

A Novel Adaptive Fuzzy Controller for Application in Autonomous Vehicles

Dai Xiaohui*, C.K. Li*, and A.B. Rad**

(Department of Electronic and Information Engineering,

**Department of Electric Engineering,

The Hong Kong Polytechnic University, Hong Kong SAR, CHINA)

Abstract: Adaptive control for a class of nonlinear systems is discussed in this paper. We use fuzzy systems to approximate the ideal optimal controller by adjusting the parameters of fuzzy systems. In order to tune these parameters, linear relationship between approximation error and parameters is established first. Then we design the adaptive laws of these parameters based on Lyapunov synthesis approach. The advantage of our method is that we can tune not only the parameters of the consequences of fuzzy rules, but also the parameters of the membership functions. As a result, a stable and more flexible controller is achieved. The performance of the adaptive scheme is demonstrated through the longitudinal vehicle control.

Keywords: Fuzzy logic system, adaptive fuzzy, fuzzy approximation, Lyapunov synthesis approach, vehicle longitudinal controller.

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1 INTRODUCTION

Most of the current research on adaptive fuzzy control only tunes the parameters of the consequences of fuzzy rules. This may cause the approximation property of fuzzy systems not to be good, and affect the performance of the controller. Aiming at this problem, we hope to tune all parameters of fuzzy rules. In order to tune these parameters, linear relationship between approximation error and all parameters of fuzzy rules is established first. Then we design the adaptive laws of these parameters based on Lyapunov synthesis approach. The advantage of our method is that we can tune not only the parameters of the consequences of fuzzy rules, but also the parameters of the membership functions. As a result, a stable and more flexible controller is achieved.

2 DESCRIPTION OF FUZZY LOGIC SYSTEMS

Before the fuzzy adaptive controller is proposed, we discuss the structure and the approximation error of fuzzy logic systems we adopted.

Structure of Fuzzy Logic Systems

Consider a multiple-input single-output (MISO) fuzzy controller which performs a mapping from an state vector $\underline{x}=(x_1, x_2, \dots, x_n)^T \in \mathfrak{R}^n$ to a control input $u \in \mathfrak{R}$. Using the Takagi-Sugeno model, the IF-THEN rules of the fuzzy

controller may be expressed as:

$$R_l: \text{IF } x_1 \text{ is } F_1^l \text{ and } \dots \text{ and } x_n \text{ is } F_n^l \\ \text{THEN } u = K_1^l g_1(\underline{x}) + K_2^l g_2(\underline{x}) + \dots + K_m^l g_m(\underline{x}) \quad (1)$$

where F_i^l is the label of the fuzzy set in x_i , for $l=1, 2, \dots, M$. $g_1(x), g_2(x), \dots$, and $g_m(x)$ are any known function of the state vector. K_1^l, K_2^l, \dots and K_m^l are the constant coefficients of the consequent part of the fuzzy rule.

In this paper, we would use product inference for the fuzzy implication and t norm, singleton fuzzifier and centre average defuzzifier, consequently, the final output value is:

$$u(\underline{x}) = \frac{\sum_{l=1}^M \left(\prod_{i=1}^n \mu^{F_i^l}(x_i) \right) \cdot (K_1^l g_1(\underline{x}) + \dots + K_m^l g_m(\underline{x}))}{\sum_{l=1}^M \left(\prod_{i=1}^n \mu^{F_i^l}(x_i) \right)} \quad (2)$$

Here, we adopt Gaussian function as the membership function of the fuzzy system because its excellent approximation properties [1], i.e.,

$$\mu^{F_i^l}(x_i) = \exp\left[-\left(\frac{x_i - c_i^l}{\sigma_i^l}\right)^2\right] \quad (3)$$

for $i=1,2,\dots,n$ and $l=1,2,\dots,M$.

And we can rewrite the equation (2) as:

$$u(\underline{x}) = \theta^T \xi(\underline{x}) = \theta^T \xi(\underline{x} | c, \sigma) \quad (4)$$

where $\theta=(K_1^1, \dots, K_m^1, K_1^2, \dots, K_m^2, \dots, K_m^M)^T$ is a parameter vector, c, σ are vectors with the elements of c_i^l and σ_i^l in equation

(5) respectively, and $\xi(x)=(\xi_1^1(x), \dots, \xi_m^1(x), \xi_1^2(x), \dots, \xi_m^2(x), \dots, \xi_m^M(x))^T$ is a regressive vector with the regressor $\xi_j^l(x)$ defined as

$$\xi_j^l(x) = \frac{\left(\prod_{i=1}^n \mu^{F_i^l}(x_i) \right) \cdot g_j(x)}{\sum_{l=1}^M \left(\prod_{i=1}^n \mu^{F_i^l}(x_i) \right)} \quad (5)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ -k_n & -k_{n-1} & \dots & \dots & \dots & \dots & -k_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ b \end{bmatrix} \quad (9)$$

A. Bounding Control

Define a function:

$$V_{bd} = \frac{1}{2} e^T P e \quad (10)$$

And because all roots of the polynomial $h(s) = s^n + k_n s^{n-1} + \dots + k_1$ are in the open left half plane ($k = (k_n, \dots, k_1)^T$ is user defined, which has been mentioned earlier), we can find which is a symmetric positive definite matrix satisfying the Lyapunov equation

$$A^T P + P A = -Q \quad (11)$$

where $Q > 0$. Differentiate the V_{bd} with respect to t , we have

$$\begin{aligned} \dot{V}_{bd} &= -\frac{1}{2} e^T Q e + e^T P B [u^* - u - u_{co} - u_{bd}] \\ &\leq -\frac{1}{2} e^T Q e + |e^T P B| [|u^*| + |u| + |u_{co}|] - e^T P B u_{bd} \end{aligned} \quad (12)$$

Assumption: We can determine a function $f^u(x)$ and constant b_L such that

$$|f(x)| \leq f^u(x) \quad \text{and} \quad 0 < b_L \leq b \quad (13)$$

This means we should have some knowledge of the system, but this is not very difficult to get.

Under the above assumption and equation (2), we could construct the bounding control u_{bd} as:

$$u_{bd} = I \operatorname{sgn}(e^T P B) \left[|u| + |u_{co}| + \frac{1}{b_L} (f^u(x) + |y_m^{(n)}| + |k^T e|) \right] \quad (14)$$

where $I=1$ if $V_{bd} > \bar{V}$ (\bar{V} is a constant specified by the designer) and $I=0$ if $V_{bd} \leq \bar{V}$. And due to $g(x) > 0$ (equation (13)), we can evaluate the value of $(e^T P B)$. So, when $V_{bd} \leq \bar{V}$, we have

$$\dot{V}_{bd} \leq -\frac{1}{2} e^T Q e \leq 0 \quad (15)$$

So, using the bounding control u_{bd} , we always have $V_{bd} \leq \bar{V}$. This means we can restrict the state of the system in a desired range using the bounding control.

B. Compensating Control

We use the compensating control to compensate for the approximation error in modelling u^* by a fuzzy system.

From the equation (10), we know that $d_u(x)$ is a residual term of the approximation error $\varepsilon(x)$, and $d_u(x)$ cannot be expressed by linear combination of parameter (θ, c, σ) . To reduce the negative effect of $d_u(x)$ to our defined Lyapunov functions, we consider the compensating control as:

$$u_{co} = \operatorname{sgn}(e^T P B) w^T Y \quad (16)$$

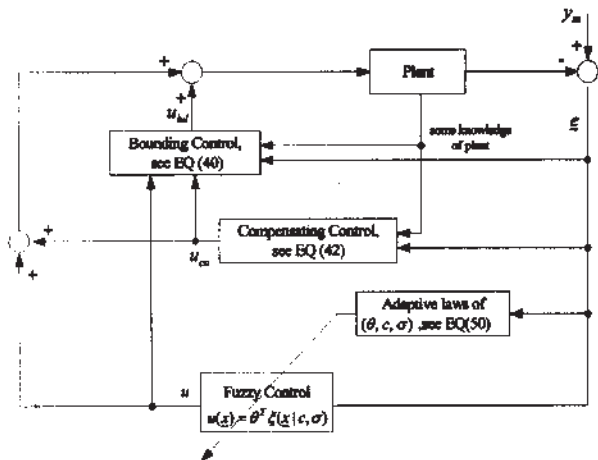


Fig.1 Structure of proposed Fuzzy controller.

3 DEVELOPMENT OF A DIRECT FUZZY ADAPTIVE CONTROLLER

Inspired from [2], we construct the control law as:

$$u_p = u + u_{co} + u_{bd} \quad (6)$$

The above equation means that the direct adaptive control law is comprised of a bounding control term, u_{bd} , a compensating control term u_{co} , and an adaptive fuzzy control term, u , which is mentioned in section III and is used to approximate the ideal controller. The compensating controller is used to compensate for approximation errors in representing the actual nonlinear dynamics by fuzzy systems with ideal parameter values. The bounding control is used to restrict the output trajectory of the system so that fuzzy systems may be defined for a small range of states. The bounding controller in this manner is similar to the supervisory control (described in the following sections). The structure of the proposed control is shown in Fig. 1.

After substituting the control law into the system, we will have

$$\dot{x}^{(n)} = f(x) + b[u + u_{co} + u_{bd}] \quad (7)$$

After some straightforward manipulation, we can obtain the error equation of the closed-loop system

$$\begin{aligned} \dot{e} &= A e + B [u^* - u - u_{co} - u_{bd}] \\ &= A e + B [\phi_0^T \xi(x | c, \sigma) + \phi_c^T u_c + \phi_\sigma^T u_\sigma + d_u(x) - u_{co} - u_{bd}] \end{aligned} \quad (8)$$

where

And also, due to $g(x) > 0$, we can evaluate the value of $\text{sgn}(e^T P B)$. However, in order to avoid chattering of the system response around the equilibrium point where the system error is zero, we can simply modify the equation of u_{co} to:

$$u_{co} = w^T Y \text{sat}((e^T P_n) / \varepsilon) \quad (17)$$

where

$$\text{sat}(x) = \begin{cases} 1, & x \geq 1 \\ x, & -1 < x < 1 \\ -1, & x \leq -1 \end{cases} \quad (18)$$

ε is a constant specified by the designer and $\varepsilon > 0$.

C. Adaptive Laws

Consider the following Lyapunov function candidate:

$$V = \frac{1}{2} e^T P e + \frac{b}{2\gamma_1} \phi_\theta^T \phi_\theta + \frac{b}{2\gamma_2} \phi_c^T \phi_c + \frac{b}{2\gamma_3} \phi_\sigma^T \phi_\sigma + \frac{b}{2\gamma_4} \phi_w^T \phi_w \quad (19)$$

Based on equation (34), taking the derivative with respect to t yields:

$$\begin{aligned} \dot{V} = & -\frac{1}{2} e^T Q e + e^T P B [\phi_\theta^T \xi(x|c, \sigma) + \phi_c^T u_c + \phi_\sigma^T u_\sigma + d_u(x) \\ & - \text{sgn}(e^T P B) w^T Y - u_{bd}] + \frac{b}{\gamma_1} \phi_\theta^T \dot{\phi}_\theta + \frac{b}{2\gamma_2} \phi_c^T \dot{\phi}_c \\ & + \frac{b}{2\gamma_3} \phi_\sigma^T \dot{\phi}_\sigma + \frac{b}{2\gamma_4} \phi_w^T \dot{\phi}_w \end{aligned} \quad (20)$$

Hence,

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2} e^T Q e + e^T P B [\phi_\theta^T \xi(x|c, \sigma) + \phi_c^T u_c + \phi_\sigma^T u_\sigma \\ & + \text{sgn}(e^T P B) w^T Y - \text{sgn}(e^T P B) w^T Y - u_{bd}] \\ & + \frac{b}{\gamma_1} \phi_\theta^T \dot{\phi}_\theta + \frac{b}{2\gamma_2} \phi_c^T \dot{\phi}_c + \frac{b}{2\gamma_3} \phi_\sigma^T \dot{\phi}_\sigma + \frac{b}{2\gamma_4} \phi_w^T \dot{\phi}_w \\ = & -\frac{1}{2} e^T Q e + e^T P B [\phi_\theta^T \xi(x|c, \sigma) + \phi_c^T u_c + \phi_\sigma^T u_\sigma \\ & + \text{sgn}(e^T P B) \phi_w^T Y - u_{bd}] + \frac{b}{\gamma_1} \phi_\theta^T \dot{\phi}_\theta + \frac{b}{2\gamma_2} \phi_c^T \dot{\phi}_c \\ & + \frac{b}{2\gamma_3} \phi_\sigma^T \dot{\phi}_\sigma + \frac{b}{2\gamma_4} \phi_w^T \dot{\phi}_w \end{aligned} \quad (21)$$

From (14), we have

$$\begin{aligned} e^T P B u_{bd} = & I(e^T P B) \text{sgn}(e^T P B) [|u| + |u_{co}|] \\ & + \frac{1}{b_i} (f^U(x) + |y_m^{(n)}| + |k^T e|) \\ \geq & 0 \end{aligned} \quad (22)$$

then we obtain:

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2} e^T Q e + \frac{b}{\gamma_1} \phi_\theta^T [\gamma_1 e^T P_n \xi(x) + \dot{\phi}_\theta] \\ & + \frac{b}{\gamma_2} \phi_c^T [\gamma_2 e^T P_n u_c + \dot{\phi}_c] + \frac{b}{\gamma_3} \phi_\sigma^T [\gamma_3 e^T P_n u_\sigma + \dot{\phi}_\sigma] \\ & + \frac{b}{\gamma_4} \phi_w^T [\gamma_4 \text{sgn}(e^T P B) e^T P_n Y + \dot{\phi}_w] \end{aligned} \quad (23)$$

where P_n is the last column of P .

We could choose the adaptive laws as:

$$\begin{aligned} \dot{\theta} &= \gamma_1 e^T P_n \xi(x) \\ \dot{c} &= \gamma_2 e^T P_n u_c \\ \dot{\sigma} &= \gamma_3 e^T P_n u_\sigma \\ \dot{w} &= \gamma_4 \text{sgn}(e^T P B) e^T P_n Y \end{aligned} \quad (24)$$

Using the facts $\dot{\phi}_\theta = -\dot{\theta}$, $\dot{\phi}_c = -\dot{c}$, $\dot{\phi}_\sigma = -\dot{\sigma}$ and $\dot{\phi}_w = -\dot{w}$, we obtain

$$\dot{V} \leq -\frac{1}{2} e^T Q e \quad (25)$$

Using the above fuzzy adaptive laws, we cannot guarantee that the parameters are bounded. We can use a projection algorithm [3] to modify the above adaptive laws.

4 EXAMPLE: VEHICLE LONGITUDINAL CONTROLLER

Automated highway systems (AHS) have drawn more and more attention in recent years because full automation can greatly increase highway capacity while improving safety[4]-[9]. In each platoon of AHS, every vehicle (except the leading car) tightly follows the preceding vehicle, and can react quickly to the preceding car and emergency due to its automated electronic device, which automatically controls the throttle and brake of the vehicle.

Recently, some soft computing technology, such as fuzzy logical control and neural network, has been applied in the controller design of automated vehicle control system [10]-[14]. And some learning or optimization methods are also proposed. The main advantages of these methods are that they don't need the exact model of vehicles and may be not sensible to imprecise data from sensor. But these approaches also bring some drawbacks. These model-free methods require some operator experience or training data. And the performance of the controller depends much on these a priori knowledge.

Within this section, we will apply our proposed adaptive fuzzy controller into vehicle longitudinal control. The objective of the adaptive fuzzy controller is to maintain a safe distance between the preceding car and the following car. The strength of our approach is that we don't require the training data, and fuzzy rules can be updated on-line according to the performance of the controller. And our approach needs little knowledge about the car. As a result, it can be transported to any vehicles regardless of the nonlinear and often unobservable dynamics.

A. Vehicle Longitudinal Dynamics

A lot of vehicle models have been proposed for different purposes. For vehicle longitudinal control design, we only consider throttle and brake control for longitudinal control, and don't consider the steering wheel. The vehicle dynamics may be expressed as the following mathematical model [2]-[8]:

$$\ddot{x} = \frac{F - cx^2 - d}{M} \quad (26)$$

$$\dot{F} = \frac{1}{\tau}(-F + u_p) \tag{27}$$

where, in the first equation, x, F, c, d, M are the position, the engine traction force, effective aerodynamic drag coefficient, rolling resistance friction, and effective inertia respectively. If we consider the engine dynamics, we have the additional equation (27), where the engine traction force F can be modeled as a first order system, and u_p is the control input.

We should say here that, although we present the exact vehicle longitudinal model, we use only some knowledge of this model to design our controller, and we may not know the exact values of all parameters in equations (26) and (27), instead, we should only know the bound of the parameters (see the next subsection). In other words, the controller design doesn't require a complete model.

B. Fuzzy Adaptive Controller Design

The main objective of vehicle longitudinal control is to maintain a constant safe spacing between the preceding car and following car.

We consider the output of the vehicle as

$$y = x_f - x_p \tag{28}$$

Based on the objective mentioned in the beginning of section 4, we simply select $y_m = 0$, then we obtain:

$$e = y_m - y = x_p - x_f \tag{29}$$

If we neglect the time lag of u , i.e., $\tau = 0$ in vehicle model, then

$$y^{(2)} = -(\dot{v}_p - \dot{v}_f) = -\dot{v}_p - \frac{1}{m} A_p v^2 - \frac{1}{m} d + \frac{1}{m} u$$

So, we have:

$$f(x) = -\frac{1}{m} A_p v^2 - \frac{1}{m} d - \dot{v}_p, \quad b = \frac{1}{m} > 0 \tag{30}$$

And we obtain:

$$e = [e, \dot{e}]^T = [x_p - x_f, v_p - v_f]^T \tag{31}$$

Let,

$$\delta = x_p - x_f, \quad v_r = v_p - v_f \tag{32}$$

Next, from the equation (30), we may determine the upper bound of $f(x)$ and the lower bound of b .

$$f(x) \leq \frac{1}{m} A_p |v|^2 + \frac{1}{m} |d| + |a_p|$$

Based on the information provided in [12], $A_p = 0.44 \text{kg/m}$, $d = 352 \text{kgm/s}^2$, and we assume the minimum mass of vehicle $1000 \leq m \leq 2000 \text{kg}$, the acceleration of vehicle $-3 \leq a \leq 1.5 \text{m/s}^2$. We get

$$f^u(x) = \frac{1}{m_{\min}} A_p |v|^2 + \frac{1}{m_{\min}} |d| + 3$$

$$g(x) \geq \frac{1}{m_{\max}} = 0.0005 = b_L \tag{33}$$

Here, we consider the following fuzzy rules of the adaptive fuzzy controller.

$$R_i: \text{IF } \delta \text{ is } F_1^i \text{ and } v_r \text{ is } F_2^i$$

$$\text{THEN } u = K_0^i + K_1^i \delta + k_2^i v_r \tag{34}$$

The detailed implementation procedure of the car longitudinal controller is as follows:

- 1) Initialize the parameters (θ, c, σ) of fuzzy rules.
- 2) Obtain the relative speed and relative distance through the sensors of the car, and then we can get system error $e = [\delta, v_r]^T$.
- 3) Calculate the membership of δ, v_r , based on equation (3).
- 4) Calculate $\xi(x)$ based on equation (5) and fuzzy rule (34), and obtain the fuzzy controller output u based on θ and $\xi(x)$ (equation (4)).
- 5) Obtain the compensating controller output u_{co} based on equation (17).

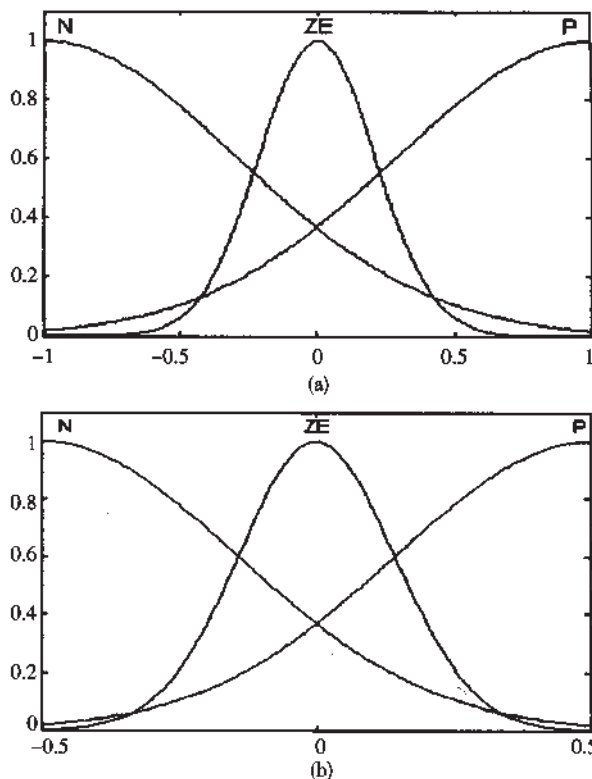


Fig. 2. Fuzzy membership function of (a) δ and (b) v_r .

- 6) Calculate the bounding controller output u_{bd} by equation (40) based on u and u_{co} , which have been obtained in step 4 and 5, respectively.
- 7) Calculate the total control input u_p of vehicle based on equation (32).
- 8) Update the parameter (θ, c, σ) based on the adaptive laws (equation (50)).
- 9) Back to step 2.

C. Simulation Results

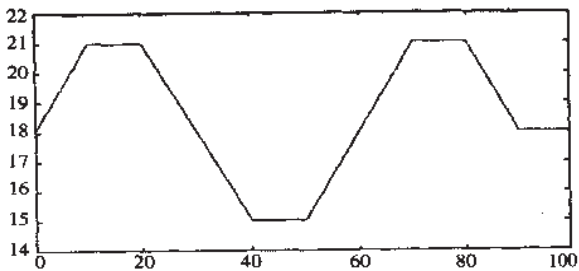
We select $k = (k_2, k_1)^T = (2, 1)^T$ (so that $s^2 + k_1 s^2 + k_2$ are in the open left half plane, i.e., stable), $Q = \text{diag}(10, 10)$, and we get symmetric positive definite matrix $P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$.

We adopt the fuzzy rules like eqn. (34) and totally have

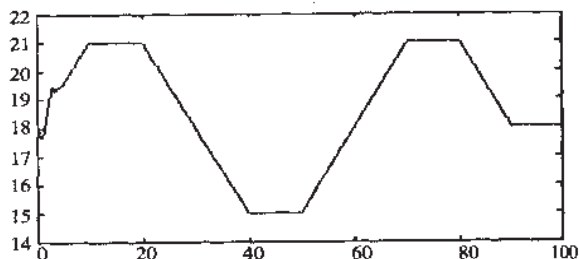
9 rules in our simulation. Initially we define three fuzzy sets over the interval $[-1,1]$ for δ , three fuzzy sets over the interval $[-0.5,0.5]$ for v_r , which are shown in Fig. 2.

In our simulations, the velocity profile of the preceding vehicle is shown in Fig. 3.

We should notice here, there is no acceleration of the preceding car to provide and we can only obtain the information of the relative speed and relative distance between the preceding car and the following car.



(a) The velocity profile of the preceding vehicle



(b) the velocity response of the following vehicle with our proposed adaptive fuzzy control.

Fig. 3

5 CONCLUSIONS

We have proposed adaptive fuzzy control in this paper. We want the fuzzy system to have a good approximation property, so we tune all parameters in the fuzzy rules including parameters of Gaussian membership functions. In order to construct the adaptive laws of these parameters, linear relationship between approximation error and parameters is established. Our proposed controller includes bounding control, compensating control and fuzzy control, which is used to approximate the optimal ideal control. The advantage of our approach is that we don't require the complete model, and fuzzy rules can be adjusted according to the performance of the controller. Moreover, our controller is more flexible because more tuned parameters are considered. Finally, we apply our approach to vehicle longitudinal control; simulation results show that it provides satisfactory performances in car-following.

While our approach presents significant advantages, there are several aspects for us to consider. First, our control scheme is only for a class of specific continuous time SISO nonlinear system. Extension to other nonlinear systems is an important direction. Second, if we can provide simpler

or more desirable approach from the implementation aspect such as real-time consideration? Last but not least, how to tune the parameters of fuzzy rules appropriately is very important. Besides Lyapunov synthesis approach, if we can combine other optimization techniques.

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