4 095 MVar increased generations, we can see that the voltage stability margin of mode 1 when expressed in MW is similar to that of mode 2, but the relative voltage stability margin is quite different. Another observation is that reactive power losses are much larger than real power losses, because voltage stability and reactive power have close correlation.

4 Conclusions

Investigations in the example show the effectiveness and efficiency of the proposed algorithm. Using a set of appropriate initial values, the algorithm can calculate the voltage collapse point reliably and computational efficiently. Compared with the standard point of collapse method, the proposed algorithm requires less memory and can make use of sparse matrix technique easily. Its program code can be obtained with proper modifications from a conventional power flow program. All these features make it a promising algorithm for voltage collapse point calculation for large systems.

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