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Investigation of Minimal Reactive Power Support of Generators

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发电机最小无功支持服务的研究

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摘要：随着竞争电力市场的出现，无功辅助服务或无功电力市场吸引了研究人员和系统运行人员越来越多的关注。无功是支持电力系统安全可靠运行的一个重要服务，不恰当的无功管理将损害其它电力市场的运行效率。最近的研究显示发电机无功出力具有无功负荷供给、系统安全保障和发电机有功输电支持等作用。因此很自然地，电力市场应给予用来支持发电机有功输电的无功出力以经济补偿，据此，这一无功出力分量可认为是发电机的最小无功出力。文章提出一种基于最优潮流模型的定量研究发电机无功出力基本分量的方法，以5母线系统算例结果阐释了相关概念，并讨论了影响发电机最小无功出力的运行约束。研究结果表明最小发电机无功出力分量确实存在，这证实了所提模型的合理性。

关键词：无功支持；无功辅助服务；输电开放

Abstract: With the emergence of competitive electricity power markets, reactive power ancillary services or reactive power markets have attracted more and more attention from researchers and system operators all over the world. Reactive power is an important system support service for the secure and reliable operation of power systems. Improper management of reactive power can also hinder the operational efficiency of other power markets. It has recently been recognized that the reactive power of a generator has several roles, namely, supplying reactive demand, maintaining system security and supporting its real power transmission. It is rational that the minimal reactive power used to support its real power transmission should not receive financial compensation in power markets. Hence this component of reactive power can be regarded as the minimal reactive power support of a generator. An optimal power flow (OPF) based method is proposed to study quantitatively basic components of a generator's reactive power. A simple 5-bus system is used to illustrate the proposed concept. The test results verify the rationality of the model and demonstrate the existence of minimal reactive power support of generators. The operational constraints that influence the generator's minimal reactive power support are also discussed.

Key words: reactive power support; reactive power ancillary service; transmission open access

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0 Introduction

The traditional regulated monopoly structure of electric utilities throughout the world is eroding and competitive wholesale energy markets are now operating in different parts of the world. In the market, buyers and sellers of power can enter into a contract transaction at their own discretion and the system operator will be responsible for dispatching these transactions while maintaining system security. Reactive power support from generators is regarded as an important ancillary service. The aims of this service are to maintain open access transmission, support system security, supply reactive demand, and control system voltage[1]. A system operator procures reactive power support services and pays the reactive power suppliers a certain monetary amount based on the marginal reactive power price or the allocated reactive power cost, or using some other rules[2~8]. In nowadays market practice, the amount of a generator's reactive power that can be traded is usually considered as the generator's actual reactive power output, or the reactive power output be-

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yond certain mandatory operational ranges[9].

After further speculating on the roles of reactive power support from generators, references [10] and [11] proposed that the reactive power support of a generator has two components: the component that helps to ship real power and the component that improves the reliability of the system. It was suggested that only the second part of the reactive power output should be compensated. Reference [12] demonstrated the concept of minimizing reactive power support by two simple test systems, but encountered difficulties when applying the method to complex systems. By using the concept and method from static voltage stability theory, reference [13] proposed a method suitable for application to large-scale power systems. The least reactive power support needed from a generator was evaluated as the least amount of reactive power needed from this generator to maintain the same degree of system security or margin. The reactive demands of load are included in the studies in references [11~13]. Similar treatment is also found in reference [14] where all components of the reactive power flowing in a transmission branch are treated as a unity and is allocated to generators according to certain rules.

This paper proposes a new method to analyze the minimal reactive power support of a generator that is used to support its real power transmission. The reactive power of a generator that is used to supply reactive demand is studied specifically. The organization of this paper is as follows: Some background knowledge is presented in Section 1, followed by a discussion of a mathematical model to this problem. All related aspects of the model are discussed. A brief introduction to the primal-dual interior point method, which is used to solve the model, is also provided. The proposed model is tested on a simple 5-bus system. The numerical results in Section 2 reveal the impact of voltage magnitude limits on the minimal reactive supports of generators and the relationship between the generators' minimal reactive power support and load level/pattern. It is found that the reactive demand of loads should be excluded from the proposed model. Some important observations on the minimal reactive support of generators are also made. Section 3 concludes the paper.

1 Mathematical model and solution method

1.1 Some background knowledge

In order to quantitatively analyze the minimal reactive power support of generators, the following issues about power markets should be clarified:

(1) The reactive power of a generator has several roles, namely, supporting its real power transmission, supplying reactive demand of load and supporting the system. From the principle of free market, these reactive power components should be equitably compensated by corresponding entities. In other words, the generators themselves provide reactive power to support their own real power transmission, the loads pay for their reactive power consumptions and all market participants share the cost of system supporting reactive power. Therefore, when we analyze the problem of minimal reactive power support of generator, the influence of the reactive power that is used to supply reactive demands should be eliminated. To achieve this goal, the reactive demand of the load is set to zero. The case studies in section 2 verify the rationality of this treatment in our model.

(2) It is rational that generators providing security support should receive monetary compensation in power markets. However there is still no generally accepted consensus on whether a system should have the same level of security before and after a generator is connected to the system and generating real power. If the reactive power supporting system security is included in the study of minimal reactive power support, a minimal level of system security should be provided, which all generators should be made to respect. Moreover, system security depends largely on the power flow pattern. With respect to voltage stability, in particular, further information about the power changing mode before carrying out the analysis is needed. The issue is complex in the power market and difficult to assess. This adds the difficulties to take into account the reactive power of system support when analyzing minimal reactive power support of a generator. Therefore, it is proper to exclude this part of reactive power from the minimal reactive power support.

(3) The concepts of open transmission access and transmission support service must be clarified. It is not necessary for a generator to provide all of the reactive demand incurred by its real power transmission from the generator terminal to the load bus. Otherwise, the minimal reactive power support will be very large for remote generators and this also will contradict with the fact that the long-distance transmission of reactive power is impractical in the electrical power industry. In fact, only a small amount of reactive output from a generator is needed to satisfy the reactive demand induced from the generator terminal to the boundary (i. e. the intersecting surface between the market and the generators) of the pool market and to respect basic system operation constraints, such as high and low voltage magnitude constraints. The remaining reactive power demand is supplied by transmission support services that are spread throughout the whole transmission system and will subsequently be allocated to each generator. This is the practice of power markets nowadays. Only by doing so can open access transmission and sufficient competition be realized in generation side. With the above reasoning, the problem of minimal reactive power support is trivial and can be easily solved if we are clear about the boundaries of the market. But when the boundary is obscure, there has been no obvious approach to tackling this situation. One approach is proposed in this paper.

From the above discussions, we can see that there are some subtle obstacles to determining the minimal reactive power support of generators and assessing different components of reactive power. This paper focuses on the component that is used to support a generator' s real power transmission. Our proposed methodology and some of its explanations are presentee in the next section.

1.2 Mathematical model

The basic idea of the model is to minimize the total reactive power generations subjected to the equality constraints of the power flow equation and the inequality constraints of basic system operating constraints, including voltage magnitude limits. This model can be expressed mathematically as follows:

$$\min \sum_{i \in S_G} \left| Q_{gi} \right| \qquad (1)$$

$$\text{s. t.}: P_{gi} - P_{1i} - V_i \sum_j V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} = 0 \qquad i \in S_B \qquad (2)$$

$$Q_{gi} - Q_{1i} - V_i \sum_j V_j (G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij} = 0 \qquad i \in S_B \qquad (3)$$

$$\underline{Q}_{gi} < Q_{gi} < \overline{Q}_{gi} \qquad i \in S_G \qquad (4)$$

$$\underline{V}_i < V_i < \overline{V}_i \qquad i \in S_B \qquad (5)$$

where S_B is the set of all nodes and S_G is the set of all generators; P_{gi} and Q_{gi} are the real power output and reactive power output of the generator at bus i respectively; P_{1i} and Q_{1i} are the real power demand and reactive power demand of the load at bus i respectively; V_i is the voltage magnitude of bus i; $\underline{V}_i$ and $\overline{V}_i$ are the lower and upper voltage magnitude limits of bus i respectively; δ_{ij} is the difference in the voltage angles of bus i and bus j; $G_{ij} + \mathrm{j}\, B_{ij}$ is the admittance between bus i and bus j. Equations (2) and (3) are power flow equations. Inequalities (4) and (5) represent the constraints of generator reactive outputs and bus voltage magnitudes, respectively. At a given energy contracted setting dispatched by the system operator, all real power outputs of the generators are fixed except for one generator, which is chosen as the slack generator to make good transmission losses. As a result, the real power flow pattern is obtained and, thus, we can study the reactive power of a generator that is used to support its real power transmission. The control variables are the reactive output of the generator. Solving this model, the minimal reactive power support of the generators can be assessed.

The objective function of the model is the summation of the absolute value of all reactive power produced or absorbed. The voltage magnitude limits are regarded as compulsory constraints. The limits in the model may be looser than the ordinary voltage limit, which may be more stringent so as to embody static security constraints to some extent. Line current limits were not considered because the system operator took the limits into account when dispatching the real power energy transactions.

1.3 Solution method

The optimization problem above can be regarded as a nonlinear programming problem and can be written in the following well-known form:

$$\min_{x} f(\boldsymbol{x}) \tag{6}$$

$$\text{s. t.}: \quad h(\boldsymbol{x}) = 0 \tag{7}$$

$$g < g(\boldsymbol{x}) < \overline{g}) \tag{8}$$

where $\boldsymbol{x}$ is a vector of variables. We use the Predictor-Corrector Primal-Dual Interior Point Method (PCPDIPM)[15~16] to solve the nonlinear programming problem.

2 Numerical experiment

2.1 A simple sample system

A sample 5-bus test system depicted in Fig. 1, which was also used in references [10~13], is used in this paper to evaluate the minimum reactive power support of generators and to gain some insights into the characteristics of the proposed model. All parameters and values are in per unit. The voltages of generators 1 to 3 are all set at 1.03 p. u. in the base state. Generator 4, as the reference generator, is located far away from the load at bus 5. The impedances of the lines are shown in Fig. 1. The load at bus 5 is 3.0+j1.5 p. u. and the real demand is shared equally by generators 1 to 3.

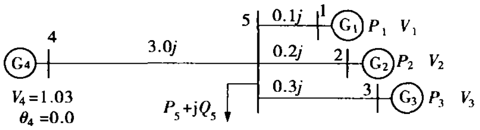


Fig. 1 A simple 5-bus sample system

Tab. 1 lists the results of the case study. Case Base-1 is the results of power flow. Cases A1, B1, and C1 are the results when the upper voltage magnitude limits of generators 1 to 3 are all set at 1.03 p. u., 1.04 p. u., and 1.07 p. u., respectively, and the load remains unchanged. Q_{total} is the sum of the local generators' reactive power outputs Q_1, Q_2, and Q_3.

Tab. 1 Case study with reactive load

p. u.

Case	Base-1	A1	B1	C1
V_1	1.030 0	1.030 0	1.021 9	1.016 9
V_2	1.030 0	1.030 0	1.039 0	1.034 0
V_3	1.030 0	1.030 0	1.0400	1.061 9
Q_1	1.139 2	1.139 2	1.048 8	0.992 4
Q_2	0.649 5	0.649 5	0.699 9	0.671 7
Q_3	0.523 8	0.523 8	0.560 4	0.642 8
Q_{total}	2.312 5	2.312 5	2.309 1	2.306 9

From Tab. 1, it can be observed that the nearest local generator 1 produces most of the reactive power, while the most remote local generator 3 produces the least reactive power. Moreover, the results of case A1 are the same as those of case Base-1. The voltage magnitude at bus 5 and the reactive output of generator 4 are the same in all four cases, at 0.924 5 p. u. and 0.036 2 p. u., respectively. When the upper voltage limits of cases A1, B1, and C1 are relaxed, the corresponding values of Q_{total} also become smaller. The reason for this is that the feasibility domain of the optimization problem is getting larger.

Because the reactive power of the load is included in the above cases, the reactive power outputs of the generators are used not only to support their real power transmission but also to satisfy the reactive load demand. To exclude the components of the reactive load, the reactive power of the load is set to zero. The other conditions of cases Base-2, A2, B2, and C2 are the same as their counterparts in the first group. The results show again that the voltage magnitude at bus 5 and the reactive output of generator 4 are the same in all of these cases, at 1.014 6 p. u. and 0.005 3 p. u., respectively. Similarly, the results of case A2 are the same as those of case Base-2, as shown in Tab. 2. When the upper voltage limits are relaxed, the values of Q_{total} decrease correspondingly for the same reason mentioned above.

Tab. 2 Case study without reactive load

p. u.

Case	Base-2	A2	B2	C2
V_1	1.030 0	1.030 0	1.022 9	1.019 1
V_2	1.030 0	1.030 0	1.037 1	1.033 3
V_3	1.030 0	1.030 0	1.040 0	1.056 6
Q_1	0.206 3	0.206 3	0.133 4	0.094 3
Q_2	0.175 7	0.175 7	0.212 4	0.192 9
Q_3	0.199 4	0.199 4	0.233 1	0.290 5
Q_{total}	0.584 1	0.584 1	0.578 9	0.577 7

An interesting point to note is the reactive outputs of generators 1 to 3. In case C2, where the terminal voltage constraints of the local generators are not violated, the results are consistent with our intuition; that is, generator 1 produces the least reactive power while generator 3 produces the most. However, when the upper voltage limits of these three generators are set lower, the reactive output of generator 3 decreases and the reactive output of generator 1 increases progressively while the reactive output of generator 2 first increases and then decreases. This pattern is also observed in the previous group. It should be noted that the remote generator 3 produces less reactive power than the local generator 1 in case A2 where the voltage limits are stringent.

The following are discussions of the above studies.

(1) The results of Tab. 1 are obscure because the generators have to generate more reactive power to satisfy the load demand. This component will conceal the reactive power component that is used to support the real power transmission of the generators. From the evidence of the above studies, the reactive components of the loads should be set to zero when evaluating the minimal reactive power support service needed to support the transportation of real power.

(2) The upper voltage limits of the buses can influence the minimal reactive outputs of generators. When these limits are ignored, the results are consistent with our intuition that remote generators produce more reactive power than local generators to support their real power transmission. However, when the voltage limits are considered, remote generators will need to reduce their reactive outputs, while local generators will be mandated to produce more.

(3) From the above two groups of studies, the following characteristic of reactive power supply can be found: (a) The reactive load is mainly supplied by the nearest local generator. Comparing case C1 with case C2, the reactive output of generator 1 increases by 0.898 1 p.u. while generator 3 only increases by 0.352 3 p.u.; (b) The corresponding increase of Q_{total} is 1.729 2 p.u. The figure includes the reactive load component and the reactive loss component.

2.2 Studies on systems with bilateral transactions

In order to gain further insights into the characteristics of the proposed model, we study the variations in the reactive power support of generators under different conditions when the amount of a bilateral transaction in the system is changed. The results with and without reactive demand are compared to show that the model possesses promising properties that can equitably assess the reactive power support of each generator.

Assume that generator 1 and an additional demand at bus 5 form a bilateral transaction. With the reactive demand of the loads taken as zero, Fig. 2 gives the results. Obviously, when the amount of bilateral trading increases, the change in the reactive power supports needed from generators 2 and 3 is small and the reactive power support needed from generator 1 increases nearly linearly. This is the feature that we expect the model to have. As for the system voltages, they all drop except the voltage of generator 1. It should be noted that each point along the curves stands for an optimization solution of the model (1).

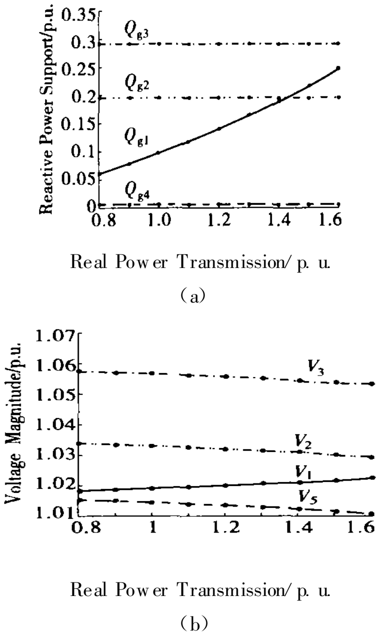
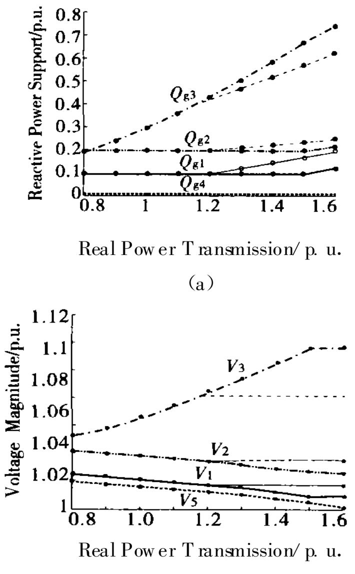


Fig. 2 Relationship between real power transmission of generator 1 and (a) reactive power support of generators (b) system voltage magnitudes when reactive demand is zero

Fig. 3 gives the results when generator 3 and an additional demand at bus 5 form a bilateral transaction. There are two sets of curves in the figure. The thicker curves have an upper voltage limit of 1.10 p.u. while the thinner curves have an upper voltage limit of 1.07 p.u. Each pair of curves bifurcates when the bilateral transaction amount is 1.2 p.u. This can be seen clearly in Fig. 3 (b). From Fig. 3 (a), we can see that the reactive power supports of generators 1 and 2 remain nearly constant before the voltage limit of generator 3 is reached. Beyond that point, the reactive power support of all generators will increase.

The situation where the reactive demand of the load is reserved and the reactive power of the bilateral transaction is changed proportionally to its real power is

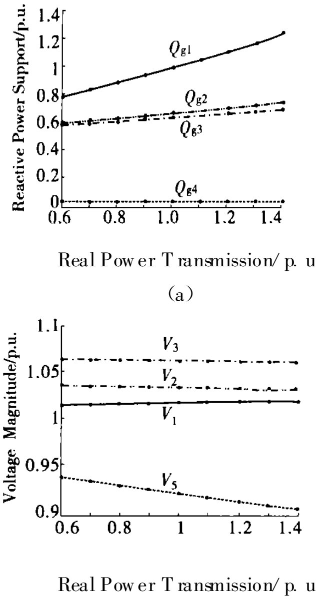


(a)

(b)

Fig. 3　Relationship between the real power transmission of generator 3 and (a) reactive power support of generators, (b) system voltage magnitudes when reactive demand is zero

studied in Fig. 4 and Fig. 5. In Fig. 4, generator 1 and an additional demand at bus 5 form a bilateral transaction. Obviously, as the amount of trading increases, all the reactive power supports of the generators will increase, while the increase is most significant for generator 1. The reason for this is that the reactive power support in this situation contains certain reactive load components, not solely the reactive component that is used to support the transmission of real power.

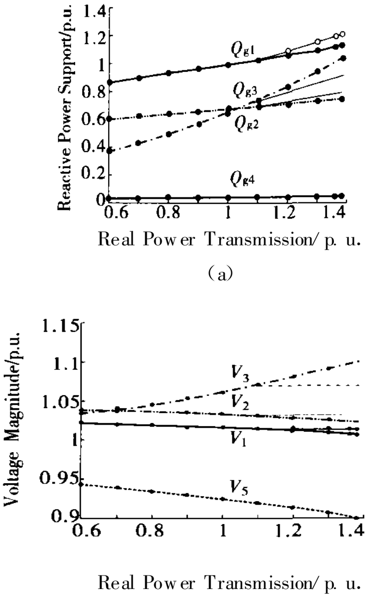


(a)

(b)

Fig. 4　Relationship between real power transmission of generator 1 and (a) reactive power supports of generators, (b) system voltage magnitudes when reactive demand is reserved

Fig. 5 illustrates the situation when generator 3 and an additional demand at bus 5 form a bilateral transaction. There are two sets of curves. The thinner curves have an upper voltage limit of 1.07 p.u. The corresponding figure for the thicker curve is 1.10 p.u. As can be seen from Fig. 5 (b), each pair of curves bifurcates when the bilateral transaction amount is 1.1 p.u. Fig. 5 (a) shows that as the amount of trading increases, in addition to generator 3, the reactive power support of generator 1 also increases noticeably. As trading increases, the reactive power support of generator 1 increases even more remarkably after the voltage limit of generator 3 is reached. It should be mentioned here that the power flow will diverge beyond a bilateral trading point of 1.38 p.u., i.e., the system will undergo a voltage collapse beyond that point.



(a)

(b)

Fig. 5　Relationship between real power transmission of generator 3 and (a) reactive power supports of generators (b) system voltage magnitudes when reactive demand is reserved

The above studies clearly show that the reactive demand of loads should be removed when evaluating the minimal reactive power component of a generator that is used to support its real power transportation. This is done by setting reactive demand to zero. It can also be seen that there are some rational factors within the proposed model. This model possesses some promising properties that we would wish to see in the model of minimal reactive support; i. e., when one generator increases its real power output, only its reactive power needs to increase while the other generators hold their reactive power outputs constant. Moreover, the increase in output of reactive power has a fairly linear relationship with the increase in the output of real power. The study also shows that the minimal reactive power support of generators is affected by operational constraints.

3 Conclusions

This paper presents a method for assessing the minimal reactive power support of generators. Theoretical analysis and explanations are given. The numerical experiments of the proposed model to a sample system yield some interesting observations. First of all, generator does have a minimal reactive power component that is used to support its real power transmission. This part of reactive power is regarded as the minimal reactive power support of the generator and should not be paid in the reactive power market. Second, the minimal reactive power support of a generator that is used to support its real power transmission should not include the reactive demand of the load. Third, the proposed OPF-based model has promising characteristics that can equitably assess the minimal reactive power support of each generator. The equitable pricing of reactive power ancillary services in competitive electricity markets is a complex issue. By identifying quantitatively the components of the reactive power of a generator, we believe that the approach proposed in this paper goes some way towards achieving this goal.

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