

活动标形及其在点啮合齿轮传动误差分析中的应用*

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摘要 活动标形法的论述首推苏步青译, 佐佐木重夫著《微分几何学》, 还有的著作从外微分形式引入活动标形. 本文论证取曲面的正交曲率网为参数网时, 曲面论的基本公式就是活动标形的微分形式, 并用其分析了点啮合齿轮传动误差.

关键词 活动标形 接触分析 曲率网

Moving Frame Aand Its Application In The Error Analysis Of Point Contact Gearing

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Abstract This paper demonstrates that the basic formulas of surface theory are just the differential formation of the moving frame, if the vertical curvature web is chosen as the parameter web. And applied the theory into the analysis of the transmission error of the point contact gearing.

Keywords moving frame contact analysis curvature web

1 The moving frame and the fundamental theory of the differential geology

Suppose that the two parameter curves u and v have been chosen as the curvature web, then we will have the following equation $F=0, M=0$. On the surfaces there are 3 vectors which are perpendicular with each other,

$$e_1 = \mathbf{r}_u / \sqrt{E}, e_2 = \mathbf{r}_v / \sqrt{G}, e_3 = e_1 \times e_2$$

The differentiate e_i can be written in the form of linear combination of e_j

$$(e_i)_u = \sum a_{ij} e_j, (e_i)_v = \sum b_{kj} e_j$$

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a_{ij} and b_{kj} formed a dissymmetric matrix respectively, that is $a_{ji} = 0, b_{ji} = 0$

$$\begin{aligned} e_{1u} &= r_{uu} \overline{E} + (1 / \overline{E})_u r_u, e_{1v} = r_{vv} / \overline{E} + (1 / \overline{E})_v r_v \\ e_{2u} &= r_{uv} \overline{G} + (1 / \overline{E})_u r_v, e_{2v} = r_{vv} / \overline{G} + (1 / \overline{G})_v r_v \end{aligned}$$

Since that $F = r_u r_v = 0, M = n r_{uv} = 0$

$$\begin{aligned} a_{12} &= -a_{21} = e_{2u} e_{1v} = r_{uv} r_u / \overline{EF} = -E_v \overline{EF} \\ a_{13} &= e_{1u} e_{3v} = r_{uv} n / \overline{E} = L / \overline{E} \\ a_{23} &= e_{2u} e_{3v} = r_{uv} n / \overline{G} = 0 \end{aligned}$$

by the same reason, $b_{12} = G_u / \overline{GE}, b_{13} = 0, b_{23} = N / \overline{G}$

$$\begin{aligned} de &= (e^1)_u du + (e^1)_v dv = \sum d k_{ij} e_j \\ d k_{ij} &= a_{ij} du + b_{jv} dv = -d k_{ji} \\ d k_{12} &= (-E_v du + g_u dv) / 2 \quad \overline{eg} = k_{gq} d k_{1v} + k_{g2} d k_2 \end{aligned}$$

In the equation, k_{g1} and k_{g2} are the geodetic distort curvatures of the curve u and v respectively.

$$\begin{aligned} \text{Since that, } d k_1 &= \overline{E} du \quad d k_2 = \overline{G} dv \\ d k_{23} &= N / \overline{G} = k_2 d k_2, d k_{13} = L / \overline{E} = k_d k_1 \end{aligned}$$

In the equation, k_1 and k_2 are the main curvatures of the curve u and v respectively. In the moving coordinate system that is fixed on the surface, the moving frame has the following differential form

$$\begin{aligned} d e_1 &= d k_{12} e_2 + d k_{13} e_3 \\ d e_2 &= -d k_{12} e_1 + d k_{23} e_3 \\ d e_3 &= -d k_{13} e_1 - d k_{23} e_2 \\ d r &= d k_1 e_1 + d k_2 e_2 \end{aligned} \tag{1}$$

What should be stressed is, the curvature web as parameter web exists in theory, but it probably can't be found in general situation. What's more, it is difficult to express in the form of apparent function. However, we needn't use such parameters in practical calculation in the example below.

2 The analysis of the movement of the point contact meshing by the Moving Frame

Suppose the $\sum^{(1)}$ and $\sum^{(2)}$ are two surfaces, establish the moving frames on the two surfaces respectively, according to equ. (1)

$$\begin{aligned} d e^{(i)} &= d k_{12}^{(i)} e_2^{(i)} + d k_{13}^{(i)} e_3^{(i)} \\ d e_2^{(i)} &= -d k_{12}^{(i)} e_1^{(i)} + d k_{23}^{(i)} e_3^{(i)} \\ d e_3^{(i)} &= -d k_{13}^{(i)} e_1^{(i)} + d k_{23}^{(i)} e_2^{(i)} \\ d r^{(i)} &= d k_1^{(i)} e_1^{(i)} + d k_2^{(i)} e_2^{(i)} \quad i = 1, 2, \\ d k_1^{(i)} &= \overline{E}^{(i)} du^{(i)} \end{aligned} \tag{2}$$

$$\begin{aligned} dk_2^{(i)} &= \overline{G}^{(i)} dv^{(i)} \\ dk_{(12)}^{(i)} &= kg_1^{(i)} dk_1^{(i)} + kg_2^{(i)} dk_2^{(i)} \\ dk_{(23)}^{(i)} &= k_2^{(i)} dk_2^{(i)} \\ dk_{13}^{(i)} &= k_1^{(i)} dk_1^{(i)} \end{aligned}$$

In the equation, $kg_1^{(i)}$ and $kg_2^{(i)}$ are the geodetic distort curvatures of the curve $u^{(i)}$ and $v^{(i)}$ respectively and $k_1^{(i)}$ and $k_2^{(i)}$ are the main curvatures of the curve $u^{(i)}$ and $v^{(i)}$ respectively.

When the two bodies turn about their own axes, the differential form of the two moving frame in the fixed coordinate systems are respectively (that is, the relationship between relative differentiation and absolute differentiation).

$$dE_j^{(i)} = de_j^{(i)} + a^{(i)} \times edH^{(i)} \quad dR^{(i)} = dr^{(i)} + a^{(i)} \times R^{(i)} dH^{(i)} \quad (3)$$

The condition for the two surfaces to contact is

$$dR^{(1)} = dR^{(2)} \quad e_3^{(1)} = e_3^{(2)} \quad (4)$$

Put the equation (3) into the equation (4), both sides is multiplied by e_3 , considering $dr^{(i)} e_3 = 0$, the instantaneous ratio is

$$dH^{(1)} / dH^{(2)} = (a^{(1)}, R^{(1)}, e_3) / (a^{(2)}, R^{(2)}, e_3) \quad (5)$$

Differentiate once more, considering equation (4), we have

$$\begin{aligned} d^2H^{(1)} / (dH^{(2)})^2 = & \frac{(a^{(1)}, dR^{(1)} / dH^{(1)}, e_3) + (a^{(1)}, R^{(1)}, de_3 / dH^{(1)}) - [(a^{(2)}, dR^{(2)} / dH^{(1)}, e_3) + (a^{(2)}, R^{(2)}, de_3 / dH^{(1)})] dH^{(1)} / dH^{(2)}}{(a^{(2)}, R^{(2)}, e_3)} \end{aligned} \quad (6)$$

Put equation (2) into equation (3), we have

$$dR^{(i)} / dH^{(1)} = dk_1^{(i)} / dH^{(1)} e_1^{(i)} + dk_2^{(i)} / dH^{(1)} e_2^{(i)} + a^{(i)} \times R^{(i)} \quad (7)$$

$$dE_3 / dH^{(1)} = -k_1^{(1)} dk_1^{(1)} / dH^{(1)} e_1^{(1)} - dk_2^{(1)} / dH^{(1)} e_2^{(1)} + a^{(1)} \times e_3 \quad (8)$$

The equation (4) can be written as

$$dk_1^{(1)} e_1^{(1)} + dk_2^{(1)} e_2^{(1)} + a^{(1)} \times R^{(1)} dH^{(1)} = dk_1^{(2)} e_1^{(2)} + dk_2^{(2)} e_2^{(2)} + a^{(2)} \times R^{(2)} dH^{(2)} \quad (9)$$

The angle bwtween the main directions of the two surfaces is ψ , then

$$\begin{aligned} e_1^{(2)} &= \cos \psi e_1^{(1)} + \sin \psi e_2^{(1)} \\ e_2^{(2)} &= -\sin \psi e_1^{(1)} + \cos \psi e_2^{(1)} \end{aligned} \quad (10)$$

Put equation (10) into equation (9), we have

$$\begin{aligned} dk_1^{(2)} / dH^{(1)} + (a^{(1)}, R^{(1)}, e_1^{(2)}) dH^{(2)} / dH^{(1)} \\ = dk_1^{(1)} / dH^{(1)} \cos \psi + dk_2^{(1)} / dH^{(1)} \sin \psi + (a^{(1)}, R^{(1)}, e_1^{(2)}) \end{aligned} \quad (11)$$

$$\begin{aligned} dk_2^{(2)} / dH^{(1)} + (a^{(2)}, R^{(2)}, e_1^{(2)}) dH^{(2)} / dH^{(1)} \\ = -dk_1^{(1)} / dH^{(1)} \sin \psi + dk_2^{(1)} / dH^{(1)} \sin \psi + (a^{(1)}, R^{(1)}, e_2^{(2)}) \end{aligned} \quad (12)$$

Differentiate the second equation of (4)

$$\begin{aligned} -dk_{13}^{(1)} e_1^{(1)} - dk_{23}^{(1)} e_2^{(1)} + a^{(1)} \times e_3 dH^{(1)} \\ = -dk_{13}^{(2)} e_1^{(2)} - dk_{23}^{(2)} e_2^{(2)} + a^{(2)} \times e_3 dH^{(2)} \end{aligned}$$

Finally, put the equation (10) into it

$$\begin{aligned}
 & -k_1^{(2)} dk_1^{(2)} / dH^{(1)} + a^{(2)} e_2^{(2)} dH^{(2)} / dH^{(1)} \\
 = & -k_1^{(1)} dk_1^{(1)} / dH^{(1)} \cos j - k_2^{(1)} dk_2^{(1)} / dH^{(1)} \sin j + a^{(1)} e_2^{(2)} \quad (13)
 \end{aligned}$$

$$\begin{aligned}
 & -k_2^{(2)} dk_2^{(2)} / dH^{(1)} + a^{(2)} e_1^{(2)} dH^{(2)} / dH^{(1)} \\
 = & k_1^{(1)} dk_1^{(1)} / dH^{(1)} \sin j - k_2^{(1)} dk_2^{(1)} / dH^{(1)} \cos j - a^{(1)} e_1^{(2)} \quad (14)
 \end{aligned}$$

Solve the equations (1), (12), (13) and (14) instantaneously, we can determine

$$dk_1^{(1)} / dH^{(1)},$$

$$dk_2^{(2)} / dH^{(1)},$$

$$dk_1^{(2)} / dH^{(1)},$$

$$dk_2^{(2)} / dH^{(1)}.$$

Then take them into equation (7), (8), and (6), we can find the acceleration of the driven gear.

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